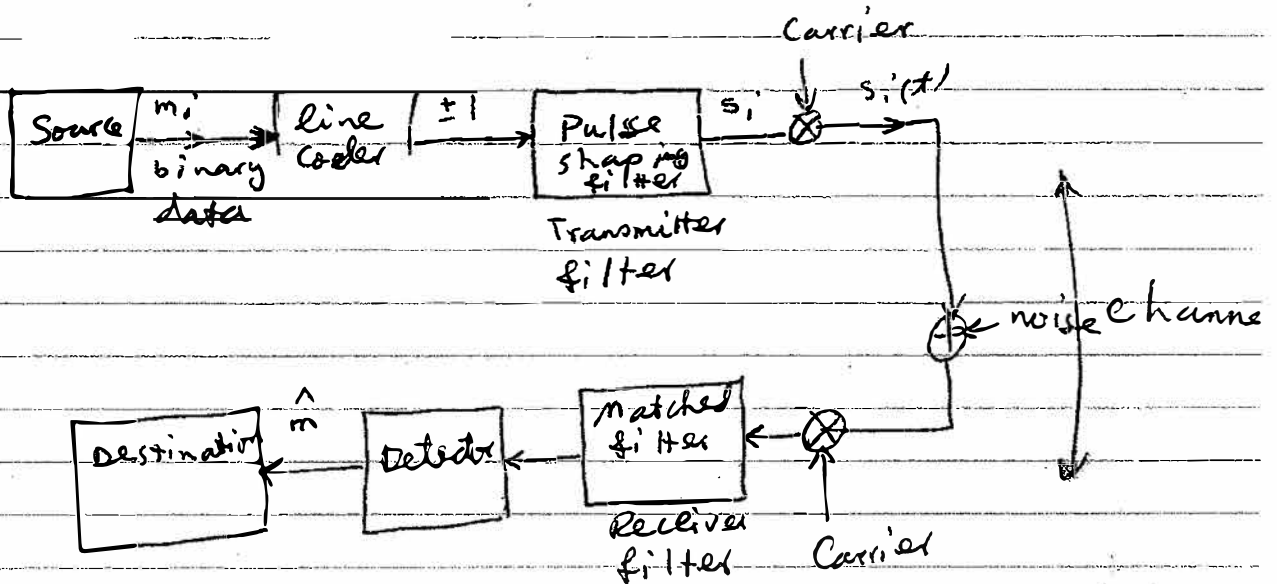


# Lecture 6:

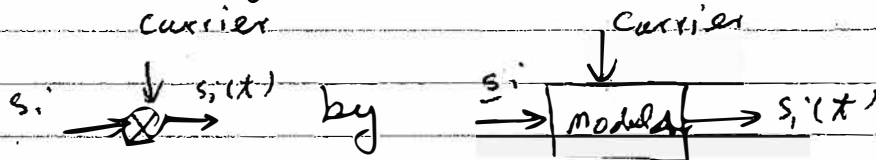
## Passband Data Transmission

(Carrier Modulation)

In carrier (Passband) data transmission, the output of the pulse shaping filter is being frequency shifted by multiplying it by a sinusoidal carrier:



To be more general, we may represent carrier



to allow for ~~modification~~ varying any arbitrary parameter of the carrier waveform.

Basic carrier Modulation Schemes:

Phase Shift Keying (PSK)

Amplitude Shift Keying (ASK)

Frequency Shift Keying (FSK)

Phase Shift Keying (PSK)

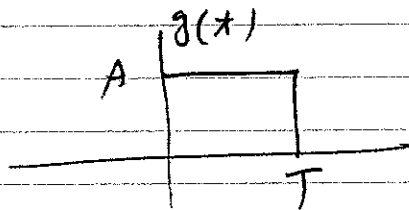
binary case:

$$s_1(x) = g(x) \cos(2\pi f_c t)$$

$$s_2(x) = g(x) \cos(2\pi f_c t + \pi) = -g(x) \cos(2\pi f_c t)$$

$g(x)$  is the pulse shaping filter, usually raised cosine filter.

For simplicity assume that  $g(x)$  is an square pulse, i.e.,



Then

$$s_1(x) = A \cos(2\pi f_c t)$$

$$s_2(x) = -A \cos(2\pi f_c t)$$

$$E_b = \int_0^T s_1^2(t) dt = \frac{A^2}{2} T_b = \int_0^T s_2^2(t) dt$$

$$\text{So: } A = \sqrt{\frac{2E_b}{T_b}}$$

So:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

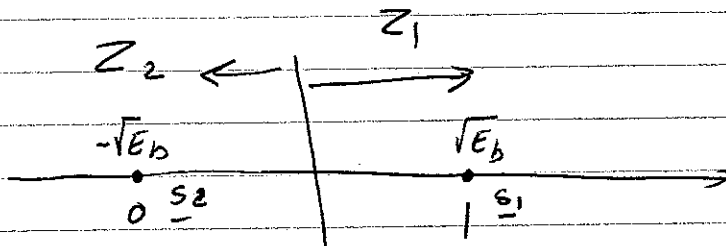
we only need one basis function for this modulation scheme, i.e.,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t < T_b$$

and, then:

$$s_1(t) = \sqrt{E_b} \phi_1(t) \quad 0 \leq t < T_b$$

$$s_2(t) = -\sqrt{E_b} \phi_1(t) \quad 0 \leq t < T_b$$



$$P_e = p_0 P(1|0) + p_1 P(0|1)$$

for equally probable message bits:

$$P_e = \frac{1}{2} P(1|0) + \frac{1}{2} P(0|1)$$

$$P(1|0) = \int_0^{\infty} f_x(x|0) dx = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0} (x + \sqrt{E_b})^2} dx$$

let  $z = (x + \sqrt{E_b}) / \sqrt{N_0/2}$

Then

$$P(1|0) = \int_{\frac{\sqrt{2E_b}}{N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Similarly

$$P(0|1) = \int_{-\infty}^0 \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x - \sqrt{E_b})^2}{N_0}} dx = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

So:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

The same as bipolar NRZ signalling.

Power Spectral Density of the PSK signal

We can represent binary PSK as:

$$S_1(x) = g(x) \cos(2\pi f_c t)$$

$$S_2(x) = -g(x) \cos(2\pi f_c t)$$

where

$$g(x) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} = A & 0 \leq x < T_b \\ 0 & \text{otherwise} \end{cases}$$

