3.2 Determine the signal space representation of the four signals $s_k(t)$, $k = 1, 2, 3, 4$, s in Figure P3.2, by using as basis functions the orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ the signal space diagram, and show that this signal set is equivalent to that for a four-phase PSK signal.

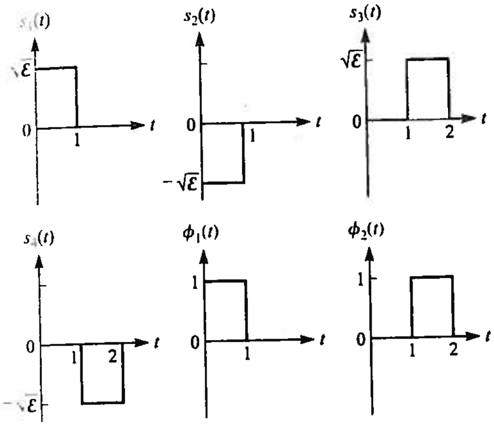


FIGURE P3.2

3.3 $\pi/4$-QPSK may be considered as two QPSK systems offset by $\pi/4$ rad.

1. Sketch the signal space diagram for a $\pi/4$-QPSK signal.
2. Using Gray encoding, label the signal points with the corresponding data bits.

3.4 Consider the octal signal point constellations in Figure P3.4.

1. The nearest-neighbor signal points in the 8-QAM signal constellation are separ in distance by A units. Determine the radii a and b of the inner and outer cir respectively.
2. The adjacent signal points in the 8-PSK are separated by a distance of A units. Deterr the radius r of the circle.

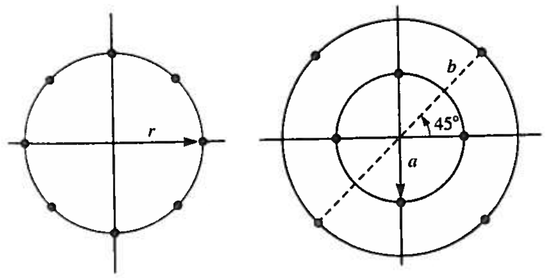


FIGURE P3.4

3. Determine the average transmitter powers for the two signal constellations, and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume that all signal points are equally probable.)

3.5 Consider the 8-point QAM signal constellation shown in Figure P3.4.

1. Is it possible to assign 3 data bits to each point of the signal constellation such that the nearest (adjacent) points differ in only 1 bit position?
2. Determine the symbol rate if the desired bit rate is 90 Mbits/s.

3.6 Consider the two 8-point QAM signal constellations shown in Figure P3.6. The minimum distance between adjacent points is $2A$. Determine the average transmitted power for each constellation, assuming that the signal points are equally probable. Which constellation is more power-efficient?

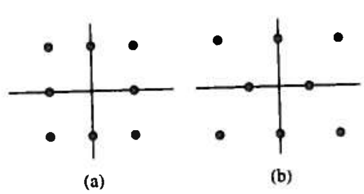


FIGURE P3.6

3.7 Specify a Gray code for the 16-QAM signal constellation shown in Figure P3.7.

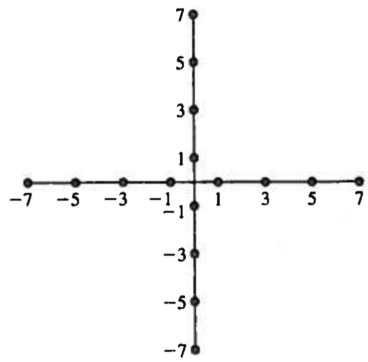
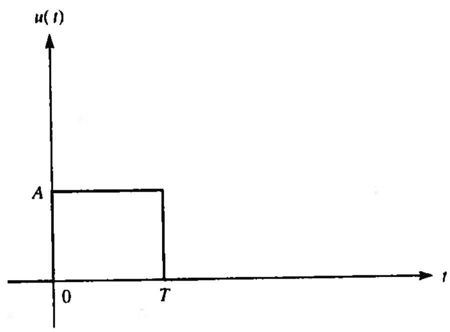


