a) *Mininum SIR*

In order to compute the minimun *SIR* at the mobile, we need to determine the number of interfering base stations in each possible configuration, which can be done by inspecting Figures 1 and 2. Table 1 shows the number of interfering base stations in the first tier, when 3 sectors ($BW = 120^o$) and 6 sectors ($BW = 60^o$) are used, for cluster sizes $N = 3$ and 4.

Using expression (1), we determine the minimum *SIR* (approximation) in each configuration (path loss exponent $n = 4$). Results are shown in Table 2.

Therefore, cluster size $N = 3$ cannot be used, since the minimum *SIR* achieved is below $SIR = 18.7$ dB. On the other hand, both configurations using cluster size $N = 4$ are feasible, regarding co-channel interference (assuming that a difference of 0.1 dB is negligible).

b) *Maximum carried traffic per cell*

Let us now computer the carried traffic per cell, when sectoring is used. As we know, each sector is assigned a subset of the set of channels assigned to the cell.

For example, for cluster size $N = 3$, each cell is assigned $300/3 = 100$ channels. If six sectors are employed, each sector is assigned $100/6 \approx 16$ channels. Using Erlang B formula, we find that each sector carries a maximum traffic of 9.83 Erlangs at a blocking probability of 0.02. Therefore, the maximum traffic carried by each cell is $9.83 \times 6 = 58.97$ Erlangs. Repeating this procedure, we can compute the maximum carried traffic per cell for other beamwidths and cluster sizes. Table 3 presents the results.

Table 1: Number of interfering base stations in the first tier (i_0) when 3 sectors ($BW = 120^o$) and 6 sectors ($BW = 60^o$) are used.

N	$BW = 60^o$	$BW = 120^o$
3	2	3
4	1	2

Table 2: Minimum SIR achieved when sectoring is used, for cluster sizes $N = 3$ and 4.

N	$BW = 60^o$	$BW = 120^o$
3	16.1 dB	14.3 dB
4	21.6 dB	18.6 dB

Table 3: Maximum carried traffic per cell (in Erlangs) when sectoring is used, for cluster sizes $N = 3$ and 4. 300 channels available in the system, $P_b = 2\%$

