## ELEC 6131 – Error Detecting and Correcting Codes Final Exam April 28, 2015

1)

- a) List all elements of GF(2<sup>3</sup>) generated by  $p(x) = x^3 + x + 1$  (1 Mark).
- b) Find the generating polynomial of (7, 5) RS code over  $GF(2^3)$  (3 Marks).
- c) Encode the binary sequence 010101010101010 in systematic form using the above code (3 Marks).
- d) Decode 0000000010100000000 (3 Marks)
- 2) Derive the generating polynomial of double-error correcting primitive BCH code of length 15 (7 Marks). Draw the systematic encoder for this code (2 Marks). What is the rate of the code (1 Mark)?
- 3) Consider the following convolutional encoder:



- a) Draw the trellis diagram for the code (2 Marks).
- b) What is the minimum free distance of the code (2 Marks).
- c) Encode 1101011 staring from state zero (2 Marks).
- d) Using the Viterbi Algorithm decode 0101001010 (4 Marks). Note: The encoding has started from an unknown state.
- 4) Let  $x_1$  and  $x_2$  be two independent binary random variables and  $y = x_1 \oplus x_2$ . Let  $\lambda_1$  and  $\lambda_2$  be the *Log-Likelihood Ratio* (LLR) of  $x_1$  and  $x_2$ , respectively.
  - a) Find the LLR of *y* (5 Marks).
  - b) Find the LLR of y for  $\lambda_1 = 3$  and  $\lambda_2 = -1$  (2 Marks). What is the probability that y is equal to zero? (1 Mark)
  - c) For a given  $\lambda_1$ , find  $\lambda_2$  such that P(y=0) = P(y=1) = 0.5 (2 Mark)
- 5) Consider a code with the parity check matrix:

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

- a) Draw the bi-partite (Tanner) graph for this code (2 Marks).
- b) Find the rate of the code (3 Marks).

- c) Is 010111 a codeword? (1 Mark).
- d) Decode elee11 where e is an erasure (2 Marks).
  e) Find a very SIMPLE encoding rule for this code (2 Marks). (by simple, I mean intuitive).