Lecture 13

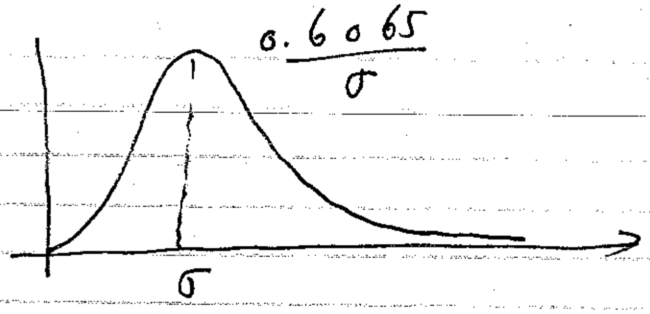
- Performance of digital modulation schemes in fading channel:

$$r(t) = \alpha(t)\, e^{-j\theta(t)}\, s(t) + n(t)$$

Where $n(t)$ is AWGN and

$\alpha(t)$ is a Rayleigh Fading component.

$$p(\alpha) = \begin{cases} \dfrac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2}, & 0 \le \alpha < \infty \\ 0 & \alpha < 0 \end{cases}$$



$$\sigma_{mean} = \int_0^\infty \alpha \, p(\alpha)\, d\alpha = \sigma\sqrt{\frac{\pi}{2}} = 1.2583\,\sigma$$

$$\sigma_\alpha^2 = \sigma^2\left(2 - \frac{\pi}{2}\right) = 0.4292\,\sigma^2$$

$$\overline{\alpha^2} = 2\sigma^2$$

To find the probability of error, we find P_e for a given α and average over all α. For any α, the effective SNR will be

$$X = \alpha^2 \frac{E_b}{N_o}$$

and then average over all α (equivalently all x will be:

$$P_e = \int_0^\infty P_e(X)\, p(X)\, dX$$

$$p(\alpha) = \frac{2\alpha}{\overline{\alpha^2}} e^{-\frac{\alpha^2}{\overline{\alpha^2}}} \Rightarrow p(X) = \frac{1}{\Gamma} e^{-\frac{X}{\Gamma}}$$

where $\Gamma = \frac{E_b}{N_0}\overline{\alpha^2}$ is the average value of the SNR.

For different modulation schemes, we have

$$P_{e,PSK} = \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right] \quad \text{Binary, Coherent}$$

$$P_{e,FSK} = \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{2+\Gamma}}\right] \quad \text{" \quad "}$$

for large E_b/N_0

$$P_{e,PSK} = \frac{1}{4\Gamma} \quad \text{Binary, Coherent}$$

$$P_{e,FSK} = \frac{1}{2\Gamma} \quad \text{" \quad "}$$

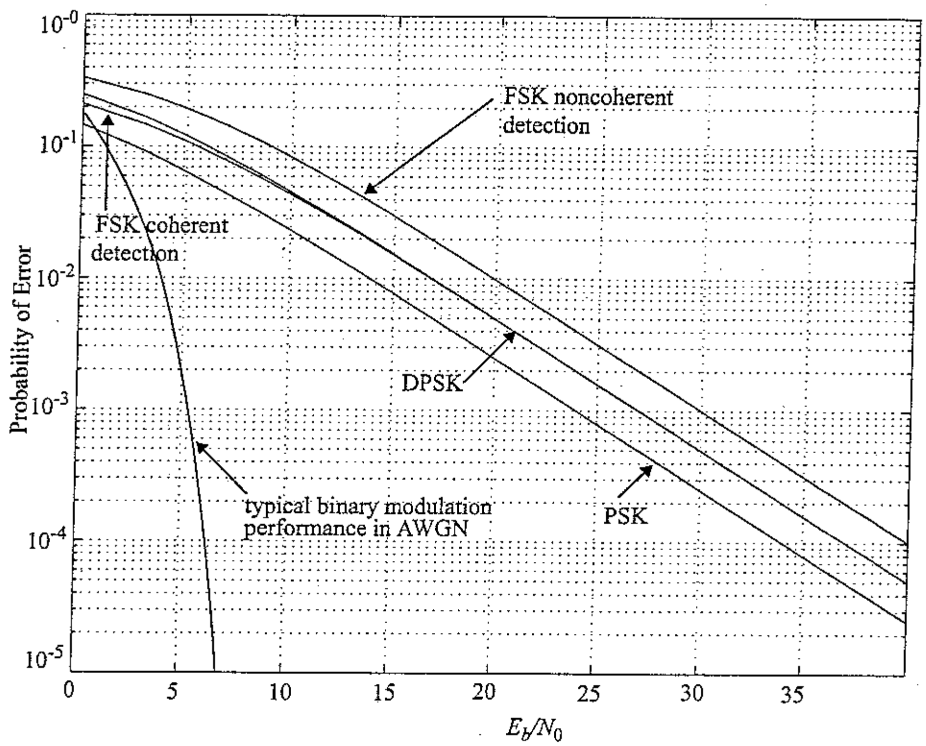
$$P_{e,DPSK} = \frac{1}{2\Gamma} \quad \text{" \quad non-coherent (differential)}$$

