Multiple-Input-Multiple-Output (MIMO)

We have already talked about antenna diversity at the receiver, i.e., the case when there are N_R receiver chains (antenna, down converter, demodulator, etc.), but one transmitter. This is called a Single-Input-Multiple-Output (SIMO) system.

More recently systems with transmit diversity have been suggested and used. These include Multiple-Input-Single-Output (MISO) and, more generally, Multiple-Input-Multiple-Output (MIMO).

In a MIMO system, there are N_T transmit antennas and N_R receive antennas. If we denote the impulse response between j-th transmitter and i-th receiver by $h_{ij}(\tau; t)$ where τ is the delay variable and t is the time variable, the channel will be characterised by an $N_R \times N_T$ matrix:

$$H(\tau;t) = \begin{bmatrix} h_{11}(\tau;t) & h_{12}(\tau;t) & \cdots & h_{1,N_T}(\tau;t) \\ h_{21}(\tau;t) & h_{22}(\tau;t) & \cdots & h_{2,N_T}(\tau;t) \\ \vdots & \vdots & & \vdots \\ h_{N_R,1}(\tau;t) & h_{N_R,2}(\tau;t) & & h_{N_R,N_T}(\tau;t) \end{bmatrix}$$

The signal received at i-th antenna is:

$$r_i(t) = \sum_{j=1}^{N_T} \int_{-\infty}^{\infty} h_{ij}(\tau;t)\, s_j(t-\tau)\, d\tau$$

$$= \sum_{j=1}^{N_T} h_{ij}(\tau;t) * s_j(\tau) \qquad i=1,2,\cdots,N_R$$

or

$$\underline{r}(t) = H(\tau;t) * s(\tau)$$

For a flat (frequency-nonselective) channel

$$H(t) = \begin{bmatrix} h_{11}(t) & h_{12}(t) & \cdots & h_{1,N_T}(t) \\ h_{21}(t) & h_{22}(t) & \cdots & h_{2,N_T}(t) \\ \vdots & \vdots & & \vdots \\ h_{N_R,1}(t) & h_{N_R,2}(t) & \cdots & h_{N_R,N_T}(t) \end{bmatrix}$$

In this case:

$$r_i(t) = \sum_{j=1}^{N_T} h_{ij}(t)\, s_j(t) \qquad i=1,2,\cdots,N_T$$

or

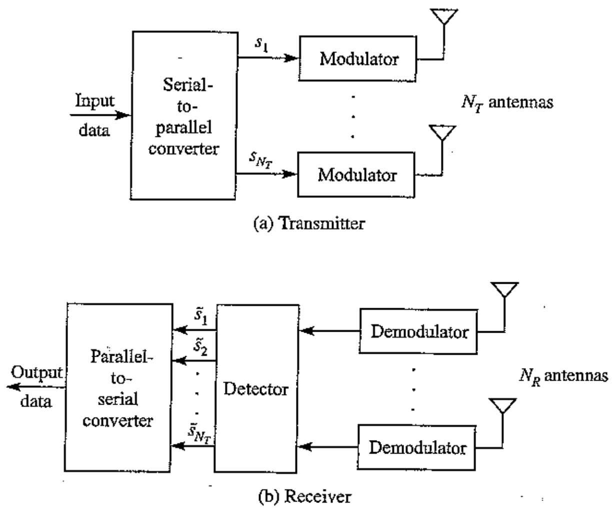
$$\underline{r}(t) = H(t)\, \underline{s}(t)$$

Furthermore, if the channel is slow fading, i.e., the impulse responses can be considered constant over a given time interval, say, $0 \leq t \leq T$:

$$\underline{r}(t) = \underline{H}\, s(t) \qquad 0 \leq t \leq T$$

Signal transmission in MIMO

The input stream is divided into N_T sub-streams (demultiplexed) and each sent over one antenna.



A communication system with multiple transmitting and receiving antennas.

The received signal at the i-th antenna is:

$$r_i(t) = \sum_{j=1}^{N_T} s_j h_{ij}\, g(t) + z_i(t) \qquad 0 \leq t \leq T, \quad i = 1, 2, \cdots, N_R$$

After demodulation (match filtering):

$$y_i = \sum_{j=1}^{N_T} s_j h_{ij} + \eta_i \qquad i = 1, 2, \cdots, N_R .$$

or in matrix form

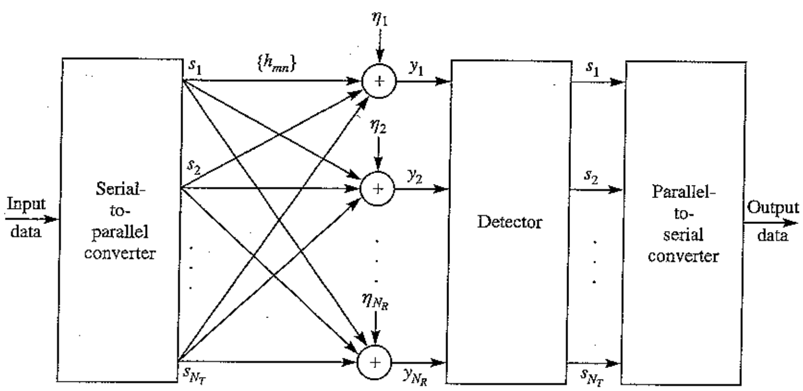
$$\underline{y} = \underline{H}\,\underline{s} + \underline{\eta}$$

where $\underline{y} = [y_1, y_2, \cdots, y_{N_R}]^T$

$\underline{s} = [s_1, s_2, \cdots s_{N_T}]^T$

and $\eta = [\eta_1, \eta_2, \cdots, \eta_{N_R}]^T$

$\underline{H}$ is the $N_R \times N_T$ matrix of channel gains.