FIGURE P3.7

3.21 The elements of the sequence $\{I_n\}_{n=-\infty}^{+\infty}$ are independent binary random variables taking values of ± 1 with equal probability. This data sequence is used to modulate the basic pulse $u(t)$ shown in Figure P3.21(a). The modulated signal is

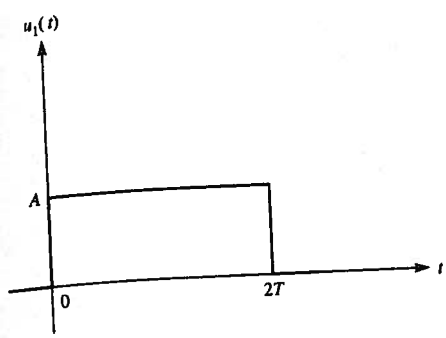
$$X(t) = \sum_{n=-\infty}^{+\infty} I_n u(t - nT)$$

FIGURE P3.21(a)



1. Find the power spectral density of $X(t)$.
2. If $u_1(t)$, shown in Figure P3.21(b), were used instead of $u(t)$, how would the power spectrum in part 1 change?
3. In part 2, assume we want to have a null in the spectrum at $f = \frac{1}{3T}$. This is done by a precoding of the form $b_n = I_n + \alpha I_{n-1}$. Find the value of α that provides the desired null.

FIGURE P3.21(b)



4. Is it possible to employ a precoding of the form $b_n = I_n + \sum_{i=1}^{N} \alpha$ N such that the final power spectrum will be identical to zero for yes, how? If no, why? (*Hint:* Use properties of analytic functions.

4.5 A communication system transmits one of the three messages m_1, m_2, and m_3 using signals $s_1(t)$, $s_2(t)$, and $s_3(t)$. The signal $s_3(t) = 0$, and $s_1(t)$ and $s_2(t)$ are shown in Figure P4.5. The channel is an additive white Gaussian noise channel with noise power spectral density equal to $N_0/2$.

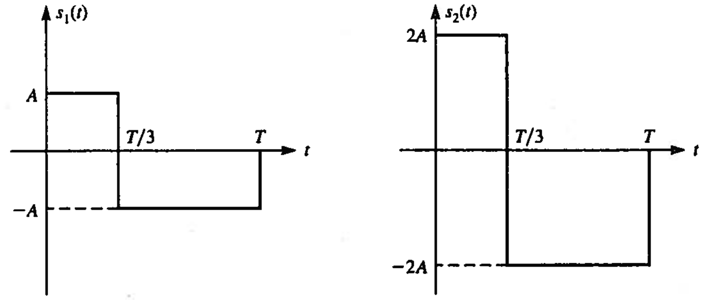


FIGURE P4.5

1. Determine an orthonormal basis for this signal set, and depict the signal constellation.
2. If the three messages are equiprobable, what are the optimal decision rules for this system? Show the optimal decision regions on the signal constellation you plotted in part 1.
3. If the signals are equiprobable, express the error probability of the optimal detector in terms of the average SNR per bit.
4. Assuming this system transmits 3000 symbols per second, what is the resulting transmission rate (in bits per second)?

4.6 Suppose that binary PSK is used for transmitting information over an AWGN with a power spectral density of $\frac{1}{2}N_0 = 10^{-10}$ W/Hz. The transmitted signal energy is $\mathcal{E}_b = \frac{1}{2}A^2T$, where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6} when the data rate is

1. 10 kilobits/s
2. 100 kilobits/s
3. 1 megabit/s

4.7 Consider a signal detector with an input

$$r = \pm A + n$$

where $+A$ and $-A$ occur with equal probability and the noise variable n is characterized by the (Laplacian) PDF shown in Figure P4.7.

1. Determine the probability of error as a function of the parameters A and σ.
2. Determine the SNR required to achieve an error probability of 10^{-5}. How does the SNR compare with the result for a Gaussian PDF?