N	$BW = 60^o$	$BW = 120^o$
3	58.97	73.88
4	39.69	52.51

3.28 Cont'd

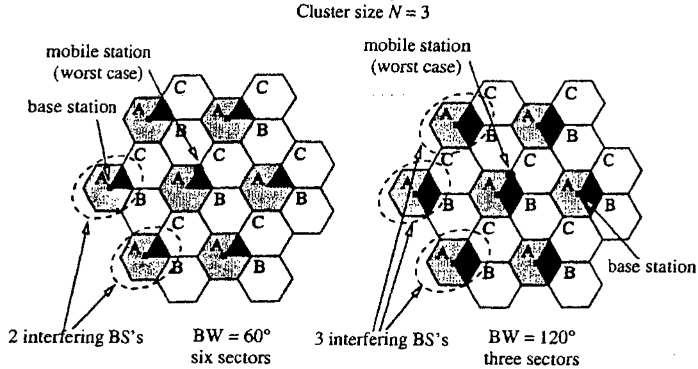


Figure 1 : Cluster size $N=3$, three & six sectors

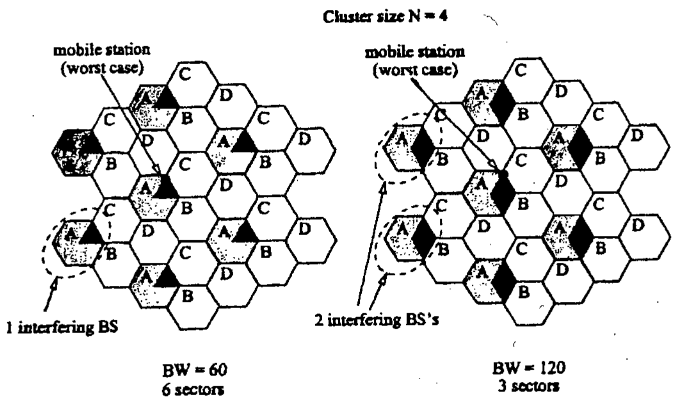


Figure 2: Cluster size $N=4$, three & six sectors

4.4

$$
\begin{aligned}
G_t &= G_r = 29 \text{ dB} && (10)\\
P_t &= 30 \text{ dBm} && (11)\\
\lambda &= \frac{c}{f} = 0.005 \text{ m} && (12)\\
d_0 &= 1\ m && (13)\\
d_1 &= 100 \text{ m} && (14)\\
d_2 &= 1000 \text{ m} && (15)\\
PL(d_0) &= 20\log_{10}\frac{4\pi d_0}{\lambda} = 20\log_{10}\frac{4\pi}{0.005} = 68 \text{ dB} && (16)\\
PL(d_1) &= PL(d_0) + 20\log_{10}\frac{d_1}{d_0} = 108 \text{ dB} && (17)\\
PL(d_2) &= PL(d_0) + 20\log_{10}\frac{d_2}{d_0} = 128 \text{ dB} && (18)\\
& && (19)\\
P_r &= P_t + G_t + G_r - PL = 30 + 29 + 29 - PL = 88 - PL && (20)\\
P_r(d_0) &= 88 - 68 = 20 \text{ dBm} && (21)\\
P_r(d_1) &= 88 - 108 = -20 \text{ dBm} && (22)\\
P_r(d_2) &= 88 - 128 = -40 \text{ dBm} && (23)\\
& && (24)\\
V &= \sqrt{4\ P_r\ R_{ant}} && (25)\\
V(d_1) &= 0.0447 \text{ v} && (26)\\
V(d_2) &= 0.0045 \text{ v} && (27)
\end{aligned}
$$

4.5

$$
\begin{aligned}
\Gamma_{\parallel} &= \frac{-\epsilon_2 \sin\theta_i + \sqrt{\epsilon_2 - \cos^2\theta_i}}{\epsilon_2 \sin\theta_i + \sqrt{\epsilon_2 - \cos^2\theta_i}}\\
\Gamma_{\perp} &= \frac{\sin\theta_i - \sqrt{\epsilon_2 - \cos^2\theta_i}}{\sin\theta_i + \sqrt{\epsilon_2 - \cos^2\theta_i}}
\end{aligned}
$$

At $\theta_i = 30^0$

	Ground	Brick	Limestone	Glass	Water
ϵ_r	15	4.44	7.51	4	81
$\Gamma_{\parallel}$	-0.33	-0.07	-0.18	-0.05	-0.64
$\Gamma_{\perp}$	-0.77	-0.59	-0.68	-0.57	-0.89

4.7 When $\theta = \theta_i$, $\Gamma_{\parallel} = 0$

$$\Rightarrow \Gamma_{\parallel} = \frac{-\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}{\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}} = 0$$

$$\Rightarrow -\varepsilon_r \cdot \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i} = 0$$

$$\Rightarrow \sqrt{\varepsilon_r - \cos^2\theta_i} = \varepsilon_r \cdot \sin\theta_i$$

$$\Rightarrow (\varepsilon_r^2 - 1)\sin^2\theta_i = \varepsilon_r - 1$$

$$\Rightarrow \sin\theta_i = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r^2 - 1}}$$

4.8 (a) The advantages of the two-ray ground reflection model in the analysis of path loss is that it considers both the direct path and a ground reflected propagation path between transmitter and receiver. The disavantage is that this model is oversimplified in that it does not include important factors such as terrain profile, vegetation and buildings.

(b) Generally, when $d > 10(h_t + h_r)$, we can say that $d \gg h_t + h_r$, and thus may apply the two ray model.

For $h_t = 35m$, $h_r = 3m$, $d = 250m$

$$d < 10(h_t + h_r) = 380\ m$$

Hence the two ray model could not be applied.

For $h_t = 30m$, $h_r = 1.5m$, $d = 450m$

$$d > 10(h_t + h_r) = 315\ m$$

Hence the two ray model could be applied.

