Lecture 7

Frequency Shift Keying (FSK)

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_i t) & 0 \le t \le T_s \\ 0 & \text{elsewhere} \end{cases}$$

for having continuous phase we require

$$f_c = \frac{n_c + i}{T_s}$$

That is frequencies should be apart by multiples of $\frac{1}{T_s}$. For orthogonality the min. spacing is $\frac{1}{2T_s}$

These signals are orthogonal also. The basis functions are:

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_s}} \cos(2\pi f_i t) & 0 \le t \le T_s \\ 0 & \text{elsewhere} \end{cases}$$

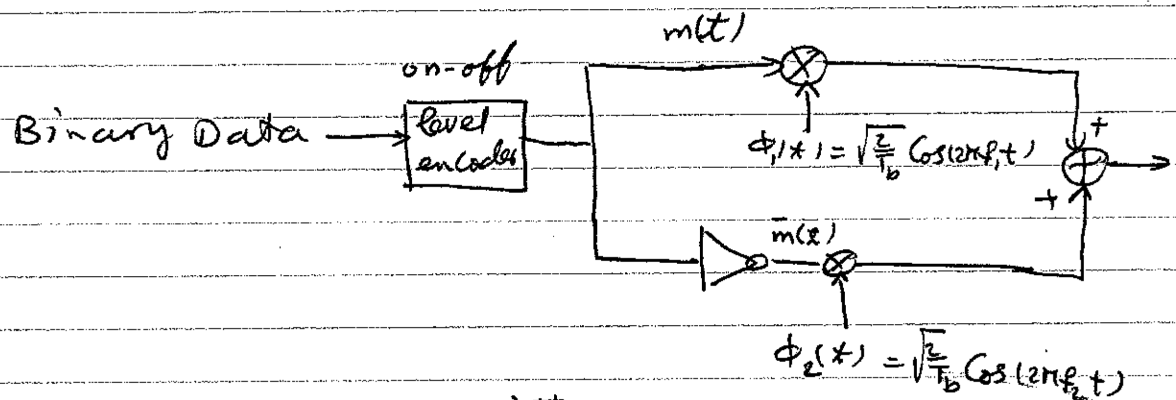
$$\underline{s}_1 = \begin{bmatrix} \sqrt{E_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \underline{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \cdots \quad \underline{s}_M = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \sqrt{E_s} \end{bmatrix}$$

For the BFSK

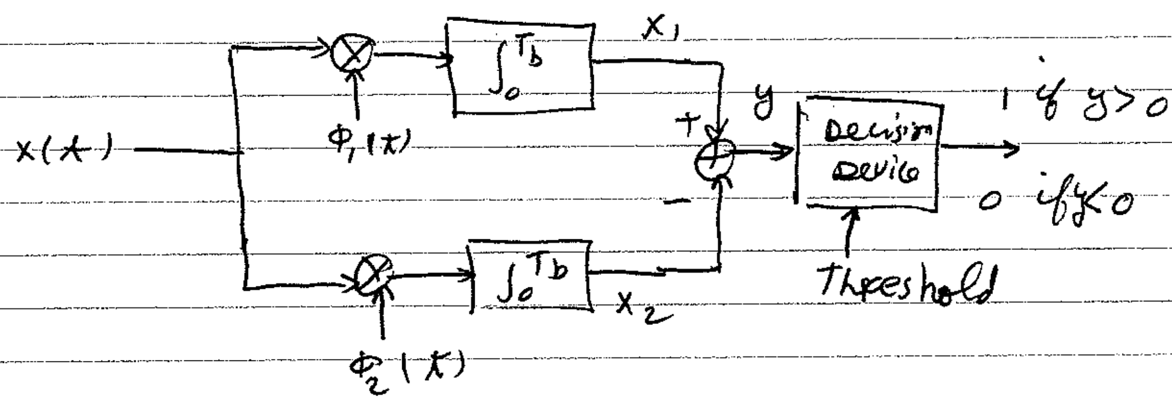
$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases}$$

$i = 1, 2$.

Transmitter and Receiver for (Coherent) BPSK:



Transmitter



Receiver

Probability of error

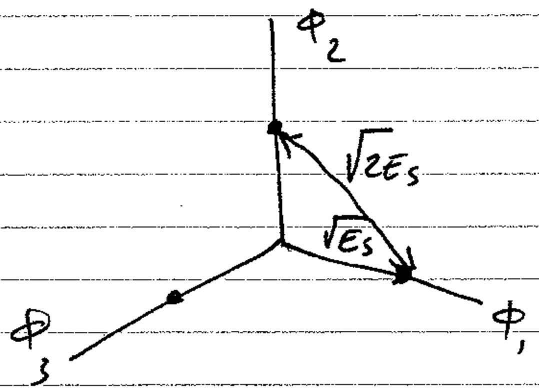
$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \text{ for BFSK}$$

Comparison with BPSK

Hilroy

Mary FSK

Probability of error:



$$P_e \leq (M-1)\, Q\left(\frac{d}{\sqrt{2N_0}}\right) = (M-1)\, Q\left(\frac{\sqrt{2E_s}}{\sqrt{2N_0}}\right) = M-1 \,.\, Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

But $E_s = E_b \log_2 M$

So:

$$P_e \leq (M-1)\, Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

This is the probability of symbol error.

To find the probability of bit error, note that the probability of bit error given a symbol error is $\frac{M/2}{M-1}$ so

$$P_B \leq \frac{M}{2} Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

BER improves with increase in M.

