Multi-user Communications

In a communications link a user called transmitter sends information to another user named the receiver and vice versa. In actual communication systems, usually, multiple users ar involved. Many transmitters can transmit information to one or many receiver(s).

Two particular type of archisechtures are ~~of~~ usually dealt with more due to theim being the building blocks of many modern communicatio networks. These two setups are: the multiple Access channel and broadcast channel.

In a multiple access many transmitters transmit their data to a central point, say a hub. We have seen multiple-antenna systems in the previous lecture. While in a MIMO (or MISO) scheme many antennas are used to transmit data, their data is provided by a single user. So, it is important to make distinction between multiple-transmit an multiple access channel. In a multiple access system, different transmitters (located at geographically different locations) transmit their data which is not known to other transmitters and, possibly, of no interest to them, to ~~a common~~ a single receiving node.

In a <u>Broadcast Network</u> on the other hand one transmitter sends its data to many receivers. The examples of broadcast channels are TV and radio stations as well as the downlink in many present day communication systems.

Consider a cellular network; each mobile station (cellular phone) transmits its data to a base station (a multiple access system) and the base station sends the data destined to the mobiles in its cell in a broadcast fashion.

Similarly in satellite communication systems each Earth Terminal sends its informations (uploads its data) to ~~the~~ a <u>hub</u> or to the satellite (in a regenerative system), and the hub (or the satellite) broadcast to all user and each of the user terminals takes the part of data destined for it (down load).

In this lecture, we will talk about <u>multiple access</u>.

Assume that K users want to transmit to a base station. We can divide to frequency spectrum available into K frequency bands (of equal or not equal width) and give band to one of the users. This scheme is called Frequency Division Multiple Access (FDMA). Assuming equal frequency bands, each user will have $\frac{W}{K}$ Hz. of bandwidth out of W Hz. of total bandwidth. So, each user can transmit at $\frac{1}{K}$-th of the rate a single user having access to whole W Hz. of bandwidth could transmit.

Another method is to divide the time into frames and in each frame have K time-slots. Each user will have one of the K time-slots. Denoting the frame duration by T_f, each user will have $\frac{T_f}{K}$ seconds every T_f seconds to transmit its information. This scheme is called <u>Time Division Multiple Access</u>. Given the fact that each user has only $\frac{T_f}{K}$ seconds to transmit all data it has buffered in T_f seconds, it is clear that the user has to transmit at a rate K times its rate in order not to have buffer overflow.

Note that in both FDMA and Time Division Multiple Access (TDMA), users are transmitting in non-overlapping channels. Therefore, the point-to-point communication principles we have seen so far work.

The problem with TDMA and FDMA is that they are inefficient when it comes to bursty traffic. Take the voice communication for example. When two parties talk, each party talks (on the average) half the time. Given this and the fact that there are pauses for reflection or taking breath, more than half the time the channel is idle. The issue is more pronounced in ~~an~~ Internet traffic. People type, think, do other things and from time to time transmit something.

In the above cases, i.e., when the traffic is bursty fixed assignment of a frequency band (in FDMA) or a time-slot (TDMA) is not efficient.

A more efficient technique is Code Division Multiple Access (CDMA). In this spectrum is given to all users and they can transmit at all times. The only thing is that each user is assigned a <u>Code</u> (a pseudo random sequence). Each user spreads its bit stream using its assigned Code and then transmits the spreaded signal. The receiver separates the data from different transmitters by Correlation what it receives with the Code of each user.

Capacity of Multiple Access Schemes:

We consider K users each with a power $P_i = P$
$i = 1, \ldots, K$.