4.8 Cont'd

(c) Using the two ray model, we can see that at large distances, the received power falls off with distance raised to the fourth power or at a rate of 40dB/decade, and the received power and path loss are independent of frequency

$\boxed{4.9}$ $\Delta = d'' - d'$

$$= \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$= d\left[1 + \left(\frac{h_t + h_r}{d}\right)^2\right]^{\frac{1}{2}} - d\left[1 + \left(\frac{h_t - h_r}{d}\right)^2\right]^{\frac{1}{2}}$$

For $d \gg h_t + h_r$, $\left(\frac{h_t + h_r}{d}\right)^2 \ll 1$, $\left(\frac{h_t - h_r}{d}\right)^2 \ll 1$.

Using Taylor series approximation, we have

$$\Delta \approx d\left[1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right] - d\left[1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right]$$

$$= d \cdot \frac{1}{2}\left[\left(\frac{h_t + h_r}{d}\right)^2 - \left(\frac{h_t - h_r}{d}\right)^2\right]$$

$$= d \cdot \frac{1}{2} \cdot \frac{4 h_t \cdot h_r}{d^2}$$

$$= \frac{2 h_t \cdot h_r}{d}$$

$\boxed{4.10}$ When $d \gg h_t + h_r$, we have $\theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2 h_t \cdot h_r}{d}$

$$\Rightarrow d = \frac{4\pi}{\lambda} \cdot \frac{h_t \cdot h_r}{\theta_\Delta}$$

$$\tan\theta_i = \frac{h_t + h_r}{d} < \tan 5^\circ \Rightarrow \frac{h_t + h_r}{\frac{4\pi}{\lambda} \cdot \frac{h_t \cdot h_r}{\theta_\Delta}} < \tan 5^\circ$$

$$\Rightarrow \frac{1 + \frac{h_r}{h_t}}{\frac{4\pi}{\lambda} \cdot \frac{h_r}{\theta_\Delta}} < \tan 5^\circ \Rightarrow h_t > \frac{h_r}{\frac{4\pi \cdot h_r \cdot \tan 5^\circ}{\lambda \cdot \theta_\Delta} - 1}$$

For $h_r = 2m$, $\theta_\Delta = 6.261$, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^2} \doteq 0.333m$

$$\Rightarrow h_t > \frac{2}{\frac{4\pi \times 2 \times \tan 5^\circ}{\frac{1}{3} \times 6.261} - 1} \Rightarrow \underline{\underline{h_{t\,min} = 37.7m}}$$

$\boxed{4.15}$ We need to find a d_f such that $\Delta = d'' - d = \frac{\lambda}{2}$.

$$\Delta = d'' - d = \sqrt{(h_t+h_r)^2+d_f^2} - \sqrt{(h_t-h_r)^2+d_f^2}$$

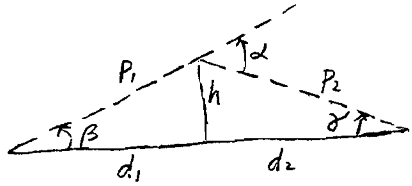
$$\Rightarrow \sqrt{(h_t+h_r)^2+d_f^2} - \sqrt{(h_t-h_r)^2+d_f^2} = \frac{\lambda}{2}$$

$$\Rightarrow (h_t+h_r)^2+d_f^2 = (h_t-h_r)^2+d_f^2+\frac{\lambda^2}{4}+\lambda\cdot\sqrt{(h_t-h_r)^2+d_f^2}$$

$$\Rightarrow d_f = \sqrt{\frac{16h_t^2\cdot h_r^2}{\lambda^2} - (h_t^2+h_r^2) + \frac{\lambda^2}{16}}$$

$\boxed{4.16}$ (a) $P_1 = \sqrt{d_1+h^2} = d_1\sqrt{1+\left(\frac{h}{d_1}\right)^2}$

