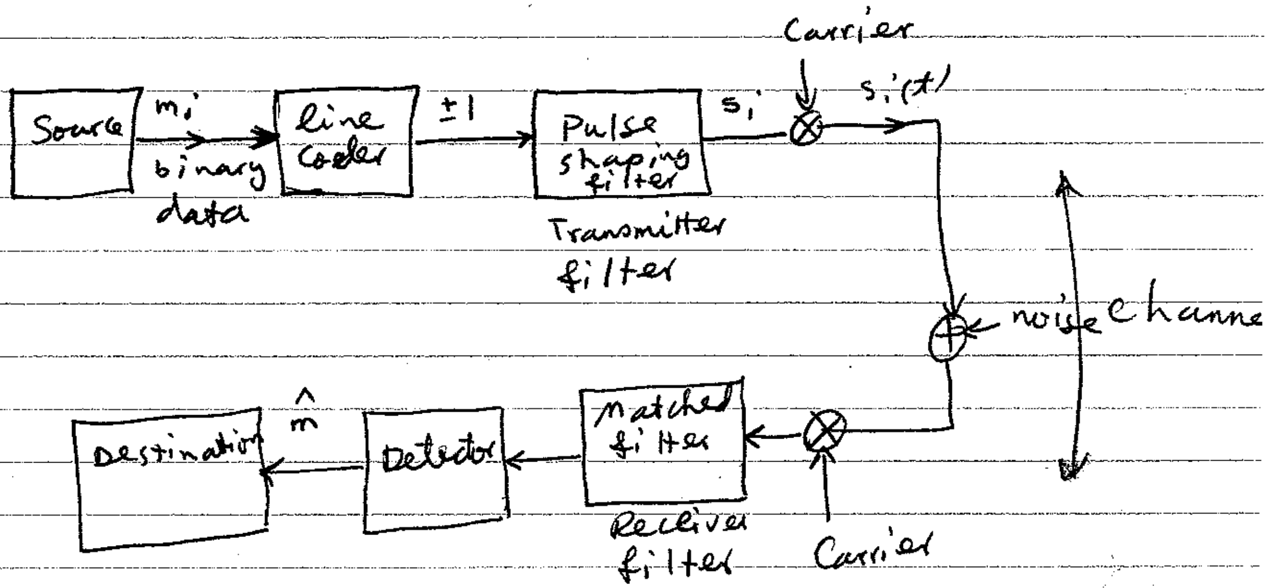
×Lecture 6, May 19, 2011

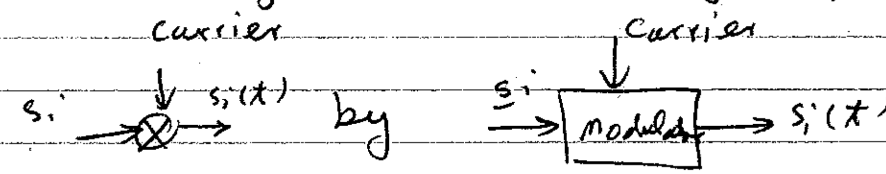
Passband Data Transmission

(carrier modulation)

In Carrier (Passband) data transmission, the output of the pulse shaping filter is being frequency shifted by multiplying it by a sinusoidal carrier:



to be more general, we may represent



to allow for ~~modification of~~ varying any abitrary parameter of the carrier waveform.

Basic carrier Modulation shemes:

Phase Shift Keying (PSK)

Amplitude Shift Keying (ASK)

Frequency Shift Keying (FSK)

Phase Shift Keying (PSK)

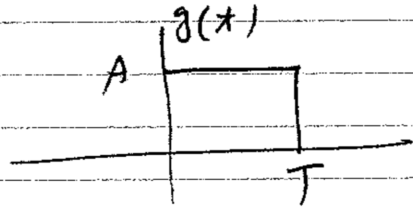
binary case:

$$s_1(t) = g(t)\cos(2\pi f_c t)$$

$$s_2(t) = g(t)\cos(2\pi f_c t + \pi) = -g(t)\cos(2\pi f_c t)$$

$g(t)$ is the pulse shaping filter, usually raised cosine filter.

For simplicity assume that $g(t)$ is an square pulse, i.e.,



Then

$$s_1(t) = A\cos(2\pi f_c t)$$

$$s_2(t) = -A\cos(2\pi f_c t)$$

$$E_b = \int_0^T s_1^2(t)\,dt = \frac{A^2}{2}T_b = \int_0^T s_2^2(t)\,dt$$

So: $A = \sqrt{\frac{2E_b}{T_b}}$

So:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

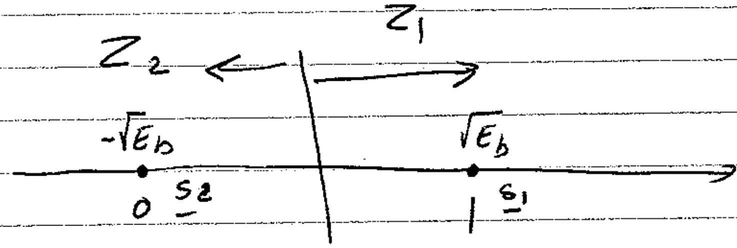
we only need one basis function for this modulation scheme, i.e.,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \qquad 0 \leq t \leq T_b$$

and, then:

$$s_1(t) = \sqrt{E_b}\,\phi_1(t) \qquad 0 \leq t \leq T_b$$

$$s_2(t) = -\sqrt{E_b}\,\phi_1(t) \qquad 0 \leq t \leq T_b$$



$$P_e = p_0 P(1|0) + P_1 P(0|1)$$

for equally probable message bits:

$$P_e = \frac{1}{2}P(1|0) + \frac{1}{2}P(0|1)$$

$$P(1|0) = \int_0^{\infty} f_X(x|0)\,dx = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(x+\sqrt{E_b})^2} dx$$

let $z = (x+\sqrt{E_b})/\sqrt{N_0/2}$

Then

$$P(1|0) = \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Similarly

$$P(0|1) = \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x-\sqrt{E_b})^2}{N_0}} dx = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

So:

$$\boxed{P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

The same as bipolar NRZ signalling.

Power Spectral Density of the PSK Signal

We can represent binary PSK as:

$$s_1(t) = g(t)\cos(2\pi f_c t)$$

$$s_2(t) = -g(t)\cos(2\pi f_c t)$$

where

$$g(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} = A & 0 \leq t < T_b \\ 0 & \text{otherwise} \end{cases}$$