If the whole bandwidth, W, was allocated to a single user, the capacity would have been:

$$C = W \log\left(1 + \frac{P}{N_0 W}\right)$$

Now consider FDMA, where each of K users ~~was~~ is given $\frac{W}{K}$ Hz. of frequency. The capacity of each use would be:

$$C_K = \frac{W}{K} \log_2\left(1 + \frac{P}{(W/K) N_0}\right)$$

The total capacity is K times this:

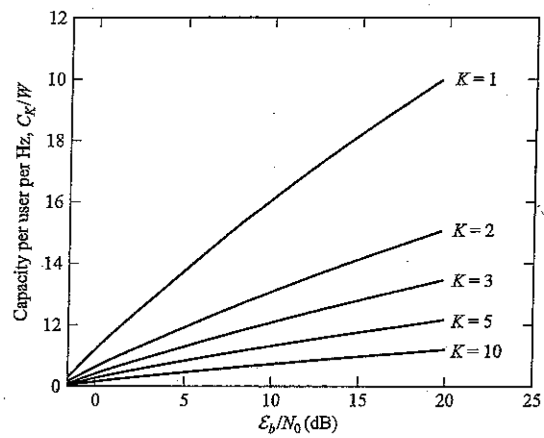
$$C = K C_K = W \log_2\left(1 + \frac{KP}{W N_0}\right)$$

As the number of users, K, increases, the total capacity increases and tends to infinity as $K \to \infty$. At the same time, the capacity per user decreases as the frequency allocated to each user $\left(\frac{W}{K}\right)$ decreases with increase of K.

The capacity per user normalized by the frequency W (the bandwidth efficiency of each user), $\frac{C_K}{W}$ is given by:

$$\frac{C_K}{W} = \frac{1}{K} \log_2\left[1 + K\frac{P}{WN_0}\right] = \frac{1}{K}\log_2\left[1 + K\frac{C_K}{W}\left(\frac{E_b}{N_0}\right)\right]$$

This per user bandwidth efficiency is shown in the figure below.



Normalized capacity as a function of $\mathcal{E}_b/N_0$ for FDMA.

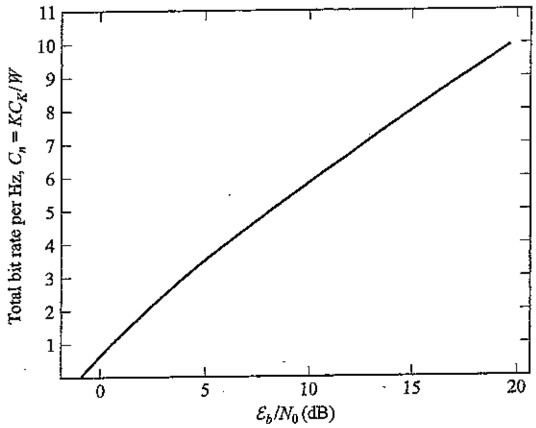
The overall bandwidth efficiency $C_n = K\frac{C_K}{W}$ is related to $\frac{E_b}{N_0}$ as,

$$C_n = \log_2\left(1 + C_n\frac{E_b}{N_0}\right)$$

or,

$$\frac{E_b}{N_0} = \frac{2^{C_n} - 1}{C_n}.$$

The following figure shows the increase in overall bandwidth efficiency with $\frac{E_b}{N_0}$.



Total capacity per hertz as a function of $\mathcal{E}_b/N_0$ for FDMA.

In TDMA each user transmits with a power KP for $\frac{1}{K}$ of time. So, the capacity per user is:

$$C_K = \left(\frac{1}{K}\right) W \log_2\left(1 + \frac{KP}{WN_0}\right).$$

This is identical with the per user capacity of FDMA. The disadvantage of TDMA, however, is that for large K, each transmitter has to transmit at a power KP which is large and difficult to sustain.

Next, we discuss the capacity of CDMA scheme. First, we consider CDMA with single user detection. Next, we talk about

CDMA with multi-user detection.

Capacity of CDMA with single user detection:

In this case, for each user, the other $K-1$ users act as interferer. In the AWGN channel, their effect is like adding an extra $(K-1)P$ of noise to $N_0 W$.