6-5

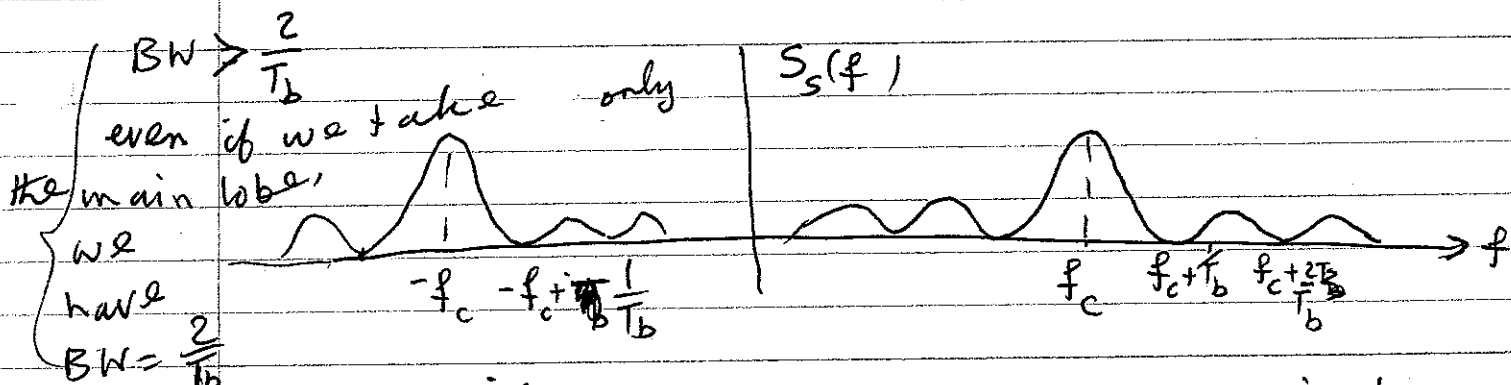
The power Spectral Density of  $s(t)$ ,

$S_s(f)$  is given as

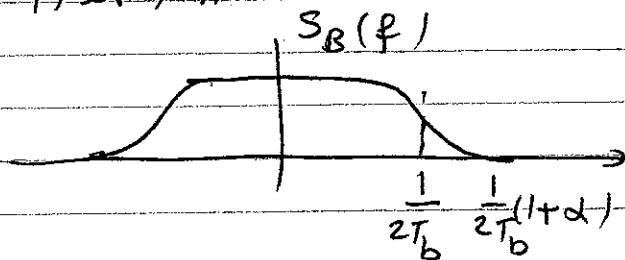
$$S_s(f) = \frac{1}{4} [S_B(f-f_c) + S_B(f+f_c)]$$

where  $S_B(f)$  is the power spectral density of the  $g(t)$ . (Remember: we know this from early lectures  $\} : \text{random binary sequence}$ ).

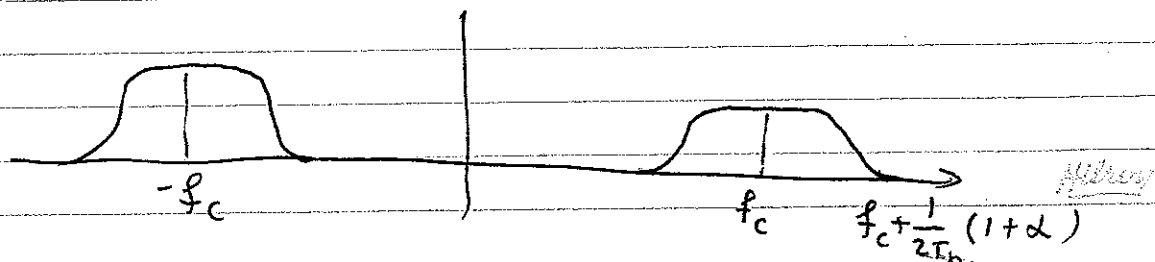
$$S_B(f) = \frac{2 E_b \sin^2(\pi f T_b)}{(\pi f T_b)^2} = 2 E_b \text{Sinc}^2(f T_b)$$



However, if we use for  $g(t)$  the raised cosine filter, then



and



6-6

So, the require Bandwidth would ~~have been~~ <sup>be</sup>:

$$BW = \frac{1}{T_b} (1+\alpha) = R_b (1+\alpha) \text{ Hz.}$$

Bandwidth efficiency is

$$\eta = \frac{R_b}{BW} = \frac{1}{1+\alpha} \text{ bits/sec./Hz.}$$

Quaternary - Phase-Shift Keying (QPSK).

$$s_i(x) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1) \frac{\pi}{4}\right] & 0 \leq x < T \\ 0 & \text{elsewhere} \end{cases}$$

for  $i = 1, 2, 3, 4$

Signal space representation

$$s_i(x) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t) \cos\left((2i-1) \frac{\pi}{4}\right) - \sqrt{\frac{2E}{T}} \sin(2\pi f_c t) \times \sin\left((2i-1) \frac{\pi}{4}\right)$$

Two orthonormal basis

$$\phi_1(x) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad 0 \leq x < T$$

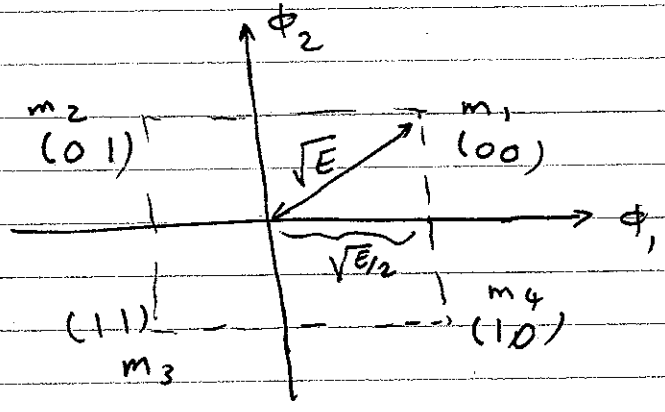
and

$$\phi_2(x) = -\sqrt{\frac{2}{T}} \sin(2\pi f_c t) \quad 0 \leq x < T$$

are required to represent QPSK.

6-7

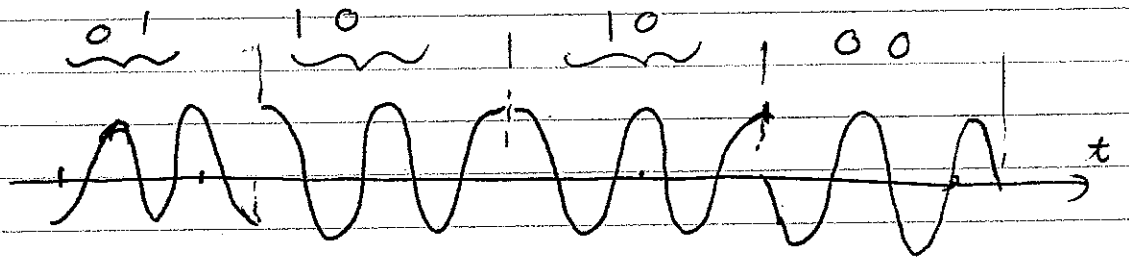
$i$		$\phi_i$	$S_{i1}$	$S_{i2}$
$i=1$	00	$\pi/4$	$+\sqrt{E}/2$	$+\sqrt{E}/2$
$i=2$	01	$3\pi/4$	$-\sqrt{E}/2$	$+\sqrt{E}/2$
$i=3$	11	$5\pi/4$	$-\sqrt{E}/2$	$-\sqrt{E}/2$
$i=4$	10	$7\pi/4$	$+\sqrt{E}/2$	$-\sqrt{E}/2$



Example:

Output corresponding to the sequence:

0 | 1 | 0 | 0 | 0 | 0



6-8

Probability of error for QPSK

With Gray coding, the probabilities of error for the two bits in a symbol are independent from each other.

Prob. of 2nd bit being in error is the probability that the noise in  $\Phi_1$  direction is greater than (less than)  $\sqrt{E}/2$  ( $-\sqrt{E}/2$ ) for 1 (0), respectively. Similarly, for the

1st bit

$$P_B = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x + \sqrt{E}/2)^2}{N_0}} dx$$

$$\text{let } z = \frac{x + \sqrt{E}/2}{\sqrt{N_0}/2}$$

to get

$$P_B = \int_{\frac{\sqrt{E}}{2\sqrt{N_0}}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

But,  $E = 2E_b$  So:

