Assignment # 4 (Solution):

1) A TV station transmit video with a rate of 12 Mbps. Assume that the station wishes to have error free transmission for half an hour.
   a. What is the required BER?

The requirement is error free transmission, so each bit in error is considered as an error. So the Maximum acceptable BER could be calculated as:

$$BER \le \frac{1}{R_b \times time} = \frac{1}{12 \times 10^6 \times 60 \times 30} = 4.6 \times 10^{-11}$$

   b. Assume that the station has 6 MHz of bandwidth and uses an MPSK modulation scheme with roll-off factor of 0.2. What is the required E_b/N_o?

Considering the roll-off factor, we have:

$$\mathrm{W} \ge \mathrm{R_s}(1+\beta) \to 6 \ge \frac{12}{\log_2 \mathrm{M}} \times 1.2$$

$$\log_2 \mathrm{M} \ge \frac{12}{6} \times 1.2 = 2.4 \to \text{round up} \to \mathrm{M} = 8 \to \text{8PSK modulation scheme.}$$

$$\mathrm{BER} \to \mathrm{P_b}(8\mathrm{PSK}) = \frac{2}{3}\mathrm{Q}(\sqrt{6 \times \frac{\mathrm{E_b}}{\mathrm{N_o}}} \times \sin\frac{\pi}{8})$$

$$\text{Note that } \mathrm{E_b} = \frac{\mathrm{E_s}}{\log_2 \mathrm{M}}$$

From the Q-function table we find the arg of Q for the required BER. That is :

$$6.42 = \sqrt{6 \times \frac{\mathrm{E_b}}{\mathrm{N_o}}} \times \sin\frac{\pi}{8} \to \frac{\mathrm{E_b}}{\mathrm{N_o}} = 46.9532$$

   c. What would be the required E_b/N_o if the DVB Reed Solomon code is used?

Please consider that when the FEC is in use, we either transmitting with higher rate (using more bandwidth), or the information bit rate should be decrease to keep the bandwidth constant. In this case for each 188 bytes, we transmit 204 bytes. However, when counting errors, we count 188 information bytes not to have errors.

$$Number\ of\ packets = \frac{Bit\ rate}{Size\ of\ packet} = \frac{12 \times 10^6 \times 60 \times 30}{188 \times 8} = 14361702.13$$

For P_{packet}:

$$P_{packet} = \frac{1}{14361702.13} = 6.96 \times 10^{-8}$$

Reed Solomon (204,188) is able to correct up to 8 packets. So we count more than 9 packets.

$$P_{packet} = \sum_{i=9}^{204} \binom{204}{i} . {p_c}^{i} . (1 - p_c)^{204-i}$$

The above series is could be approximated by considering only the first term, as the rest of terms are decreasing sharply.

$$6.96 \times 10^{-8} = \binom{204}{9} . {p_c}^{9} . (1 - p_c)^{204-9}$$

Please note that the term $\frac{\mathrm{i+t}}{2\times 204}$ shouldn't be considered. Because we are considering packets to be in error, and we don't care how many bytes or bits of a packet is in error.

By testing different values on the above equation we found $p_c = 3.54 \times 10^{-3}$. Now we are able to find the BER:

$$p_c = 1 - (1 - P_b)^8 \approx 8 \times P_b \rightarrow P_b = 4.4 \times 10^{-4}$$

Note that p_c is the probability that a byte is error. BER or P_b is Bit Error Rate.

Please note that using RS code (or any other FEC) we transmit some over heads (Parity bits). So we consume more bandwidth. Let's see how many bandwidth we are consuming using the 8PSK and Reed Solomon code.

$$B = \frac{\frac{204}{188} \times R_b}{\log_2 M}(1 + \beta) \rightarrow B = \frac{13.02}{3}(1.2) \rightarrow B = 5.2 < 6\ MHz$$

Note that we are consuming less than available bandwidth. So we are good to go. However, in cases where the calculated bandwidth is more than the available bandwidth, we should increase the modulation order to compensate that.