$P_2 = \sqrt{d_2^2+h^2} = d_2\sqrt{1+\left(\frac{h}{d_2}\right)^2}$



Since $d_1, d_2 \gg h \gg \lambda$, $\frac{h}{d_1}, \frac{h}{d_2} \ll 1$. using Taylor series approximation, we have

$$P_1 \doteq d_1\left[1+\frac{1}{2}\left(\frac{h}{d_1}\right)^2\right] = d_1 + \frac{1}{2}\frac{h^2}{d_1}$$

$$P_2 \doteq d_2\cdot\left[1+\frac{1}{2}\left(\frac{h}{d_2}\right)^2\right] = d_2 + \frac{1}{2}\frac{h^2}{d_2}$$

$$\Rightarrow \Delta = P_1 + P_2 - (d_1+d_2)$$

$$\doteq \left(d_1 + \frac{1}{2}\frac{h'}{d_1}\right) + \left(d_2 + \frac{1}{2}\frac{h^2}{d_2}\right) - (d_1+d_2)$$

$$= \frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)$$

and $\phi = \frac{2\pi\Delta}{\lambda} \doteq \frac{2\pi}{\lambda}\left[\frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)\right]$.

(b) From the definition of ν, $\frac{\nu^2\pi}{2} = \phi$

$$\Rightarrow \nu = \sqrt{\phi\cdot\frac{2}{\pi}}$$

$$\Rightarrow \nu = \sqrt{\frac{2\pi}{\lambda}\left[\frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)\right]\cdot\frac{2}{\pi}} = h\cdot\sqrt{\frac{2}{\lambda}\cdot\frac{d_1+d_2}{d_1 d_2}}$$

Since $\tan\beta = \frac{h}{d_1} \ll 1$, $\tan\gamma = \frac{h}{d_2} \ll 1$, we have

$$\beta \approx \tan\beta = \frac{h}{d_1}, \quad \gamma \approx \tan\gamma = \frac{h}{d_2}$$

4.16 Cont'd

$$\Rightarrow \alpha = \beta + \delta = \frac{h}{d_1} + \frac{h}{d_2} = h\left(\frac{d_1+d_2}{d_1 \cdot d_2}\right)$$

$$\Rightarrow h = \alpha \cdot \frac{d_1 \cdot d_2}{d_1+d_2}$$

$$\Rightarrow \nu = h \cdot \sqrt{\frac{2}{\lambda} \cdot \frac{d_1+d_2}{d_1 \cdot d_2}} = \alpha \cdot \frac{d_1 \cdot d_2}{d_1+d_2} \cdot \sqrt{\frac{2}{\lambda} \cdot \frac{d_1+d_2}{d_1 \cdot d_2}}$$

$$= \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}}$$

4.18 Diffracted power decreases with the increase of the frequency as shown in Figure 1.

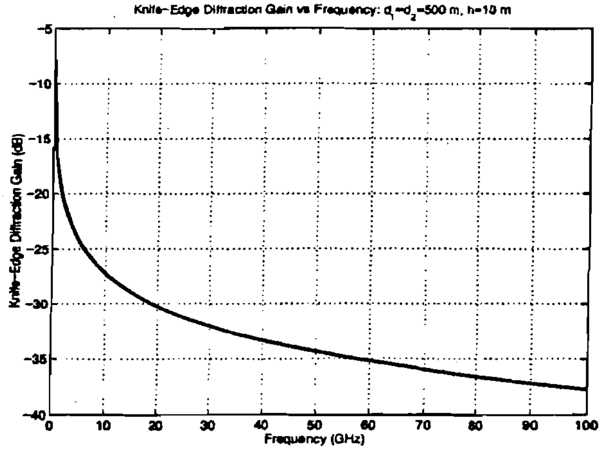


Figure 1: Diffracted power vs. frequency.