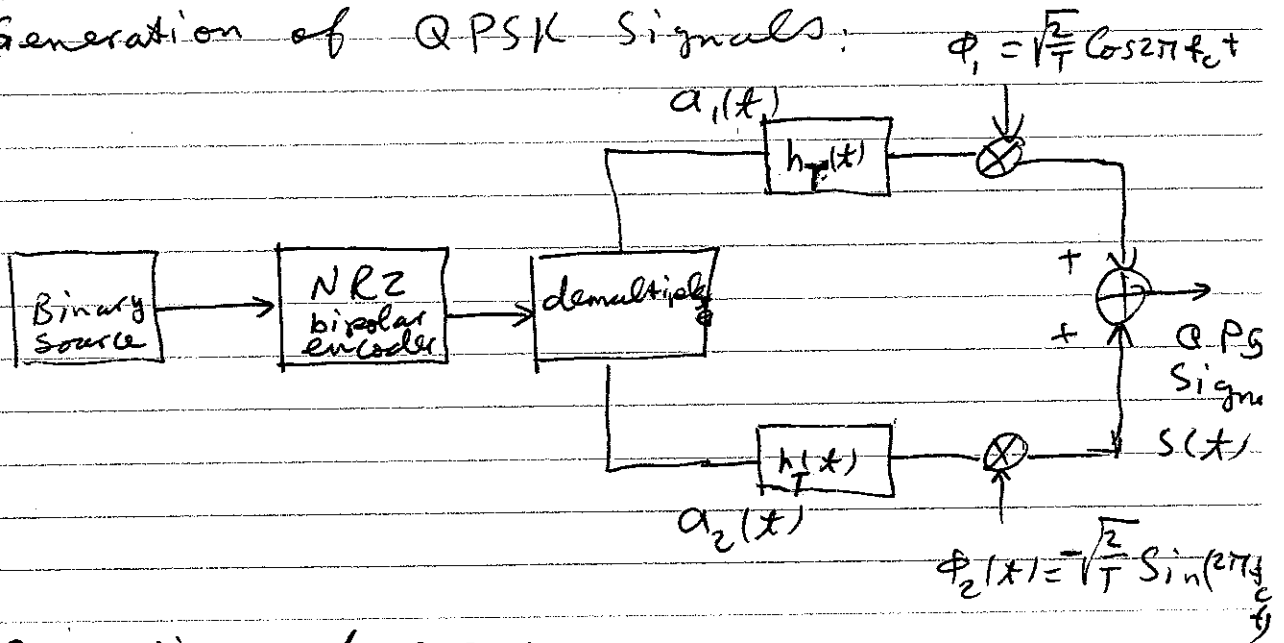
$$P_B = Q\left(\sqrt{2E_b/N_0}\right)$$
$$P_B = \dots$$



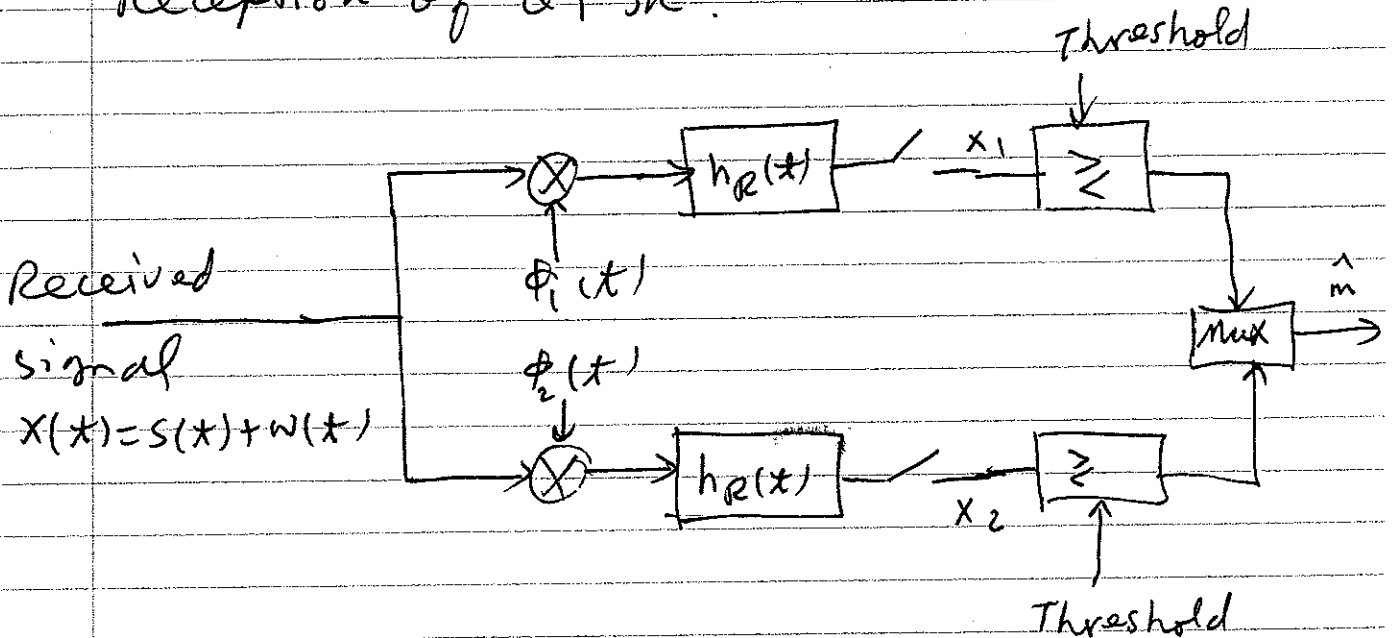
## 6-9

So the, BER of BPSK and QPSK is the same.

Generation of QPSK Signals:



Reception of QPSK:

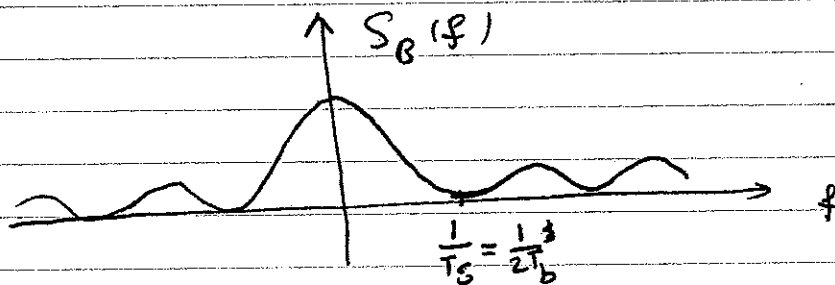


## Power Spectra of QPSK signal

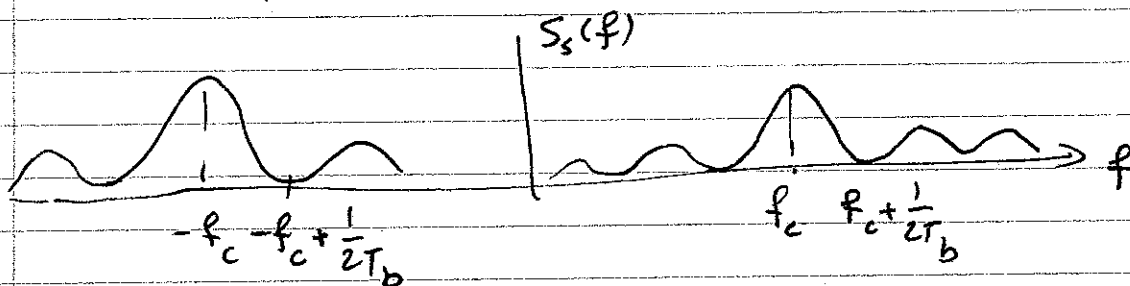
for unfiltered QPSK:

$$g(t) = \begin{cases} \sqrt{\frac{E_s}{T}} & 0 \leq t \leq T_s \\ 0 & \text{elsewhere} \end{cases}$$