$$P_{e,FSK} = \frac{1}{\Gamma} \quad \text{non-Coherent.}$$

For GMSK

$$P_{e,GMSK} = \frac{1}{2}\left(1 - \sqrt{\frac{\delta\Gamma}{\delta\Gamma+1}}\right) \simeq \frac{1}{4\delta\Gamma}$$

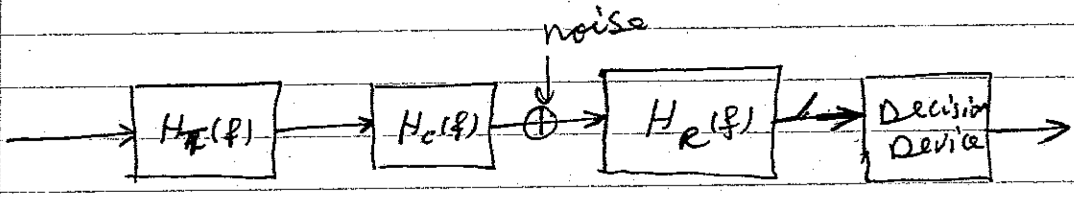
where $\delta = \begin{cases} 0.68 & \text{for } BT = 0.25 \\ 0.85 & \text{for } BT = \infty \end{cases}$



FSK noncoherent detection
FSK coherent detection
DPSK
typical binary modulation performance in AWGN
PSK
Probability of Error
E_b/N_0

Equalization:

ISI either as a result of band limitation or multipath can be overcome by compensating non-idealness of the channel $\Rightarrow$ equalization



$$F(f)\ \cancel{H_{eff}(f)} = H_T(f)\,H_c(f)\,H_R(f)$$

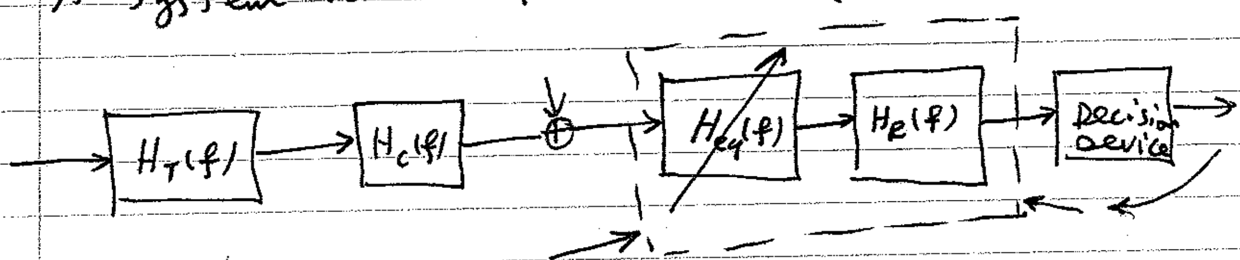
We require that the overall transfer function $F(f)$ or $H_{eff}(f)$ or equivalently $\overset{f(t)}{h_{eff}(t)}$ have a certain shape, e.g., raised cosine.