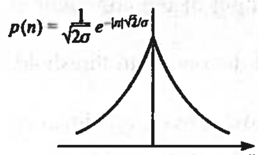


FIGURE P4.7

4.8 The signal constellation for a communication system with 16 equiprobable symbols is shown in Figure P4.8. The channel is AWGN with noise power spectral density of $N_0/2$.

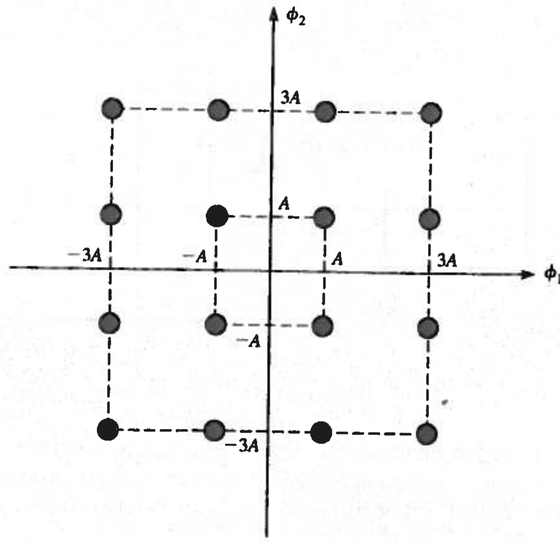


FIGURE P4.8

1. Using the union bound, find a bound in terms of A and N_0 on the error probability for this channel.
2. Determine the average SNR per bit for this channel.
3. Express the bound found in part 1 in terms of the average SNR per bit.
4. Compare the power efficiency of this system with a 16-level PAM system.

4.20 For the QAM signal constellation shown in Figure P4.20, determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high that errors occur only between adjacent points.

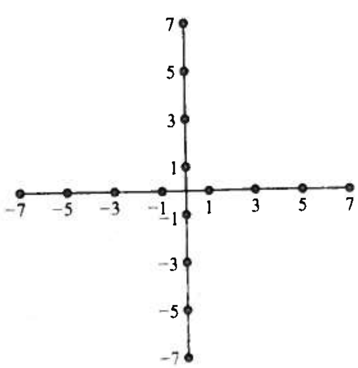


FIGURE P4.20

4.28 Consider the four-phase and eight-phase signal constellations shown in Figure P4.28. Determine the radii r_1 and r_2 of the circles such that the distance between two adjacent points in the two constellations is d. From this result, determine the additional transmitted energy required in the 8-PSK signal to achieve the same error probability as the four-phase signal at high SNR, where the probability of error is determined by errors in selecting adjacent points.

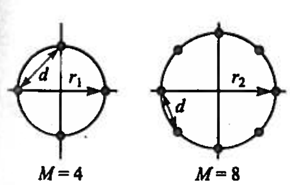


FIGURE P4.28

4.44 Let X denote a Rayleigh distributed random variable, i.e.,

$$f_X(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

1. Determine $\mathrm{E}[Q(\beta X)]$, where β is a positive constant. (*Hint*: Use the definition of the Q function and change the order of integration.)
2. In a binary antipodal signaling, let the received energy be subject to a Rayleigh distributed attenuation; i.e., let the received signal be $r(t) = \alpha s_m(t) + n(t)$, and therefore, $P_b = Q\left(\sqrt{\frac{2\alpha^2 \mathcal{E}_b}{N_0}}\right)$, where α^2 denotes the power attenuation and α has a Rayleigh PDF similar to X. Determine the average error probability of this system.
3. Repeat part 2 for a binary orthogonal system in which $P_b = Q\left(\sqrt{\frac{\alpha^2 \mathcal{E}_b}{N_0}}\right)$.
4. Find approximations for the results of parts 2 and 3 with the assumption that $\sigma^2 \frac{\mathcal{E}_b}{N_0} \gg 1$, and show that in this case both average error probabilities are proportional to $\frac{1}{\overline{\mathrm{SNR}}}$ where $\overline{\mathrm{SNR}} = 2\sigma^2 \frac{\mathcal{E}_b}{N_0}$.
5. Now find the average of $e^{-\beta\alpha^2}$, where β is a positive constant and α is a random variable distributed as $f_X(x)$. Find an approximation in this case when $\beta\sigma^2 \gg 1$. We will later see that this corresponds to the error probability of a noncoherent system in fading channels.

