# ELEC6151: Information Theory and Source Coding
# Midterm
# March 27, 2013

1) Consider a source with output +1 and -1. Assume that $P(X=-1)=0.25$.

a) Find $H(Y)$ for $Y = X^3 + 2$ in bits (2 Mark).

b) Find $H(Z)$ for $Z = X^2 + 1$ (2 Mark).

2) A source generates character $x$ taking values from the set $\{0,1,2,\ldots,9,a,b,c,\ldots,y,z\}$. The character $x$ is a numeral, $\{0,1,2,\ldots,9\}$, with probability 1/3, is a vowel, $\{a,e,i,o,u\}$ with probability 1/3 and is otherwise one of the 21 consonants. Find the maximum entropy of $X$ (4 Marks).

3) a) For the source with five symbols with probability distribution $\mathbf{p} = (0.3, 0.3, 0.2, 0.1, 0.1)$ construct a binary Huffman code and find its average length (2 Marks).

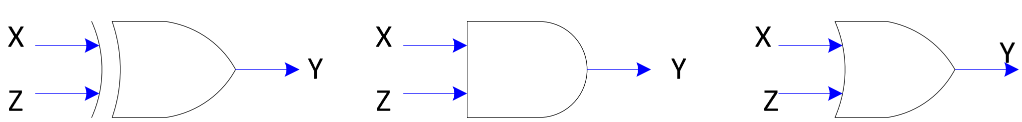
b) Find the probability distribution $\mathbf{q}$ on five symbols such that the code you constructed in part (a) is optimum, i.e., its average length under $\mathbf{q}$ is equal to its entropy $H(\mathbf{q})$ (3 Marks).

4) Find the capacity of the channel given by $p(y|x)$,

$$p(y|x) = \begin{bmatrix} 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \\ 1/3 & 0 & 2/3 \end{bmatrix}.$$

(5 Marks).

5) The following diagrams show, the three basic gates (XOR, AND, OR).




with the following truth tables:

| X | Z | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

a) XOR Gate

| X | Z | Y |
|---|---|---|

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

b) AND Gate

| X | Z | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

c) OR Gate

Consider each gate as a noisy channel with $X$ being the input, $Y$ the output and $Z$ the noise. Find the capacity of the following channels:

a) XOR gate with $Z$ being a binary memoryless random variable with parameter $p$ (probability of $Z$ being one is $p$ ) (4 Marks).
b) AND gate with $Z$ being a binary memoryless random variable with parameter ½ (4 Marks).
c) OR gate with $Z$ being a binary memoryless random variable with parameter ½ (4 Marks).