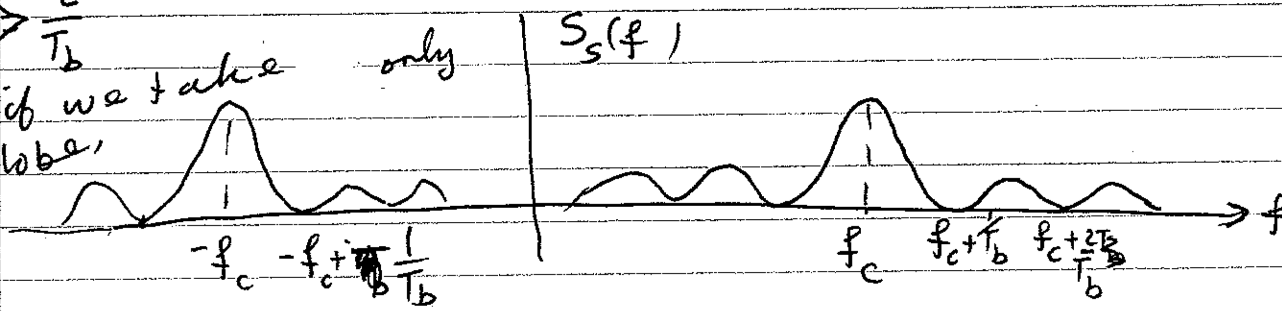
The power Spectral Density of $s(t)$, $S_s(f)$ is given as

$$S_s(f) = \frac{1}{4}\left[S_B(f-f_c) + S_B(f+f_c)\right]$$

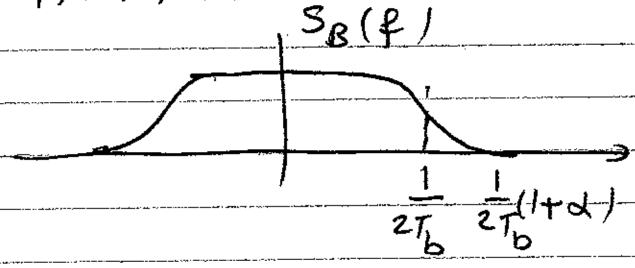
where $S_B(f)$ is the power spectral density of the $\pm g(t)$. (Remember: we know this from early lectures: random binary sequence).

$$S_B(f) = \frac{2E_b \sin^2(\pi f T_b)}{(\pi f T_b)^2} = 2E_b \operatorname{sinc}^2(f T_b)$$

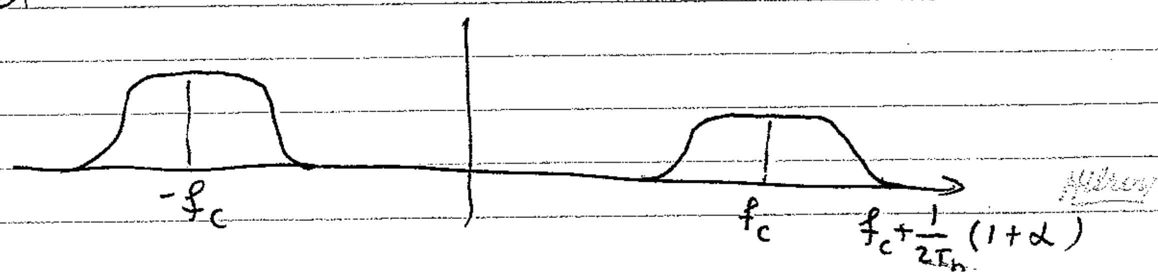
$BW > \frac{2}{T_b}$ even if we take only the main lobe, we have $BW = \frac{2}{T_b}$



However, if we use for $g(t)$ the raised cosine filter, then



and



So, the require Bandwidth would ~~have been~~ be:

$$BW = \frac{1}{T_b}(1+\alpha) = R_b(1+\alpha) \quad Hz.$$

Bandwidth efficiency is

$$\eta = \frac{R_b}{BW} = \frac{1}{1+\alpha} \quad \text{bits/sec./Hz.}$$

Quaternary Phase-Shift Keying (QPSK).

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

for $i = 1, 2, 3, 4$

Signal space representation

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t) \cos(2i-1)\frac{\pi}{4} - \sqrt{\frac{2E}{T}} \sin(2\pi f_c t) \times \sin(2i-1)\frac{\pi}{4}$$

Two orthonormal basis

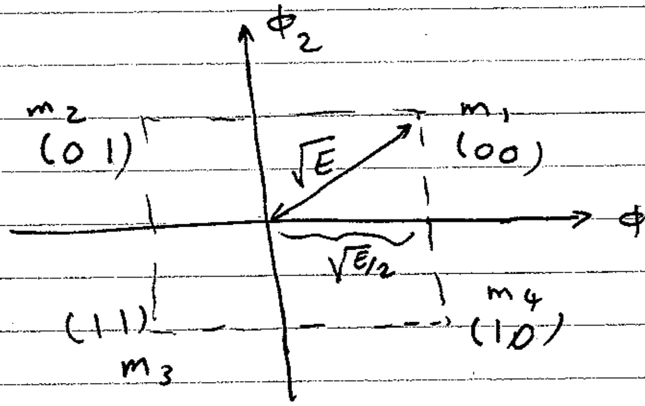
$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \le t \le T$$

and

$$\phi_2(t) = -\sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \le t \le T$$

are required to represent QPSK.

i		ϕ_i	s_{i1}	s_{i2}
$i=1$	0 0	$\pi/4$	$+\sqrt{\frac{E}{2}}$	$+\sqrt{E/2}$
$i=2$	0 1	$\frac{3\pi}{4}$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
$i=3$	1 1	$\frac{5\pi}{4}$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
$i=4$	1 0	$\frac{7\pi}{4}$	$\sqrt{E/2}$	$-\sqrt{E/2}$

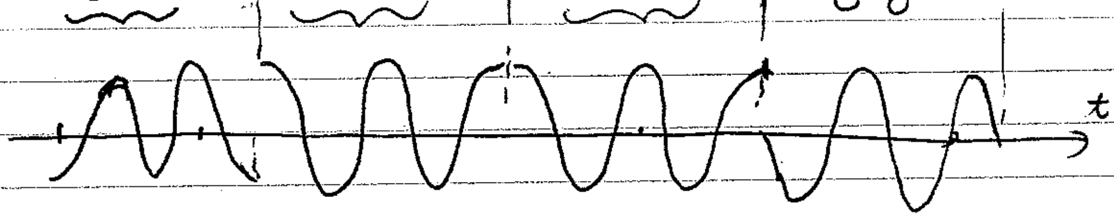


Example:

Output corresponding to the sequence:

0 1 1 0 1 0 0 0

0 1 | 1 0 | 1 0 | 0 0



Probability of error for QPSK

With Gray coding, the probability of error for the two bits in a symbol are independent from each other.

Prob. of 2nd bit being in error is the probability that the noise in ϕ_1 direction is greater than (less than) $\sqrt{E/2}$ $(-\sqrt{E/2})$ for 1 (0), respectively. Similarly, for the 1st. bit

$$P_B = \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x+\sqrt{E/2})^2}{N_0}} dx$$

let $z = \dfrac{x + \sqrt{E/2}}{\sqrt{N_0/2}}$

to get

$$P_B = \int_{\sqrt{\frac{E}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

