

ELEC6151: Information Theory and Source Coding
Midterm
Oct. 25, 2008

1) a) Find the entropy rate of the two-state Markov chain with transition matrix (3 Marks),

$$P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

b) What values of α and β maximize the entropy rate of part (a)? (2 Marks)

2) Consider a source whose output takes three values A, B and C with probabilities $3/4, 3/16$ and $1/16$, respectively.

a) Design an optimum prefix code for this source if its output is to be encoded one symbol at a time (3 Marks). Find the average length of the code and compare with the entropy of the source (1 Marks).

b) Assume that the source output is being encoded two symbols at a time, i.e., as AA, AB, etc. Design an optimum prefix code for this case (5 Marks). Find the Average length of the code and compare with the entropy of the source (1 Mark).

3) Consider the channel with the transition matrix

$$P = \begin{bmatrix} 2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & 2/3 \end{bmatrix}.$$

Show that the capacity of this channel is achieved by setting the probability of using one of the input letters to zero. What is the channel capacity? Why is this letter not being used? (5 Marks)

4) Assume that two BSC channels with crossover probabilities $\epsilon_1 = 0.1$ and $\epsilon_2 = 0.2$ are connected to form a composite channel. A decoder is used to decode the output of the first channel to be encoded for transmission over the second channel. What is the capacity of the resulting channel? (5 Marks)

5) Consider a discrete additive noise channel with output $Y = X \oplus Z$ where \oplus represents modulo K addition, the input $X \in \{0, 1, \dots, K-1\}$ and the noise Z takes values $0, 1, \dots, L$ with equal probability. Find the capacity of this channel. (5 Marks).