$$S_B(f) = 2E_s \text{Sinc}^2(T_s f) = 4E_b \text{Sinc}^2(2T_b f)$$



$$S_s(f) = \frac{1}{4} [S_B(f - f_c) + S_B(f + f_c)]$$

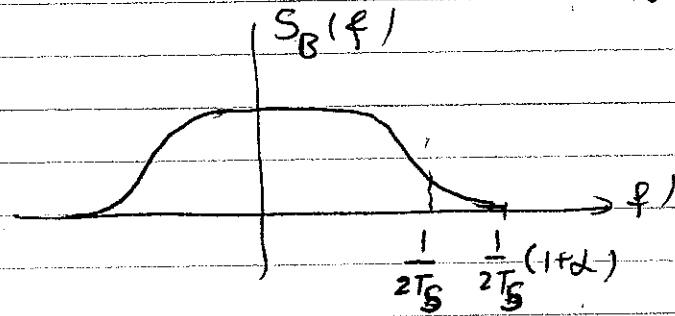


$BW = \frac{1}{T_b}$  if only ~~one~~ main lobe is considered

6-11

if, instead, we use raised cosine filter

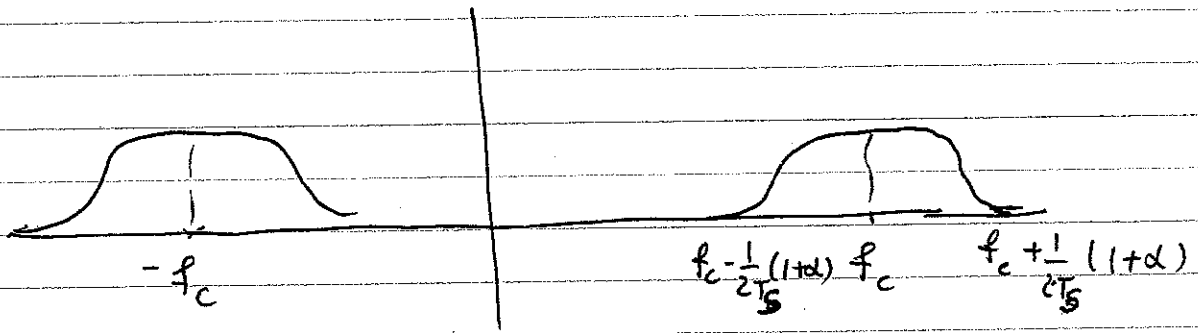
then



~~$g(f) = \frac{\sin(\pi f T)}{\pi f T} \cos(\pi f \alpha T)$~~

$$g(f) = \frac{\sin(2\pi W T)}{2\pi W T} \times \frac{\cos(2\pi \alpha W T)}{1 - 16\alpha^2 W^2 T^2}$$

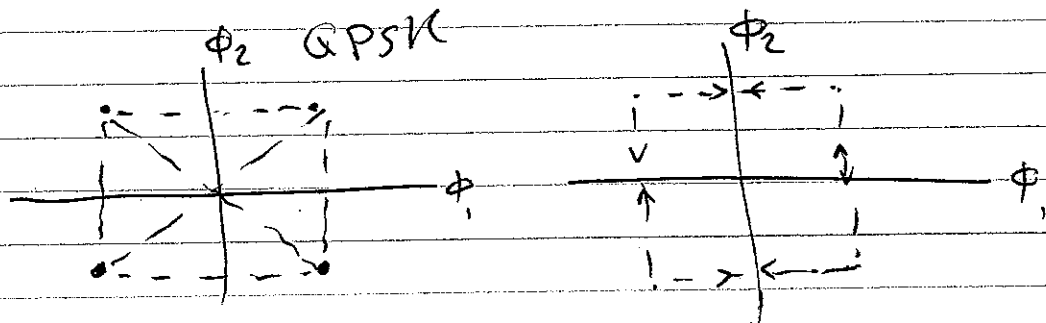
and



and  $BW = \frac{1}{T_S}(1+\alpha) = \frac{1}{2T_b}(1+\alpha)$

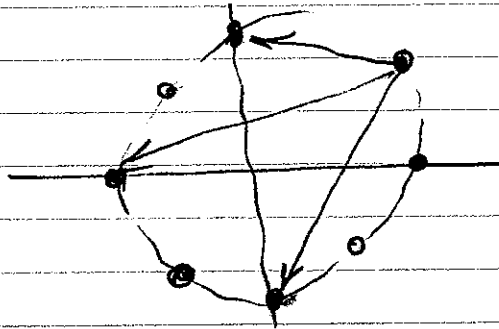
offset QPSK (OQPSK)

in order to avoid  $\pm 180^\circ$  phase jumps in QPSK, one may delay I or Q data stream by half a symbol (1 bit).



6-12

$\frac{\pi}{4}$  - QPSK



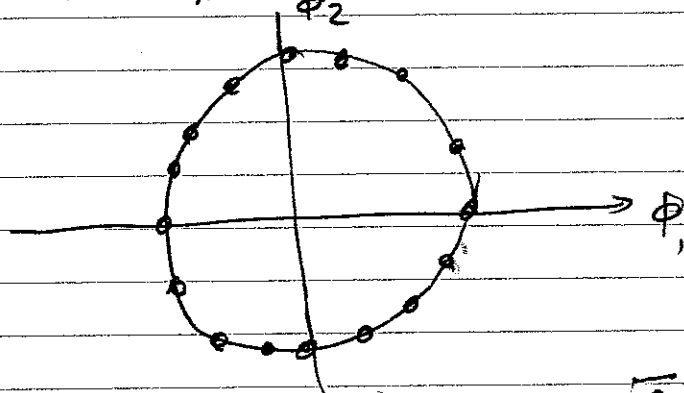
In this scheme, the signal points are alternatively taken from one of the two QPSK constellations.

So, the phase change is  $\pm \frac{\pi}{4}$  and  $\pm \frac{3\pi}{4}$  instead of  $\pm \frac{\pi}{2}$  and  $\pm \pi$ .

M-ary PSK (MPSK)

$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos(2\pi f_c t + \frac{2\pi}{M}(i-1)) \quad i=1, 2, \dots, M$$

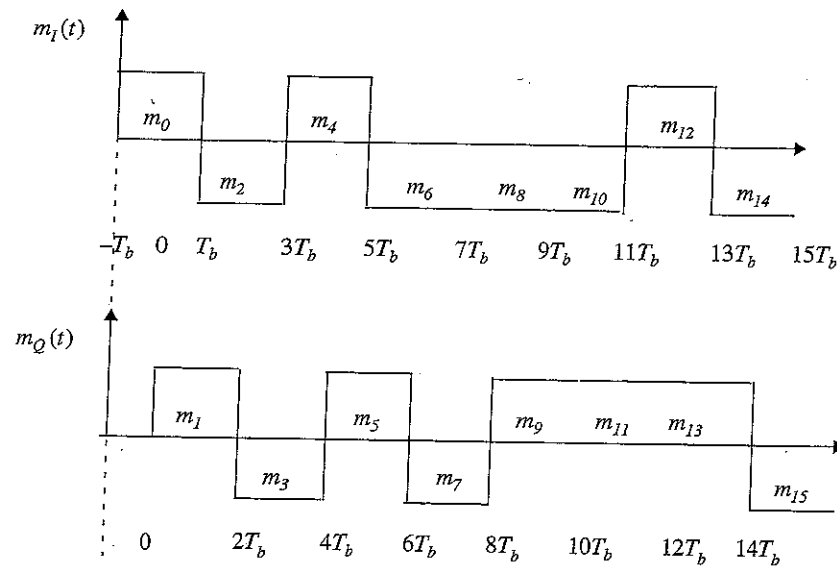
$$= \sqrt{\frac{2E_s}{T}} \cos \frac{2\pi}{M}(i-1) \cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T}} \sin \frac{2\pi}{M}(i-1) \sin(2\pi f_c t)$$



$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

*filmy*



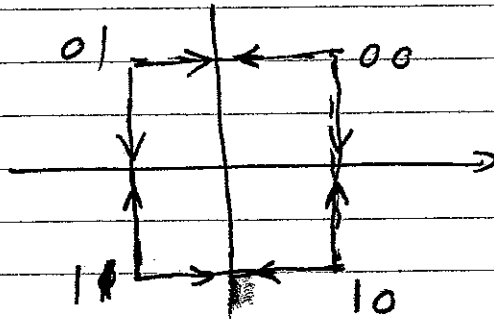
**Figure 6.30** The time offset waveforms that are applied to the in-phase and quadrature arms of an OQPSK modulator. Notice that a half-symbol offset is used.

