Question 1: A stream of digital data with data rate of 20 Mb/s is to be transmitted in an additive white Gaussian noise channel with single sided power spectral density of 2×10^{-10} Watts/Hz where the available bandwidth is 4 MHz. The required system performance is a bit error rate of 10^{-5}. If the received signal power is 25 milli-watts, find the best modulation scheme (MQAM or MPSK). Justify your solution and explain why your result is the best you can get. If it is not possible to find any modulation scheme, explain why.

Solution:

The modulation scheme is either MQAM or MPSK. We should choose one of these modulation schemes and the value of "M" such that the transmitted signal bandwidth is at most 4 MHz and the system bit error rate is at most 10^{-5}.

First we find value of "M" for MQAM and MPSK such that the transmitted signal bandwidth is at most 4 MHz as shown below:

$$BW(MQAM) = BW(MPSK) = (1+\beta)\frac{R_b}{\log_2 M} = (1+\beta)\frac{20}{\log_2 M} \le 4 \Rightarrow$$
$$\Rightarrow \log_2 M \ge 5(1+\beta) \Rightarrow M \ge 2^{5(1+\beta)}$$

We know that $0 < \beta \le 1$. Therefore, to satisfy the above inequality, the minimum value for M would be equal to $M = 64$ with $\beta = 0.2$. Note that β should not be close to zero to have a realizable pulse shaping and matched filters and therefore the solution of $M = 32$ with $\beta = 0$ is not acceptable.

Since by increasing M, the transmitted signal bandwidth would decrease, therefore any signaling level of larger than or equal to $M = 64 = 2^6$ satisfies the bandwidth requirement as long as M is a power of 2, i.e., $M = 2^L$ and $L \ge 6$ is an integer .

Therefore, $BW(MQAM) = BW(MPSK) \le 4MHz$ for $M \ge 2^6 = 64$. We may have a solution if the bit error rate satisfies 10^{-5}.

Now, we check whether the bit error rate is less than or equal to 10^{-5}. We consider 64QAM system since we know the bit error rate of 64PSK is more than 64QAM. From formula sheet, we know that the symbol error rate of 64QAM can be found as shown below:

$$P_E(MQAM) = 4\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)Q\left(\sqrt{\left(\frac{3}{M-1}\right)\frac{E_s}{N_0}}\right)$$

We also know that Gray coding can be found if the MQAM system has a square constellation diagram. Since 64QAM has 64 points and has a square structure of 8×8. From the formula sheet, we can find the relation between bit error rate and symbol error rate and therefore, the overall system bit error rate would be:

$$P_B(MQAM) = \frac{P_E(MQAM)}{\log_2 M} = \frac{4}{\log_2 M}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)Q\left(\sqrt{\left(\frac{3}{M-1}\right)\frac{E_s}{N_0}}\right)$$

$$P_B(64QAM) = \frac{4}{\log_2 64}\left(\frac{\sqrt{64}-1}{\sqrt{64}}\right)Q\left(\sqrt{\left(\frac{3}{64-1}\right)\frac{E_b \log_2 64}{N_0}}\right) = \frac{7}{12}Q\left(\sqrt{\left(\frac{3}{64-1}\right)\frac{P_r \log_2 64}{N_0 R_b}}\right)$$

$$P_B(64QAM) = \frac{7}{12}Q\left(\sqrt{\left(\frac{3}{63}\right)\frac{25\times10^{-3}\times 6}{2\times10^{-10}\times 20\times10^{6}}}\right) = \frac{7}{12}Q\left(\sqrt{\frac{25}{14}}\right) = \frac{7}{12}Q(1.336) = 0.9\times10^{-1}$$

Therefore, the bit error rate of the 64QAM system is 0.9×10^{-1} which does not satisfy the requirement of 10^{-5}. We cannot find another value for the modulation level of M satisfying both bandwidth requirement of 4 MHz and bit error rate requirement of 10^{-5} since for $M < 64$ the bandwidth is not satisfied and for $M \geq 64$ bit error rate requirement is not satisfied. Note that by increasing $M > 64$, the bit error rate of MQAM system will not satisfy 10^{-5}.

$P_B(64QAM) = 0.9\times10^{-1}$: $P_B(MQAM) > 10^{-5}$ for $M \geq 2^6 = 64$
Therefore, no modulation level M can be found for MQAM to be acceptable.

If we calculate the bit error rate of 64PSK, obviously it will be worse than the bit error rate of 64QAM and therefore, we cannot find a modulation level M for any of these modulation schemes which satisfy both the bandwidth requirement of 4 MHz and bit error rate requirement of 10^{-5}. The solution is to use coding to achieve the required BER.