4.45 In a binary communication system two equiprobable messages $s_1 = (1, 1)$ and $s_2 = (-1, -1)$ are used. The received signal is $\boldsymbol{r} = \boldsymbol{s} + \boldsymbol{n}$, where $\boldsymbol{n} = (n_1, n_2)$. It is assumed that n_1 and n_2 are independent and each is distributed according to

$$f(n) = \frac{1}{2} e^{-|n|}$$

Determine and plot the decision regions D_1 and D_2 in this communication scheme.

4.59 Consider a transmission line channel that employs $n - 1$ regenerative repeaters plus the terminal receiver in the transmission of binary information. Assume that the probability of error at the detector of each receiver is p and that errors among repeaters are statistically independent.

1. Show that the binary error probability at the terminal receiver is

$$P_n = \frac{1}{2}\left[1 - (1 - 2p)^n\right]$$

2. If $p = 10^{-6}$ and $n = 100$, determine an approximate value of P_n.

4.60 A digital communication system consists of a transmission line with 100 digital (regenerative) repeaters. Binary antipodal signals are used for transmitting the information. If the overall end-to-end error probability is 10^{-6}, determine the probability of error for each repeater and the required $\mathcal{E}_b/N_0$ to achieve this performance in AWGN.

4.61 A radio transmitter has a power output of $P_T = 1$ W at a frequency of 1 GHz. The transmitting and receiving antennas are parabolic dishes with diameter $D = 3$ m.

1. Determine the antenna gains.
2. Determine the EIRP for the transmitter.
3. The distance (free space) between the transmitting and receiving antennas is 20 km. Determine the signal power at the output of the receiving antenna in decibels.

4.62 A radio communication system transmits at a power level of 0.1 W at 1 GHz. The transmitting and receiving antennas are parabolic, each having a diameter of 1 m. The receiver is located 30 km from the transmitter.

1. Determine the gains of the transmitting and receiving antennas.
2. Determine the EIRP of the transmitted signal.
3. Determine the signal power from the receiving antenna.

4.64 A spacecraft located 100,000 km from the earth is sending data at a rate of frequency band is centered at 2 GHz, and the transmitted power is 10 W. Th uses a parabolic antenna, 50 m in diameter, and the spacecraft has an anten of 10 dB. The noise temperature of the receiver front end is $T_0 = 300$ K.

1. Determine the received power level.
2. If the desired $\mathcal{E}_b/N_0 = 10$ dB, determine the maximum bit rate that the s transmit.

4.65 A satellite in geosynchronous orbit is used as a regenerative repeater in a di nication system. Consider the satellite-to-earth link in which the satellite a gain of 6 dB and the earth station antenna has a gain of 50 dB. The downli at a center frequency of 4 GHz, and the signal bandwidth is 1 MHz. If the re for reliable communication is 15 dB, determine the transmitted power fo downlink. Assume that $N_0 = 4.1 \times 10^{-21}$ W/Hz.

5.16 The probability of error for binary PSK demodulation and detection when there is a carrier phase error ϕ_e is

$$P_2(\phi_e) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}\cos^2\phi_e}\right)$$

Suppose that the phase error from the PLL is modeled as a zero-mean Gaussian random variable with variance $\sigma_\phi^2 \ll \pi$. Determine the expression for the average probability of error (in integral form).

7.13 The generator matrix for a linear binary code is

$$G = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

a. Express G in systematic $[I|P]$ form.
b. Determine the parity check matrix H for the code.
c. Construct the table of syndromes for the code.
d. Determine the minimum distance of the code.
e. Demonstrate that the codeword c corresponding to the information sequence 101 satisfies $cH^t = 0$.