Bandwidth efficiency of M-ary FSK

for ~~continuous phase~~ orthogonal FSK:

$$BW = \frac{M}{2T_s}$$

(for non-coherent $BW = \frac{M}{T_s}$)

$T_s = T_b \log_2 M$

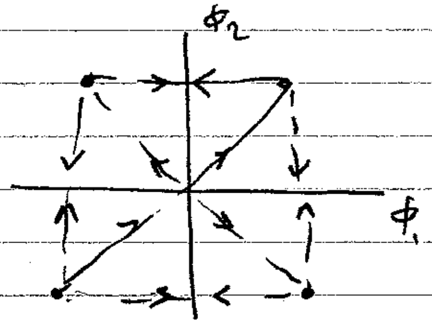
So

$$BW = \frac{M}{2T_b \log_2 M} = \frac{M R_b}{2 \log_2 M}$$

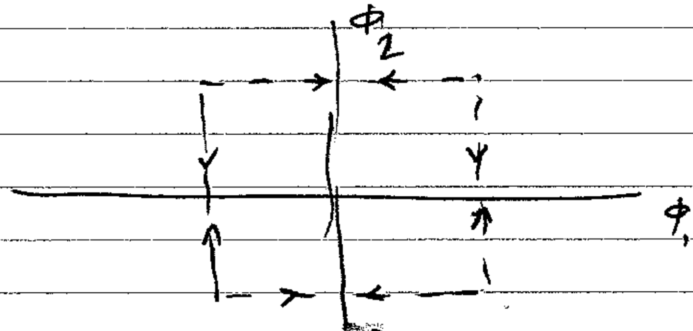
$$\eta = \frac{R_b}{BW} = \left(\frac{M}{2 \log_2 M}\right)^{-1}$$

the Bandwidth efficiency deteriorates as M increases.

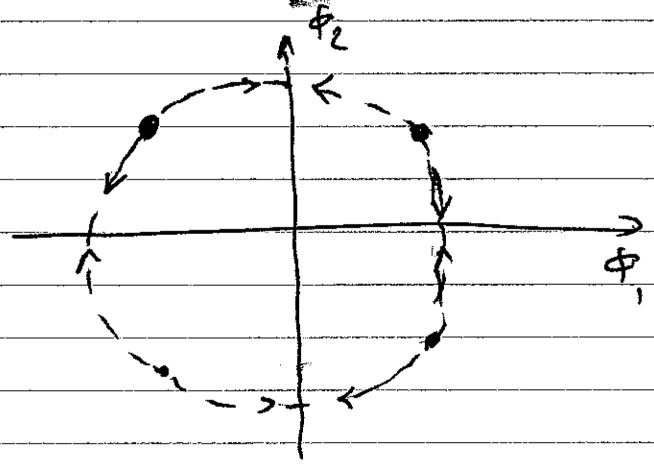
Minimum (Frequency) Shift Keying (MSK)



