# ELEC 6131 – Error Detecting and Correcting Codes

**Midterm**

**March 8, 2016**

1) Consider the polynomial $p(x) = x^8 + x^5 + x + 1$ in GF(2). Is $p(x)$ primitive? Why? (2 Marks).

2) Consider a (7, 4) Hamming code with the following generating matrix:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   a. Find the generator polynomial the code. (2 Marks).
   b. Find the generator matrix of dual code of this code (2 Marks).
   c. Is the dual code a perfect code? Why? (2 Marks).
   d. Find the generator polynomial of the dual code. (2 Mark).

3) Consider the polynomial $p(x) = x^3 + x + 1$
   a. Prove that $p(x)$ is a primitive polynomial (2 Marks).
   b. List all elements of GF($2^3$) generated by $p(x)$ (3 Marks).
   c. List the minimal polynomials of all elements of GF($2^3$) (3 Marks).

4) Consider a code with the following generating matrix:

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   a. How many error patterns can this code correct? (2 Marks).
   b. Find the parity check matrix of the code (2 Marks).
   c. The received bits are $\begin{bmatrix} 1 & e_1 & 0 & e_3 & e_4 & 0 & e_7 & 1 \end{bmatrix}$ where $e_1, e_3, e_4$ and $e_7$ are erased bits. Find the transmitted codeword (3 Marks).

5) Draw the block diagram of the encoder and decoder for the (15,11) cyclic Hamming code with generator polynomial $g(x) = x^4 + x + 1$ (5 Marks)