So, for each user the noise is $WN_0+(K-1)P$ and the capacity is:

$$C_K = W \log_2\left[1+\frac{P}{WN_0+(K-1)P}\right]$$

or,

$$\frac{C_K}{W} = \log_2\left[1+\frac{C_K}{W}\frac{E_b/N_0}{1+(K-1)\left(\frac{C_K}{W}\right)\frac{E_b}{N_0}}\right]$$

$$= \frac{\ln\left[1+\frac{C_K}{W}\frac{E_b/N_0}{1+(K-1)(C_K/W)E_b/N_0}\right]}{\ln 2}$$

We can use the inequality $\ln(1+x) \leq x$

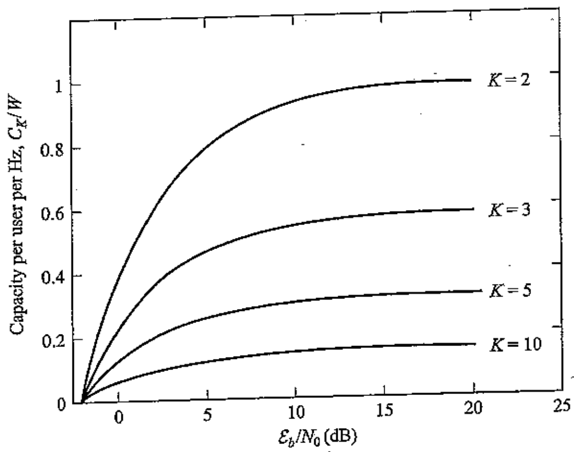
$$\frac{C_K}{W} \leq \frac{C_K}{W}\frac{E_b/N_0}{1+K\left(\frac{C_K}{W}\right)\left(\frac{E_b}{N_0}\right)}\cdot\frac{1}{\ln 2}$$

or $$1+K\left(\frac{C_K}{W}\right)\left(\frac{E_b}{N_0}\right) \leq \frac{E_b/N_0}{\ln 2}$$

The overall bandwidth efficiency $C_n = K \frac{C_K}{W}$ is upper bounded as:

$$C_n \leq \frac{1}{\ln 2} - \frac{1}{\frac{E_b}{N_0}} < \frac{1}{\ln 2}$$

So, in CDMA with single user detection the total capacity does not increase with K.



Normalized capacity as a function of $\mathcal{E}_b/N_0$ for noncooperative CDMA. (CDMA with single user detection)

CDMA with multi user detection

Denote the received signal by Y:

$$Y = \sum_{i=1}^{K} X_i + Z$$

where X_i, $i=1,\ldots,K$ is the signal from the i-th transmitter and Z is the noise.

Each use transmit at a fixed power P and at rate R_i, $i=1,\ldots,K$.

It is obvious that if $K-1$ of users stay idle and only one transmits with Power P, the rate will be limited as:

$$R_i < W\log_2(1+\frac{P}{WN_0}) \qquad 1\leq i\leq K$$

if two of the users transmit and the other $K-2$ stay silent, the rate of one, say user i can be

$$R_i < W\log_2(1+\frac{P}{WN_0})$$

the other user, say j-th user, will see the noise WN_0+P, i.e., the channel noise plus the i-th user's signal, so, the rate of user j, can be:

$$R_j < W\log_2(1+\frac{P}{WN_0+P})$$

So,

$$R_i+R_j < W\log_2(1+\frac{P}{WN_0})+W\log(1+\frac{P}{WN_0+P})$$

$$= W\log_2(1+\frac{2P}{WN_0})$$

$$1\leq i,j\leq K$$

Similarly if three users transmit and $K-3$ be silent

$$R_i + R_j + R_k < W \log_2 \left(1 + \frac{3P}{WN_0}\right)$$

and so on.

The total rate will be upper bounded as

$$R_{sum} = \sum_{i=1}^{K} R_i < W \log_2 \left(1 + \frac{KP}{WN_0}\right)$$

The rate per user will be upper bounded as:

$$R = \frac{R_{sum}}{K} < \frac{W}{K} \log_2 \left(1 + \frac{KP}{WN_0}\right)$$

which the same as the per user rate of the FDMA and TDMA.