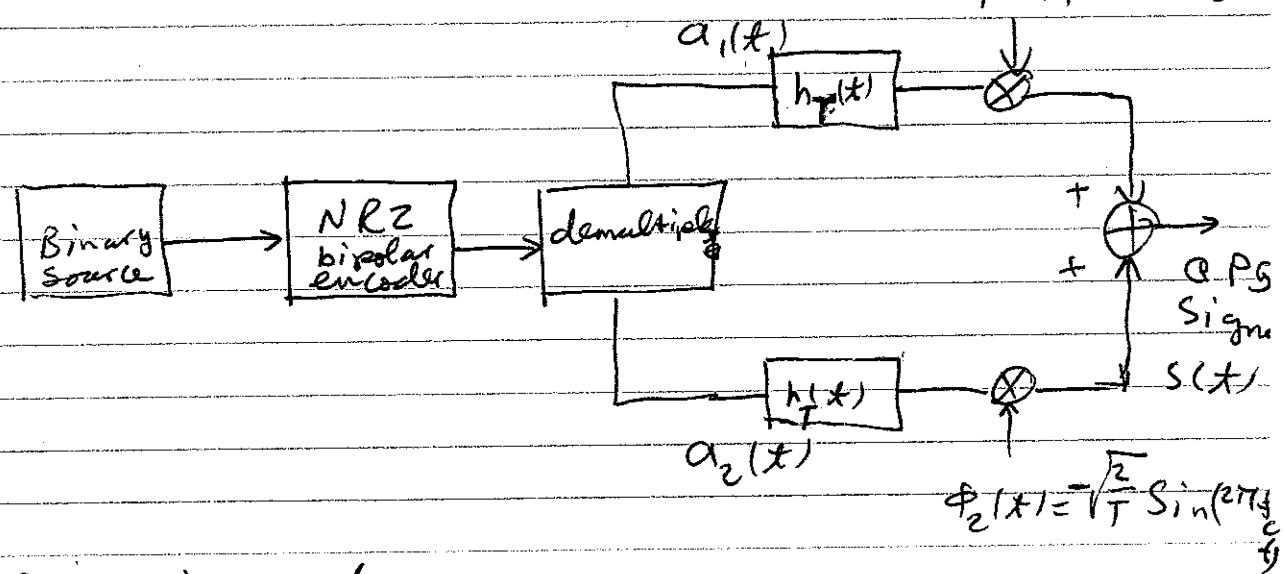
But, $E = 2E_b$ So:

$$P_B = Q\left(\sqrt{2E_b/N_0}\right)$$

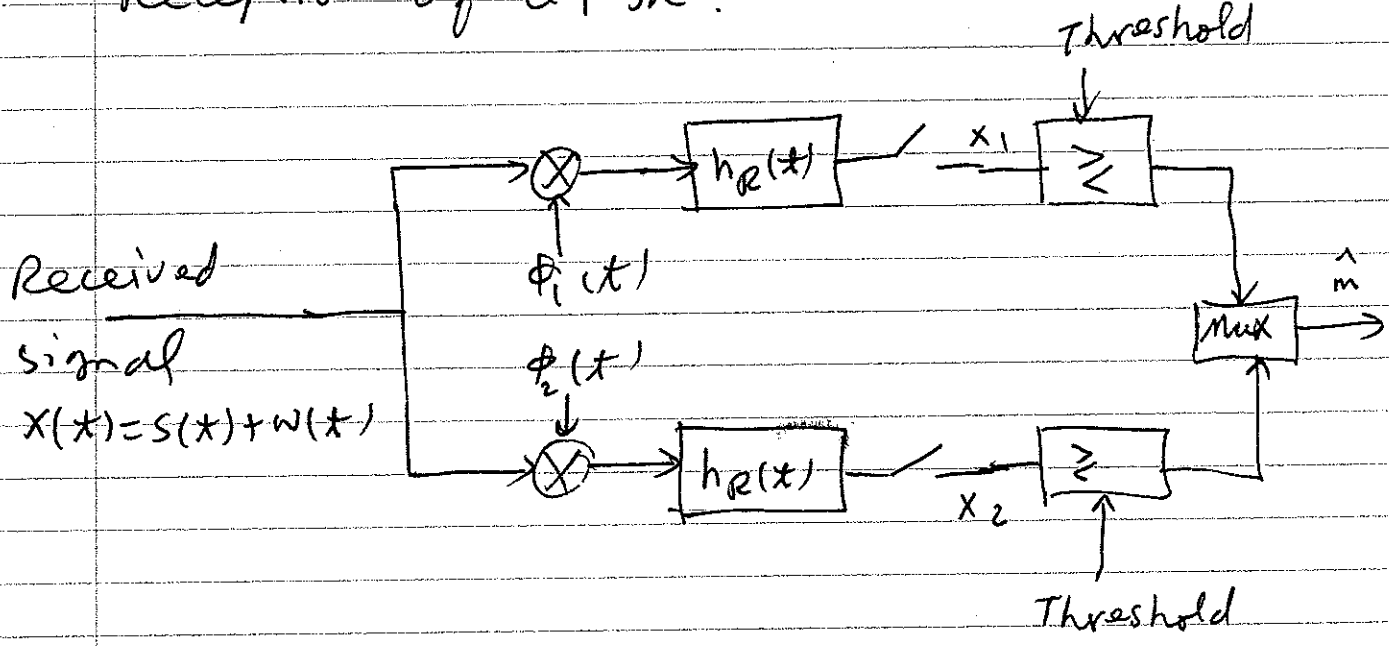
$P_B =$

So the, BER of BPSK and QPSK is the same.

Generation of QPSK Signals:



Reception of QPSK:

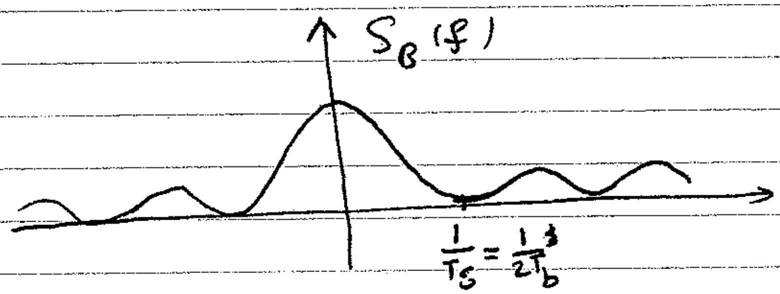


Power Sepectra of QPSK Signal

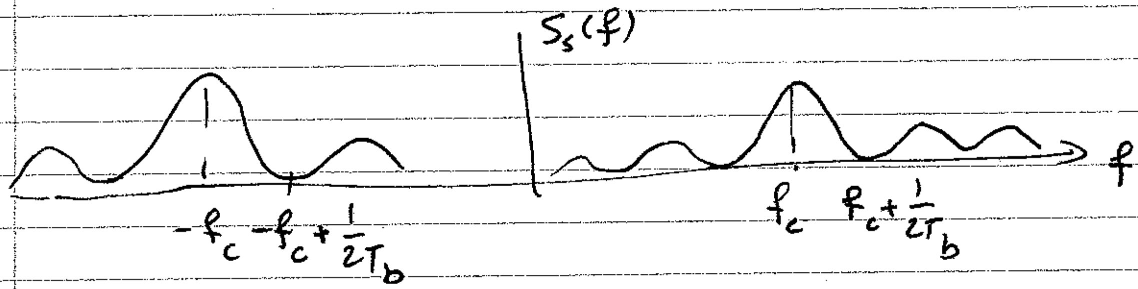
for unfiltered QPSK:

$$g(t) = \begin{cases} \sqrt{\frac{E_s}{T}} & 0 \le t \le T_s \\ 0 & \text{elsewhere} \end{cases}$$

$$S_B(f) = 2E_s \operatorname{Sinc}^2(T_s f) = 4E_b \operatorname{Sinc}^2(2T_b f)$$

