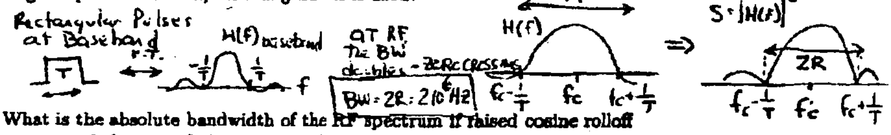
6.12

Assume a binary bit stream is to be modulated on an RF carrier. If the baseband bit stream has a data rate of 1 Megabit per second, then:

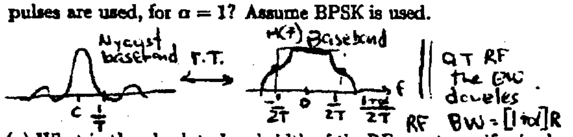
$T = 10^{-6}\,s$

$R = 10^{6}\,s$

(a) What is the first-zero crossing bandwidth of the RF spectrum if simple rectangular pulses are used, assuming BPSK is used?



(b) What is the absolute bandwidth of the RF spectrum if raised cosine rolloff pulses are used, for $\alpha = 1$? Assume BPSK is used.



$\alpha = 1$

$BW = (1+1)\,10^6 = \boxed{2\,10^6\,Hz}$

(c) What is the absolute bandwidth of the RF spectrum if raised cosine rolloff pulses are used, for $\alpha = \frac{1}{3}$? Assume BPSK is used.

$$RF\ BW = \left[1 + \tfrac{1}{3}\right] 10^6 = \boxed{1.333\ 10^6\ Hz}$$

(d) If a timing jitter of 10^{-6} seconds exists at the receiver and raised cosine rolloff pulses are used, will the detector experience intersymbol interference from the adjacent symbols? Explain.

Nyquist pulses produce NO ISI for all k, t such that $t = kT_s$ except for $k=0$ (the desired pulse). If the jitter is exactly $10^{-6}\,s$, then the timing offset is exactly one symbol. Thus, there will be NO ISI except for the next symbol, which will be perfectly received, i.e. the receiver will be off sync by T.

(e) If GMSK modulation is to be used and a 3 dB bandwidth of 500 kHz is used for the Gaussian low pass filter, what is the proper choice for the FM peak frequency deviation, ΔF?

$$\Delta F = \frac{1}{4T_b} = \frac{R_b}{4} = \frac{10^6}{4} = \boxed{250\ kHz}$$

(NOT A function of 3 dB BW!)

(f) For GMSK modulation using $BT \le 0.5$, how many spectral sidelobes occur?

No sidelobes occur at $BT < \frac{1}{2}$

I will accept two sidelobes (one on each side of carrier)

6.13

For $SNR = 30\,dB = 1000$, $B = 200\,KHz$, the maximum possible data rate, $C = B \cdot \log_2\left(1 + \frac{S}{N}\right) = 200 \times 10^3 \times \log_2(1+1000) \doteq 1.99\ Mbps$

The GSM data rate is 270.833 bps, which is only about $0.136\,C$

6.14 For IS-54, $R = 48.6$ Kbps, $B = 30$ KHz.

$$\Rightarrow \eta_B = \frac{R}{B} = \frac{48.6}{30} = 1.62 \text{ bps/Hz}$$

For GSM, $R = 270.833$ Kbps. $B = 200$ KHz

$$\Rightarrow \eta_B = \frac{270.833}{200} = 1.35 \text{ bps/Hz}$$

For PDC, $R = 42$ kbps, $B = 25$ KHz $\Rightarrow \eta_B = \frac{42}{25} = 1.68$ bps/Hz

For IS-95, the bandwidth efficiency depends on the number of users K. For $R = 9.6$ Kbps/s, $B = 1.2288$ MHz.

$$\Rightarrow \eta_B = \frac{K \cdot R}{B} = \frac{K \cdot 9.6 \times 10^3}{1.2288 \times 10^6} \doteq K \cdot 7.8 \times 10^{-3} \text{ bps/Hz}$$

