# ELEC 6131 - Error Detecting and Correcting Codes <br> Final Exam <br> April 28, 2015 

1) 

a) List all elements of $\operatorname{GF}\left(2^{3}\right)$ generated by $p(x)=x^{3}+x+1(1$ Mark).
b) Find the generating polynomial of $(7,5)$ RS code over $\operatorname{GF}\left(2^{3}\right)(3$ Marks).
c) Encode the binary sequence 010101010101010 in systematic form using the above code (3 Marks).
d) Decode 000000000101000000000 (3 Marks)
2) Derive the generating polynomial of double-error correcting primitive BCH code of length 15 ( 7 Marks). Draw the systematic encoder for this code (2 Marks). What is the rate of the code (1 Mark)?
3) Consider the following convolutional encoder:

a) Draw the trellis diagram for the code (2 Marks).
b) What is the minimum free distance of the code ( 2 Marks).
c) Encode 1101011 staring from state zero (2 Marks).
d) Using the Viterbi Algorithm decode 0101001010 (4 Marks).

Note: The encoding has started from an unknown state.
4) Let $x_{1}$ and $x_{2}$ be two independent binary random variables and $y=x_{1} \oplus x_{2}$. Let $\lambda_{1}$ and $\lambda_{2}$ be the Log-Likelihood Ratio (LLR) of $x_{1}$ and $x_{2}$, respectively.
a) Find the LLR of $y$ ( 5 Marks).
b) Find the LLR of $y$ for $\lambda_{1}=3$ and $\lambda_{2}=-1$ ( 2 Marks). What is the probability that $y$ is equal to zero? ( 1 Mark)
c) For a given $\lambda_{1}$, find $\lambda_{2}$ such that $P(y=0)=P(y=1)=0.5$ (2 Mark)
5) Consider a code with the parity check matrix:

$$
\mathbf{H}=\left(\begin{array}{llllll}
1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0
\end{array}\right)
$$

a) Draw the bi-partite (Tanner) graph for this code (2 Marks).
b) Find the rate of the code (3 Marks).
c) Is 010111 a codeword? (1 Mark).
d) Decode e1ee11 where e is an erasure (2 Marks).
e) Find a very SIMPLE encoding rule for this code (2 Marks). (by simple, I mean intuitive).

