ELEC 6131 – Error Detecting and Correcting Codes
Solution to Assignment 9

1) The error locating polynomial of double error correcting RS code over GF(2^4) is $\sigma(X) = \alpha^5 X^2 + \alpha^2 X + 1$. Find the location of errors.

$$\sigma(X) = \alpha^5(X^2 + \alpha^{12}X + \alpha^{10}) = \alpha^5(X + \alpha^4)(X + \alpha^6).$$

The roots of $\sigma(X)$ are α^4 and α^6. So, the errors are at locations:

$$\beta_1 = \frac{1}{\alpha^6} = \alpha^9 \text{ and } \beta_2 = \frac{1}{\alpha^4} = \alpha^{11}.$$

2) Derive the parity check matrix of (15, 11) Hamming code (7 Marks). Decode the received stream. Decode the stream

$r(X) = X^{14} + e_{12}X^{12} + e_1 X + 1$.

$H = [1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, \alpha^7, \alpha^8, \alpha^9, \alpha^{10}, \alpha^{11}, \alpha^{12}, \alpha^{13}, \alpha^{14}]$.

$$\mathrm{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The first equation gives:

$$1 + e_1 + 1 = 0 \Rightarrow e_1 = 0.$$

The second equation gives:

$$0 = 0 \Rightarrow no\ information$$

The third equation gives:

$$e_{12} + e_1 = 0 \Rightarrow e_1 = e_{12} = 0.$$

The fourth equation gives:

$$e_1 + 1 = 0 \Rightarrow e_1 = 1.$$

So, decoder fails.

3) Consider the convolutional encoder with generating function

$$G(D) = \left[\frac{1 + D^2}{1 + D + D^2}, \frac{D}{1 + D + D^2}\right].$$

Is this encoder catastrophic? Why?

Answer: No, the encoder is not catastrophic since GCD of the two generating polynomials is one.

4) Consider the Galois field GF (2^4) generated by the polynomial $p(x) = x^4 + x + 1$. Find the generator polynomial of a primitive binary BCH code with n = 15 and t = 2 (7 Marks). What is the minimum distance and the rate of the resulting code?

$$g(x) = \phi_1(x)\phi_2(x) = (x^4 + x + 1)(x^4 + x^3 + x^2 + x + 1)$$

or,

$$g(x) = (x^8 + x^7 + x^6 + x^4 + 1)$$

So, $n - k = 8$ and $k =$ 15-8 $= 7 \Rightarrow$ rate $= \frac{7}{15}$.

We have,

$$d_{min} = 5 \Rightarrow t = 2.$$

5) Find the expression for the LLR of $x \oplus y$ in terms of the LLR's of x and y.

$$\mathrm{L}(x) = Ln\frac{P(x=0)}{P(x=1)} \Rightarrow P(x=0) = \frac{e^{L(x)}}{1+e^{L(x)}}$$

and

$$L(y) = Ln\frac{P(y=0)}{P(y=1)} \Rightarrow P(y=0) = \frac{e^{L(y)}}{1+e^{L(y)}}$$

The LLR of $x \oplus y$ is:

$$\mathrm{L}(x \oplus y) = Ln\frac{\mathrm{P}(x \oplus y = 0)}{P(x \oplus y = 1)}$$

$$P(x \oplus y = 0) = P(x=0)P(y=0) + P(x=1)P(y=1)$$

Substituting probabilities of x and y, we get:

$$P(x \oplus y = 0) = \frac{e^{L(x)+L(y)} + 1}{(1+e^{L(x)})(1+e^{L(y)})}$$

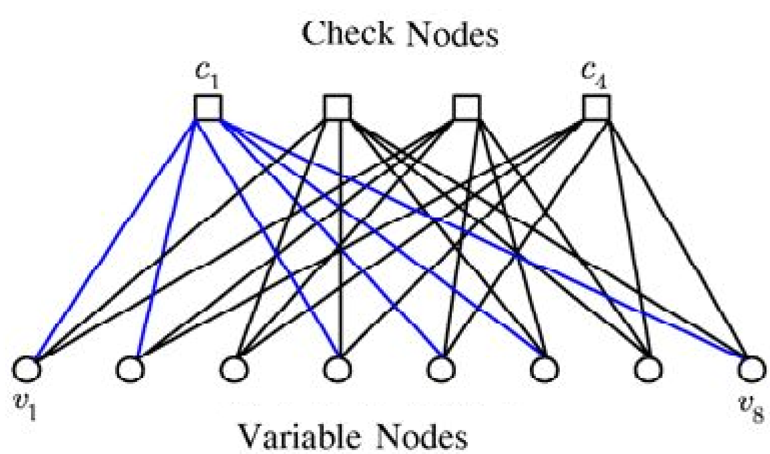
Similarly,

$$P(x \oplus y = 1) = \frac{e^{L(x)} + e^{L(y)}}{(1+e^{L(x)})(1+e^{L(y)})}$$

Therefore,

$$L(x \oplus y) = Ln\frac{e^{L(x)+L(y)} + 1}{e^{L(x)} + e^{L(y)}}$$

6) Consider a code with the following Tanner graph:



a) Write the parity check matrix of the code.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

b) What are the row and column degree distribution functions?
   $\lambda(x) = x^2$ and $\rho(x) = x^5$
c) Find the rate of the code.
   Rate=1/2.
d) Is 01011001 a codeword?
   All check nodes a re zero. So, it is a codeword.
e) Decode $e_1 1 e_3 0 1 e_6 1 0$ where e_1, e_3, e_6 are erasures.
   The first check node gives:

$$e_1 + 1 + 1 + e_6 = 0 \Rightarrow e_1 = e_6$$

The second check node gives:

$$e_1 + e_3 + e_6 + 1 = 0 \Rightarrow e_3 = 1$$

The third and fourth check node give $e_3 = 1$.

So, we know that e_3 and $e_1 = e_6$. Therefore, there are two valid codewords:01101010 and 11101110.