If $SNR = 20dB = 100$, the theoretical spectral efficiency

$$\eta_{B\max} = \log_2(1+SNR) = \log_2(1+100) \doteq 6.66 \text{ bps/Hz}$$

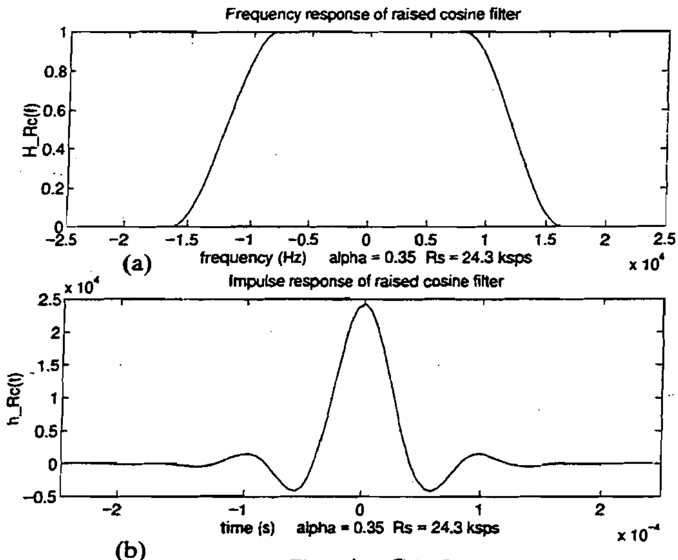
6.15 See the MATLAB program p6_15.m and Fig. p6-15 (a) and (b)

$$H_{RC}(f) = \begin{cases} 1 & 0 \le |f| \le \frac{1-\alpha}{2T_s} \\ \frac{1}{2}\left\{1 + \cos\left[\frac{\pi \cdot (|f| \cdot 2T_s - 1 + \alpha)}{2\alpha}\right]\right\} & \frac{(1-\alpha)}{2T_s} < |f| \le \frac{(1+\alpha)}{2T_s} \\ 0 & |f| > \frac{(1+\alpha)}{2T_s} \end{cases}$$

$$h_{RC}(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\pi t} \cdot \frac{\cos\left(\frac{\pi \alpha t}{T_s}\right)}{1 - \left(\frac{4\alpha t}{2T_s}\right)^2}$$

Fraction of the total radiated energy that will fall out-of-band

$$= 1 - \frac{\int_{-15K}^{15K} H_{RC}^2(f) \cdot \left(\frac{\sin(\pi f T_s)}{\pi f T_s}\right)^2 df}{\int_{-\frac{(1+\alpha)R_s}{2}}^{\frac{(1+\alpha)R_s}{2}} H_{RC}^2(f) \cdot \left(\frac{\sin(\pi f T_s)}{\pi f T_s}\right)^2 df} \doteq 3 \times 10^{-5} = 0.003\%$$



Frequency response of raised cosine filter
H_Rc(f)
1
0.8
0.6
0.4
0.2
0
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
(a)
frequency (Hz) alpha = 0.35 Rs = 24.3 ksps
x 10^4
Impulse response of raised cosine filter
x 10^4
2.5
2
1.5
1
0.5
0
-0.5
h_Rc(t)
-2
-1
0
1
2
time (s) alpha = 0.35 Rs = 24.3 ksps
x 10^-4
(b)

Fig. p6_15(a), (b)

6.16 $\underline{BT_s = 0.5}$, $T_s = \frac{1}{19.2 KSPS}$ (See also problem 6.27)

$\Rightarrow B = \frac{0.5}{T_s} = 0.5 \times 19.2 \times 10^3 = 9.6\ KHz$

$\Rightarrow \alpha = \frac{1.1774}{2 \cdot B} = \frac{1.1774}{2 \times 9.6 \times 10^3} \doteq 6.13 \times 10^{-5}$

$\Rightarrow H_G(f) = \exp(-\alpha^2 f^2) = \underline{\underline{\exp(-3.75 \times 10^{-9} f^2)}}$

$h_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} t^2\right) \doteq \underline{\underline{28907.08 \exp(-2.62 \times 10^9 t^2)}}$

$\underline{BT_s = 0.2}$, $\Rightarrow B = \frac{0.2}{T_s} = 0.2 \times 19.2 \times 10^3 = 3.84\ KHz$

$\Rightarrow \alpha = \frac{1.1774}{2 \cdot B} = \frac{1.1774}{2 \times 3.84 \times 10^3} \doteq 1.533 \times 10^{-4}$