QPSK
phase-trajectory



O-QPSK
phase Trajectory



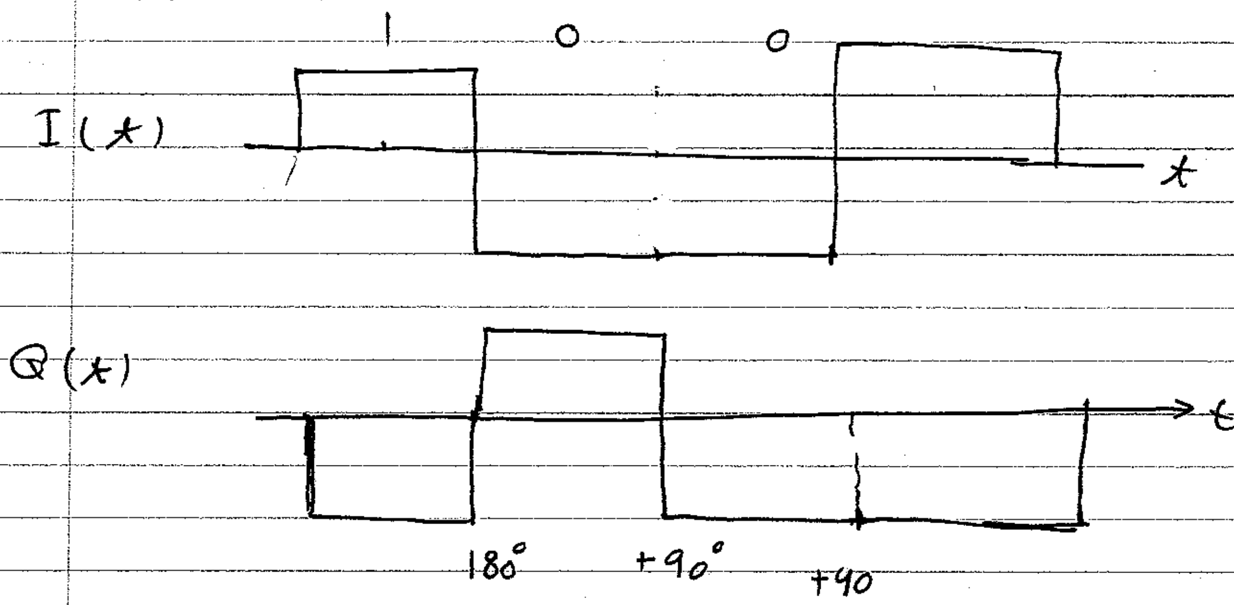
MSK
phase-trajector

example

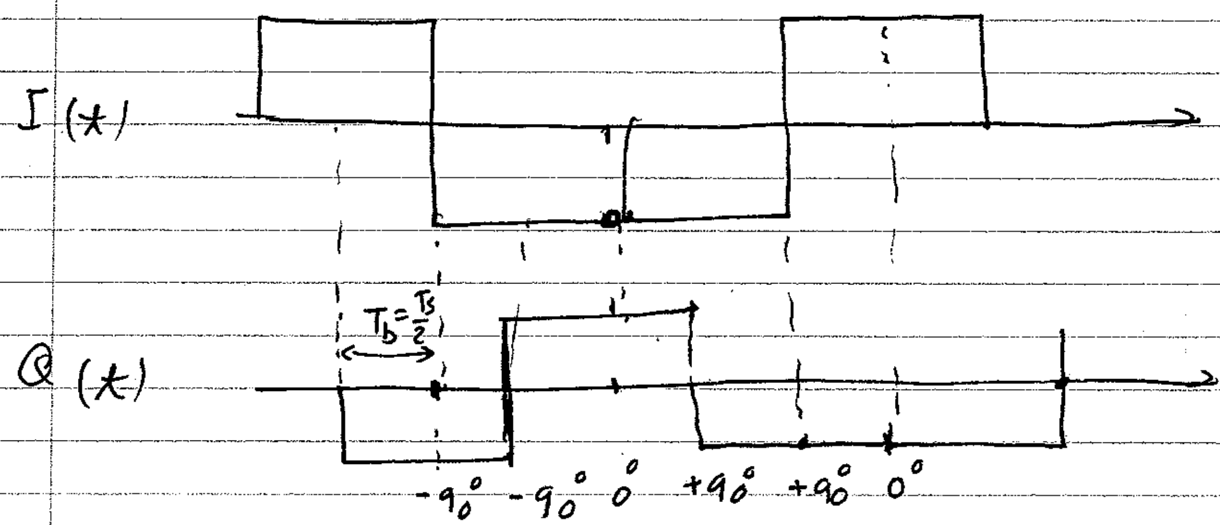
1 0 0 1 0 0 1 0

I	1	0	0	1
Q	0	1	0	0

For QPSK



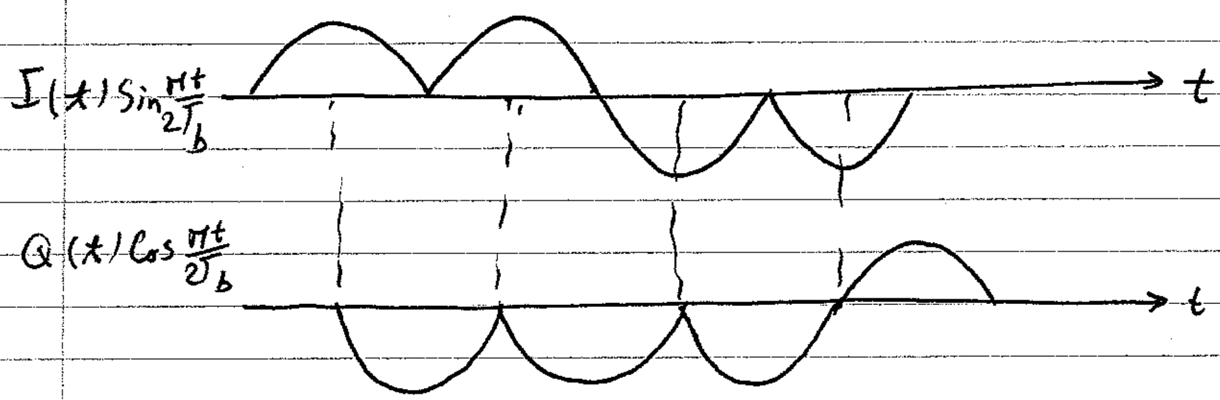
For OQPSK



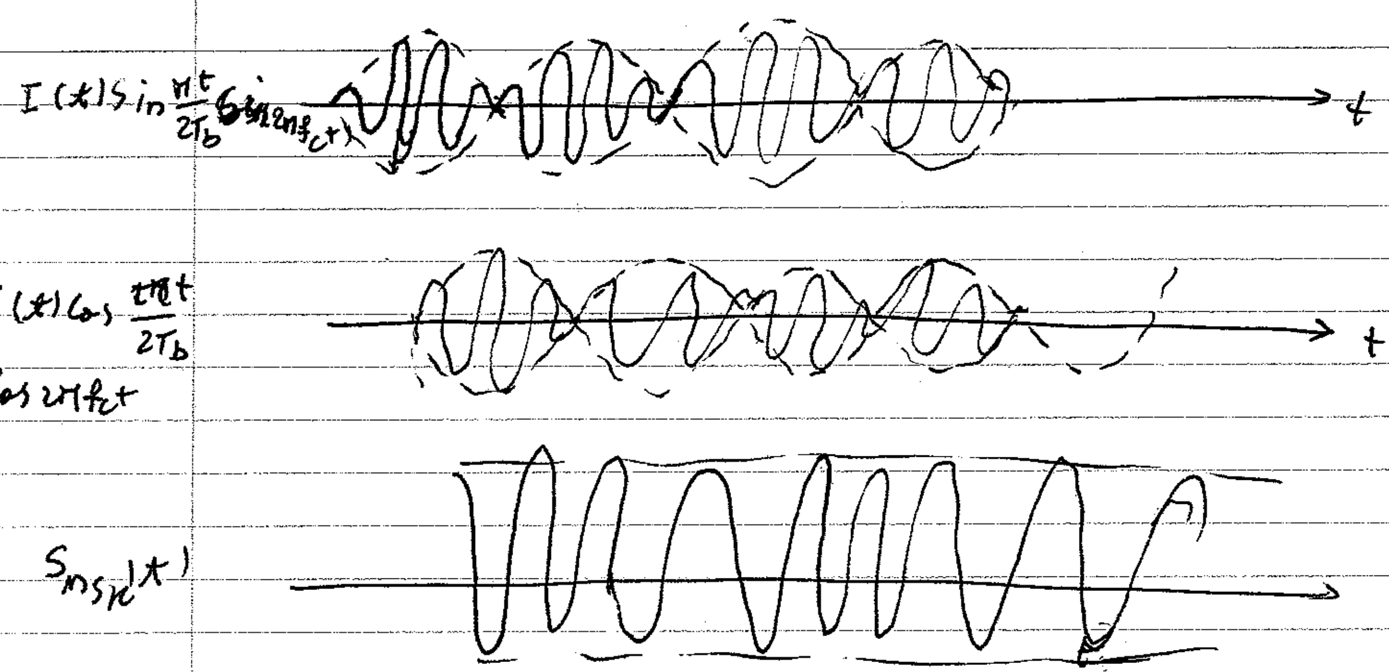
For MSK

we multiply $I(t)$ of OQPSK by $\sin\left(\frac{\pi t}{2T_b}\right) = \sin\left(\frac{\pi t}{T_s}\right)$

