CHAPTER 5

5.1 $c = \lambda f \Rightarrow \lambda = \frac{c}{f} = \frac{3\,10^8\,m/s}{1.95\,10^9\,Hz} = 0.154\,m$

$f_d = \frac{V}{\lambda}\cos\theta \qquad f_{d_{max}} = \frac{V}{\lambda}\ ;\ -f_{d_{max}} = -\frac{V}{\lambda}$

$V = 1\,km/hr \Rightarrow V = 0.278\,m/s \Rightarrow f_d = 1.8\,Hz$

$V = 5\,km/hr \Rightarrow V = 1.39\,m/s \Rightarrow f_d = 9.03\,Hz$

$V = 100\,km/hr \Rightarrow V = 27.8\,m/s \Rightarrow f_d = 180.5\,Hz$

$V = 1000\,km/hr \Rightarrow V = 278\,m/s \Rightarrow f_d = 1805\,Hz$

∴ @ 1 km/hr, spectral edges are 1949.9999982 mHz and 1950.0000018 mHz

@ 5km/hr, spectral edges are 1949.99999097 mHz and 1950.00000903 mHz

@ 100km/hr, edges are 1949.9998195 mHz and 1950.0001805 mHz

@ 1000 km/hr, edges are 1949.998195 mHz
1950.001805 mHz

$\boxed{5.5}$ $B_c \approx \frac{1}{5\sigma_\tau} \approx 2\ B_{baseband} \geq \frac{2}{T_s}$

for flat fading

$$\therefore \frac{2}{T_s} \leq \frac{1}{5\sigma_\tau}$$

$$\therefore T_s \geq 10\sigma_\tau \text{ for flat fading}$$

5.6 For (a), $\overline{\tau} = \dfrac{1\times0+1\times50+0.1\times75+0.01\times100}{1+1+0.1+0.01} \doteq 27.725\,(ns)$

$$\overline{\tau^2} = \frac{1\times0+1\times50^2+0.1\times75^2+0.01\times100^2}{1+1+0.1+0.01} \doteq 1498.8\,(ns^2)$$

$\Rightarrow$ the rms delay spread $\sigma_\tau = \sqrt{1498.8-27.725^2} \doteq 27\,(ns)$

Since $\frac{\sigma_\tau}{T_s} \leq 0.1$, $T_s \geq 10\sigma_\tau = 270\,ns$

$\Rightarrow$ Smallest symbol period $T_{S_{min}} = \underline{\underline{270ns}}$

greatest data rate $R_{max} = \frac{1}{T_{S_{min}}} \doteq \underline{\underline{3.7\,Mbps}}$

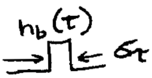
For (b), $\overline{\tau} = \dfrac{0.01\times0+0.1\times5+1\times10}{0.01+0.1+1} \doteq 9.46\,(\mu s)$

$$\overline{\tau^2} = \frac{0.01\times0+0.1\times5^2+1\times10^2}{0.01+0.1+1} = 92.34\,(\mu s^2)$$

$$\sigma_\tau = \sqrt{\overline{\tau^2}-(\overline{\tau})^2} = \sqrt{92.34-(9.46)^2} \doteq 1.688\,(\mu s)$$

$T_{S_{min}} = 10\,\sigma_\tau = \underline{\underline{16.88\,(\mu s)}}$ $\quad R_{max} = \frac{1}{T_{S_{min}}} \doteq \underline{\underline{59.25\,kbps}}$