$\Rightarrow H_G(f) = \exp(-\alpha^2 f^2) = \underline{\underline{\exp(-2.35 \times 10^{-8} f^2)}}$

$h_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} \cdot t^2\right) \doteq \underline{\underline{11559 \exp(-4.195 \times 10^8 t^2)}}$

$\underline{BT_s = 0.75}$, $\Rightarrow B = \frac{0.75}{T_s} = 0.75 \times 19.2 \times 10^3 = 14.4\ KHz$

$\Rightarrow \alpha = \frac{1.1774}{2 \cdot B} = \frac{1.1774}{2 \times 14.4 \times 10^3} \doteq 4.088 \times 10^{-5}$

$\Rightarrow H_G(f) = \exp(-\alpha^2 f^2) = \underline{\underline{\exp(-1.67 \times 10^{-9} \cdot f^2)}}$

$h_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} \cdot t^2\right) \doteq \underline{\underline{4.334 \times 10^4 \cdot \exp(-5.8998 \times 10^9 t^2)}}$

The impulse response and frequency response are shown in Fig. p6_16 (a) and (b), respectively. Using the MATLAB program p6_16.M, We can calculate the fraction, F_{out}, of the total radiated energy that would fall out-of-band. $\underline{\text{For } BT_s = 0.5}$, $F_{out} \doteq \underline{\underline{2.32 \times 10^{-3}}}$

$\underline{\text{For } BT_s = 0.2}$, $F_{out} \doteq \underline{\underline{2.21 \times 10^{-7}}}$

$\underline{\text{For } BT_s = 0.75}$, $F_{out} \doteq \underline{\underline{9.91 \times 10^{-3}}}$

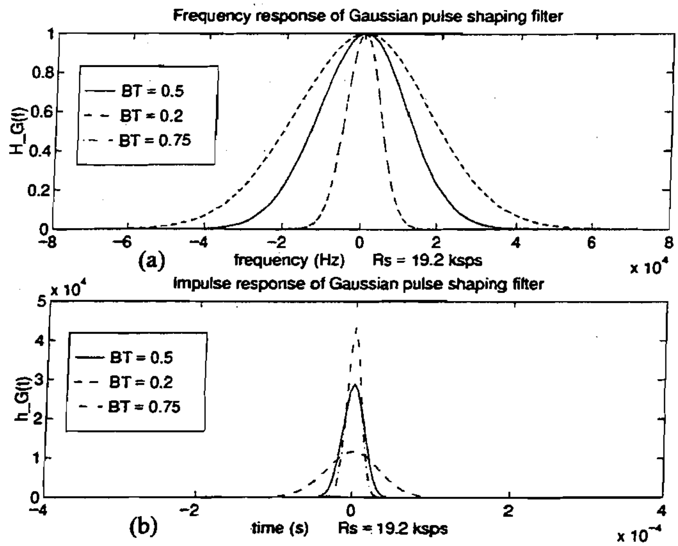

Frequency response of Gaussian pulse shaping filter
H_G(f)
BT = 0.5
BT = 0.2
BT = 0.75
frequency (Hz)   Rs = 19.2 ksps
x 10^4
(a)
Impulse response of Gaussian pulse shaping filter
h_G(t)
x 10^4
BT = 0.5
BT = 0.2
BT = 0.75
time (s)   Rs = 19.2 ksps
x 10^-4
(b)

Fig. p6-16 (a), (b)

6.19 See the MATLAB program p6_19.m and Fig. p6-19(a), (b), (c), (d), (e), (f), (g), (h) and (i).

For a binary message stream 01100101, the serial data stream is converted to two parallel data streams, each with symbol rate as one half of the bit rate. The even data bits $m_I(t)$ (the first bit of data stream is labeled as bit 0) 0100 are first offset by one bit period and then multiplied by $x(t)$ (See Fig. 6.39 in section 6.9.2), the odd data bit $m_Q(t)$ are multiplied by $y(t)$. The sum of these two multiplication results is the MSK signal. The signals $m_I(t)\cdot x(t)$, $m_I(t)$, $m_Q(t)\cdot y(t)$, $m_Q(t)$, and $S_{MSK}(t)$ are shown in Fig. p6-19 (a), (b), (c), (d), (e), respectively. In the receiver, the input of the integrator, $S_{MSK}(t)\cdot x(t)$ in the I channel is shown in Fig. p6-19(f) and the output of the integrator is shown in Fig. p6-19(g). For the Q channel, $S_{MSK}(t)\cdot y(t)$ is shown in Fig. p6-19(h) and the output of the integrator is shown in Fig. p6-19(i). In Fig. p6.19(g) and (i), the sampled signals as the input of the threshold detectors are also illustrated.