In this case, CDMA does not provide higher rate as FDMA. However for the case where the rates of users are chosen to be different, ~~other~~ it is possible to achieve higher sum rate.

Example: To see how CDMA can provide higher sum rate, let's consider a $K=2$ multiple access system.

Assume that users 1 and 2 transmit with power P_1 and P_2 respectively.

First, assume that the receiver uses two single user detectors to work independently and detect data from transmitters 1 and 2.

The signal is:

$$Y = X_1 + X_2 + n$$

So, user 1's signal X_1 is corrupted by a noise with power $P_2 + N_0 W$ and, therefore, its SNR is $\frac{P_1}{N_0 W + P_2}$. If the rate of user 1 is not exceeding

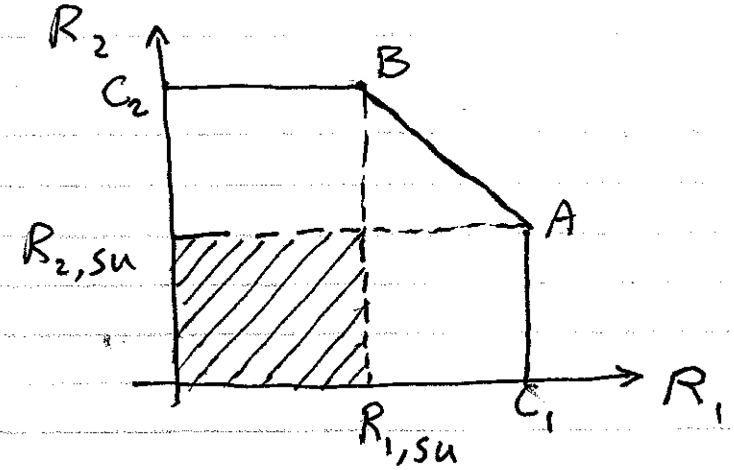
$$R_{1,SU} = W \log\left(1 + \frac{P_1}{N_0 W + P_2}\right)$$

its data can be detected reliably.

Similarly, data from user 2 is corrupted with a noise with power $P_1 + N_0 W$ and the maximum rate it can have is:

$$R_{2,su} = W \log\left(1 + \frac{P_2}{N_0 W + P_1}\right).$$

The achievable rate region for the single user detection is shown in the figure below as the dashed region.



Interference cancellation:

If after detecting the information from user 2 (at rate $R_{2,u}$), we remove it from the received signal Y, we can give to the detector for user 1, the signal

$$Y_1 = Y - X_2 = X_1 + n$$

So, the signal from user 1 is now corrupted by channel noise with power $N_0 W$ only and it can have the rate

$$C_1 = W \log\left(1 + \frac{P_1}{N_0 W}\right)$$

So, now user 2 transmits at rate $R_{2,su}$ and user 1 transmits at rate C_1. This corresponds to point A in the figure. Note that the sum rate is:

$$R_{2,su} + C_1 = W \log\left(1 + \frac{P_2}{N_0 W + P_1}\right) + W \log\left(1 + \frac{P_1}{N_0 W}\right)$$

$$= W \log\left(1 + \frac{P_1 + P_2}{N_0 W}\right)$$

Of course, at the receiver side, one could detect user 1 first with user 2 acting as interferer. In this case the rate of user 1, would have been $R_{1,su}$. After detecting user one's data it could have been subtracted from Y to get

$$Y_2 = Y - X_1 = X_2 + n$$

Now, X_2 could be detected as long as the rate of user 2 did not exceed:

$$C_2 = W \log\left(1 + \frac{P_2}{N_0 W}\right).$$

This point, $(R_{1,su}, C_2)$ corresponds to point B on the figure. Now the sum rate is

$$R_{1,su} + C_2 = W \log\left(1 + \frac{P_1}{N_0 W + P_2}\right) + W \log\left(1 + \frac{P_2}{N_0 W}\right)$$

which is, as before, equal to:

$$W \log\left(1+\frac{P_1+P_2}{N_0 W}\right).$$

Any point on the line connecting A to B can also be achieved by time sharing, i.e., a fraction α of times user 1 is detected first and user 2 second and $(1-\alpha)$ of time we detect data from user 2 and then we detect user 1's data.