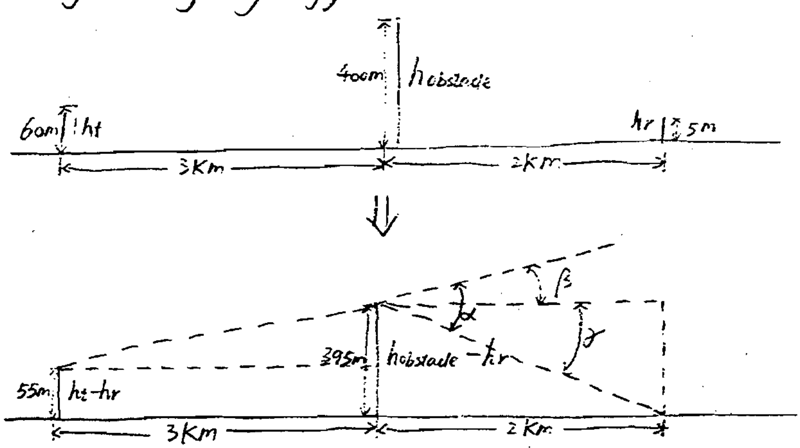
4.19 Given $P_t = 10W$, $G_t = 10dB = 10$, $L = 1dB \doteq 1.259$,
$G_r = 3dB = 2$, $f_c = 900\,MHz$, $d = 3000 + 2000 = 5000m$,

We have $\lambda_c = \frac{c}{f_c} \doteq 0.333(m)$ and free space received power

$$P_{freespace} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda_c^2}{(4\pi)^2 d^2 \cdot L} = \frac{10 \times 10 \times 2 \times 0.333^2}{(4\pi)^2 \times (5000)^2 \times 1.259}$$

$$\doteq 4.48 \times 10^{-9}(w) \doteq \underline{\underline{-53.5\ dBm}}$$

For the geometry shown below, we can redraw it in another geometry by approximation.



From the figure above we have

4.19 Cont'd

$$\tan\beta = \frac{h_{obstacle} - h_t}{d_1} = \frac{400-60}{3000} \doteq 0.1133 \Rightarrow \beta \doteq 0.11285\ (rad)$$

$$\tan\gamma = \frac{h_{obstacle} - h_r}{d_2} = \frac{400-5}{2000} = 0.1975 \Rightarrow \gamma \doteq 0.195\ (rad)$$

$$\Rightarrow \alpha = \beta + \gamma = 0.11285 + 0.195 \doteq 0.3078\ (rad)$$

and $v = \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}} = 0.3078 \times \sqrt{\frac{2 \times 3000 \times 2000}{0.333 \times (3000+2000)}} \doteq 26.12$

Using the approximation equation (3.59.e), we obtain

$$G_d(dB) = 20 \cdot \log_{10}\left(\frac{0.225}{v}\right) \qquad v > 2.4$$

$$= 20 \cdot \log_{10}\left(\frac{0.225}{26.12}\right)$$

$$\doteq -41.3\ dB.$$

$$\Rightarrow P_{received} = P_{freespace}\ (dBm) + G_d$$

$$= -53.5 - 41.3$$

$$= \underline{\underline{-94.8\ dBm}}$$

$\Rightarrow$ loss due to diffraction $L_d = P_{freespace} - P_{received} = \underline{\underline{41.3\ dB}}$

4.25

a) Handoff $\rightarrow$ two independent events: $P_{r,1} < P_{r,HO}$ and $P_{r,2} > P_{r,min}$. Therefore, the probability that a handoff occurs is given by

$$Pr[\text{ handoff}] = Pr[P_{r,1} < P_{r,HO}] \times Pr[P_{r,2} > P_{r,min}], \tag{2}$$

where $P_{r,1}$ and $P_{r,2}$ are the received signals at BS_1 and BS_2, respectively. $P_{r,1}$ is given by

$$\begin{aligned} P_{r,1} &= \underbrace{P_0 - 10n\log_{10}(d_1/d_0)}_{m_1} + \chi_1 \\ &= m_1 + \chi_1. \end{aligned} \tag{3}$$

Likewise for $P_{r,2}$

$$\begin{aligned} P_{r,2} &= \underbrace{P_0 - 10n\log_{10}[(D - d_1)/d_0]}_{m_2} + \chi_2 \\ &= m_2 + \chi_2. \end{aligned} \tag{4}$$

Therefore, for a given distance d_1, $P_{r,1}$ and $P_{r,2}$ are Gaussian variables with standard deviation σ and means m_1 and m_2, respectively.