So to calculate the required E_b/N_o we use the 8-PSK formula.

$$4.4 \times 10^{-4} = \frac{2}{3} \mathrm{Q}\left(\sqrt{6 \times \frac{\mathrm{E_b}}{\mathrm{N_o}}} \times \sin\frac{\pi}{8}\right)$$

$$\sqrt{6 \times \frac{\mathrm{E_b}}{\mathrm{N_o}}} \times \sin\frac{\pi}{8} = 3.22 \rightarrow \boldsymbol{\frac{E_b}{N_o} = 11.7999}\ or\ \frac{\mathrm{E_b}}{\mathrm{N_o}} = 10.72\ dB$$

d. Assume that carrier frequency is 521 MHz and the station would like to cover viewers having antenna with 5 dBi gain and receivers with overall noise figure of 15 dB up to the distance of 50 Km. What should be the station's ERIP without and with RS coding? (Assume LOS signal propagation model).

i. Without coding;

$$\boldsymbol{\frac{E_b}{N_o}|_{req} = 46.9532}$$

$$\mathrm{T_{eq}} = 290.\left(10^{\frac{\mathrm{NF_{eq}}}{10}} - 1\right) \rightarrow \mathrm{T_{eq}} = 290.\,(10^{1.5} - 1) = 8880\ °\mathrm{K}$$

$$\mathrm{N_o} = \mathrm{K_B T_{eq}} \rightarrow \mathrm{N_o} = 1.38 \times 10^{-23} \times 8880 = 1.22 \times 10^{-19}$$

$$\mathrm{E_b} = \mathrm{N_o} \times \left(\frac{\mathrm{E_b}}{\mathrm{N_o}}\right) \rightarrow \mathrm{E_b} = 46.9 \times 1.22 \times 10^{-19} = 5.74 \times 10^{-18}$$

$$(\text{Received Signal Power}) \rightarrow \mathrm{P_r} = \mathrm{E_b} \times \mathrm{R_b} \rightarrow \mathrm{P_r} = 5.74 \times 10^{-18} \times 12 \times 10^{6} = 6.89 \times 10^{-11}$$

Note that this is the minimum required power at the receiver.

Now the EIRP of the transmitter could be calculated using the range equation.

$$P_r = \frac{P_t \times G_t \times G_r}{L_s} \quad \overset{dB}{\Rightarrow} \quad P_r = P_t + G_t + G_r - L_s$$

Note that EIRP is $P_t \times G_t$ or in dB $P_t + G_t$.

It is mentioned to use the Line of sight (LOS) propagation model, also known as free space propagation model. So the path loss (L_s) could be calculated from:

$$L_s = \left(\frac{4\pi d}{\lambda}\right)^2 \quad \xrightarrow{in\ dB} \quad L_s = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right), dB$$

$$L_s = 1.19 \times 10^{12}$$

Solving for EIRP we have:

$$EIRP = \frac{P_r \times L_s}{G_r} \rightarrow EIRP = 25.94\ Watts$$

Note that this is the required EIRP to be transmitted from the station when we have no coding scheme.

ii. With Reed Solomon code.

$$\boldsymbol{\frac{E_b}{N_o}|_{req} = 11.7999}$$

Because the approach is the same as before, here we use a shortcut the answer. But in the exams you should avoid that and write the full approach.

Eb/No(i)	EIRP(i)
Eb/No(ii)	EIRP(ii)

$$\mathrm{EIRP(ii)} = \frac{25.94}{46.9} \times 11.79 \rightarrow \mathrm{EIRP} = \mathbf{6.5\ Watts}$$

e. Repeat part (d) if a convolutional code with coding gain of 4 dB is used as the inner code and the RS code as the outer code.

If we use inner coding scheme with the gain of 4 dB, we increase the performance of the system by 4 dB. 4 dB translates to 2.5 non-dB value. That means instead of requiring $\frac{\mathrm{E_b}}{\mathrm{N_o}} = 11.7999$, we require $\frac{11.799}{2.5} = 4.69$.

f. Repeat part d and e considering two-ray propagation model. Assume that $h_t = 50\ m$ and $h_r = 10\ m$.