Due to the time alignment of  $m_I(t)$  and  $m_Q(t)$  in standard QPSK, phase transitions occur only once every  $T_s = 2T_b$  s, and will be a maximum of  $180^\circ$  if there is a change in the value of both  $m_I(t)$  and  $m_Q(t)$ . However, in OQPSK signaling, bit transitions (and, hence, phase transitions) occur every  $T_b$  s. Since the transition instants of  $m_I(t)$  and  $m_Q(t)$  are offset, at any given time only one of the two bit streams can change values. This implies that the maximum phase shift of the transmitted signal at any given time is limited to  $\pm 90^\circ$ . Hence, by switching phases more frequently (i.e., every  $T_b$  s instead of  $2T_b$  s) OQPSK signaling eliminates  $180^\circ$  phase transitions.

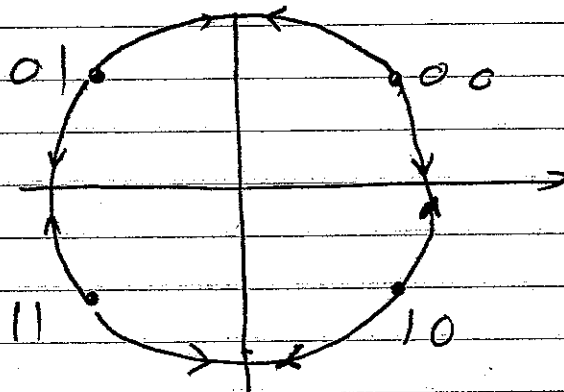
Since  $180^\circ$  phase transitions have been eliminated, bandlimiting of (i.e., pulse shaping) OQPSK signals does not cause the signal envelope to go to zero. Obviously, there will be some amount of ISI caused by the bandlimiting process, especially at the  $90^\circ$  phase transition points. But the envelope variations are considerably less, and hence hardlimiting or nonlinear amplification of OQPSK signals does not regenerate the high frequency sidelobes as much as in QPSK. Thus, spectral occupancy is significantly reduced, while permitting more efficient RF amplification.

The spectrum of an OQPSK signal is identical to that of a QPSK signal, hence both signals occupy the same bandwidth. The staggered alignment of the even and odd bit streams does not change the nature of the spectrum. OQPSK retains its bandlimited nature even after nonlinear amplification, and therefore is very attractive for mobile communication systems where bandwidth efficiency and efficient nonlinear amplifiers are critical for low power drain. Further, OQPSK signals also appear to perform better than QPSK in the presence of phase jitter due to noisy reference signals at the receiver [Chu87].

Now, we have made the <sup>allowed</sup> changes as shown below:



We can make the changes smoother by changing the phase gradually,



That is, instead of making the phase changes abruptly at  $kT_b$  time instants make it gradually between  $kT_b$  and  $T(k+1)T_b$ .

This will be done by multiplying  $m_I$  by

$$\cos\left(\frac{\pi t}{2T_b}\right) \text{ and } m_Q \text{ by } \sin\left(\frac{\pi t}{2T_b}\right).$$

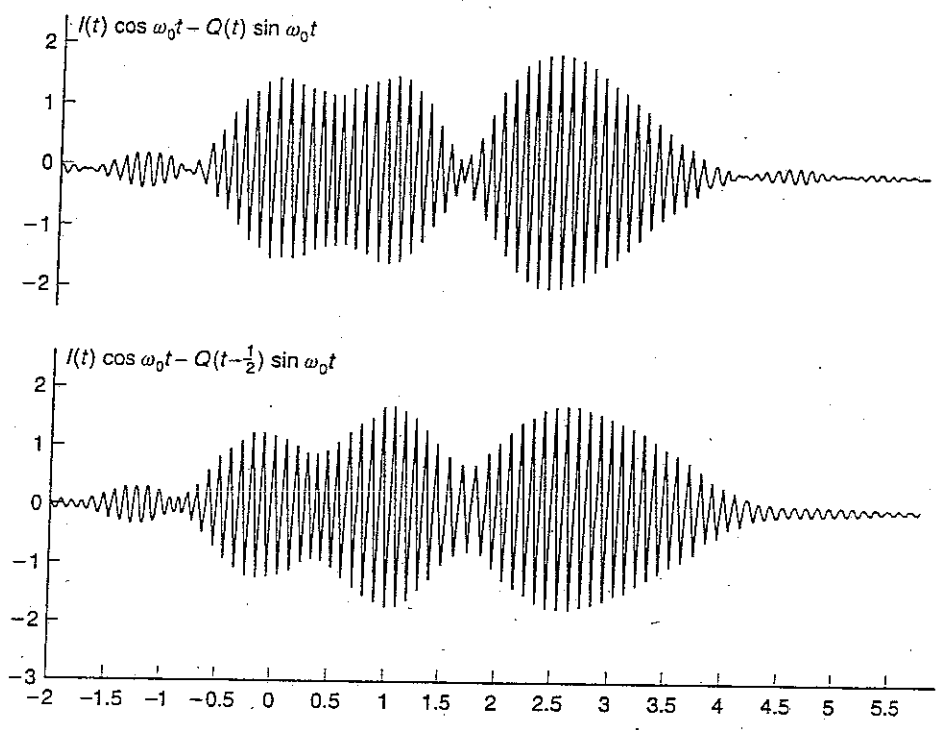


Figure 3.12 Comparison of QPSK (top) and offset QPSK (bottom) with same data and 30% root RC pulses. Data as in Figs. 3.1, 3.2, and 3.11.