and multiply $Q(t)$ by $\cos\left(\frac{\pi t}{2T_b}\right)$



then modulate these



$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ the same as BPSK or QPSK.

Non- Coherent Communication

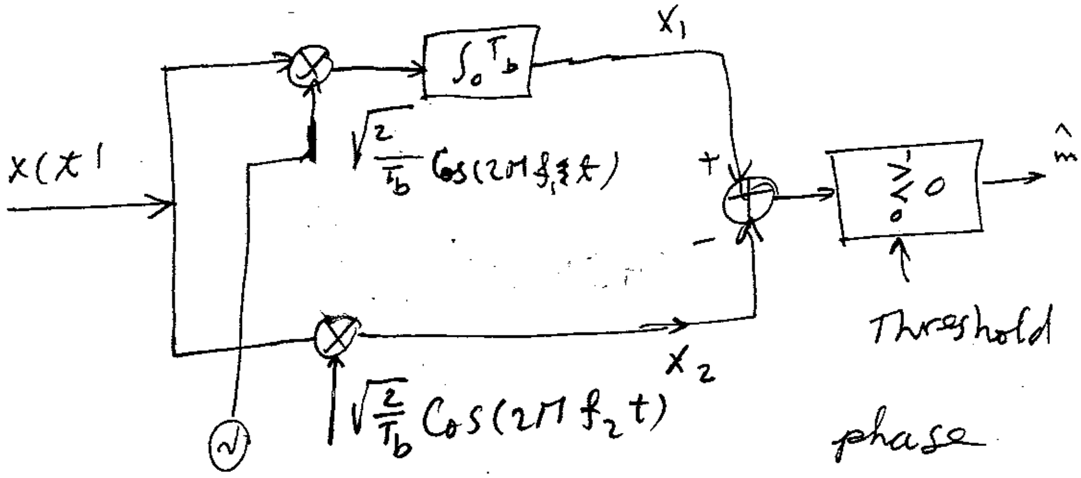
Non- Coherent BFSK

We have

$$S_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases}$$

for $i = 1, 2$

For Coherent receiver



Say, the receiver reference phase is off by φ

then

$$X_1 = \frac{2}{T_b}\sqrt{E_b}\int_0^{T_b} \cos(2\pi f_1 t)\cos(2\pi f_1 t + \varphi)\, dt$$

$$= \frac{2}{2T_b}\sqrt{E_b}\int_0^{T_b} \cos(\varphi)\, dt + \frac{2}{2T_b}\sqrt{E_b}\int_0^{T_b} \cos(2\pi(2f_1)t)\, dt \rightarrow 0$$

$$= \sqrt{E_b}\cos\varphi$$

and

$$X_2 = \int_0^{T_b} \frac{2}{T_b}\sqrt{E_b}\,\cos(2\pi f_2 t+\varphi)\cos(2\pi f_1 t)\,dt$$