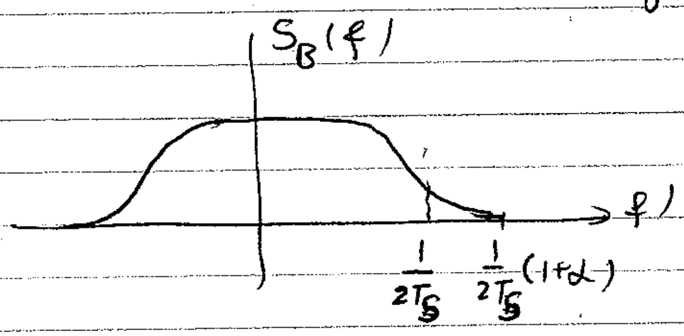


$$S_s(f) = \frac{1}{4}\left[S_B(f-f_c) + S_B(f+f_c)\right]$$



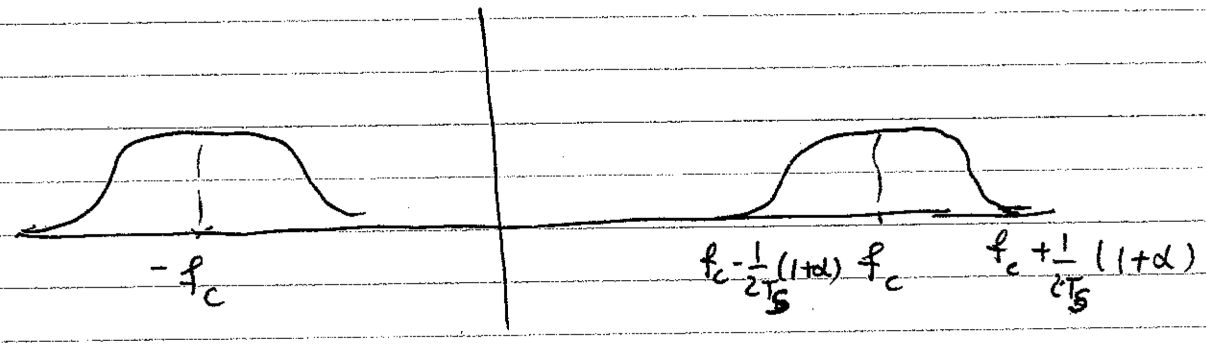
$BW = \frac{1}{T_b}$ if only ~~main~~ main lobe is considera if, insteadd, we use raised cosine filter

then



$$g(t) = \frac{\sin(2\pi W t)}{2\pi W t} \times \frac{\cos(2\pi\alpha W t)}{1 - 16\alpha^2 W^2 t^2}$$

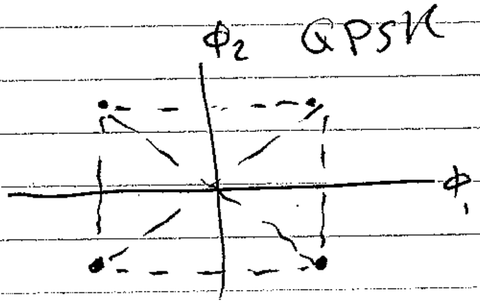
and

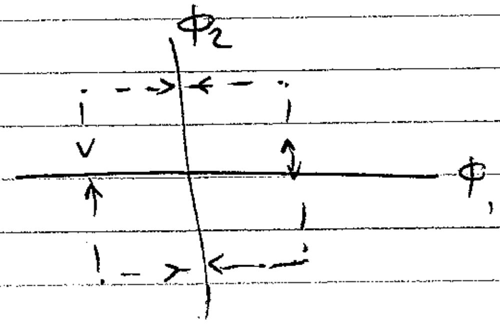


and $BW = \frac{1}{T_S}(1+\alpha) = \frac{1}{2T_b}(1+\alpha)$

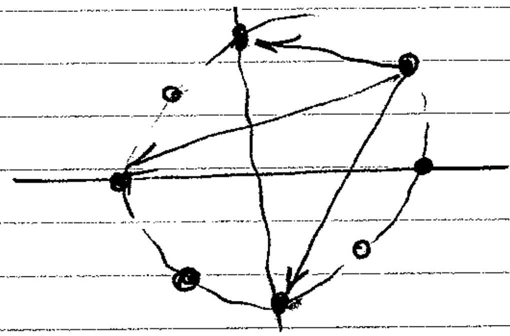
Offset QPSK (OQPSK)

in order to avoid $\pm 180°$ phase jumps in QPSK, one may delay I or Q data stream by half a symbol (1 bit).





$\frac{\pi}{4}$ - QPSK



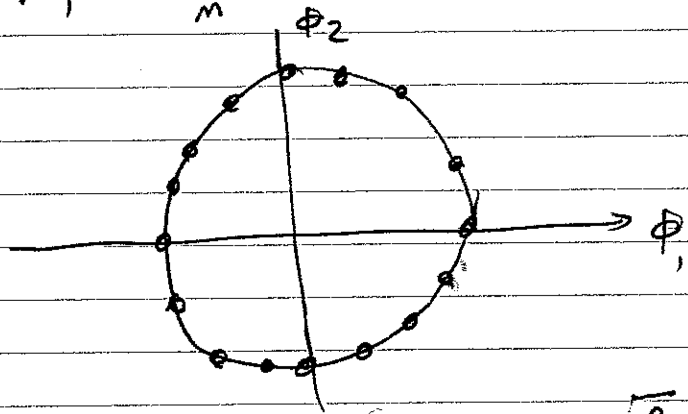
In this scheme, the signal points are alternatively taken from one of the two (QPSK constellations.

So, the phase change is $\pm\frac{\pi}{4}$ and $\pm\frac{3\pi}{4}$ instead of $\pm\frac{\pi}{2}$ and $\pm\pi$.

M-ary PSK (MPSK)

$$s_i(t) = \sqrt{\frac{2E_s}{T}}\cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right) \quad i = 1, 2, \ldots, M$$

$$= \sqrt{\frac{2E_s}{T}}\cos\frac{2\pi}{M}(i-1)\cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T}}\sin\frac{2\pi}{M}(i-1)\sin(2\pi f_c t)$$



$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ $\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$

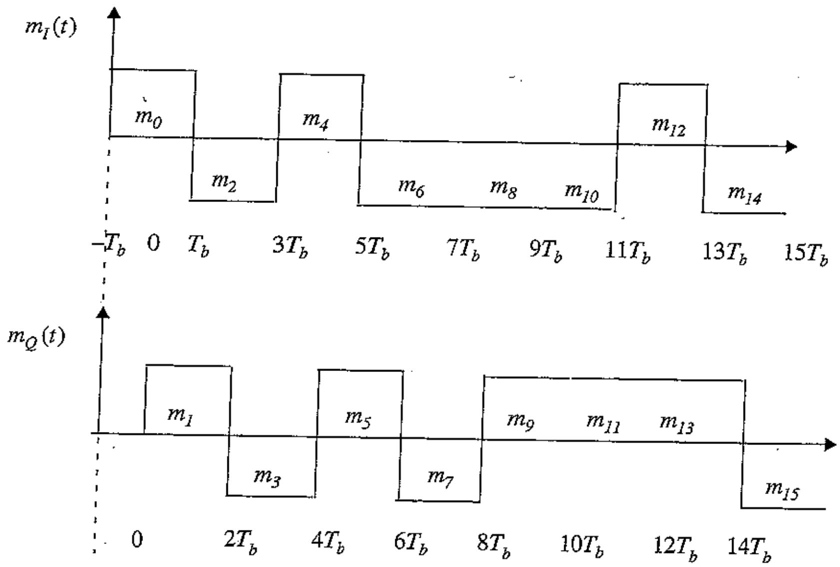


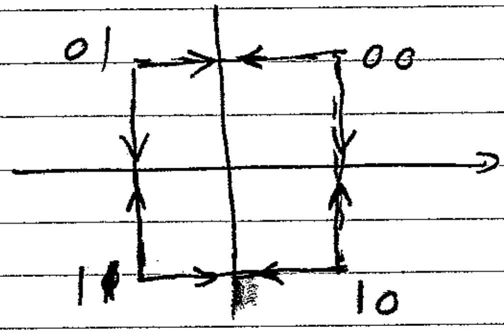
Figure 6.30 The time offset waveforms that are applied to the in-phase and quadrature arms of an OQPSK modulator. Notice that a half-symbol offset is used.