7.14 A code is self-dual if $C = C^\perp$. Show that in a self-dual code the block length is always even and the rate is $\frac{1}{2}$.

7.15 Consider a linear block code with codewords {0000, 1010, 0101, 1111}. Find the dual of this code and show that this code is self-dual.

7.16 List the codewords generated by the matrices given in Equations 7.9–13 and 7.9–15, and thus demonstrate that these matrices generate the same set of codewords.

7.17 Determine the weight distribution of the (7, 4) Hamming code, and check your result with the list of codewords given in Table 7.9–2.

7.25 Show that when a binary sequence x of length n is transmitted over a BSC with crossover probability p, the probability of receiving y, which is at Hamming distance d from x, is given by

$$P(y|x) = (1-p)^n \left(\frac{p}{1-p}\right)^d$$

From this conclude that if $p < \frac{1}{2}$, $P(y|x)$ is a decreasing function of d and hence ML decoding is equivalent to minimum-Hamming-distance decoding. What happens if $p > \frac{1}{2}$?

7.43 For the (7, 4) cyclic Hamming code with generator polynomial $g(X) = X^3 + X^2 + 1$, construct an (8, 4) extended Hamming code and list all the codewords. What is d_{min} for the extended code?

7.44 An (8, 4) linear block code is constructed by shortening a (15, 11) Hamming code generated by the generator polynomial $g(X) = X^4 + X + 1$.

a. Construct the codewords of the (8, 4) code and list them.

b. What is the minimum distance of the (8, 4) code?

8.1 A convolutional code is described by

$$g_1 = [101], \quad g_2 = [111], \quad g_3 = [111]$$

1. Draw the encoder corresponding to this code.
2. Draw the state-transition diagram for this code.
3. Draw the trellis diagram for this code.
4. Find the transfer function and the free distance of this code.
5. Verify whether or not this code is catastrophic.

8.2 The convolutional code of Problem 8.1 is used for transmission over an AWGN channel with hard decision decoding. The output of the demodulator detector is (101001011110111 ⋯). Using the Viterbi algorithm, find the transmitted sequence, assuming that the convolutional code is terminated at the zero state.

8.3 Repeat Problem 8.1 for a code with

$$g_1 = [110], \quad g_2 = [101], \quad g_3 = [111]$$

8.4 The block diagram of a binary convolutional code is shown in Figure P8.4.

1. Draw the state diagram for the code.
2. Find the transfer function of the code $T(Z)$.
3. What is d_{free}, the minimum free distance of the code?

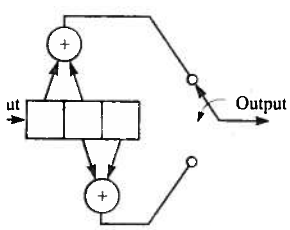


FIGURE P8.4

4. Assume that a message has been encoded by this code and transmitted over a bin[a]
symmetric channel with an error probability of $p = 10^{-5}$. If the received sequence

$$r = (110, 110, 110, 111, 010, 101, 101)$$

using the Viterbi algorithm, find the most likely information sequence, assuming t[h]
the convolutional code is terminated at the zero state.

5. Find an upper bound to the bit error probability of the code when the above bina[ry]
symmetric channel is employed. Make any reasonable approximation.

8.17 Consider the $K = 3$, rate 1/2, convolutional code shown in Figure P8.17. Suppose that the code is used on a binary symmetric channel and the received sequence for the first eight branches is 0001100000001001. Trace the decisions on a trellis diagram, and label the survivors' Hamming distance metric at each node level. If a tie occurs in the metrics required for a decision, always choose the upper path (arbitrary choice).

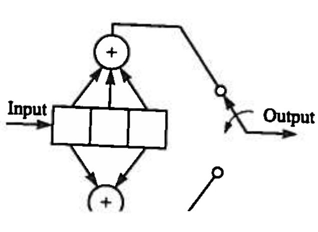


FIGURE P8.17