Discrete-time model of the communication system with multiple transmit and receive antennas in a frequency-nonselective slow fading channel.

Detection of MIMO Signals:

1) Maximum-Likelihood Detector (MLD).

2) Linear Receivers:

- MMSE Detector.

- Inverse Channel Detector (ICD)

3) Non-linear detectors: Successive Cancellation.

Maximum-Likelihood Detector (MLD):

This is the optimum receiver and for a Gaussian MIMO channel, it means that we have to find $\hat{\underline{s}}$ that minimizes:

$$\mu(\underline{s}) = \sum_{i=1}^{N_R} \left| y_i - \sum_{j=1}^{N_T} h_{ij}\, s_j \right|^2$$

The complexity of MLD is high: grows exponetially with the number of transmit antennas (M^{N_T}).

Minimum Mean Sequare-Error (MMSE) detector

Here, we linearly combine the outputs of matched filters to get an estimate of the transmitted signal and try to minimize the mean-square-error between this estimate and the transmitted signal.

That is, we form:

$$\underline{\hat{s}} = W^H \underline{y}$$

where W^H is the conjugate transpose of the matrix W : an $N_R \times N_T$ weight matrix.

The objective is to minimize:

$$J(\underline{W}) = E[\|e\|^2] = E[\|\underline{s} - W^H \underline{y}\|^2]$$

This minimization results in:

$$\underline{w}_n = R_{yy}^{-1} r_{s_n}$$

where $\underline{R}_{yy} = E[\underline{y}\,\underline{y}^H] = H \underline{R}_{ss} H^H + N_o I$

is the N_R by N_R autocorrelation matrix of the received signal $\underline{y}$,

$R_{ss} = E[\underline{s}\,\underline{s}^H]$, $r_{s_n y} = E[s_n^* \underline{y}]$ and

$E[\eta \eta^H] = N_o I$.

When, the signal vector has un-correlated zero-mean components R_{ss} is a diagonal matrix.

Each component of $\hat{\underline{s}}$ is quantized to the nearest symbol value.

Inverse Channel Detector (ICD).

This is a zero-forcing approach. Here also an estimate of $\underline{s}$ is formed by linearly combining the received signals. When the number of transmit and receive antennas are equal ($N_R = N_T$), we set $W^H = H^{-1}$ so that the inter-channel interference is completely eliminated:

$$\hat{\underline{s}} = H^{-1}\underline{y} = s + H^{-1}\eta .$$

When $N_R > N_T$, we have

$$W^H = (H^H H)^{-1} H^H$$

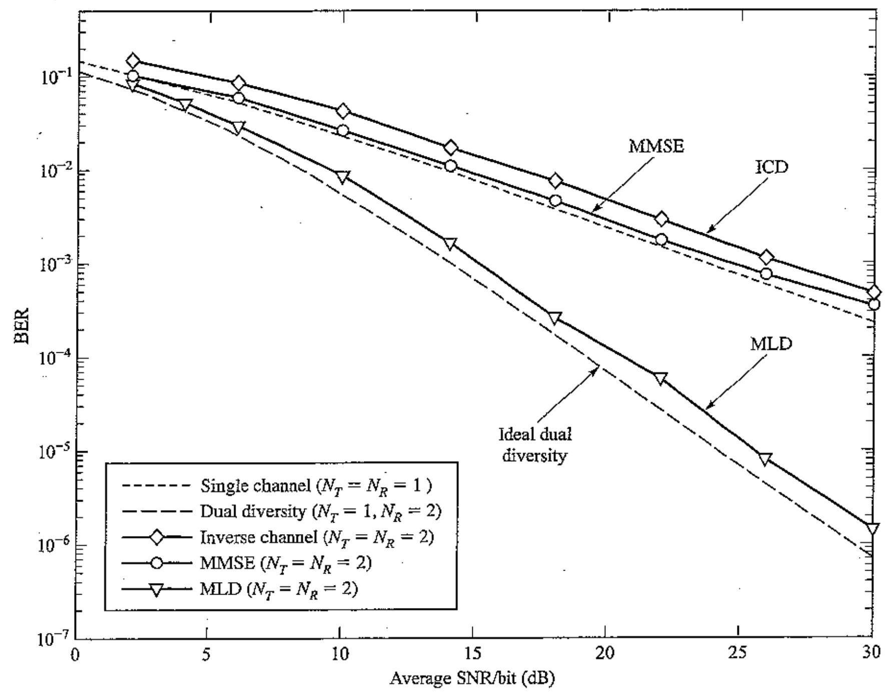
That is, W is the pseudo-inverse of the channel matrix H.

Following figures show the performance of MLD and MMSE (and ICD) for ($N_R = 2$, $N_T = 2$) and ($N_T = 2$, $N_R = 3$). It can be seen that using MMSE (or ICD) one degree of diversity is lost, i.e., $N_R = N_T = 2$ with MMSE has the diversity of $N_R = 1, N_T = 1$, and ~~with~~ for $N_T = 2$, $N_R = 3$ one gets the diversity of $(1, 2)$, i.e., the diversity of $N_R - 1$.