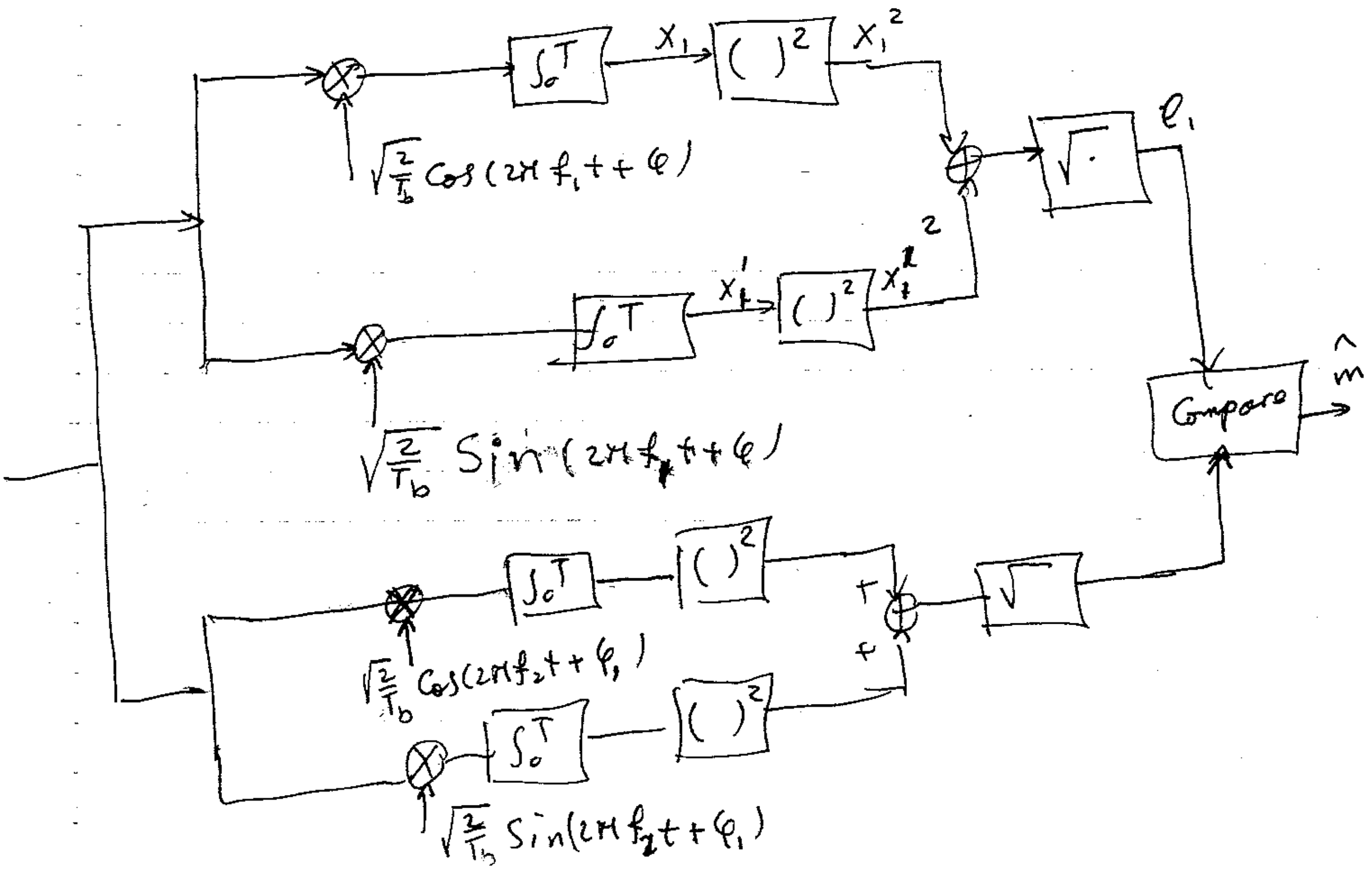
$$X_2 = \frac{\sqrt{E_b}}{T_b}\int_0^{T_b}\cos(2\pi(f_2-f_1)t+\varphi)\,dt + \frac{\sqrt{E_b}}{T_b}\int_0^{T_b}\cos(2\pi(f_2+f_1)t+\varphi)\,dt \rightarrow 0$$

$X_2 = \sqrt{E_b}\,\sin\varphi$ (using the fact that $f_2 - f_1 = \frac{1}{T_b}$)

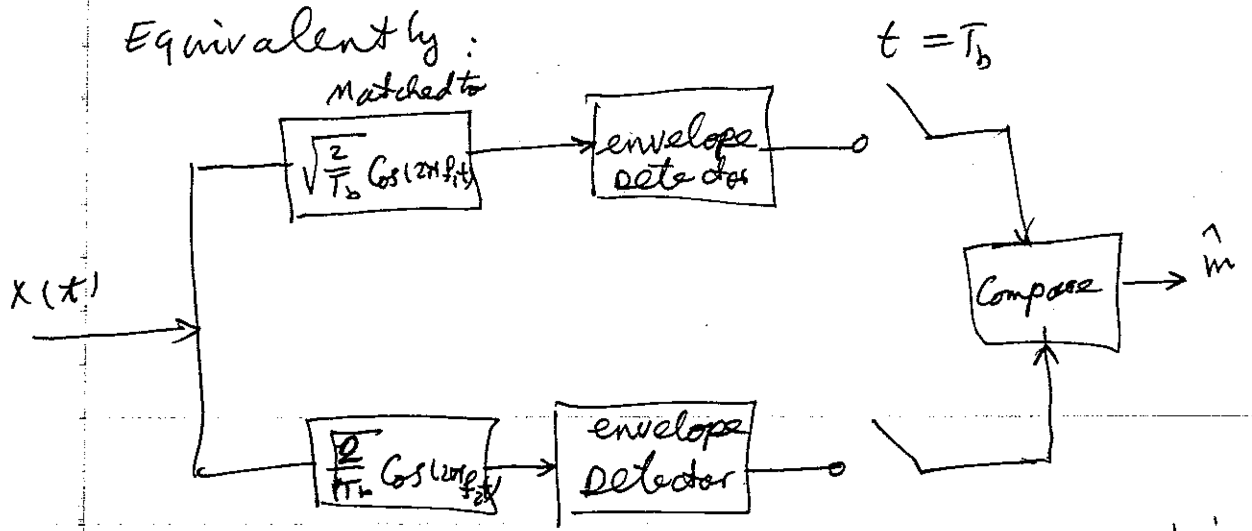
If φ is large X_1 can become smaller than X_2 resulting in erroneous decision.

Solution

use $\sqrt{X_1^2 + X_1'^2}$ as the decision parameter



Equivalently:



Bit error rate for non-coherent binary FSK

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$

~~(Not much worse than coherent)~~

for coherent case, we had:

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \approx \frac{1}{2\sqrt{\pi}}\exp\left(-\frac{E_b}{2N_0}\right)$$

Differential Phase Shift Keying (DPSK)

Instead of comparing against a reference phase, we compare with the signal in the previous signalling interval:

Assume the previous bit was "1" so:

$$s(t) = \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) \qquad 0 \leq t \leq T_b$$

Now if the present bit is "1", we have

$$s^n(t) = \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) \qquad T_b \leq t \leq 2T_b$$