Due to the time alignment of $m_I(t)$ and $m_Q(t)$ in standard QPSK, phase transitions occur only once every $T_s = 2T_b$ s, and will be a maximum of 180° if there is a change in the value of both $m_I(t)$ and $m_Q(t)$. However, in OQPSK signaling, bit transitions (and, hence, phase transitions) occur every T_b s. Since the transition instants of $m_I(t)$ and $m_Q(t)$ are offset, at any given time only one of the two bit streams can change values. This implies that the maximum phase shift of the transmitted signal at any given time is limited to ±90°. Hence, by switching phases more frequently (i.e., every T_b s instead of $2T_b$ s) OQPSK signaling eliminates 180° phase transitions.

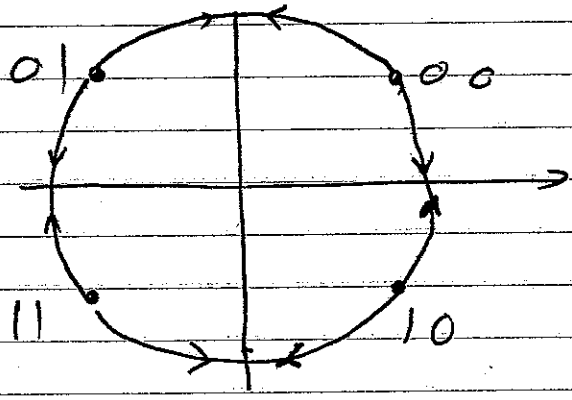
Since 180° phase transitions have been eliminated, bandlimiting of (i.e., pulse shaping) OQPSK signals does not cause the signal envelope to go to zero. Obviously, there will be some amount of ISI caused by the bandlimiting process, especially at the 90° phase transition points. But the envelope variations are considerably less, and hence hardlimiting or nonlinear amplification of OQPSK signals does not regenerate the high frequency sidelobes as much as in QPSK. Thus, spectral occupancy is significantly reduced, while permitting more efficient RF amplification.

The spectrum of an OQPSK signal is identical to that of a QPSK signal, hence both signals occupy the same bandwidth. The staggered alignment of the even and odd bit streams does not change the nature of the spectrum. OQPSK retains its bandlimited nature even after nonlinear amplification, and therefore is very attractive for mobile communication systems where bandwidth efficiency and efficient nonlinear amplifiers are critical for low power drain. Further, OQPSK signals also appear to perform better than QPSK in the presence of phase jitter due to noisy reference signals at the receiver [Chu87].

Now, we have made the allowed changes as shown below:



We can make the changes smoother by changing the phase gradually,



That is, instead of making the phase changes abruptly at kT_b time instants make it gradually between kT_b and $(k+1)T_b$. This will be done by multiplying m_I by $\cos(\frac{\pi t}{2T_b})$ and m_Q by $\sin(\frac{\pi t}{2T_b})$.

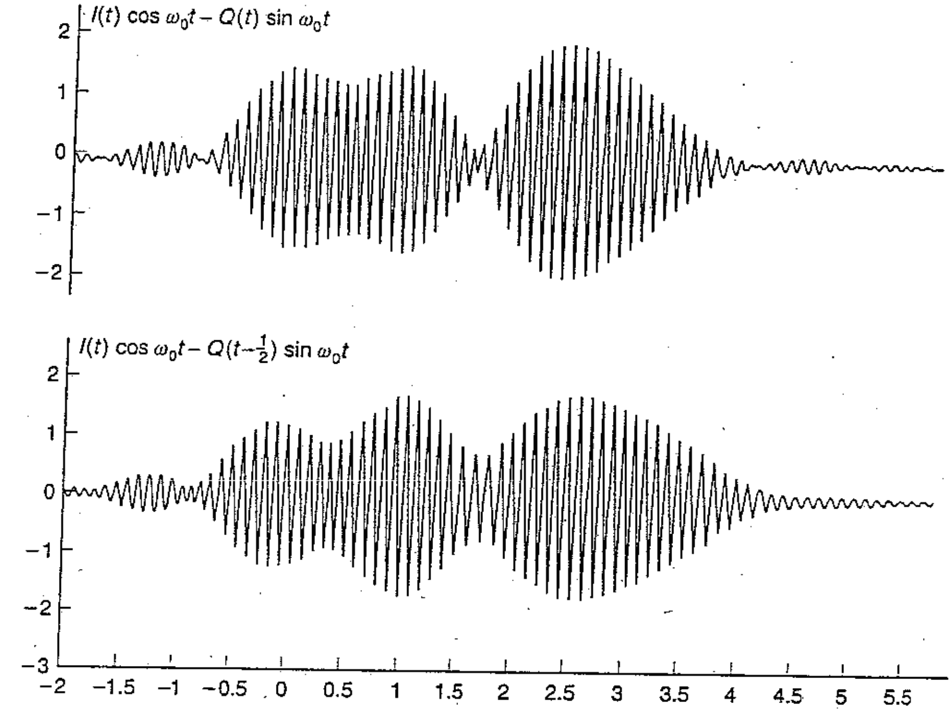


Figure 3.12 Comparison of QPSK (top) and offset QPSK (bottom) with same data and 30% root RC pulses. Data as in Figs. 3.1, 3.2, and 3.11.

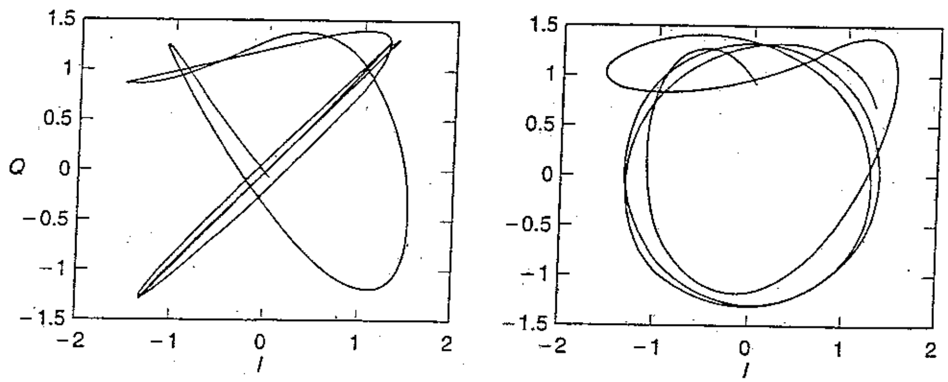


Figure 3.13 The I/Q plots for QPSK (left) and offset QPSK (right) for I data (+ − + − + − − + + −) and Q data (+ − + − + + + + − +). There are five 180 phase changes.