6.20 $P_b = Q\left(\sqrt{\frac{3N}{K-1}}\right) = Q\left(\sqrt{\frac{3\times 511}{63-1}}\right) = Q(4.9725) \doteq \underline{\underline{3.3\times10^{-7}}}$

In determine the above result, we assume that all interferers provide equal power, the same as the desired user. All users are considered orthogonal and independent, and the Gaussian approximation is assumed to be valid.

6.21 $Q\left(\sqrt{\frac{3N}{K-1}}\right) = 10 \times 3.3 \times 10^{-7} = 3.3 \times 10^{-6}$

$$\Rightarrow \sqrt{\frac{3N}{K-1}} = 4.5602 \Rightarrow \frac{3 \times 511}{K-1} = 20.3058 \Rightarrow \underline{\underline{K \doteq 76}}$$

6.22

T_c=1/1.2288Mcps, T_s=13kbps, 7.8dB ~ 6

so the processing gain is PG=T_s/T_c= 1.2288M/13k = 94

Assume BPSK,

$$BER = Q\left[\frac{1}{\sqrt{\frac{K-1}{3PG}+\frac{No}{2Eb}}}\right] = Q\left[\frac{1}{\sqrt{\frac{20-1}{3*94.52}+1/12}}\right] = Q(2.58) = 0.0049$$

For **actual IS-95 system**, some coding overhead is added. So the bit rate is 19.2kbps, and since QPSK is used, the base band rate is thus 9600bps. This gives out PG=1.2288M/9600=**128. The** corresponding BER is:

$$BER = Q\left[\frac{1}{\sqrt{\frac{K-1}{3PG}+\frac{No}{2Eb}}}\right] = Q\left[\frac{1}{\sqrt{\frac{20-1}{3*128}+1/12}}\right] = Q(2.74) = 0.0031$$

6.25 (a) number of hops per second

$= 2 \text{ hops/bit} \times R = 2 \times 25000 = \underline{\underline{5 \times 10^4 \text{ hops/sec}}}$

(b) For $\frac{E_b}{N_0} = 20dB = 100$ and a single user (assuming AWGN)

$P_b = \frac{1}{2} \cdot \exp(-\frac{E_b}{2N_0}) = \frac{1}{2} \cdot \exp(-\frac{100}{2}) \doteq \underline{\underline{9.64 \times 10^{-23} \doteq 0}}$

(c) number of possible hopping channels,

$M = \frac{20 \times 10^6}{50 \times 10^3} = 400 \Rightarrow P_b \doteq \frac{1}{2}[\frac{K-1}{M}] = \frac{1}{2} \times \frac{20}{400} = \underline{\underline{0.025}}$

(d) $P_b \simeq \frac{1}{2} P_h = \frac{1}{2}[1 - (1 - \frac{1}{M})^{K-1}]$

$= \frac{1}{2}[1 - (1 - \frac{1}{400})^{200}] \doteq \underline{\underline{0.2}}$

6.26 Hints for solving Problem 6.26

To evaluate the probability of error, P_e, of a signal in flat Rayleigh fading, simply weight the P_e by the conditional likelihood of the signal being a particular value. That is,

$$P(\text{error}) = \int_0^\infty \underbrace{P(\text{error}\,|\text{specific } E_b/N_0)}_{\text{in text books - AWGN}} \cdot P(\text{specific } E_b/N_0)\, d\left(\frac{E_b}{N_0}\right) \quad (1)$$

where the probability density of the fading E_b/N_0 is given as the square of a Rayleigh distributed r.v., which is easily shown to be exponential, eqn. (6.155).