However, if the channel is not constant, say length of wire or in general ~~the~~ path between calling and called parties is not the same for all pairs of subscribers, then, we need to compensate for this.

In the case of wireline or fixed wireless, one can adjust the equalizer at the outset of the call and keep it constant for the duration of the call. (or even once for ever)

In the case of mobile the path properties changes during the call requiring adaptive equalization.

A system with equalizer is:



$H_R(f)$ also can be conceived as a variable filter matching to the channel variances.

Implementation of equalizers using FIR filters (transversal filters)

let the received signal be:

$$y(t) = x(t) * f(t) + n(t)$$

where $x(t)$ is the original message signal

$$f(t) = h_T(t) * h_c(t) * h_R(t)$$

and $n(t)$ is the AWGN

when we insert the $h_{eq}(t)$, i.e., the equalizer, the output will be

$$\hat{x}(t) = x(t) * f(t) * h_{eq}(t) + n(t) * h_{eq}(t)$$

when there is no noise we would like

$$\hat{x}(t) = x(t)$$

i.e. $\quad f(t) * h_{eq}(t) = \delta(t)$

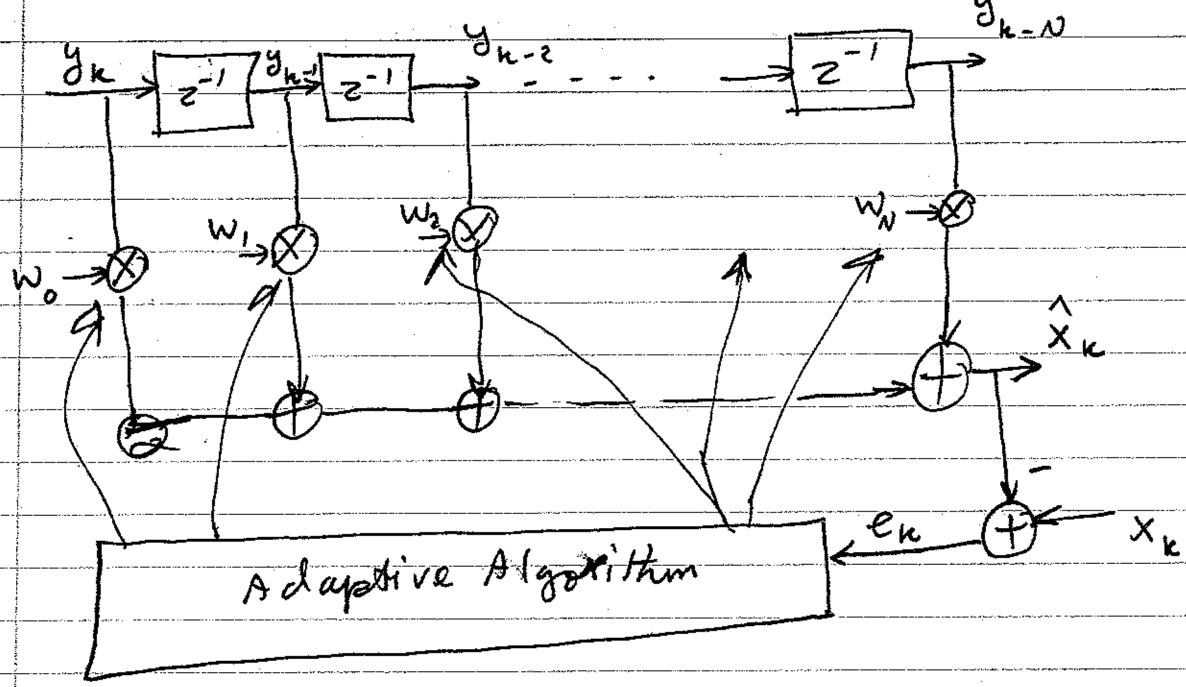
or

$$F^*(f)\, H_{eq}(f) = 1$$

or

$$H_{eq}(f) = \frac{1}{F^*(f)} \qquad \text{zero-forcing equalizer}$$

FIR filter implementation:



$$e_k = x_k - \hat{x}_k = x_k - \sum_{n=0}^{N} w_n y_{k-n}$$

Least MSE criterion, minimizing:

$$\varepsilon = E[|e_k|^2] = E\left[\left|x_k - \sum_{n=0}^{N} w_n y_{k-n}\right|^2\right]$$

with proper choice w_n's.

This can be done by equating the derivative of ε w.r.t. w_n's to zero.

$$\frac{\partial}{\partial w_i}\varepsilon = 0 \Rightarrow \sum_{n=0}^{N} \hat{w}_n R_y(i-n) = r_{xy}(i)$$

$$\hat{w}_0 R_y(0) + \hat{w}_1 R_y(1) + \cdots \hat{w}_N R_y(N) = \hat{r}_{xy}(0)$$

$$\hat{w}_0 R_y(1) + \hat{w}_1 R_y(0) + \cdots \hat{w}_N R_y(N-1) = \hat{r}_{xy}(1)$$

$$\vdots$$

This can be written as:

$$\underline{\hat{w}}\, \mathbb{R} = \mathbb{P}$$

or $\underline{\hat{w}} = \mathbb{R}^{-1} \mathbb{P}$

where

$$\mathbb{R} = E \begin{bmatrix} y_k^2 & y_k y_{k-1} & \cdots & y_k y_{k-N} \\ y_{k-1} y_k & y_{k-1}^2 & \cdots & y_{k-1} y_{k-N} \\ \vdots & & & \\ y_{k-N} y_k & \cdots & \cdots & y_{k-N}^2 \end{bmatrix}$$

and

$$\mathbb{P} = E \left[x_k y_k \quad x_k y_{k-1} \quad x_k y_{k-2} \; \cdots \; x_k y_{k-N} \right]^T$$

$$MMSE = \mathcal{E}_{min} = E\{x_k^2\} - \mathbb{P}^T \underline{\hat{w}}$$

Different types of adaptation:
- over head data (preamble)
- Feedback data.

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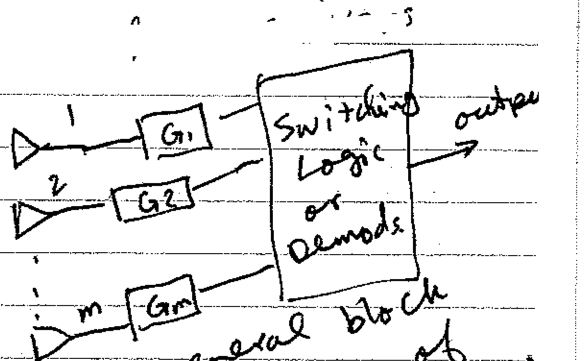
Types of diversity:

- Space Diversity (Antenna Diversity)
- Frequency Diversity
- Time Diversity ⟷ Coding
- Polarization Diversity.