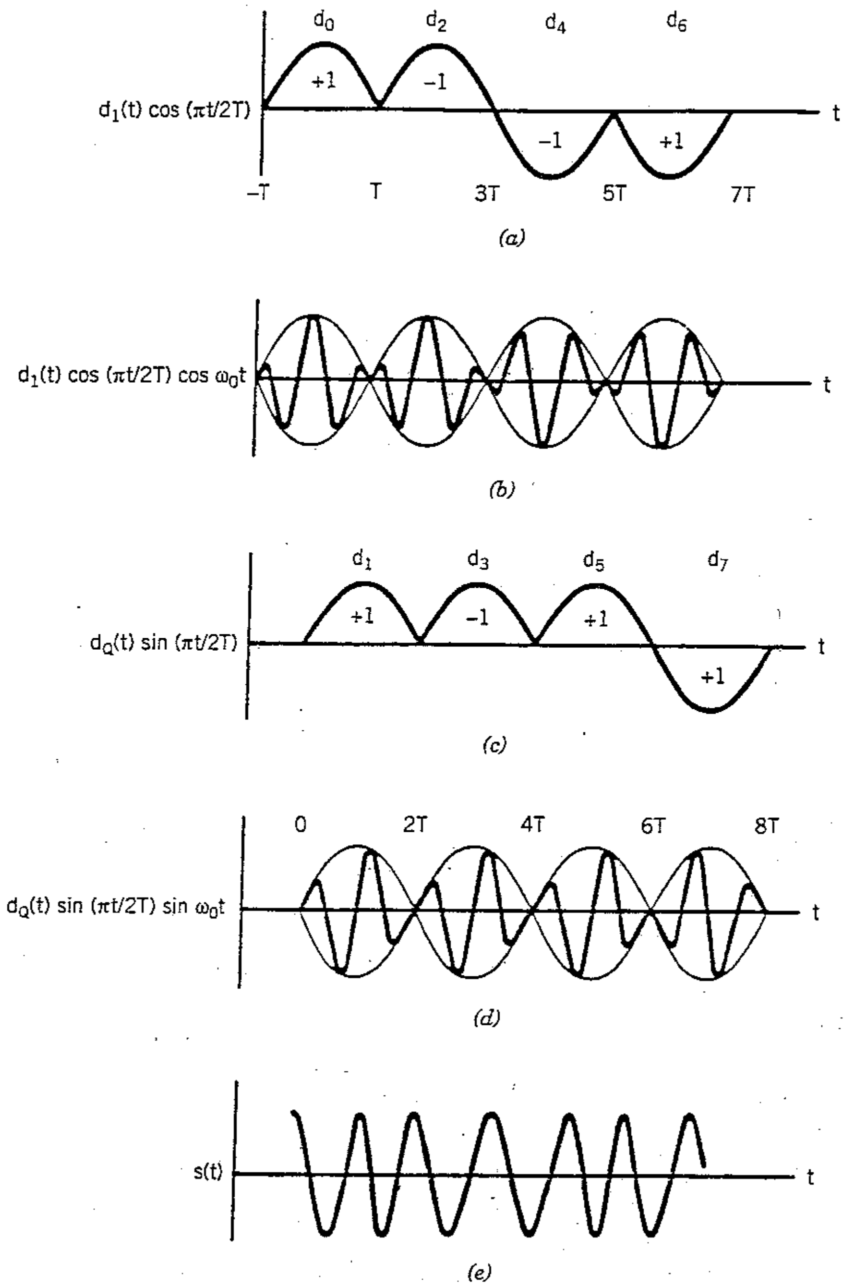


Figure 7.25 MSK waveform composition. (*a*) Modified *I* bit stream. (*b*) *I* bit stream times carrier. (*c*) Modified *Q* bit stream. (*d*) *Q* bit stream times carrier. (*e*) MSK waveforms. (From [Pas79] © IEEE.)

So, we will have

$$S_{MSK}(t) = m_I(t)\cos\left(\frac{\pi t}{2T}\right)\cancel{\cos_Q}(w_c t) + m_Q(t)\sin\left(\frac{\pi t}{T}\right)\sin w$$

if $m_I(t) = m_Q(t) = 1$

we have

$$S_{MSK}(t) = \cos\left(\frac{\pi t}{2T_b}\right)\cos(w_c t) + \sin\left(\frac{\pi t}{T}\right)\sin(w_c t)$$

$$= \cos\left(W_c - \frac{\pi}{2T_b}\right)t = \cos 2\pi\left(f_c - \frac{1}{4T_b}\right)t$$

if $m_I(t) = 1$ and $m_Q(t) = -1$

then

$$S_{MSK}(t) = \cos 2\pi\left(f_c + \frac{1}{4T_b}\right)t$$

if $m_I = m_Q = -1$

$$S_{MSK}(t) = -\cos 2\pi\left(f_c - \frac{1}{4T_b}\right)t = \cos\left[2\pi f_c t - \frac{\pi t}{2T_b} + \pi\right)$$

and for $m_I = -1$ and $m_Q = +1$

$$S_{MSK}(t) = -\cos 2\pi\left(f_c + \frac{1}{4T_b}\right)t$$

So

$$S_{MSK}(t) = \cos\left[2\pi f_c t - m_I(t)m_Q(t)\frac{\pi t}{2T_b} + \Phi_k\right]$$

