# ELEC 6131 – Error Detecting and Correcting Codes

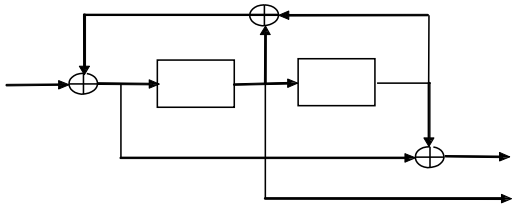
**Final Exam**

**April 12, 2016**

1) Consider the Galois field GF ($2^4$) generated by the polynomial $p(x) = x^4 + x + 1$.
   a) Find the generating polynomial of (15, 13) RS code over this field (5 Marks).
   b) What is the error correcting capability of this code? Erasure correcting capability? (1 Mark)
   c) Encode the sequence $u(x) = x^3$ in a systematic form. (3 Marks)
   d) Decode the received sequence $r(x) = e_1 x + e_2 x^3$ where $e_1$ and $e_2$ are erased symbols (3 Marks).

2) Consider the Galois field GF ($2^4$) generated by the polynomial $p(x) = x^4 + x + 1$. Find the generator polynomial of a primitive binary BCH code with n = 15 and t = 3 (7 Marks). What is the minimum distance and the rate of the resulting code (3 Marks)?

3) Consider the following convolutional encoder with generating function $G(D) = \left[ \frac{1+D^2}{1+D+D^2}, \frac{D}{1+D+D^2} \right]$



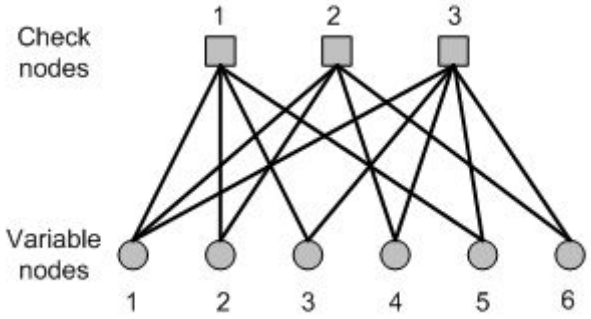
   a) Draw the trellis diagram for the code (2 Marks).
   b) What is the minimum free distance of the code (2 Marks).
   c) Encode 1101011 staring from state zero (2 Marks).
   d) Using the Viterbi Algorithm decode 0101001010 (4 Marks).
   Note: The encoding has started from an unknown state.

4) What is a catastrophic encoder? What is the condition for a convolutional encoder not to be catastrophic? (3 Marks).

5) Let $x$ and $y$ be independent binary random variables.

   a) Find the LLR of $x$ if $P(x = 0) = 0.5$ (1 Mark).
   b) Find the LLR of $x$ if $x = 1$, i.e., $P(x = 0) = 0$ (1 Mark).
   c) Find the LLR of $x$ if $x = 0$ (1 Mark).
   d) Find the LLR of $z = x \oplus y$ if $P(x = 0) = P(y = 1) = 0.25$ (3 Marks).

6) Consider a code with the following Tanner graph:




a) Write the parity check matrix of the code (2 Marks).
b) What are the row and column degree distribution functions (2 Marks)?
c) Find the rate of the code using the result of part 2 (2 Marks) and compare with the design rate.
d) Is 010111 a codeword? (1 Mark).
e) Decode e1ee11 where e is an erasure (2 Marks).