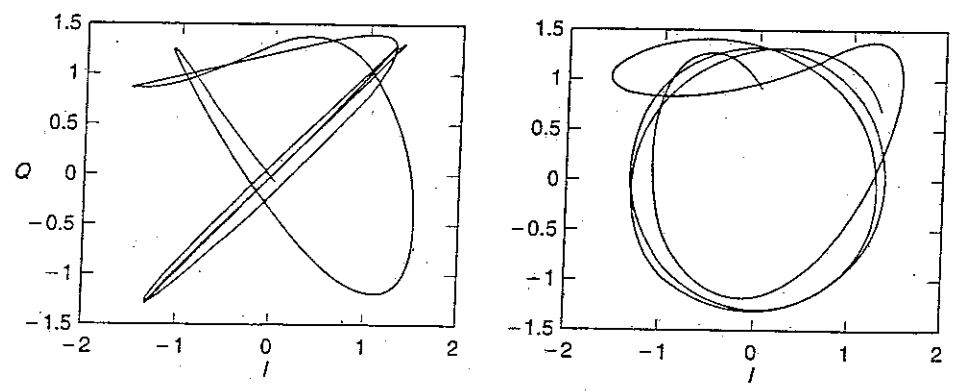
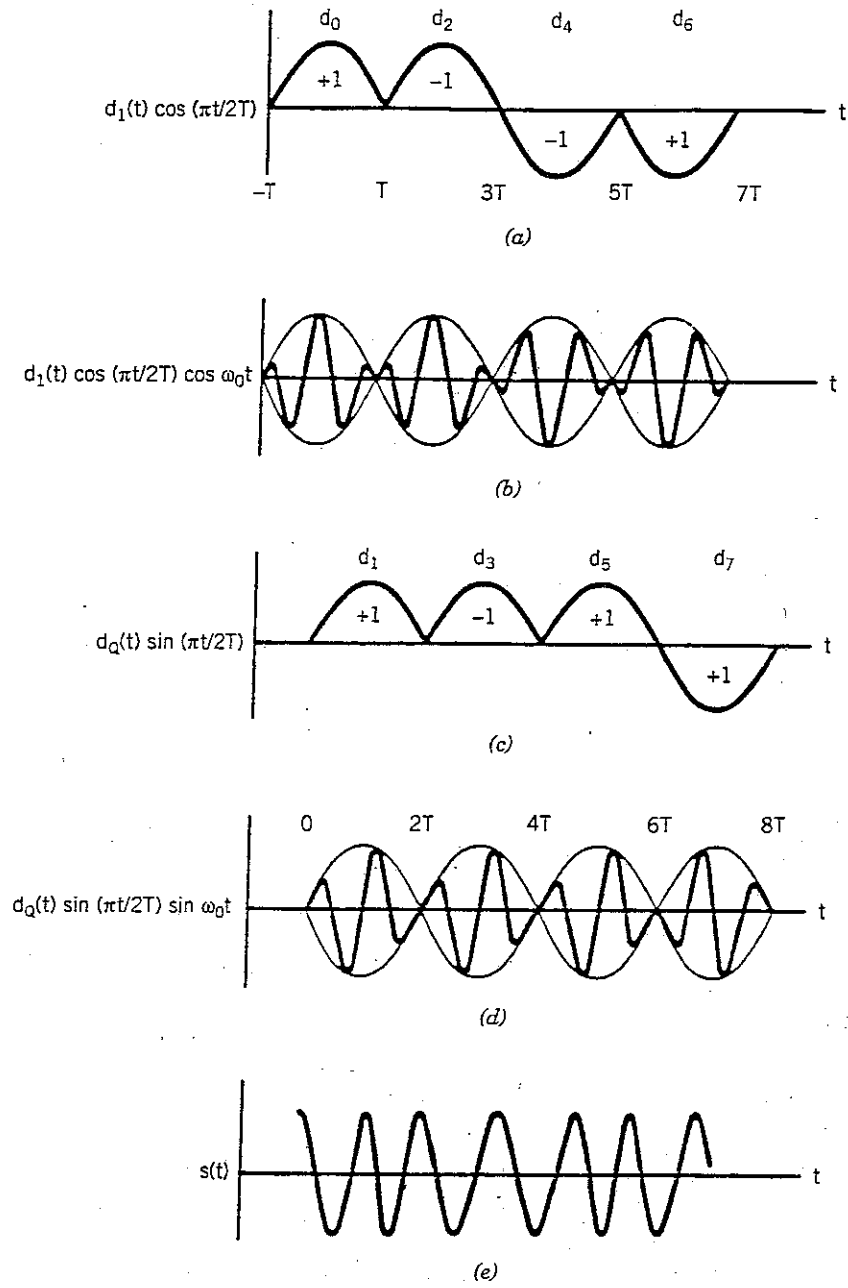


Figure 3.13 The  $I/Q$  plots for QPSK (left) and offset QPSK (right) for  $I$  data (+ - + - + - - + + -) and  $Q$  data (+ - + - + + + - +). There are five 180 degree phase changes.

11-14



**Figure 7.25** MSK waveform composition. (a) Modified I bit stream. (b) I bit stream times carrier. (c) Modified Q bit stream. (d) Q bit stream times carrier. (e) MSK waveforms. (From [Pas79] © IEEE.)

11-15



So, we will have

$$s_{MSK}(t) = m_I(t) \cos\left(\frac{\pi t}{2T_b}\right) \cos(\omega_c t) + m_Q(t) \sin\left(\frac{\pi t}{T_b}\right) \sin(\omega_c t)$$

if  $m_I(t) = m_Q(t) = 1$

we have

$$\begin{aligned} s_{MSK}(t) &= \cos\left(\frac{\pi t}{2T_b}\right) \cos(\omega_c t) + \sin\left(\frac{\pi t}{T_b}\right) \sin(\omega_c t) \\ &= \cos\left(\omega_c t - \frac{\pi}{4T_b} t\right) = \cos 2\pi\left(f_c - \frac{1}{4T_b}\right)t \end{aligned}$$

if  $m_I(t) = 1$  and  $m_Q(t) = -1$

then

$$s_{MSK}(t) = \cos 2\pi\left(f_c + \frac{1}{4T_b}\right)t$$

if  $m_I = m_Q = -1$

$$s_{MSK}(t) = -\cos 2\pi\left(f_c - \frac{1}{4T_b}\right)t = \cos\left[2\pi f_c t - \frac{\pi t}{2T_b} + \pi\right]$$

and for  $m_I = -1$  and  $m_Q = +1$

$$s_{MSK}(t) = -\cos 2\pi\left(f_c + \frac{1}{4T_b}\right)t$$

So

$$s_{MSK}(t) = \cos\left[2\pi f_c t - m_I(t)m_Q(t) \frac{\pi t}{2T_b} + \phi_k\right]$$