where $\Phi_k = 0$ $\quad m_I(t) = 1$

$\Phi_k = \pi$ $\quad m_I(t) = -1$

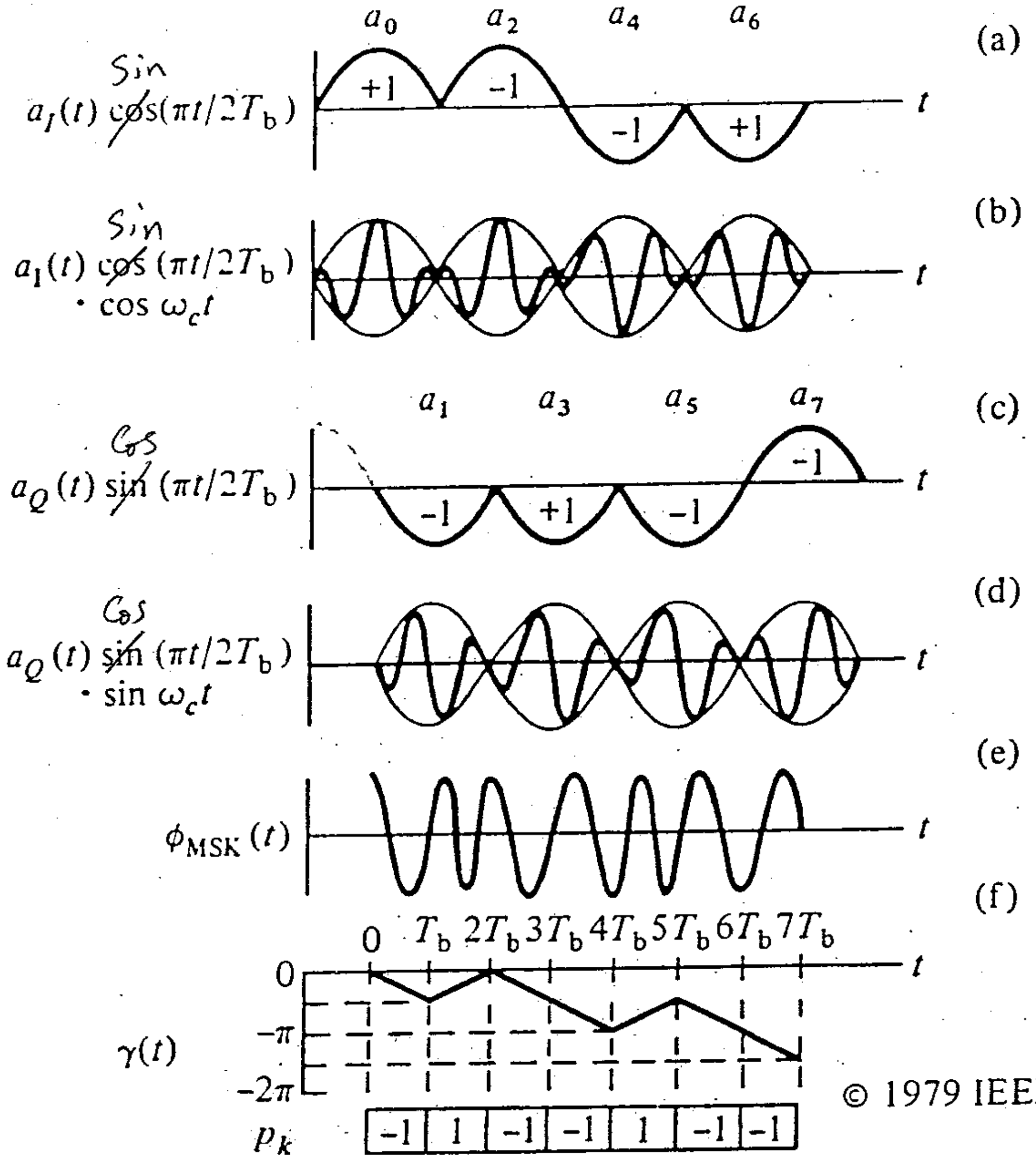


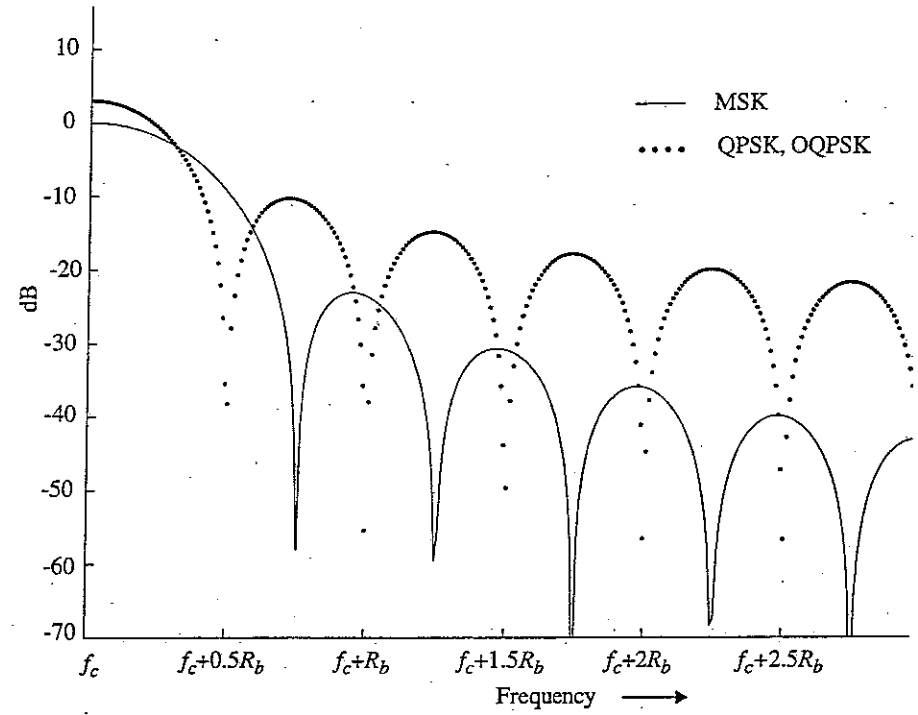
Figure 10.30 MSK waveforms.†

$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$

11-18

MSK Power Spectrum

$$P_{\text{MSK}} = \frac{16}{\pi^2}\left(\frac{\cos 2\pi(f+f_c)T}{1.16f^2T^2}\right)^2 + \frac{16}{\pi^2}\left(\frac{\cos 2\pi(f-f_c)T}{1.16f^2T^2}\right)^2$$



Power spectral density of MSK signals as compared to QPSK and OQPSK signals.

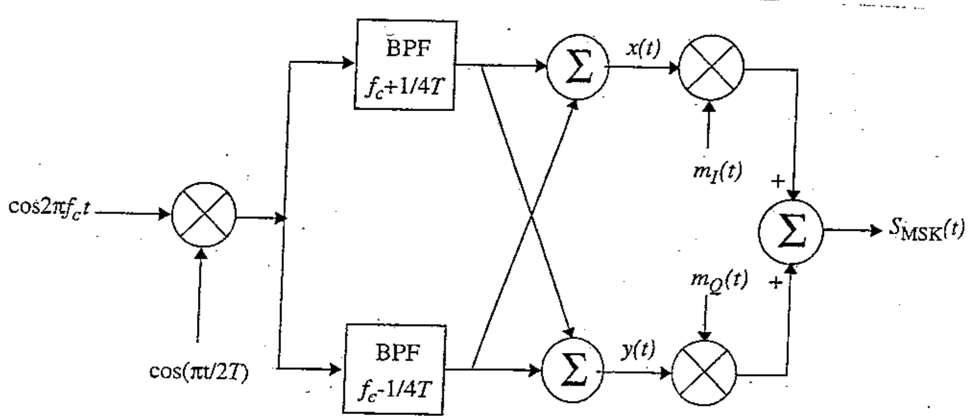


Figure 5.39
Block diagram of an MSK transmitter.

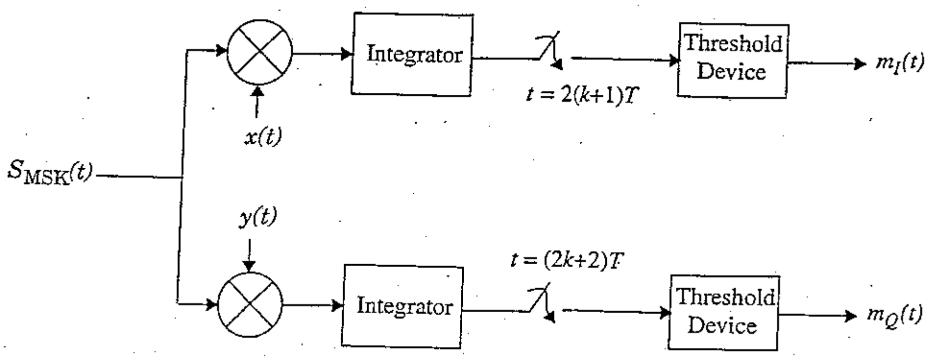
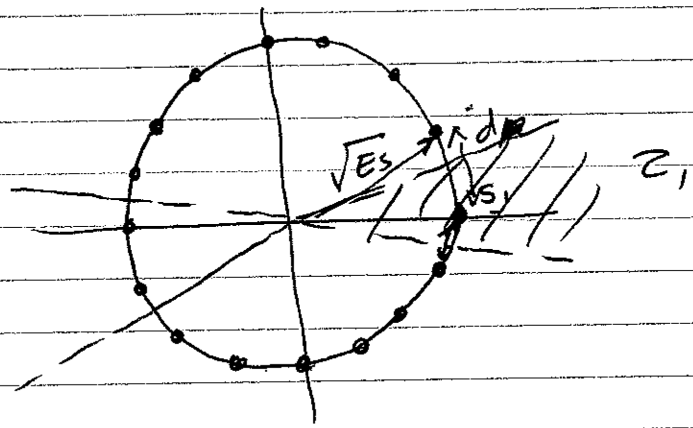


Figure 5.40
Block diagram of an MSK receiver.