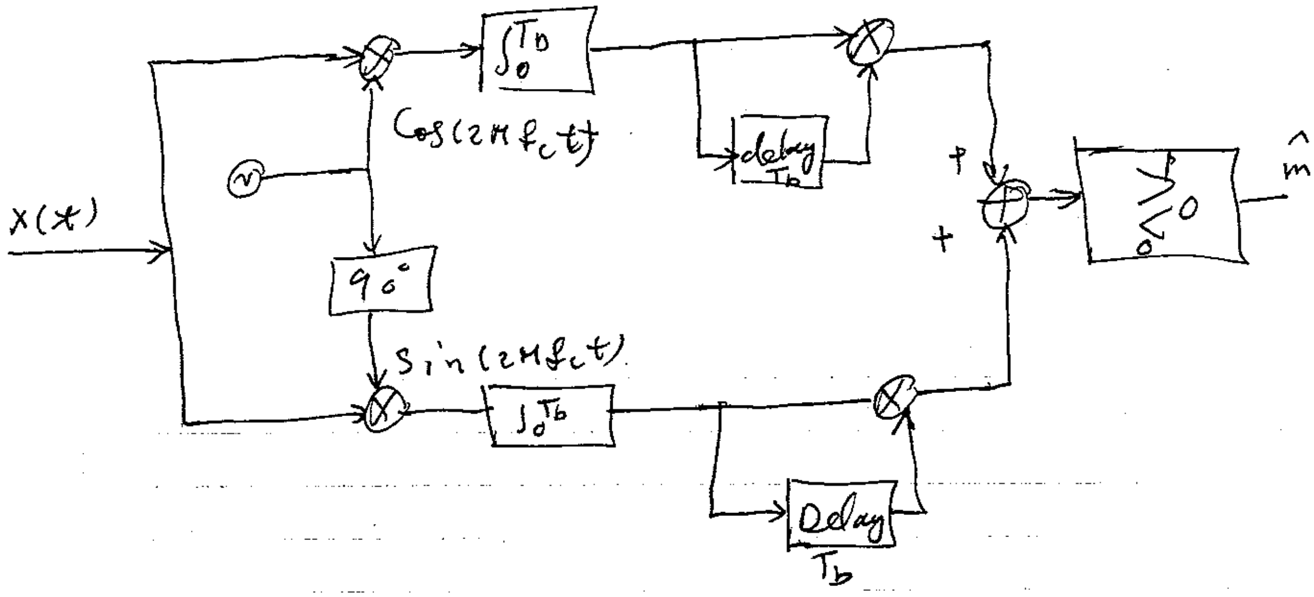
and if the present bit is "0", we have

$$s^n(t) = \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t)$$

So if, we correlate $s^n(t)$ (or noisy version of it $x^n(t)$) with the previously received signal, we get:

$$\frac{E_b}{2T_b}\int_0^{T_b} \cos^2(2\pi f_c t)\, dt = \begin{matrix} E_b & \text{if "1"} \\ -E_b & \text{if "0"} \end{matrix}$$

Alternatively, we can find the phase at time n w.r.t. a reference ~~phase~~ carrier and compare with that of the previous epoch.



For this ~~mod~~ demodulation scheme

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$$

~~But~~ for coherent PSK, we had

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \approx \frac{1}{\sqrt{\pi}\,2} \exp\left(-\frac{E_b}{N_0}\right)$$

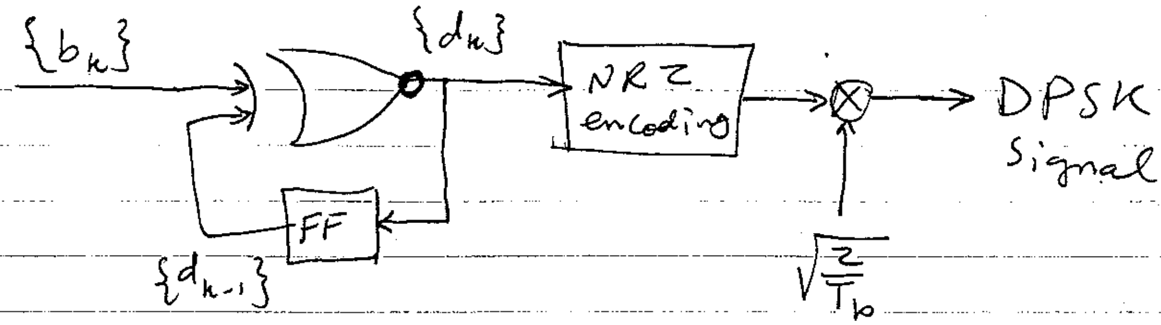
Differential encoding:

b_k → D.E. → d_k

if b_k is 1 d_k remains unchanged

if b_k is 0 d_k is inverted

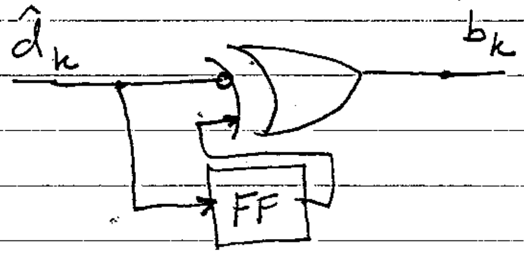
$$d_k = \overline{b_k \oplus d_{k-1}}$$



$\{b_k\}$		1	0	0	1	0	0	1	1	
$\{d_{k}\}$	1	1	0	1	1	0	1	1	1	
φ_n = phase	0	0	π	0	0	π	0	0	0	

At the receiver (for Coherent DEPSK).