Thus, the probability $Pr[P_{r,1} < P_{r,HO}]$ is

$$\begin{aligned} Pr[P_{r,1} < P_{r,HO}] &= \int_{-\infty}^{P_{r,HO}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(x-m_1)^2}{2\sigma^2}\right] dx \\ &= 1 - Q\left(\frac{P_{r,HO} - m_1}{\sigma}\right). \end{aligned} \tag{5}$$

where $Q(x)$ is the Q-function:

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-y^2}{2}\right) dy. \tag{6}$$

Likewise

$$\begin{aligned} Pr[P_{r,2} > P_{r,min}] &= \int_{P_{r,min}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(x-m_2)^2}{2\sigma^2}\right] dx \\ &= Q\left(\frac{P_{r,min} - m_2}{\sigma}\right). \end{aligned} \tag{7}$$

Substituting (3)-(7) into (2), we have the probability that a handoff occurs as a function of the distance d_1. Figure 2 shows the received area average power at both base stations (m_1 and m_2) (Not required in the homework!). Figure 3 shows the probability that a handoff occurs, along with the probabilities $Pr[P_{r,1} < P_{r,HO}]$ and $Pr[P_{r,2} > P_{r,min}]$.

b) From the plot in Figure 3, the distance d_{ho}, such that the probability that a handoff occurs is equal to 80%, is $d_{ho} \approx 1000$ m.

4.25 Cont'd.

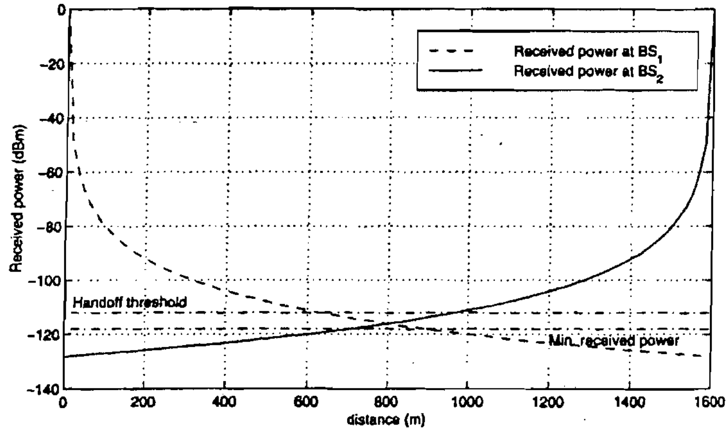


Figure 2: Received area average power levels at the mobile, from both base stations.

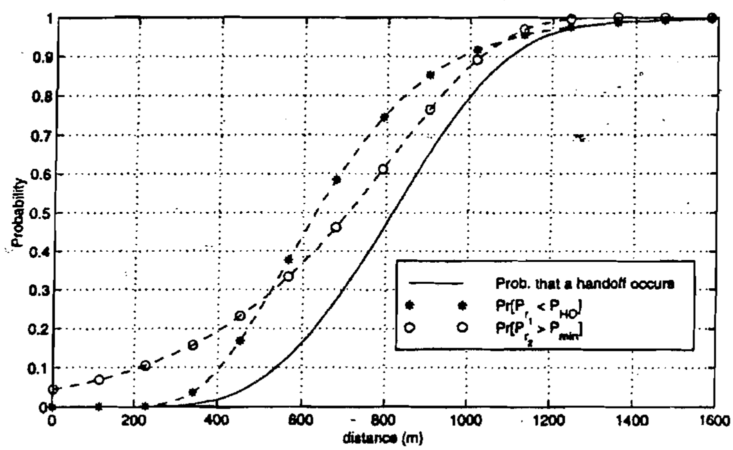


Figure 3: Probability that a handoff occurs, $Pr[P_{r,1} < P_{r,HO}]$ and $Pr[P_{r,2} > P_{r,min}]$