11-16

where  $\phi_k = 0$   
 $\phi_k = \pi$

$m_I(t) = 1$   
 $m_I(t) = -1$

Hibroy

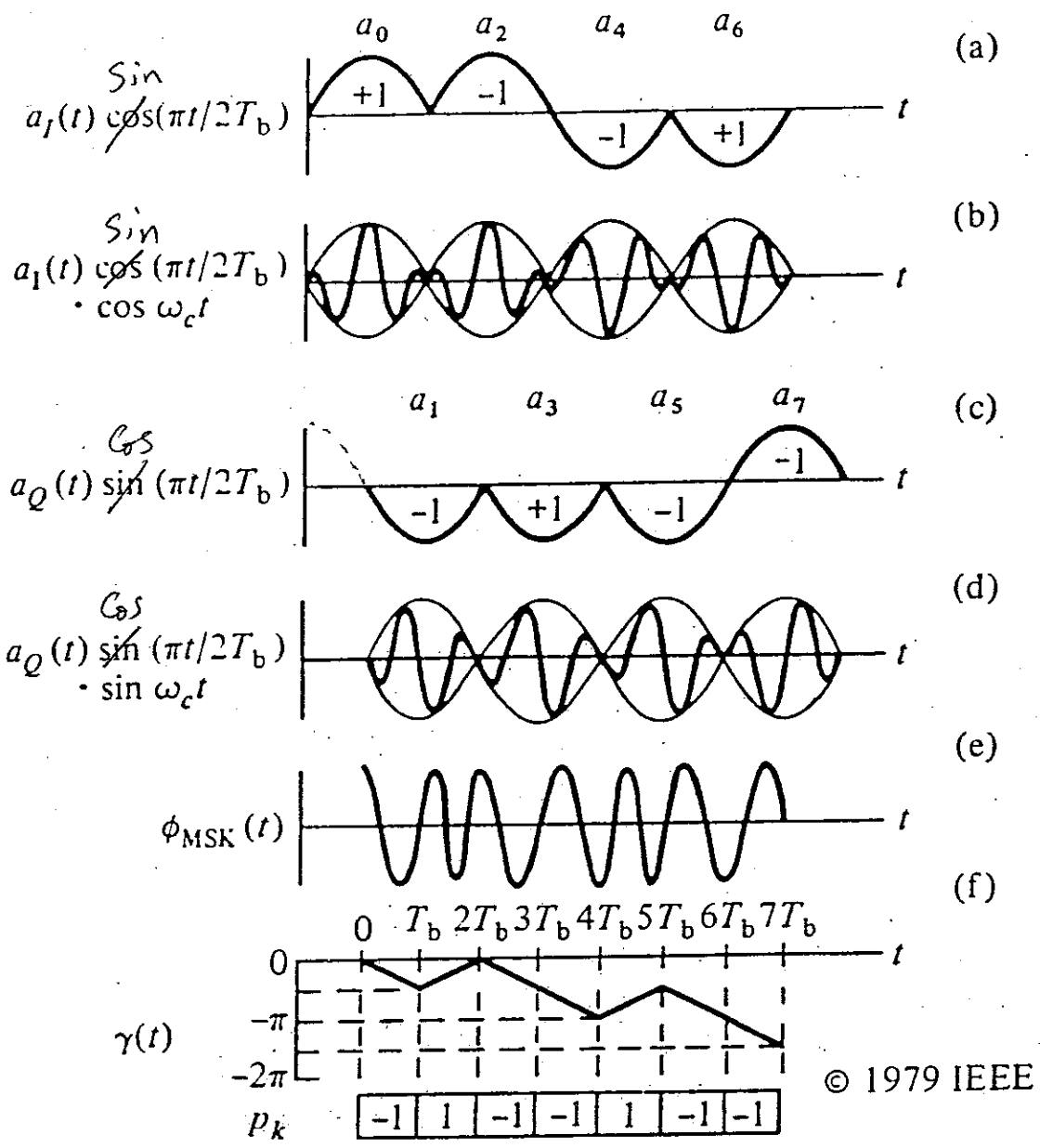


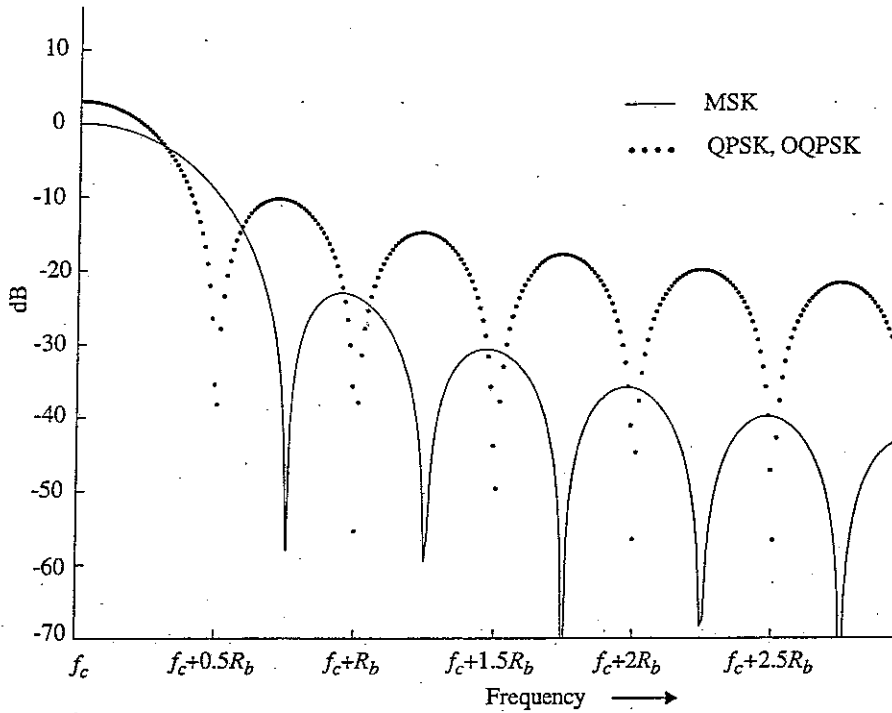
Figure 10.30 MSK waveforms.†

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

11-18

### MSK Power Spectrum

$$P_{\text{MSK}} = \frac{16}{\pi^2} \left( \frac{\cos 2\pi(f + f_c)T}{1.16f^2T^2} \right)^2 + \frac{16}{\pi^2} \left( \frac{\cos 2\pi(f - f_c)T}{1.16f^2T^2} \right)^2$$



Power spectral density of MSK signals as compared to QPSK and OQPSK signals.

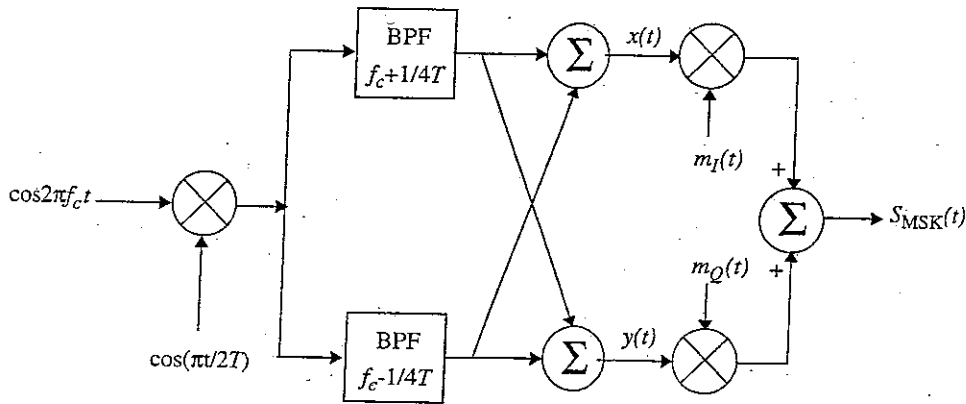


Figure 5.39  
Block diagram of an MSK transmitter.

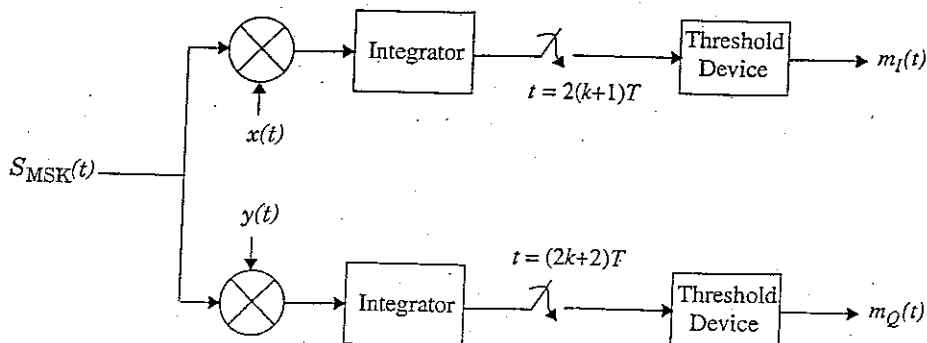
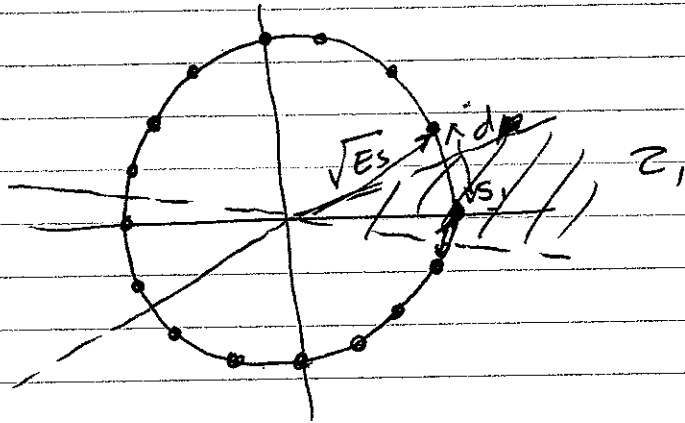


Figure 5.40  
Block diagram of an MSK receiver.

6-13

Probability of error for MPSK:



$$P_e = P_{e|s_1} = P(x \notin Z_1 | s_1) \rightarrow P(\text{noise} > \frac{d}{2}) \\ = P(\text{noise} > \sqrt{E_s} \sin \frac{\pi}{M})$$

or

$$P_e \approx P(s_2 | s_1) = Q\left(\sqrt{\frac{2E_b}{N_0}} \sin \frac{\pi}{M}\right)$$

also

$$P_e < P(s_2 | s_1) + P(s_3 | s_1) = 2Q\left(\sqrt{\frac{2E_b}{N_0}} \sin \frac{\pi}{M}\right)$$

for large (also moderate)  $M$ , the upper and lower bounds are close and

$$P_e \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin \frac{\pi}{M}\right)$$

Note that this is the probability of symbol error.