$\hat{b}_k = \bar{\hat{d}}_k \oplus \hat{d}_{k-1}$



Summary (Comparison)

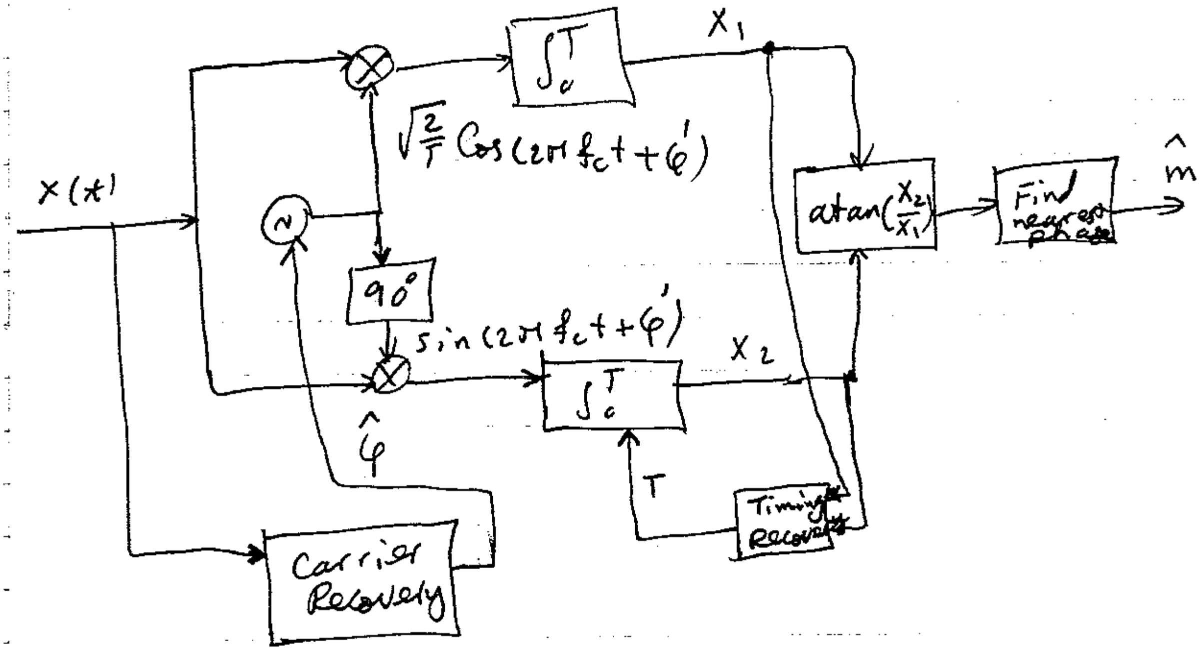
a) Coherent PSK
" QPSK $\qquad Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$

b) Coherent BFSK $\qquad Q\left(\sqrt{\frac{E_b}{N_o}}\right)$

c) DPSK $\qquad \frac{1}{2}\exp\left(-\frac{2E_b}{N_o}\right)$

d) Non-coherent BFSK $\qquad \frac{1}{2}\exp\left(-\frac{E_b}{N_o}\right)$

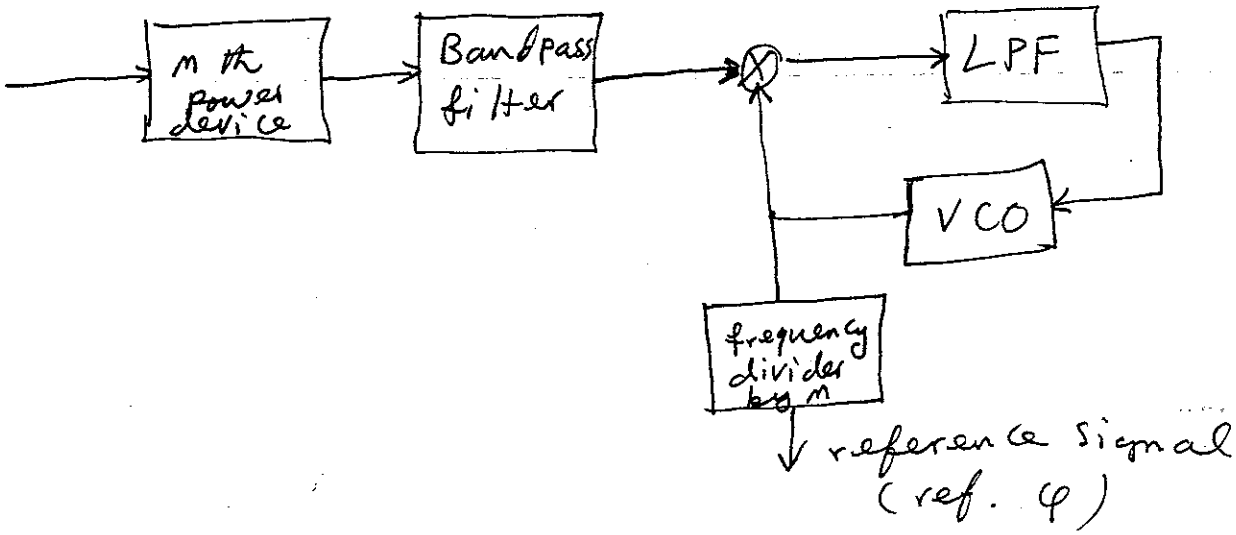
Synchronization:



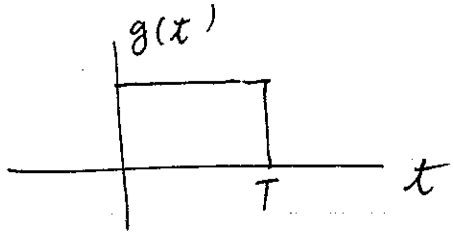
$x(t) = A\cos(2\pi f_c t + \varphi + \varphi_{data})$ + noise

$\varphi_{data} = 0, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}.$ (e.g., QPSK)