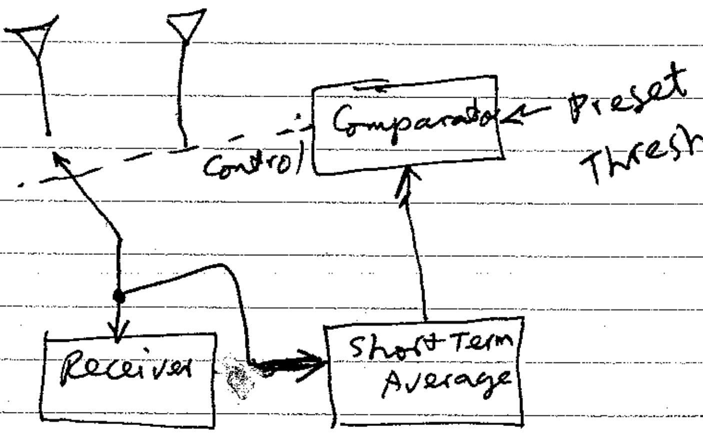


general block Diagram of Diversity Receiver

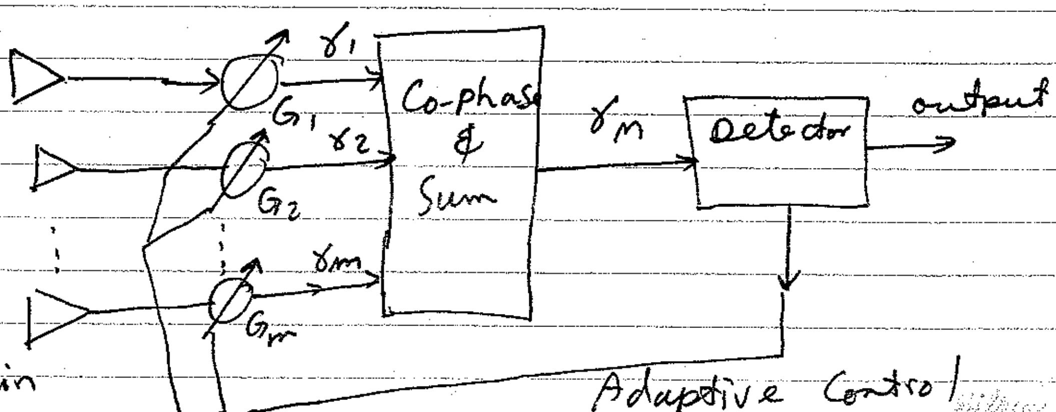
Diversity strategies:

- Selection Diversity
- Feedback or Scanning Diversity

- Simple to implement
- only one receiver required



- Maximal Ratio Combining



- equal gain combining

Diversity : selection diversity improvement:

Take the average SNR as

$$\overline{SNR} = \Gamma = \frac{E_b}{N_o}\overline{\alpha^2}$$

then the instantaneous $SNR = \gamma_i$ has the probability density:

$$p(\gamma_i) = \frac{1}{\Gamma} e^{-\frac{\gamma_i}{\Gamma}} \qquad \gamma_i \geq 0$$

The probability that the Signal-to-Noise ratio is less than a certain threshold γ is:

$$P_r(\gamma_i \leq \gamma) = \int_0^{\gamma} P(\gamma_i)\, d\gamma_i = \int_0^{\gamma} \frac{1}{\Gamma} e^{-\frac{\gamma_i}{\Gamma}} d\gamma_i$$

$$= 1 - e^{-\frac{\gamma}{\Gamma}}$$

Now, if we have M independent paths, the probability that SNR over all of them is less than γ is:

$$P_r(\gamma_1 < \gamma, \gamma_2 < \gamma, \cdots \gamma_M < \gamma) = \left(1 - e^{-\frac{\gamma}{\Gamma}}\right)^M = P_M(\gamma)$$

and Probability that at least one branch achie SNR $> \gamma$ is:

$$P_r(\gamma_i > \gamma) = 1 - P_M(\gamma) = 1 - \left(1 - e^{-\frac{\gamma}{\Gamma}}\right)^M$$