Multiuser Detection of CDMA Signals:

In FDMA and TDMA the users transmit over orthogonal channels, therefore, users can be detected individually without loss of performance. CDMA on the other hand is an interference prone technique. In CDMA, using single user detection results in loss of performance as it does not take into consideration the fact that each user, in addition to channel's white noise is affected by interference from other users. Therefore, in order to benefit from the capacity provided by CDMA, use of multiuser detection is essential.

Modeling of a CDMA System:

In CDMA each user is given a pseudo random sequence, i.e., a sequence of 0's and 1's. (or -1's or $+1$'s after level coding)
Users use their codes in order to spread their data. Denote the code for user k by $\underline{a}_k = (a_k(0), a_k(1), \cdots a_k(L-1))$. Spreading is done by multiplying each bit to be transmitted by this code. In the binary case where bits are presented by -1 and $+1$, user k, sends

$$-a_k(0), -a_k(1), \dots, -a_k(L-1)$$

for zero and,

$$+a_k(0), +a_k(1), \dots +a_k(L-1)$$

for one.

Therefore, each bit of, say, duration T is represented by L mini-bits (or chips) of duration $\frac{T}{L} = T_c$ where T_c is called chip duration.

L is called the <u>processing gain</u> or <u>spreading</u> factor.

In CDMA pulse shaping (transmit filtering) is done at the chip level. Denote the transmit filter by $p(t)$. The the signature waveform of user k is:

$$g_k(t) = \sum_{\ell=0}^{L-1} a_k(\ell)\, p(t - \ell T_c), \qquad 0 \le t \le T$$

We denote the cross correlation function of users i and j by $\rho_{ij}(\tau)$. We have

$$\rho_{ij}(\tau) = \int_{\tau}^{T} g_i(t) g_j(t-\tau)\, dt$$

and

$$\rho_{ji}(\tau) = \int_{0}^{\tau} g_i(t) g_j(t+T+\tau)\, dt$$

where

$i < j$ and $0 \le \tau \le T$ is the time-shift between $g_i(t)$ and $g_j(t)$. In the case of synchronous CDMA, $\tau = 0$ and we only deal with $\rho_{ij}(0)$.

Assume binary signaling and denote the information sequence of the k-th user by

$$\underline{b}_k = [b_k(0), \dots\dots b_k(N-1)]$$

where $b_k(n) \in \{-1, +1\}, \quad n = 0, \dots, N-1$

The waveform transmitted by user k is:

$$s_k(t) = \sqrt{E_k} \sum_{n=0}^{N-1} b_k(n)\, g_k(t - nT)$$

The total received signal at the receiver is:

$$r(t) = s(t) + n(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t)$$

$$= \sum_{k=1}^{K} \sqrt{E_s} \sum_{n=0}^{N-1} b_k(n)\, g_k(t - nT - \tau_k) + n(t)$$

The optimum receiver

Let's first consider the case of Synchronous CDMA, in this case, the bit in position n, i.e., transmitted between $(n-1)T$ and nT experiences interference from bits transmitted in this position. Therefore, in order to detect bits of users, we need to consider only the bits aligned with it. The received waveform then will be:

$$r(t) = \sum_{k=1}^{K} \sqrt{E_k}\, b_k\, g_k(t) + n(t) \qquad 0 \le t \le T$$

where we have dropped the index and written $b_k(0)$ as b_k.