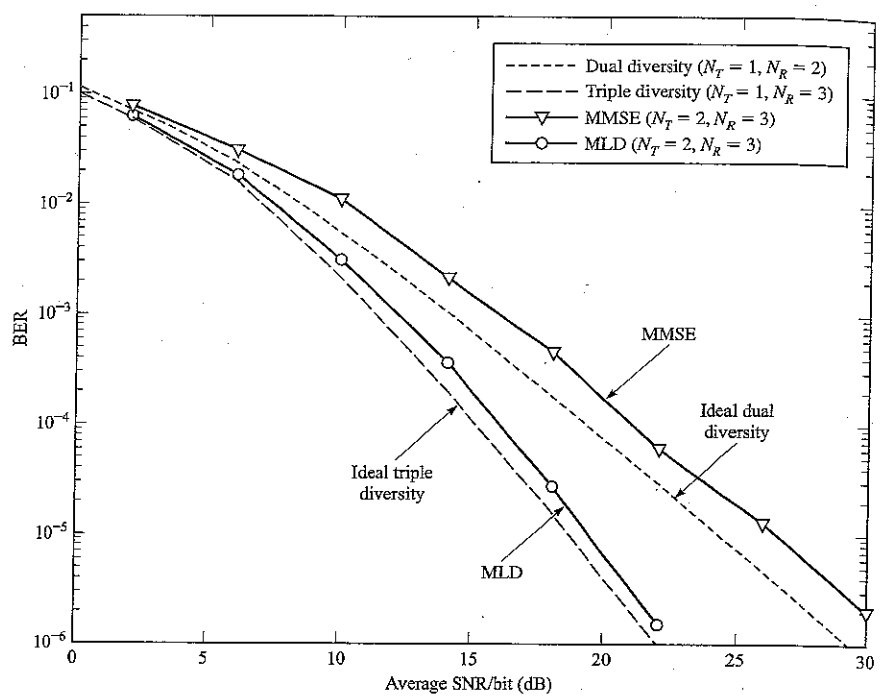
With MLD, one has the same diversity as $(1, N_R)$.

In general for an (N_T, N_R) system, MLD achieves order N_R diversity while MMSE achieves and order of $N_R - N_T + 1$ diversity when $N_R \geq N_T$. The reason is that a linear detector uses $N_T - 1$ degree of freedom to cancel interference from $N_T - 1$ interfering transmit antennas and, therefor is left with $N_R - (N_T - 1) = N_R - N_T + 1$ degrees of freedom.

Complexity: MLD has a complexity of M^{N_T}, while The lineare dectors' complexity grows ~~with~~ as N_T and N_R. For small N_T and M, say $N_T \leq 4$ and $M = 4$, MLD is feasible.



Performance of MLD, MMSE, and inverse channel detectors with $N_R = 2$ receiving antennas.



Performance of MLD and MMSE detectors with $N_R = 3$ receiving antennas.

MIMO signaling when the channel is known at the transmitter

If $\underline{H}$ is known at the transmitter, we use Singular Value decomposition (SVD) to write H as:

$$H = U \Sigma V^H .$$

where U is an N_R by R matrix, V is and $N_T \times R$ matrix and Σ is an $R \times R$ diagonal matrix and U and V are <u>orthonormal</u>, i.e., $U^H U = I_R$ and $V^H V = I_R$.

At the transmitter the signal $\underline{s}$ is linearly transformed as:

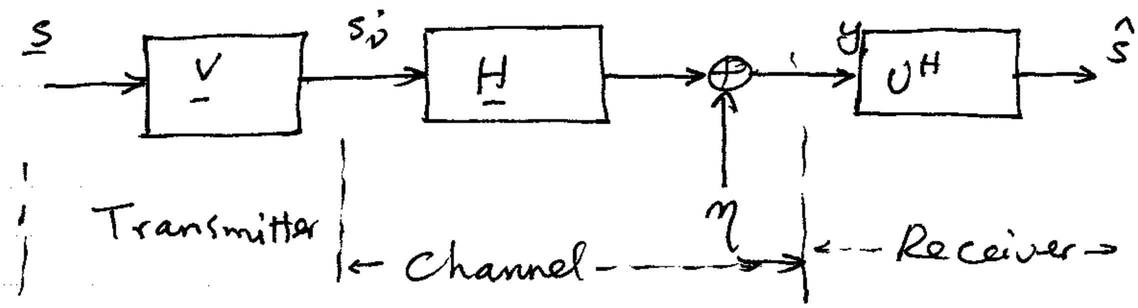
$$s_v = V \underline{s}$$

Then the received signal will be:

$$\underline{y} = H s_v + \eta = H V \underline{s} + \eta$$

At the receiver, we use the linear transformation U^H, i.e.

$$\hat{\underline{s}} = U^H \underline{y} = U^H H V \underline{s} + U^H \eta$$
$$= U^H (U \Sigma V^H) V \underline{s} + U^H \eta = \Sigma s + U^H \eta .$$



It is seen that the elements of received signal ~~have~~ become decorrelated and can be detected individually.

Capacity of MIMO channels:

H can be written as

$$H = U \Sigma V^H$$

where U is $N_R \times R$, Σ is $R \times R$ and $\underline{V}$ is $N_T \times R$ matrix. R is the rand of matrix H. U and V are orthonormal. Σ has diagonal values $\sigma_1, \sigma_2, \cdots \sigma_R$. These are called singular values of H (in case of a full-rank square matrix eigen-values). H can be written as

$$H = \sum_{i=1}^{R} \sigma_i u_i v_i^H$$

where $\{u_i\}$ and $\{v_i\}$ are columns ~~of~~ vectors of

$\underline{U}$ and $\underline{V}$, respectively,

HH^H is an $N_R \times N_R$ matrix and can be written as:

$$HH^H = Q \Lambda Q^H.$$

where $Q^H Q = I_{N_R}$ and Λ is and $N_R \times N_R$ diagonal matrix with elements λ_i, $i = 1, \cdots, N_R$.

λ_i's are the eigenvalues of HH^H, given by

$$\lambda_i = \begin{cases} \sigma_i^2 & i = 1, 2, \cdots, R \\ 0 & i = R+1, \cdots, N_R \end{cases}$$

where σ_i are singular values of H.