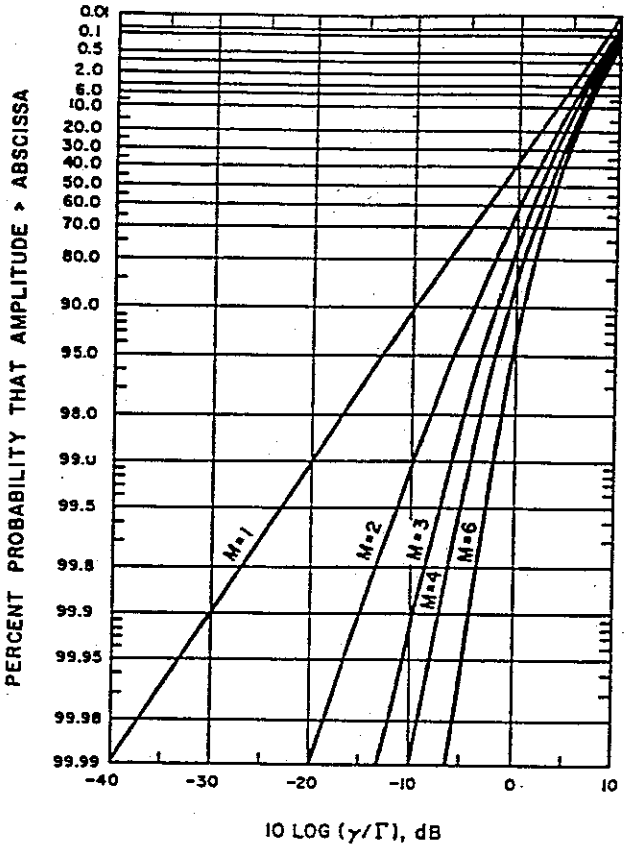


Figure 6.11
Graph of probability distributions of $SNR = \gamma$ threshold for M branch selection diversity. The term Γ represents the mean SNR on each branch [From [Jak71] © IEEE].

The pdf of γ is:

$$p_M(\gamma) = \frac{d}{d\gamma} P_M(\gamma) = \frac{M}{\Gamma}\left(1 - e^{-\frac{\gamma}{\Gamma}}\right)^{M-1} e^{-\frac{\gamma}{\Gamma}}$$

and the average SNR is:

$$\bar{\gamma} = \int_0^{\infty} \gamma p_M(\gamma)\, d\gamma = \Gamma \int_0^{\gamma} M x (1 - e^{-x})^{M-1} e^{-x} dx$$

where $x = \frac{\gamma}{\Gamma}$. The evaluation of above rsults in:

$$\bar{\gamma} = \Gamma \sum_{k=1}^{M} \frac{1}{k}$$

or the improvement as a result of M branc diversity is

$$\frac{\bar{\gamma}}{\Gamma} = \sum_{k=1}^{M} \frac{1}{k}$$

Example: For a four (4) branch diversity scheme if the average SNR is 20 dB determine the probability that the SNR drops below 10 dB.

Answer: $\Gamma = 20\,dB$ and $\gamma = 10\,dB$

$\Gamma = 100$ and $\gamma = 10$ so $\frac{\gamma}{\Gamma} = 0.1$

$P_M(10\,dB) = (1 - e^{-0.1})^4 = 0.000082 = 8.2 \times 10^{-5}$

without diversity:

$$P_1(10\,dB) = (1-e^{-0.1}) = 0.095 \cong 10\%$$

<u>Maximal ratio combining improvement:</u>

Selection diversity is simple to implement:
it only requires <u>monitoring</u> & antenna selection switch.

However, due to the fact that it does not use information from the ~~remaini~~ other branches (the ones with lower SNR) it is not optimal.

In ratio combining, the voltage signal from M diversity branches (r_i's) are co-phased and added (coherent combination), resulting in

$$r_M = \sum_{i=1}^{M} G_i r_i$$

where G_i is the gain of ith branch.

The received signal Power is then

$$\frac{r_M^2}{2} = \frac{\left(\sum_{i=1}^{M} G_i r_i\right)^2}{2}$$

13 - 16

Assume that Noise power is the same in all branches, say N, the the total Noise N_T is:

$$N_T = N \sum_{i=1}^{M} G_i^2$$

$$\gamma_p = \frac{\left(\sum r_i^2\right)^2}{2N \sum r_i^2} = \sum \frac{r_i^2}{2N}$$

and overall SNR is

$$\gamma_M = \frac{r_M^2}{2N_T} = \frac{\left(\sum_{i=1}^{M} G_i r_i\right)^2}{2N \sum_{i=1}^{M} G_i^2}$$

or

$$\gamma_M = \frac{1}{2} \frac{\left(\sum_{i=1}^{M} \frac{r_i^2}{N}\right)^2}{N \sum_{i=1}^{M} \left(\frac{r_i^2}{N^2}\right)} = \sum_{i=1}^{M} \frac{r_i^2}{2N} = \sum_{i=1}^{M} \gamma_i$$

