ELEC6151: Information Theory and Source Coding
Final
Dec. 11, 2008

1) Consider the discrete memoryless channel $Y = X + Z \pmod 4$ where $X \in \{0,1,2,3\}$, $Z \in \{0,1\}$, $p_z(0) = 1/3$, $p_z(1) = 2/3$ and Z and X are independent. Find the capacity of the channel (5 Marks) and the maximizing input distribution (2 Marks).

2) A zero mean Gaussian random variable X with variance σ_x^2 is measured with a device introducing a measurement error E, i.e., the reading is $Y = X + E$. Assume that E is zero-mean Gaussian with variance σ_e^2 and is independent of X. What is the amount of information that Y contains about X (6 Marks).

3) Consider 3 parallel Gaussian channels $Y_i = X_i + Z_i$, $i = 1,2,3$, where $Z_i \sim N(0,\ 2i)$. Find the capacity of the channel for total power P=1, 5, 7 (6 Marks).

4) Consider a source X uniformly distributed on the set $\{1,2,3,4\}$. Find the rate distortion function of this source with respect to the distortion measure:

$$d(x,\hat{x}) = \begin{cases} 0 & x = \hat{x} \\ 1 & x \neq \hat{x} \end{cases}$$

(7 Marks).

Note: The proof is required.

5) 100 CDMA mobile terminals transmit to a base station. The Signal-to-Noise-Ratio (SNR) at the receiver is 10 dB for each of these terminals. The terminals transmit at the same rate. Find the maximum transmission rate per terminal if the total bandwidth is 1 MHz. (7 Marks).
Note: Make any assumption you consider necessary.

6) Let $C(x) = \frac{1}{2}\log(1+x)$ denote the channel capacity of a Gaussian channel with signl-to-noise-ratio x. Show that:

$$C\left(\frac{P_1}{N}\right) + C\left(\frac{P_2}{P_1 + N}\right) = C\left(\frac{P_1 + P_2}{N}\right).$$

(5 Marks). What is the significance of this relationship? (what is the interpretation?) (2 Marks).