The log-likelihood function for ML receiver is:

$$\Lambda(\underline{b}) = \int_0^T \left[r(t) - \sum_{k=1}^{K} \sqrt{E_k}\, b_k\, g_k(t) \right]^2 dt$$

or,

$$\Lambda(\underline{b}) = \int_0^T r^2(t)\, dt - 2\sum_{k=1}^{K} \sqrt{E_k}\, b_k \int_0^T r(t) g_k(t)\, dt$$

$$+ \sum_{j=1}^{K} \sum_{k=1}^{K} \sqrt{E_j E_k}\, b_k b_j \int_0^T g_k(t) g_j(t)\, dt$$

where

$$r_k = \int_0^T r(t)\, g_k(t)\, dt$$

is the correlation of the received waveform $r(t)$ with k-th signature sequence and can be obtained as the output of a (chip) matched filter. The integral in the last term is:

$$\rho_{jk} = \rho_{jk}(0) = \int_0^T g_j(t)\, g_k(t)\, dt$$

Noting that the first term is the same for all possible data sequences, it can be neglected and finding the following correlation metrics;

$$C(\underline{r}_k, \underline{b}_k) = 2\sum_{k=1}^{K} \sqrt{E_k}\, b_k r_k - \sum_{k=1}^{K}\sum_{j=1}^{K} \sqrt{E_j E_k}\, b_k b_j \rho_{jk}$$

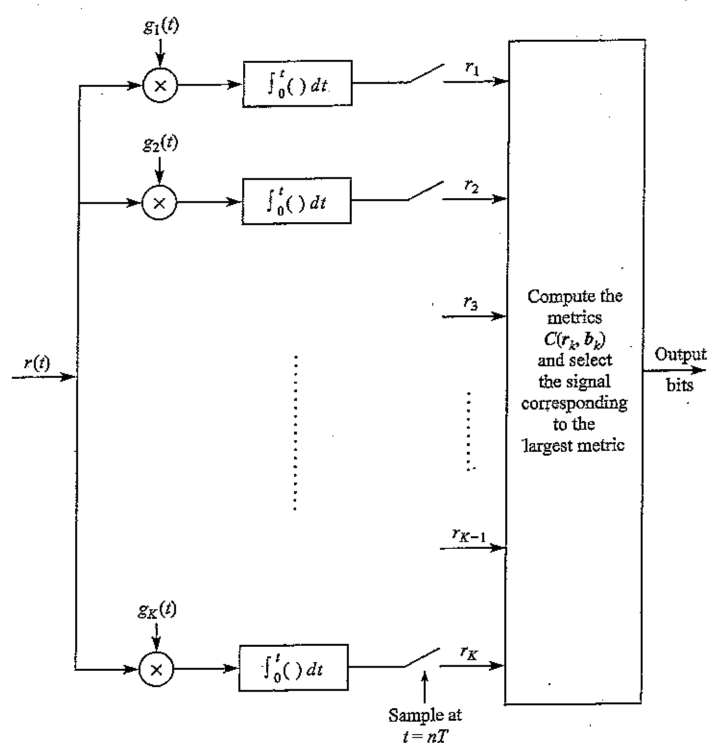
or, in matrix form

$$C(\underline{r}_k, \underline{b}_k) = 2\underline{b}_k^t \underline{r}_k - \underline{b}_k^t R_s \underline{b}_k$$

where

$$\underline{r}_k = [r_1, r_2, \ldots, r_k]^t, \quad \underline{b}_k = [\sqrt{E_1}\, b_1, \ldots \sqrt{E_k}\, b_k]^t$$

and R_{ss} is the correlation matrix,



Optimum multiuser receiver for synchronous transmission.

The complexity of maximum-likelihood (optimum) receiver grows exponentially since the metrics has to be calculated for each of the 2^K possible binary vectors ~~of~~ with K components.

In the asynchronous case, each bit experience interfference from two bits, therefore, the optimum receiver involves two correlation matrices

Sub-optimal CDMA receivers

Since the capacity of the optimum receivers grows exponentially with the number of users, for large number of users, sub-opotimal detectors are used.

The conventional single user detector is simplest detector structure, but performs rather poor. Other sub-optimal receivers can broadly divided into two categories:

- Linear receivers,
- Non-linear receivers.

Linear Receivers include:

- Decorrelating Detector

and

- MMSE Detector.

Non-linear receivers include interference cancellation schemes.