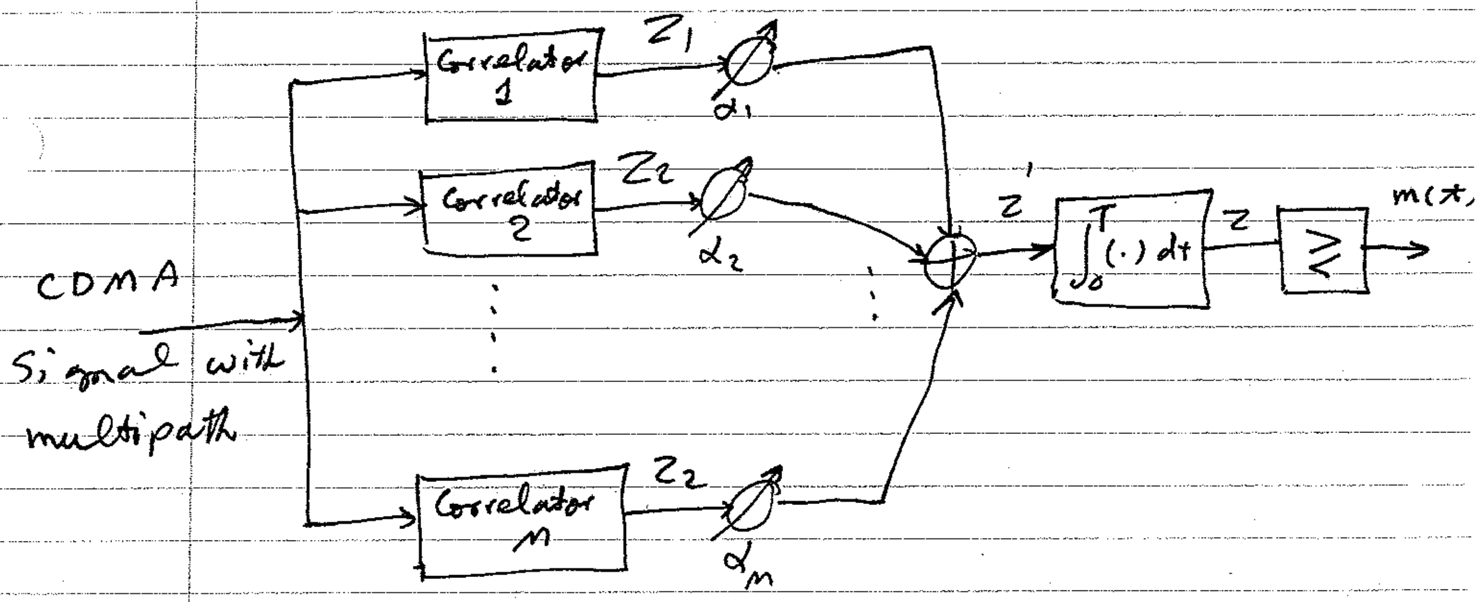
where, we have ~~thous~~ used the fact that γ_M is maximized when $\cancel{G_i = \frac{r_i}{N}}$, $G_i = k r_i$

we note that the SNR of M branch combiner is the sum of ~~all~~ SNRs of all branches.

$$\bar{\gamma}_M = \sum_{i=1}^{M} \bar{\gamma}_i = \sum_{i=1}^{M} \Gamma = M\Gamma$$

Rake Receiver

In CDMA, multipath arrivals apart by more than one chip duration are practically uncorrelated, So they do not cause ISI $\Rightarrow$ no need for equalization. However, the information in these multiple receptions can be used in type of diversity receiver called the rake receiver.