The $\underline{\text{Frobenius norm}}$ of H is defined as,

$$\|H\|_F = \sqrt{\sum_{i=1}^{N_R} \sum_{j=1}^{N_T} |h_{ij}|^2}$$

$$= \sqrt{\text{trace}(HH^H)}$$

$$= \sqrt{\sum_{i=1}^{N_R} \lambda_i}$$

When $\{h_{ij}\}$ are i.i.d. with unit variance $\|H\|_F^2$ is a chi-square variable with $2N_T N_R$ degrees of freedom with distribution

$$p(x) = \frac{x^{n-1}}{(n-1)!} e^{-x} \qquad x \geq 0$$

Capacity of flat Deterministic MIMO channel

Consider an AWGN channel with matrix H.

Then

$$\underline{y} = H\underline{s} + \eta$$

capacity is defined as:

$$C = \max_{p(\underline{s})} I(\underline{y}; \underline{s})$$

It can be shown (Telatar 1999) that:

$$\underline{C} = \max_{\text{trace}(R_{ss}) = E_s} \log_2 \det \left(I_{N_R} + \frac{1}{N_0} H R_{ss} H^H\right)$$

bps/Hz.

When the signals on N_T transmit antennas are statistically independent with energy $\frac{E_s}{N_T}$, we have

$$R_{ss} = \frac{E_s}{N_T} I_{N_T}$$

and

$$C = \log_2 \det \left(I_{N_R} + \frac{E_s}{N_T N_0} H H^H\right), \quad \text{bps/Hz.}$$

letting $HH^H = Q \Lambda Q^H$, we have

$$\begin{aligned} C &= \log_2 \det \left(I_{N_R} + \frac{E_s}{N_T N_0} Q \Lambda Q^H\right) \\ &= \log_2 \det \left(I_{N_R} + \frac{E_s}{N_T N_0} Q Q^H \Lambda\right) \\ &= \log_2 \det \left(I_{N_R} + \frac{E_s}{N_T N_0} \Lambda\right) = \sum_{i=1}^{R} \log_2\left(1 + \frac{E_s}{N_T N_0} \lambda_i\right) \end{aligned}$$

For SISO, $R=1$ and $\lambda_1 = |h_{11}|^2$, so:

$$C_{SISO} = \log_2\left(1+\frac{E_s}{N_0}|h_{11}|^2\right).$$

For <u>SIMO</u> (single-input multiple-output),

$\underline{h} = [h_{11}, h_{21}, \dots h_{N_R,1}]^T$

$$\lambda_1 = \|h\|_F^2 = \sum_{i=1}^{N_R} |h_{i1}|^2$$

$$C_{SIMO} = \log_2\left(1+\frac{E_s}{N_0}\|h\|_F^2\right)$$

$$= \log_2\left(1+\frac{E_s}{N_0}\sum_{i=1}^{N_R}|h_{i1}|^2\right)$$

For a <u>MISO</u> Channel,

$\underline{h} = [h_{11}, h_{12}, \dots, h_{1,N_T}]^T$ and

$$\lambda_1 = \|h\|_F^2 = \sum_{i=1}^{N_T} |h_{1j}|^2$$

$$C_{MISO} = \log_2\left(1+\frac{E_s}{N_T N_0}\|h\|_F^2\right)$$

$$= \log_2\left(1+\frac{E_s}{N_T N_0}\sum_{j=1}^{N_T}|h_{1j}|^2\right)$$

It is seen from the above formulas that for the same Frobenius norm capacity of SIMO (receiver antenna diversity) is higher than that of MISO. The reason is that in MISO transmitted power is divided between N_T

antennas while in SIMO it is transmitted from a single antenna.

The ergodic capacity of flat (frequency-non-selective) MIMO channel

$$\bar{C}_{MIMO} = E\left[\log_2 \det\left[I_{N_R} + \frac{E_s}{N_{0i}} H R_{ss}^{*} H^{H}\right)\right]$$

where expectation is taken over all realization of H and R_{ss}^{*} is the autocorrelation matrix of the maximizing signal vector.

In the case of SIMO for any realization of the channel, we have

$$C_{SIMO} = \log_2\left(1 + \frac{E_s}{N_0}\sum_{i=1}^{N_R} |h_{i1}|^2\right)$$

$$= \log_2\left(1 + \frac{E_s}{N_0} x\right)$$

where $X = \sum_{i=1}^{N_R} |h_{i1}|^2$ is a χ^2 random variable with $2N_R$ degrees of freedom.