<u>Single user detection</u>:

In this case the received waveform is fed to K matched filters and the output of matched filters are individually detected, e.g., in the binary case the output of the matched filter is compared to zero and if it exceeds zero, the transmitted bit is decoded as one and else it is decided that a one have been transmitted. The output of k-th filter.

$$r_k = \int_0^T r(t) g_k(t)\, dt$$

$$= \sqrt{E_k}\, b_k + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sqrt{E_j}\, b_j\, \rho_{jk} + n_k$$

where $n_k = \int_0^T n(t)\, g_k(t)\, dt$

if the signature sequences are orthogonal, the interference (the second term) vanishes and the single user detector is optimal.

However, if the sequences are not orthogonal the interference is present and in the case ~~of~~ that other users have higher power, they can overwhelm the desired signal. This can very often happen in cellular systems when the interferer is closer to the base station. This is called the <u>near-far</u> <u>problem</u>.

<u>Decorrelating Detector</u>

In vector form the received vector is:

$$\underline{r}_K = R_s \underline{b}_K + n_K$$

where

$$\underline{b}_K = [\sqrt{E_1}\, b_1, \sqrt{E_2}\, b_2, \cdots, \sqrt{E_K}\, b_K]^t$$

$$\underline{n}_K = [n_1, n_2, \cdots n_K]$$

and R_s is the correlation matrix with elements $\{\rho_{ij}\}$.

The noise vector is zero-mean Gaussian with Covariance matrix:

$$E[n_K n_K^t] = \frac{N_0}{2} R_s$$

So:

$$p(\underline{r}_K | \underline{b}_K) = \frac{1}{(\pi N_0)^{K/2}\sqrt{\det R_s}} \exp\left[-\frac{1}{N_0}(r_K - R_s b_K)^t R_s^{-1}(r_K - R_s b_K)\right)$$

The optimum estimate of $\underline{b}_K$ denoted by $\underline{b}_K^o$ can be found by minimizing the likelihood function:

$$\Lambda(b_K) = (r_K - R_s b_K)^t R_s^{-1}(r_K - R_s b_K)$$

This can be minimized by:

$$\underline{b}_K^o = R_s^{-1} \underline{r}_K .$$

The detected signals will be the sign of elements of $\underline{b}_K^o$, i.e.,

$$\hat{\underline{b}}_K = \operatorname{sgn}(\underline{b}_K^o) .$$

<u>Example</u>: to clarify the idea of decorrelating, consider the example of a two user system: $K=2$. Denot the correlation between the signature waveform by ρ. That is $\int_0^T g_1(t) g_2(t) dt = \rho$

The correlation matrix is given as

$$R_s = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

The inverse of R_s is:

$$R_s^{-1} = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}$$

The received signal is:

$$r(t) = \sqrt{E_1}\, b_1 g_1(t) + \sqrt{E_2}\, b_2 g_2(t) + n(t)$$

If we correlate $r(t)$ with $g_1(t)$ and $g_2(t)$, we get

$$r_1 = \sqrt{E_1}\, b_1 + \rho\sqrt{E_2}\, b_2 + n_1$$

and

$$r_2 = \rho\sqrt{E_1}\, b_1 + \sqrt{E_2}\, b_2 + n_2$$

or

$$\underline{r}_2 = \begin{bmatrix} \sqrt{E_1}\, b_1 + \rho\sqrt{E_2}\, b_2 + n_1 \\ \rho\sqrt{E_1}\, b_1 + \sqrt{E_2}\, b_2 + n_2 \end{bmatrix}$$

Therefore,

$$b_2^{o} = R_s^{-1}\underline{r}_2 = \begin{bmatrix} \sqrt{E_1}\, b_1 + (n_1 - \rho n_2)/(1-\rho^2) \\ \sqrt{E_2}\, b_2 + (n_2 - \rho n_1)/(1-\rho^2) \end{bmatrix}$$

We notice that the transformation R_s^{-1} has eliminated the interference.