The BER, assuming Gray coding is:

*Hilary*

$$P_B = \frac{2}{\log_2^m} \left( \sqrt{\frac{2E_s}{N_0}} \sin \frac{\pi}{m} \right)$$

let  $E_s = E_b \log_2^m$  to get:

$$P_B = \frac{2}{\log_2^m} Q \left( \sqrt{\frac{2E_b \log_2^m}{N_0}} \sin \frac{\pi}{m} \right)$$

Bandwidth requirement of MPSK:

unfiltered:

$$BW = \frac{2}{T_s} = \frac{2}{T_b \log_2^m} = \frac{2R_b}{\log_2^m}$$

with raised cosine filter

$$BW = \frac{1}{T_s} (1+d) = \frac{1}{T_b \log_2^m} (1+d) = \frac{R_b (1+d)}{\log_2^m}$$

Bandwidth efficiency:

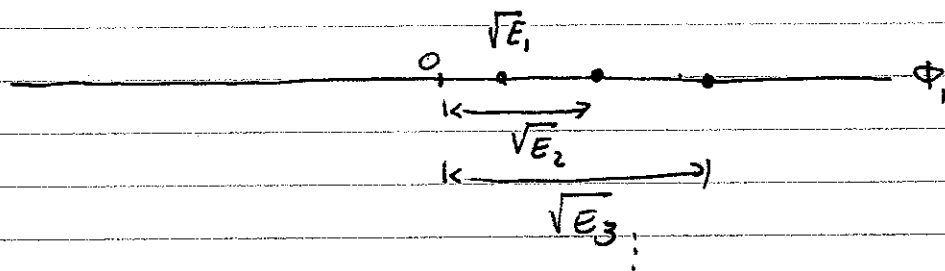
$$\eta = \frac{R_b}{BW} = \frac{\log_2^m}{1+d}$$

M-ary ASK (M-ary PAM)

$$s(x) = A_i \cos(2\pi f_c t)$$

$$E_i = \frac{A_i^2 T_s}{2}$$

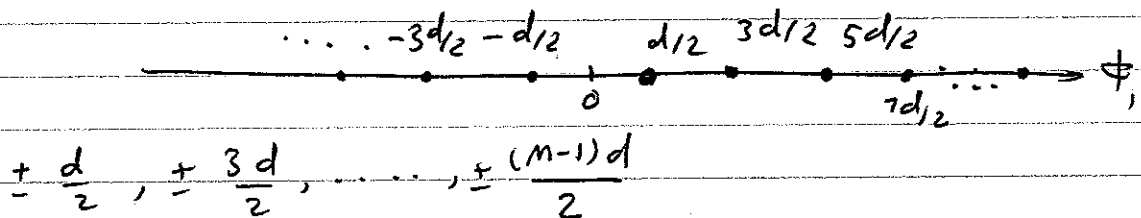
$$s(x) = \sqrt{\frac{2E_i}{T_s}} \cos(2\pi f_c t)$$



where

$$\phi_1(x) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t)$$

let the distance between signals be the same and equal to  $d$ :



The average power is

$$E_{av} = \frac{(d/2)^2 [1 + 3^2 + 5^2 + \dots + (M-1)^2] \times 2}{M}$$

$$E_{av} = \frac{d^2}{2} \times \frac{1}{M} \times \frac{M(M-1)(M+1)}{6} = \frac{d^2(M^2-1)}{12}$$

6-16

Probability of symbol error:

$$P_e = [(m-2) 2 Q\left(\frac{d}{2\sigma}\right) + 2 \times Q\left(\frac{d}{2\sigma}\right)] / m$$

$$P_e = \frac{2(m-1)}{m} Q\left(\frac{d}{2\sigma}\right)$$

$$d = \sqrt{\frac{12 E_{av}}{(m^2-1)}}$$

$$\sigma^2 = \frac{N_0}{2} \Rightarrow \sigma = \sqrt{\frac{N_0}{2}}$$

So,

$$P_e = \frac{2(m-1)}{m} Q\left(\sqrt{\frac{6 E_{av}}{(m^2-1)N_0}}\right)$$

or

$$P_e = 2\left(1 - \frac{1}{m}\right) \times Q\left(\sqrt{\frac{6 E_{av}}{(m^2-1)N_0}}\right)$$

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M-ary Quadrature Amplitude Modulation (QAM)

This is a hybrid modulation scheme, where both phase and amplitude are varied.

like MPSK, we have

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t)$$

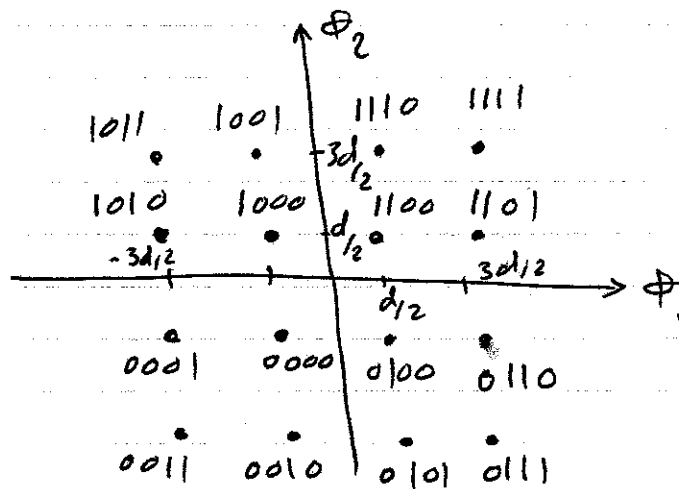
but here

$$s_k(t) = \sqrt{\frac{2E_0}{T}} a_k \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}} b_k \sin(2\pi f_c t)$$

$k = 0, \pm 1, \pm 2,$

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i.e., the amplitude in I and Q dimension is also varied according to the symbol to be transmitted ( $a_n$  and  $b_n$ ). Here  $\sqrt{E_0} = \frac{d}{2}$  as in the case of M-ASK.



Probability of symbol error

$$P_e = 1 - P_c = 1 - (1 - P_{\sqrt{m}})^2 \approx 2P_{\sqrt{m}}$$

where  $P_{\sqrt{m}}$  is the probability of error of a 1-dimensional scheme ( $\sqrt{m}$ -PAM or  $\sqrt{m}$ -ASK).

So:

$$P_e \approx 4 \left(1 - \frac{1}{\sqrt{m}}\right) Q \left( \sqrt{\frac{3E_{av}}{2(m-1)N_0}} \right)$$

Since  $= 4 \left(1 - \frac{1}{\sqrt{m}}\right) Q \left( \sqrt{\frac{3E_{av}}{(m-1)N_0}} \right)$

$$P_{\sqrt{m}} = 2 \left(1 - \frac{1}{\sqrt{m}}\right) Q \left( \sqrt{\frac{3E'_{av}}{(m-1)N_0}} \right)$$

where  $E'_{av} = \frac{1}{2} E_{av}$  is the average signal energy of



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the 1-dimensional scheme.

BER: Assuming Gray Coding

$$P_B = 4 \left(1 - \frac{1}{\sqrt{M}}\right) \frac{1}{\log_2 M} Q \left( \sqrt{\frac{3 \log_2^2 M}{M-1} \frac{E_b}{N_0}} \right)$$

Comparison with M-ary PSK

$$P_B \approx \frac{1}{\log_2 M} Q \left( \sqrt{\frac{E_b \log_2^2 M}{N_0} \sin \frac{\pi}{M}} \right)$$

for large M

$$P_B \approx \frac{1}{\log_2 M} Q \left( \sqrt{\frac{E_b \log_2^2 M}{N_0} \frac{\pi}{M}} \right)$$

while the exponent for M-QAM decreases

as  $\sqrt{\frac{\log_2^2 M}{M-1}}$  for M-PSK the decrease is

as  $\frac{\sqrt{\log_2^2 M}}{M}$  which is faster. So, the

performance QAM is better.