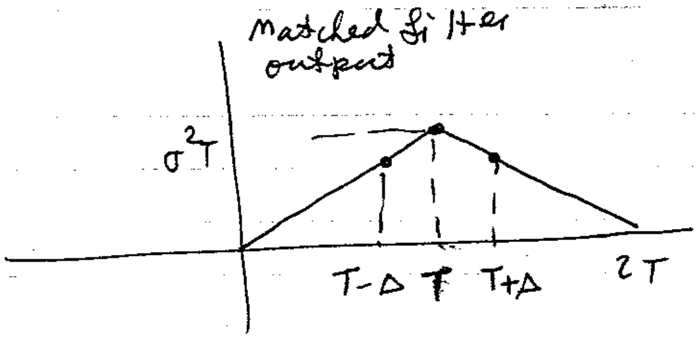
Carrier Recovery (Phase Estimation)

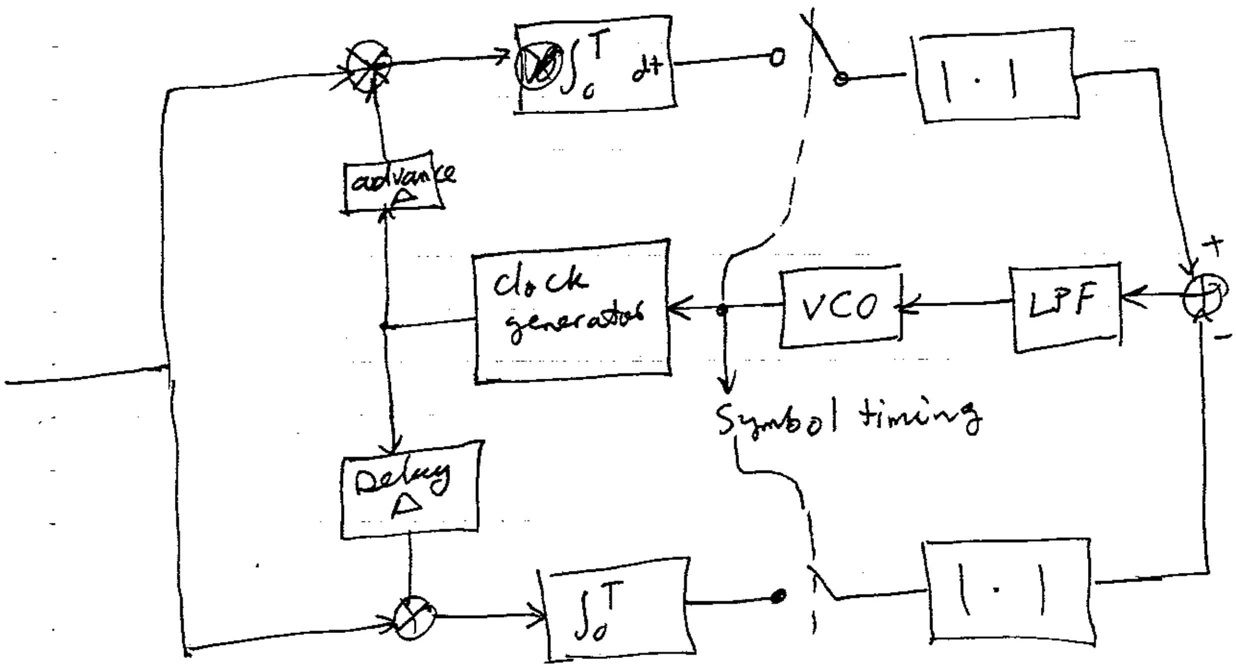


Timing Recovery (Bit Timing / Symbol Timing Estimation)



Matched filter
$g(t) = h(T-t)$





Early-late gate

TABLE 9.1 Summary of formulas for the bit error probability P_e for different data transmission systems

	P_e
Coherent PSK	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
Coherent FSK (with 1-bit decoding)	$Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
MSK	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
QPSK	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
Noncoherent FSK	$\frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$
DPSK	$\frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$

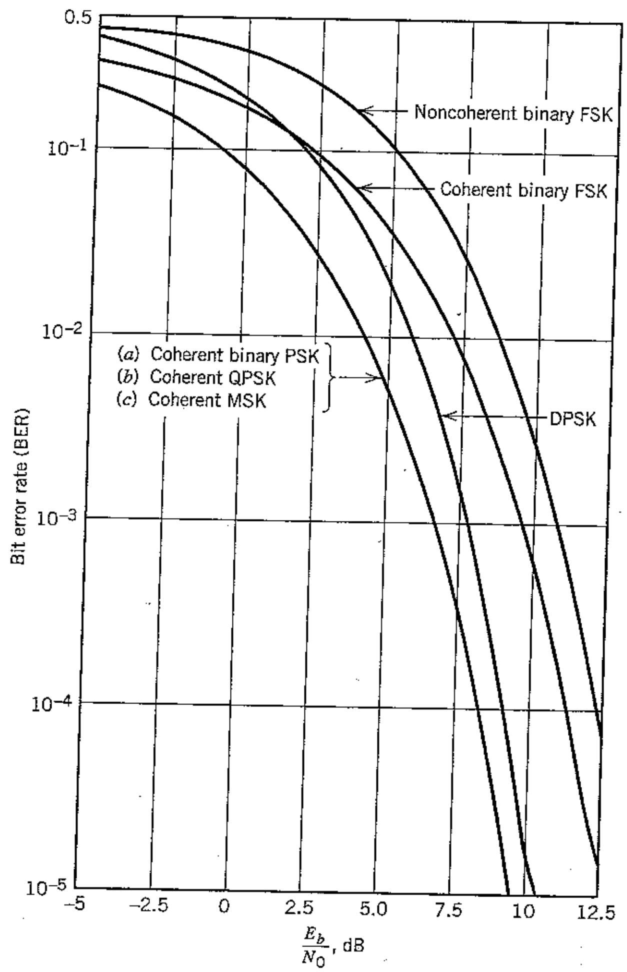


FIGURE 9.20 Comparison of the noise performances of different PSK and FSK systems.