It is interesting to note that transforming by R_s^{-1} multiplication is equivalent to using modified signature waveforms:

$$g_1'(t) = g_1(t) - \rho g_2(t)$$

$$g_2'(t) = g_2(t) - \rho g_1(t)$$

Minimum Mean-Squared Error (MMSE) detector:

This scheme is similar to MMSE Equalizer for ISI. The same way the decorrelating receiver is similar to zero-forcing equalizer.

In the MMSE detector, a transformation W is applied to the input vector $\underline{r}$ to get $\underline{b}^{\circ}$ as

$$\underline{b}^{\circ} = W \underline{r} .$$

Then, we try to minimize the MSE between $\underline{b}$ and $\underline{b}^{\circ}$ given by:

$$J(\underline{b}) = E[(\underline{b} - W\underline{r})^{t}(\underline{b} - W\underline{r})]$$

The minimization results in

$$E[(\underline{b} - W\underline{r})\underline{r}^{t}] = 0$$

or,

$$E[\underline{b}\,\underline{r}^t] = W E[\underline{r}\,\underline{r}^t] \qquad (A)$$

In synchronous CDMA case:

$$E[\underline{b}_k \underline{r}_k^t] = E[b_k b_k^t] R_s^t = \underline{D}\, R_s^t$$

and

$$E[\underline{r}_k \underline{r}_k^t] = E[(\underline{R}_s \underline{b}_k + \underline{n}_k)(\underline{R}_s \underline{b}_k + \underline{n}_k)^t]$$

$$= \underline{R}_s \underline{D}\, \underline{R}_s^t + \frac{N_0}{2} \underline{R}_s^t$$

$\underline{D}$ is the diagonal matrix with diagonal elements $\{E_k, 1 \leq k \leq K\}$, i.e., it is the matrix of gains.

Substituting the expressions for $E[\underline{b}_k \underline{r}_k^t]$ and $E[r_k r_k^t]$ in (A), we get

$$\underline{W}^o = (R_s + \frac{N_0}{2} D^{-1})^{-1}$$

Now, the detection will be as follows:

1) Transform $\underline{r}_k$ by $\underline{W}^o$ to get,

$$b_k^o = \underline{W}^o \underline{r}_k$$

2) Take the sign of each element of $\underline{b}_k^o$,

$$\hat{\underline{b}}_k = sgn(\underline{b}_k^o).$$

When SNR is high, $\frac{N_0}{2} D^{-1}$ has small elements on its diagonal (compared to diagonal elements of R_s. So, in high SNR

$$\underline{W}^o = R_s^{-1},$$

which is the same as decorrelating receiver.

So, at high SNR, MMSE receiver approaches decorrelating receiver. Here, we see another advantage of decorrelating receiver, i.e., the fact that it does not need to know the channel gains, i.e., amplitudes forming the matrix D.

In low SNR, the elements of $\frac{N_0}{2} D^{-1}$ are large and $\underline{R}_s$ is negligible. In such a case MMSE receiver approaches single user detector, i.e., it ignores the interference.

<u>Successive Interference Cancellation</u> (SIC):

In this approach, users are detected one at a time. The effect of the detected users is eliminated in detecting remaining users. Usually, the users are sorted in the order of decreased received power.

The strongest signal is demodulated first. Then the effect of this users is subtracted and the second strongest signal is detected and so on.

When detecting user k's data, assuming that users $k+1, k+2, \ldots, K$ are already detected, the detection scheme is:

$$\hat{b}_k = \mathrm{sgn}\left[r_k - \sum_{j=k+1}^{K} \sqrt{E_j}\, \rho_{jk}\, \hat{b}_j \right]$$

It is seen from the above procedure that since the users detected later are detected with the interference of users stronger than them removed, the performance will not be the same for all users, possibly, better for weaker users.

This motivates the use of <u>Multistage Interference Cancellation</u>. That is after ~~doing~~ performing the detection once (this first stage can be SIC or any other scheme), SIC be performed one or more rounds (stages) in each stage using the detected symbols from the previous stage.