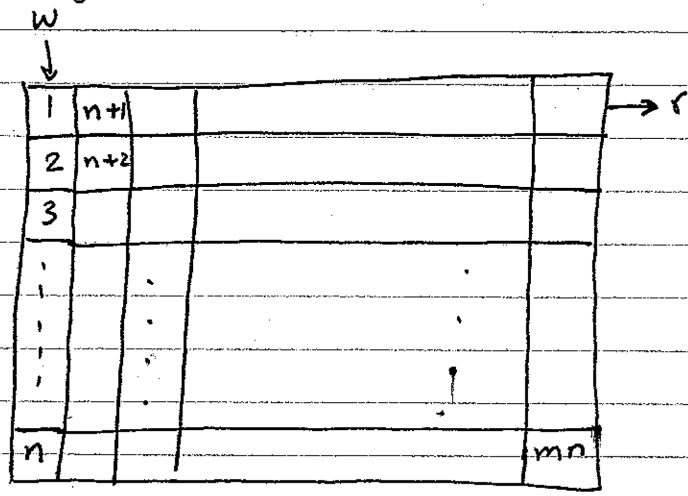


$$z' = \sum_{m=1}^{M} \alpha_m Z_m$$

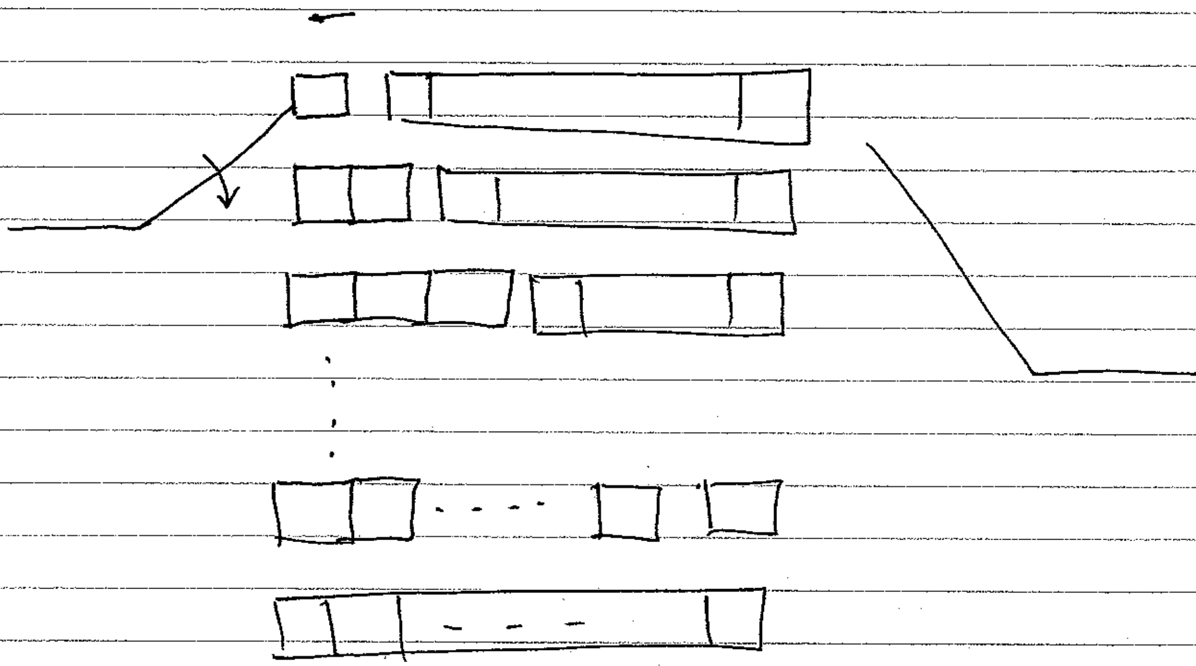
α_m's can be chosen using adaptive algorithms or simply based on the correlator output strength, i

$$\alpha_m = \frac{Z_m^2}{\sum_{m=1}^{M} Z_m^2}$$

Interleaving



Block interleaver



Convolutional interleaver.