Assignment 2

P1)

Consider the signal $s(t)$ shown in Figure P4.1.

(a) Determine the impulse response of a filter matched to this signal and sketch it as a function of time.

(b) Plot the matched filter output as a function of time.

(c) What is the peak value of the output?

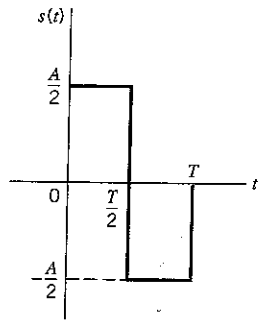

FIGURE P4.1

P2)

In this problem we explore another method for the approximate realization of a matched filter, this time using the simple resistance-capacitance (RC) low-pass filter shown in Figure P4.4. The frequency resonse of this filter is

$$H(f) = \frac{1}{1 + jf/f_0}$$

where $f_0 = 1/2\pi RC$. The input signal $g(t)$ is a rectangular pulse of amplitude A and duration T. The requirement is to optimize the selection of the 3-dB cutoff frequency f_0 of the filter so that the peak pulse signal-to-noise ratio at the filter output is maximized. With this objective in mind, show that the optimum value of f_0 is $0.2/T$, for which the loss in signal-to-noise ratio compared to the matched filter is about 1 dB.

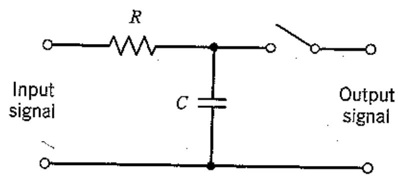

FIGURE P4.4

P3)

In a binary PCM system, symbols 0 and 1 have *a priori probabilities* p_0 and p_1, respectively. The conditional probability density function of the random variable Y (with sample value y) obtained by sampling the matched filter output in the receiver of Figure 4.4 at the end of a signaling interval, given that symbol 0 was transmitted, is denoted by $f_Y(y|0)$. Similarly, $f_Y(y|1)$ denotes the conditional probability density function of Y, given that symbol 1 was transmitted. Let λ denote the threshold used in the receiver, so that if the sample value y exceeds λ, the receiver decides in favor of symbol 1; otherwise, it decides in favor of symbol 0. Show that the optimum threshold λ_{opt}, for which the average probability of error is a minimum, is given by the solution of

$$\frac{f_Y(\lambda_{opt}|1)}{f_Y(\lambda_{opt}|0)} = \frac{p_0}{p_1}$$

P4) A continuous-time signal is sampled and then transmitted as a PCM signal. The random variable at the input of the decision device in the receiver has a variance of 0.01 volts^2.

(a) Assuming the use of polar NRZ signaling, determine the pulse amplitude that must be transmitted for the average error rate not to exceed 1 bit in 10^8 bits.

(b) If the added presence of interference causes the error rate to increase to 1 bit in 10^6 bits, what is the variance of the interference?

P5) A computer puts out binary data at the rate of 56 kb/s. The computer output is transmitted using a baseband binary PAM system that is designed to have a raised-cosine spectrum. Determine the transmission bandwidth required for each of the following rolloff factors: $\alpha = 0.25, 0.5, 0.75, 1.0$.

P6) An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of representation levels used is 128. A synchronizing pulse is added at the end of each code word representing a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth 12 kHz using a quaternary PAM system with raised-cosine spectrum. The rolloff factor is unity.

(a) Find the rate (b/s) at which information is transmitted through the channel.

(b) Find the rate at which the analog signal is sampled. What is the maximum possible value for the highest frequency component of the analog signal?