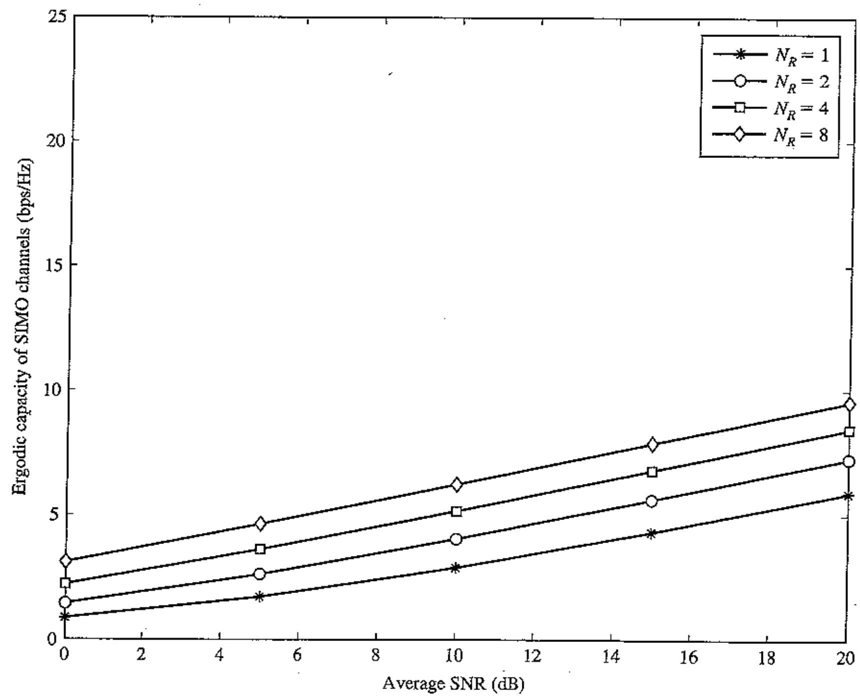
So,

$$\bar{C}_{SIM} = E\left[\log_2\left(1 + \frac{E_s}{N_0}\sum_{i=1}^{N_R}|h_{i1}|^2\right)\right.$$

$$= \int_0^{\infty} \log_2\left(1 + \frac{E_s}{N_0} x\right) p(x)\, dx \quad \text{bps/Hz.}$$

where $p(x)$ is the pdf of the variable X.

The figure below shows the capacity of SIMO for different SNR.



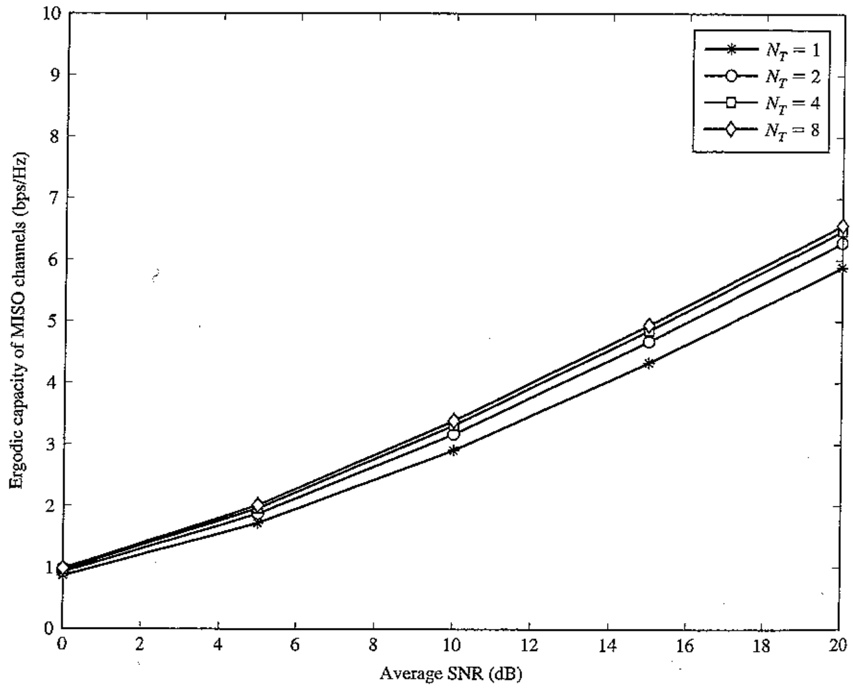
Ergodic capacity of SIMO channels.

<u>For MISO Channels</u>:

$$\bar{C}_{MISO} = E\left[\log_2\left(1 + \frac{E_s}{N_T N_0}\sum_{j=1}^{N_T}|h_{1j}|^2\right)\right]$$

$$= \int_0^\infty \log_2\left(1 + \frac{E_s}{N_T N_0}x\right)p(x)\,dx \quad \text{bps/Hz}.$$

The ergodic capacity MISO for $N_T = 2, 4$ and 8 is depicted in the following figure for different SNR.



Ergodic capacity of MISO channels.

<u>For the general case (MIMO):</u>

$$\bar{C}_{MIMO} = E\left[\sum_{i=1}^{R} \log_2\left(1+\frac{E_s}{N_0 N_T}\lambda_i\right)\right]$$

$$= \int_0^\infty \cdots \int_0^\infty \left[\sum_{i=1}^{R} \log_2\left(1+\frac{E_s}{N_0 N_T}\lambda_i\right)\right] p(\lambda_1, \cdots \lambda_R)\, d\lambda_1 \cdots d\lambda_R$$

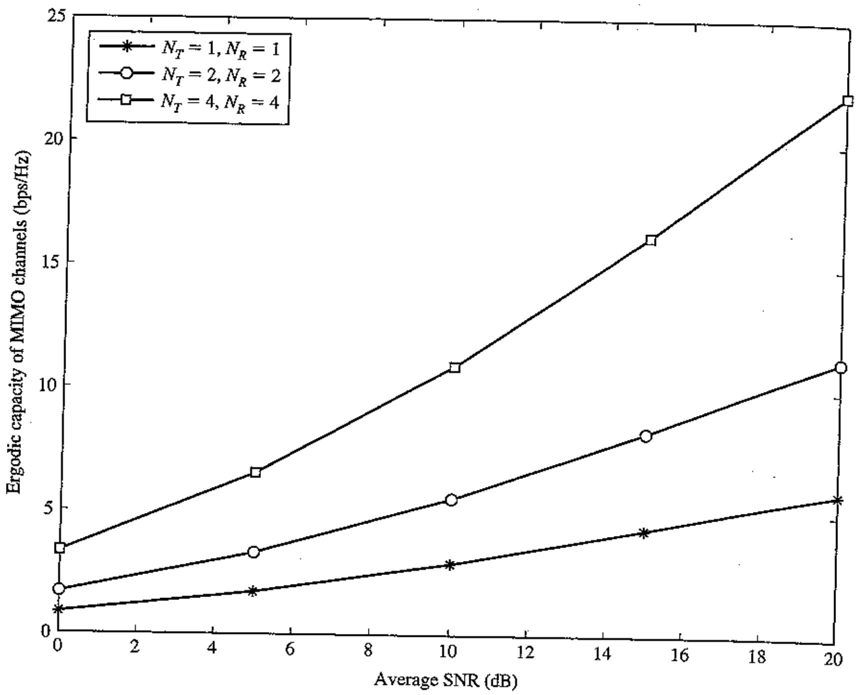
For the case where the elements of $\underline{H}$ are complex-valued zero-mean Gaussian with unit variance and $N_R = N_T = N$, the pdf of $\{\lambda_i\}$ is:

$$p(\lambda_1, \cdots \lambda_N) = \frac{(\pi/2)^{N(N-1)}}{[\Gamma_N(N)]^2} \exp\left[-\sum_{i=1}^{N}\lambda_i\right] \prod_{\substack{i,j \\ i<j}} (2\lambda_i - 2\lambda_j) \prod_{i=1}^{N} u(\lambda_i)$$

where

$$\Gamma_N(N) = \pi^{N(N-1)/2} \prod_{i=1}^{N} (N-i)!$$

Figure below shows the $\bar{C}_{MIMO}$ versus SNR for $N_R = N_T = 2$ and $N_R = N_T = 4$



Ergodic capacity of MIMO channels.

Outage Capacity of MIMO Channel:

When the channel is ergodic, the average capacity found above serves as a measure of the bit rate efficiency limit in the Shannon's sense. In the case of non-ergodic channel, we choose a realization and we have a capacity which may be lower or higher the ergodic (average) capacity. In such a case, ~~we~~ the use of <u>outage</u> concept is useful.

In this case we pick a given (acceptable in a sense) outage probability. Then,

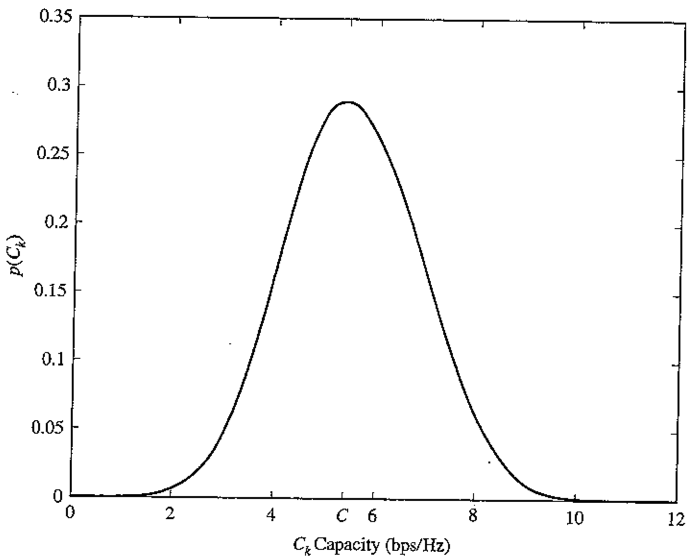
$$P(C \leq C_p) = P_{out}$$

gives use a numbe C_p which is a capacity that we ~~have~~ exceed $(1-P_{out})\%$ of the time. That is $P_{out}\%$ of the times we have to live with a channel with a capacity lower thant C_p. C_p is called the $P_{out}\%$ outage capacity of the channel.

For each channel realization, i.e., for any give $\underline{H}$, we have a capacity for a given SNR. This capacity when computed for various realizations of H, will have a CDF:

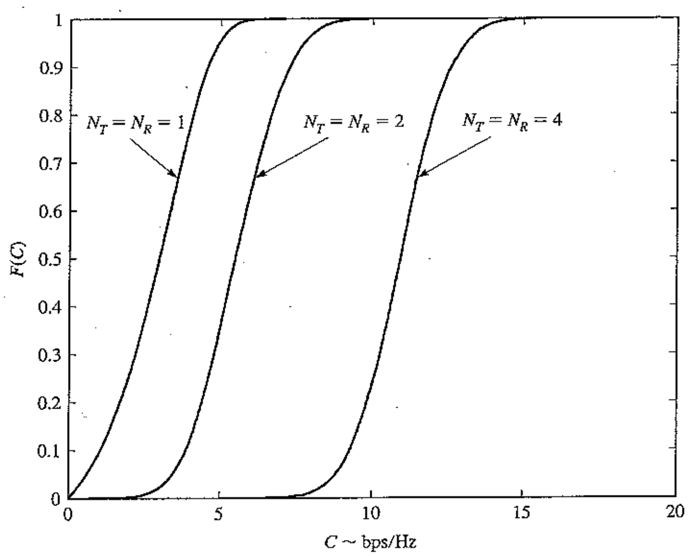
$$F(C) = P(C_k \leq C)$$

For example, for $N_T = N_R = 2$, the pdf of the capacity is given as:



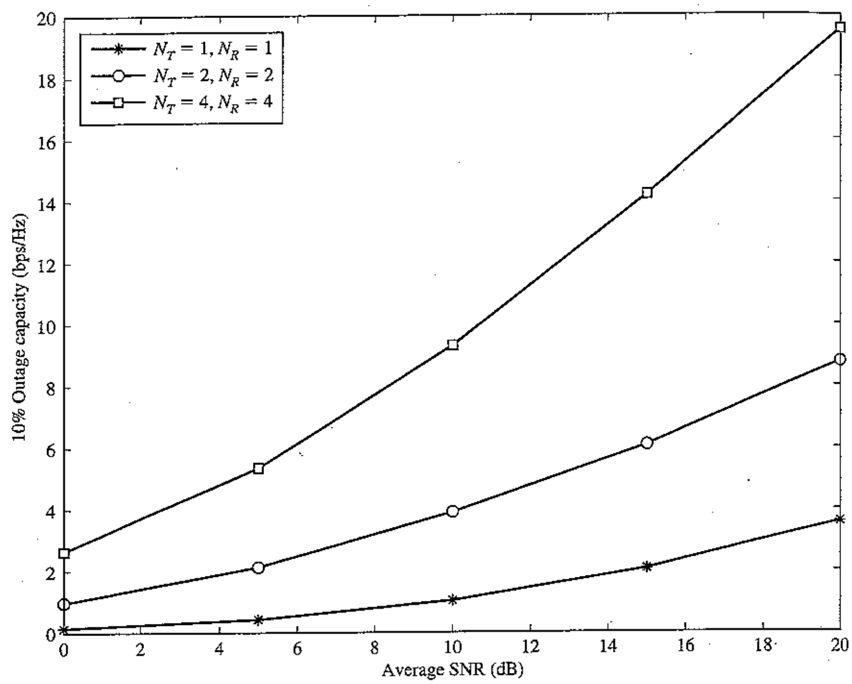
Probability density function of channel capacity for an $N_T = N_R = 2$ MIMO channel at SNR = 10 dB.

Given the probability densities (pdf) such as the one above, we can find the CDF. The figure below shows the CDF of C for $N_T = N_R = 2$ and $N_T = N_R = 4$.



CDF of MIMO channel capacity at SNR = 10 dB.

The CDF's like the one above give the outage capacity for a given SNR. The outage capacities for a given P_{out} can be collected for various SNR and ploted as below.



10% Outage capacity of MIMO channels.

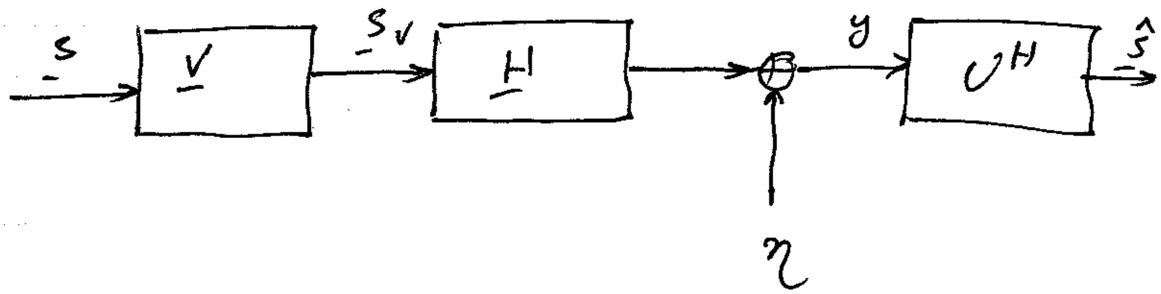
Capacity of MIMO when CSI is available at the transmitter:

In the above discussion, we ~~only~~ assumed that the channel's specif realization is only known at the receiver. In such a case, it is natural that the transmitter allocate the

Same power to the antenna elements.

In the case where the transmitter also knows the channel condition, it can better allocate its available power thus achieving a higher capacity.

As we saw earlier, in order to simplify detection, the transmitter transforms its signal vector $\underline{s}$ by multiplying it by matrix $\underline{V}$ and the receiver transforms its received signal by operating U^H on it.



Here $H = U \Sigma V^H$ where U and V are orthonormal matrices and Σ is a diagonal matrix with its diagonal elements being the singular values of H. We have:

$$E[s^H s] = \sum_{i=1}^{R} E[|s_i|^2] = \sum_{i=1}^{R} \sigma_{is}^2 = N_T$$

where σ_{is}^2 is the variance of the signal assigned to i-th antenna.

The received vector is

$$\underline{y} = \sqrt{\frac{E_s}{N_T}}\, H V_s s + \eta = \sqrt{\frac{E_s}{N_T}}\, \underline{U}\, \underline{\Sigma}\, \underline{s} + \eta$$

after transformation by U^H we get:

$$y' = U^H \underline{y} = \sqrt{\frac{E_s}{N_T}}\, \underline{\Sigma}\, \underline{s} + U^H \underline{\eta}$$

$$= \sqrt{\frac{E_s}{N_T}}\, \underline{\Sigma}\, \underline{s} + \underline{\eta}'$$

or

$$y_i' = \sqrt{\frac{E_s \lambda_i}{N_T}} + \eta_i' \qquad i = 1, 2, \ldots, R.$$

usually $R = \min(N_T, N_R)$.

So, the capacity is ~~def~~ given as:

$$C(\{\sigma_{is}^2\}) = \sum_{i=1}^{R} \log_2\left(1 + \frac{E_s \lambda_i}{N_T N_0}\sigma_{is}^2\right)$$

for a given power allocation $\{\sigma_{is}^2\}$.

It is natural that the transmitter try to optimize the capacity by proper power allocation, say, by selecting $\{\sigma_{is}^2\}$ using water filling. Then

$$C = \max_{\{\sigma_i^2\}} \sum_{i=1}^{R} \log_2\left(1 + \frac{E_s \lambda_i}{N_T N_0}\sigma_{is}^2\right)$$

Now to find the ergodic capacity, we ~~can~~ get expectation with respect to $\{\lambda_i\}$, i.e., different channel realizations H.

So:

$$\bar{C} = E\left[\max_{\{\sigma_{is}^2\}} \sum_{i=1}^{R} \log_2\left(1+ \frac{E_s \lambda_i}{N_T N_0}\sigma_{is}^2\right)\right]$$

<u>Multiplexing gain</u> versus <u>Diversity gain</u>:

Use of multiple antennas at the transmitter and/or receiver, provides with multiple sub-channels between transmit and receive sites. This diversity can be used for either sending more information (this is called the <u>multiplexing gain</u>) or sending the data more reliably (<u>diversity gain</u>).