Using two-ray model instead of LOS propagation model:

$$L_s = \frac{d^4}{{h_t}^2 {h_r}^2} \rightarrow L_s = 2.5 \times 10^{13}$$

$$EIRP = \frac{EIRP(d,i)}{L_s(d,i)} \times L_s \rightarrow EIRP_{f,i} = 544.9\ watts$$

$$EIRP_{f,ii} = \frac{EIRP_{d,ii}}{L_s(d,ii)} \rightarrow EIRP_{f,ii} = \frac{6.5}{1.19 \times 10^{12}} \times 2.5 \times 10^{13} = 136.55\ watts$$

And in dB scale we have:

$$EIRP_{f,i} = 27.36\ dBW$$

$$EIRP_{f,ii} = 21.35\ dBW$$

If assuming 4 dB inner convolutional code, we have to reduce 4 dB from RS coded to have the answers for part e, with two-ray propagation model:

$$EIRP_{f,e} = 17.35\ dBW\ with\ inner\ convolutional\ coding$$

$$EIRP_{f,e} = 54.325\ Watts$$

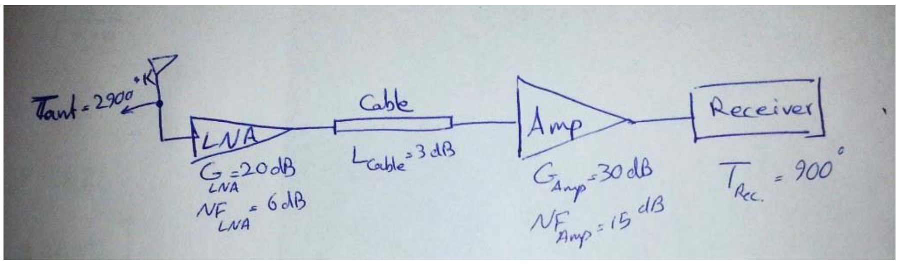
2) The uplink signal to noise ratio of a satellite link is $(E_b/N_o)_U$ 12 dB and the downlink SNR is $(E_b/N_o)_D$ 20 dB. Find the overall (E_b/N_o) in dB.

$$(E_b/N_o)_{overal}{}^{-1} = (E_b/N_o)_U{}^{-1} + (E_b/N_o)_D{}^{-1}$$

$$\frac{1}{\left(\frac{E_b}{N_o}\right)} = \frac{1}{(E_b/N_o)_U} + \frac{1}{(E_b/N_o)_D} = \frac{1}{10^{1.2}} + \frac{1}{10^2} \xrightarrow{\text{overal}} \frac{E_b}{N_o} = 11.36$$

3) A TV receiver has an antenna with noise temperature of 2900° K, an LNA with a gain of 20 dB and noise figure of 6 dB, a cable with 3 dB loss, an amplifier with gain of 30 dB and noise figure of 15 dB and a receiver with a noise temperature of 900° K. Find the overall noise figure of the system:

   a. If the LNA is connected between the antenna and cable.

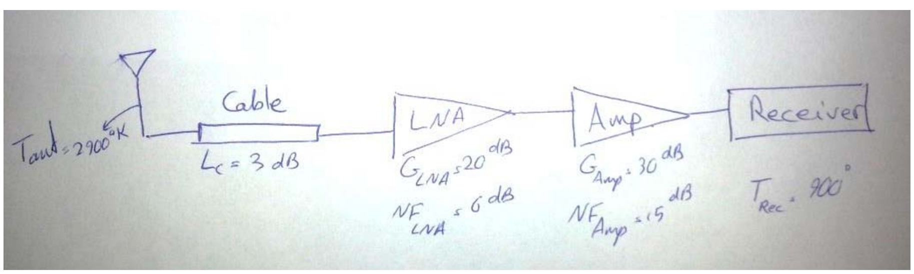


$$T_{eq} = T_{ant} + T_{LNA} + \frac{T_c}{G_{LNA}} + \frac{T_{amp}}{G_{LNA}\frac{1}{L_c}} + \frac{T_{rec}}{G_{LNA}\frac{1}{L_c}G_{amp}}$$

$$T_{eq} = 2900 + 864.5 + \frac{288}{100} + \frac{8880}{100 \times \frac{1}{2}} + \frac{900}{100 \times 0.5 \times 1000} = 3945$$

NF = T/290 + 1 = 14.60 or 11.64 dB

   b. If the LNA is connected at the end of the cable.



$$T_{eq} = T_{ant} + T_c + \frac{T_{LNA}}{\frac{1}{L_c}} + \frac{T_{amp}}{G_{LNA}\frac{1}{L_c}} + \frac{T_{rec}}{G_{LNA}\frac{1}{L_c}G_{amp}}$$

$$T_{eq} = 2900 + 288 + \frac{864.5}{0.5} + \frac{8880}{100 \times \frac{1}{2}} + \frac{900}{100 \times 0.5 \times 1000} = 5058°K$$

$NF = 18.441$ or 12.65 dB