5.7 (a) $T_s = \frac{1}{100\,kbps} = 10^{-5}s$. $\quad \boxed{0 \leq \sigma_\tau \leq 10^{-6}s}$



if $\sigma_\tau \leq \frac{1}{10}T_s \Rightarrow$ flat fading

$T_s \geq 10\,\sigma_\tau \qquad \sigma_\tau \leq \frac{T_s}{10}$

(b) $f_d = \frac{v}{\lambda}$

$c = \lambda f$

$\lambda = c/f = 3\,10^8/5.8\,10^9 \approx \underline{\underline{0.05m}}$

$$\text{velocity} = \frac{30\text{ miles}}{hr}\cdot\frac{1hr}{3600\text{ sec}}\cdot\frac{5280'}{1\text{ mile}}\cdot\frac{12''}{1'}\cdot\frac{2.54cm}{1''}\cdot\frac{1m}{100cm}$$

$$= \frac{30\cdot5280\cdot12\cdot2.54}{3600\cdot100} = 13\text{ m/s}$$

$f_d = \frac{13}{0.05} = \boxed{260\,Hz}$

5.7 Cont'd.

Coherence Time Define $\begin{bmatrix} 90\% \\ 50\% \end{bmatrix}$

$T_c \approx \frac{1}{f_m} = \boxed{0.004s}$

$T_c \approx \frac{9}{16\pi f_m} = \frac{1}{5 f_m} = \boxed{0.00125}$

(c) Here we have $f_d = 200 Hz$; $T_s \approx 10^{-5} s$; $T_c \approx 10^{-3} s$

slow fading $\Rightarrow T_s << T_c$ here $10^{-5} << 10^{-3}$

$\boxed{\text{slow fading}}$

d) pick your T_c, then

$\# \text{ bits sent} = R_b \cdot T_s = \frac{10^5 b}{s} \cdot 10^{-3} s \approx \underline{\underline{100}}$

$\boxed{5.8}$ For (a), the 90% correlation coherence bandwidth is

$B_{c0.9} \doteq \frac{1}{50\sigma_\tau} = \frac{1}{50 \times 27 (ns)} \doteq \underline{\underline{740 \, KHZ}}$

the 50% correlation coherence bandwidth is

$B_{c0.5} \doteq \frac{1}{5\sigma_\tau} = \underline{\underline{7.4 \, MHZ}}$

For (b), $B_{c0.9} = \frac{1}{50\sigma_\tau} = \frac{1}{50 \times 1.688 (\mu s)} \doteq \underline{\underline{11.85 \, KHZ}}$

$B_{c0.5} = \frac{1}{5\sigma_\tau} \doteq \underline{\underline{118.5 \, KHz}}$

$\boxed{5.11}$ $P(r<R) = \int_{-\infty}^{R} p(r)dr$

$$= \int_0^R \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr$$

$$= \frac{1}{2}\int_0^R \frac{1}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr^2$$

$$\overset{t=r^2}{=} \frac{1}{2}\int_0^{R^2} \frac{1}{\sigma^2} \exp\left(\frac{-t}{2\sigma^2}\right) dt$$

$$= \frac{1}{2}\cdot\frac{1}{\sigma^2}(-2\sigma^2)\cdot \exp\left(\frac{-t}{2\sigma^2}\right)\Big|_0^{R^2}$$

$$= 1 - \exp\left(\frac{-R^2}{2\sigma^2}\right)$$

For $\rho = -10dB = 0.316$. the percentage of time that a signal is 10dB or more below the rms value for a Rayleigh fading signal is $P_{o,1} = 1 - \exp(-\rho^2) = 1 - \exp(-0.316^2) \doteq \underline{\underline{9.5\%}}$

$\boxed{5.12}$ (a) Since $N_R = \sqrt{2\pi} f_m \cdot \rho e^{-\rho^2}$, the ratio of the desired signal level to rms signal level that maximizes N_R is the solution of the equation

$$\frac{dN_R}{d\rho} = 0 \implies \sqrt{2\pi} f_m \cdot (1-2\rho^2)\cdot e^{-\rho^2} = 0$$

$$\implies 1 - 2\rho^2 = 0 \implies \rho = \underline{\underline{\frac{1}{\sqrt{2}} = -3dB}}$$

(b) $\lambda = \frac{c}{f_c} = \frac{3\times10^8}{900\times10^6} \doteq 0.33\ (m)$ $\quad V = \frac{50\times10^3 m}{3600\ sec} \doteq 13.89\ m/s$

$\implies f_m = \frac{V}{\lambda} = \frac{13.89}{0.33} \doteq 41.67\ (HZ)$