Take for example a MISO system with N_T transmit antennas and $N_R=1$ receive antenna. Signals $s_1, s_2, \cdots s_{N_T}$ are coded (spread) using CDMA codes $c_1, \cdots c_{N_T}$ and sent over the channel. The received signal is:

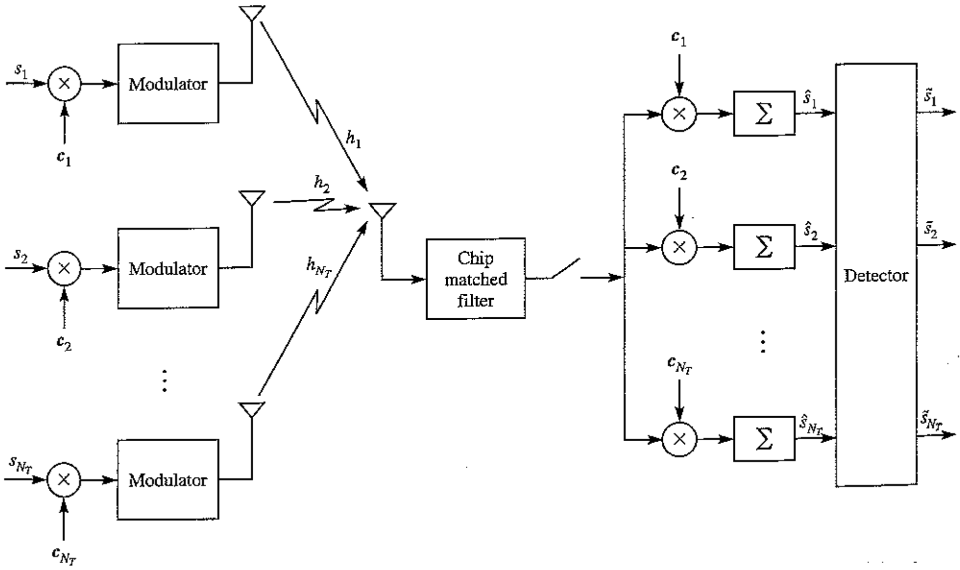
$$y = \sum_{i=1}^{N_T} s_i c_i h_i + \eta$$

after decorrelating (de spreading), we have:

$$y_i = \sqrt{\frac{E_s}{N_T}}\, s_i h_i + \eta_i \qquad i = 1, 2, \cdots, N_T$$

The receiver makes decision individually for each stream y_i by calculating $y_i h_i^*$. We have here a multiplexing gain of N_R but no diversity gain. But if we send the same data over all N_T antennas each spread with a different Code, we have a diversity gain of N_T (using maximal ratio combining). In this case we do not have any multiplexing gain. We can ~~the~~ have some other case where we send one data stream over two antennas and one stream on each of the remaining ~~or~~ or some other arrangement. In general a MIMO system can have between 1 and $\min(N_R, N_T)$

multiplexing gain and between N_R and $N_T N_R$ diversity gain.



MISO system with spread spectrum signals.

Space-time Coding

MIMO allows the user to transmit several data streams on different antenna elements. One can use part of the extra ~~capacity~~ throughput to improve the reliability by encoding each stream, i.e., doing temporal coding as is usual in SISO system.

On the other hand, we can send data symbols related to the same stream on more than one antenna. That is we can use ~~to~~ spatial redundancy. This is called Space-time Coding.

Tarokh et al. developed the idea of Space-time coding and used Trellis codes to code across elements (STTC). Later a simple block coding scheme for a (2,1) ~~##~~ MISO channel proposed by Alamouti created considerable interest in space-time block codes (STBC).

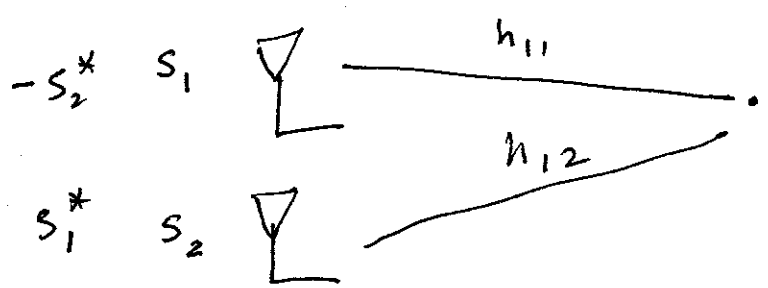
We here briefly describe Alamouti's scheme. STBC codes for larger number of antennas are covered in the text. You may also refer to papers by Tarokh and others.

Alamouti STBC

In 1998, Alamouti proposed a scheme for an $N_T = 2$ and $N_R = 1$ MISO channel. The generator matrix for Alamouti code is

$$G = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$$

That is, two consecutive s_1 and s_2 are transmitted each on one of the elements in one time instant and $-s_2^*$ and s_1^* are transmitted next time instant



Where $H = [h_{11}, h_{12}]$ is the channel matrix. At the receiver,

$$y_1 = h_{11} s_1 + h_{12} s_2 + \eta_1$$

$$y_2 = -h_{11} s_2^* + h_{12} s_1^* + \eta_2$$

we can write

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2^* \end{bmatrix}$$

Let

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{bmatrix}^H \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}$$

to get

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \begin{bmatrix} h_{11}^* & h_{12} \\ h_{12}^* & -h_{11} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} +$$

$$+ \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11} \end{bmatrix} \underline{\eta}$$

$$= \left[|h_{11}|^2 + |h_{12}|^2 \right] \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \underline{\eta}'$$