If we let X = random E_b/N_0 due to fading and let α^2 denote a chi-square (exponential) r.v. with the pdf of a squared Rayleigh distributed voltage, then

$$X = \alpha^2 \left(\frac{E_b}{N_0}\right) \quad (2)$$

Let's let

$$\Gamma = \overline{\alpha^2}\frac{E_b}{N_0}, \text{ the average value of } \frac{E_b}{N_0} \quad (3)$$

Then:

$$P_e(\Gamma) = \int_0^\infty P_e(X) \cdot \frac{1}{\Gamma} e^{-\frac{X}{\Gamma}} dX \text{ (Eqn. 1)} \quad (4)$$

is the value of P_e in flat slow Rayleigh fading.

6.26 Hints cont'd.

Hint: Derive the p.d.f. for

$$X = \alpha^2 \left(\frac{E_b}{N_0}\right) \tag{5}$$

where E_b/N_0 is a constant and α is Rayleigh, and you get (6.155), an exponential PDF.

Note a table of integrals can evaluate (Eqn. 1) where

$$P_e^{(1)}(x) = \frac{1}{2}e^{-\eta x} \tag{6}$$

$$\eta = \frac{1}{2} \text{ for noncoherent FSK} \tag{7}$$

$$\eta = 1 \text{ for DPSK} \tag{8}$$

and

$$P_e^{(2)}(x) = \frac{1}{2} erfc\sqrt{\beta x} \tag{9}$$

$$\beta = \frac{1}{2} : \text{coherent FSK} \tag{10}$$

$$\beta = 1 \text{ coherent PSK} \tag{11}$$

A table of integrals can show:

$$P_b^{(1)} = \int_0^\infty P_b^{(1)}(X)\frac{1}{\Gamma}e^{-\frac{X}{\Gamma}}dX \tag{12}$$

$$= \frac{1}{2}\left[\frac{1}{1+\eta\Gamma}\right] \tag{13}$$

$$P_e^{(2)} = \int_0^\infty P_b^{(2)}(X)\frac{1}{\Gamma}e^{-\frac{X}{\Gamma}}dX \tag{14}$$

$$= \frac{1}{2}\left[\frac{1}{1-\sqrt{1+\frac{1}{\beta\Gamma}}}\right] \tag{15}$$

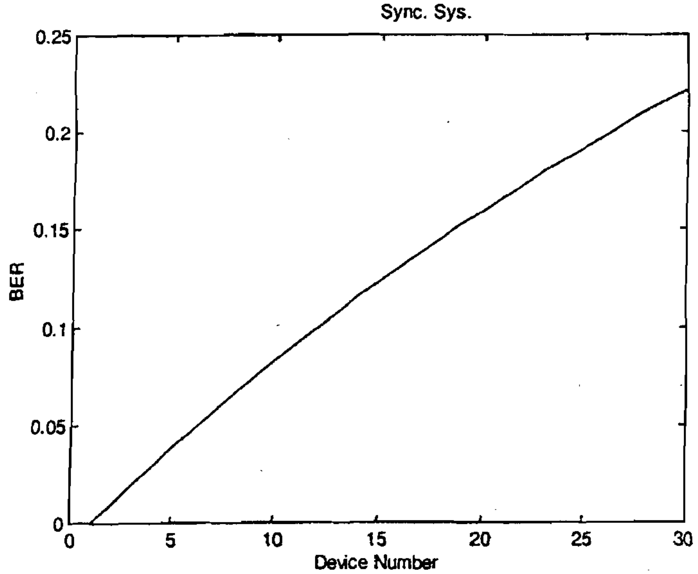
OR - if not closed form, use numerical integration for specific values of Γ.

6.26 Solution

We can treat the system as synchronized because they are in the nearby local area.

Using formula 6.147 and 6.149, we can plot the curve for BER vs. K. Please note that you should use the exact expression of 6.148, not the approximation expression, so that you can observe the change of the curve. A sample code in Matlab is enclosed.

When K=25, BER is above 0.2, which is redeemed as unacceptable because we need to apply significant coding scheme to reduce the end user BER. However, for the homework, any reasonable justification about number of acceptable users will be credited.



6.27 See the MATLAB program p6_27.m and Fig. p6-27a (a), (b). Fig. p6-27b(a), (b), Fig. p6-27c (a), (b) and Fig. p6-27d.

For a Gaussian lowpass filter with transfer function

$H_G(f) = \exp(-\alpha^2 f^2)$

the 3-dB bandwidth B is related with α by the following equation.