$\boxed{4.28}$ noise floor $= K \cdot B_W \cdot F \cdot T_o$

$= 1.38 \times 10^{-23} \times 30 \times 10^3 \times 10 \times 290 \doteq 1.2 \times 10^{-15} (W)$

$\doteq -119.2 \, (dBm)$

$\Rightarrow$ threshold $\gamma =$ noise floor $(dBm) +$ SNR(dB)

$= -119.2 + 25 = -94.2 \, (dBm)$

Given $EIRP = P_t \cdot G_t = 100W$, $G_r = 0\,dB = 1$, $d_o = 1Km$, $\lambda = \frac{c}{f} = 0.333m$

$$P_r(d_o) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_o^2} = \frac{100 \times 1 \times 0.333^2}{(4\pi)^2 \times (1000)^2} \doteq 7.04 \times 10^{-8} (W)$$

$\doteq -41.5 \, dBm$

For $d = 10\,Km$, $n = 4$.

$$\overline{P_r(d)} = P_r(d_o) - 10 \cdot n \cdot \log_{10}\left(\frac{d}{d_o}\right) = -41.5 - 40 = -81.5\,dBm$$

$$\Rightarrow Pr(P_r(d) > \gamma) = Q\left[\frac{\gamma - \overline{P_r(d)}}{\sigma}\right] = Q\left[\frac{-94.2 - (-81.5)}{8}\right]$$

$= Q(-1.5875) \doteq \underline{\underline{0.944}}$

4.29

(a) Find the minimum mean square error (MMSE) estimate for the path loss exponent, n.

First note that $P_r(100m) = 0\,dBm = P_r(d_0)$

$P_r(d) = P_r(d_0) - PL(d)$

d	P_r (dBm)
100	0
200	-25
1000	-35
2000	-38

$$J_n = \sum_{i=1}^{4} [P_i - \hat{P}_i]^2 = \underbrace{[0-0]^2}_{100m} + \underbrace{[-25-(0-10n\log(\tfrac{200}{100}))]^2}_{200m}$$

$$+[-35-(0-10n\log\tfrac{1000}{100})]^2 + [-38-(0-10n\log\tfrac{2000}{100})]^2$$

$$= 625 - 150n + 9n^2 + 1225 - 700n + 100n^2 + 1444 - 988n + 169n^2$$

$$\therefore J_n = 278n^2 - 1838n + 3294$$

$$\frac{dJ_n}{dn} = 556n - 1838 \Rightarrow \boxed{n = 3.30}$$

(b) Calculate the standard deviation of shadowing about the mean value.

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{J_n}{4}}\Bigg|_{n=3.30}$$

$$= \frac{\sqrt{278(3.3)^2 - 1838(3.3) + 3294}}{\sqrt{4}}$$

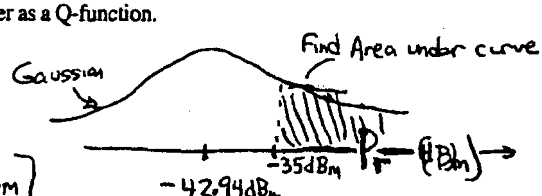
$$= \frac{\sqrt{3027 - 6065.4 + 3294}}{\sqrt{4}} = \frac{\sqrt{255.6}}{\sqrt{4}} = \boxed{8\,dB}$$

(c) Estimate the received power at $d = 2$ km using the resulting model.

$$P_r(d) = P_r(d_0) - PL(d) = 0\,dBm - 10[3.3]\log_{10}\left(\frac{2000}{100}\right)$$

$$= (0 - 42.94)\,dBm = \boxed{-42.94\,dBm}$$

(d) Predict the likelihood that the received signal level at 2 km will be greater than –35 dBm. Express your answer as a Q-function.



AVG. PWR AT 2KM using Model

$$P_r[P_r(2km) \geq -35\,dBm] = Q\left[\frac{-35-(-42.94)}{8}\right]$$

$$= Q\left(\frac{7.93}{8}\right) = Q(0.99) \approx Q(1)$$