Probability of error for MPSK:



$$P_e = P_{e|s_1} = P(x \notin Z_1|s_1) > P(\text{noise} > \tfrac{d}{2})$$

$$= P(\text{noise} > \sqrt{E_s}\sin\tfrac{\pi}{M})$$

or

$$P_e > P(s_2|s_1) = Q\left(\sqrt{\frac{2E_b}{N_0}}\sin\frac{\pi}{M}\right)$$

also

$$P_e < P(s_2|s_1) + P(s_3|s_1) = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\sin\frac{\pi}{M}\right)$$

for large (also moderate) M, the upper and lower bounds are close and

$$P_e \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)$$

Note that this is the probability of symbol error.

The BER, assuming Gray coding is:

$$P_B = \frac{2}{\log_2 M} \left(\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{M} \right)$$

let $E_s = E_b \log_2 M$ to get:

$$P_B = \frac{2}{\log_2 M} Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\frac{\pi}{M} \right)$$

Bandwidth requirement of MPSK:

unfiltered:

$$BW = \frac{2}{T_s} = \frac{2}{T_b \log_2 M} = \frac{2R_b}{\log_2 M}$$

with raised cosine filter

$$BW = \frac{1}{T_s}(1+\alpha) = \frac{1}{T_b \log_2 M}(1+\alpha) = \frac{R_b(1+\alpha)}{\log_2 M}$$

Bandwith efficiency.

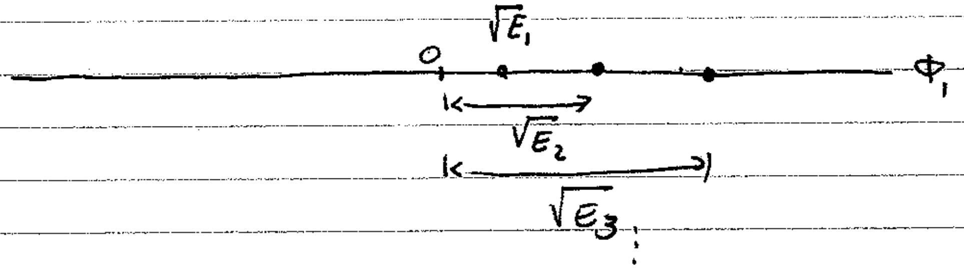
$$\boxed{\eta = \frac{R_b}{BW} = \frac{\log_2 M}{1+\alpha}}$$

M-ary ASK (M-ary PAM)

$$s(t) = A_i \cos(2\pi f_c t)$$

$$E_i = \frac{A_i^2 T_s}{2}$$

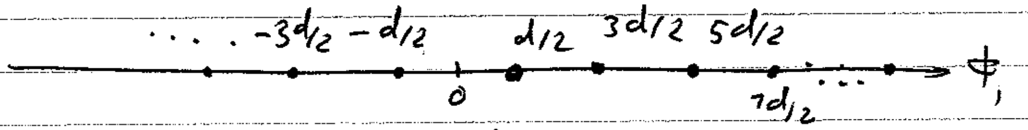
$$s(t) = \sqrt{\frac{2E_i}{T_s}} \cos(2\pi f_c t)$$



where

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t)$$

let the distance between signals be the same and equal to d:



$$\pm \frac{d}{2}, \pm \frac{3d}{2}, \ldots\ldots, \pm \frac{(M-1)d}{2}$$

The average power is

$$E_{av} = \frac{(d/2)^2 \left[1 + 3^2 + 5^2 + \cdots + (M-1)^2\right] \times 2}{M}$$

$$E_{av} = \frac{d^2}{2} \times \frac{1}{\cancel{M}} \times \frac{\cancel{M}(M-1)(M+1)}{6} = \frac{d^2(M^2-1)}{12}$$

Probability of symbol error:

$$P_e = \left[(M-2)\,2Q\left(\frac{d}{2\sigma}\right) + 2\times Q\left(\frac{d}{2\sigma}\right)\right]/M$$

$$P_e = \frac{2(M-1)}{M}\,Q\left(\frac{d}{2\sigma}\right)$$

$$d = \sqrt{\frac{12\,E_{av}}{(M^2-1)}} \qquad \sigma^2 = \frac{N_o}{2} \Rightarrow \sigma = \sqrt{\frac{N_o}{2}}$$

So,

$$P_e = \frac{2(M-1)}{M}\,Q\left(\sqrt{\frac{6E_{av}}{(M^2-1)N_o}}\right)$$

or

$$P_e = 2\left(1-\frac{1}{M}\right)\times Q\left(\sqrt{\frac{6E_{av}}{(M^2-1)N_o}}\right)$$

M-ary Quadrature Amplitude Modulation (QAM)

This is a hybrid modulation scheme, where both phase and amplitude are varied.

like MPSK, we have

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\,\cos(2\pi f_c t)$$

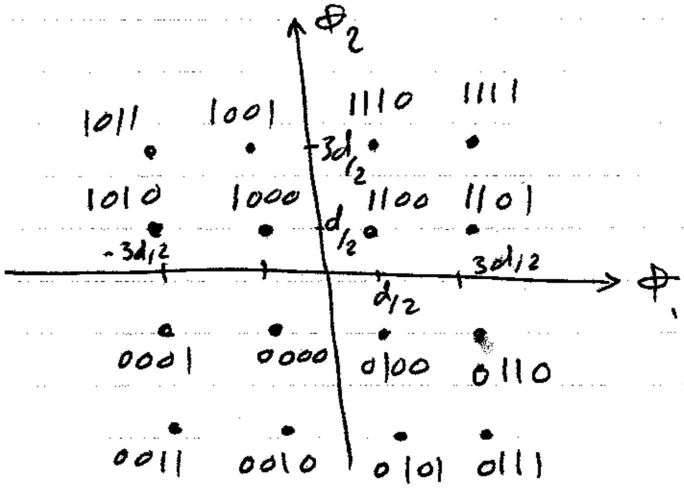
$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\,\sin(2\pi f_c t)$$

but here

$$s_k(t) = \sqrt{\frac{2E_o}{T}}\,a_k\cos(2\pi f_c t) - \sqrt{\frac{2E_o}{2}}\,b_k\sin(2\pi f_c t)$$

$$k = 0, \pm 1, \pm 2,$$

i.e., the amplitude in I and Q dimension is also varied according to the symbol to be transmitted (a_k and b_k). Here $\sqrt{E_0} = \frac{d}{2}$ as in the case of M-ASK.



Probability of symbol error

$$P_e = 1 - P_c = 1 - (1 - P_{\sqrt{M}})^2 \approx 2 P_{\sqrt{M}}$$

where $P_{\sqrt{M}}$ is the probability ~~that~~ of error of a 1-dimensional scheme ($\sqrt{M}$-PAM or $\sqrt{M}$-ASK).

So:

$$P_e \cong 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_{av}}{2(M-1)N_0}}\right)$$

$$= 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_{av}}{(M-1)N_0}}\right)$$

Since

$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E'_{av}}{(M-1)N_0}}\right)$$

where $E'_{av} = \frac{1}{2} E_{av}$ is the average signal energy of

6-18

the 1-dimensional scheme.

BER : Assuming Gray Coding

$$P_B = 4\left(1 - \frac{1}{\sqrt{M}}\right)\frac{1}{\log_2 M}\, Q\left(\sqrt{\frac{3\log_2 M}{(M-1)}\frac{E_b}{N_0}}\right)$$

Comparison with M-ary PSK

$$P_B \cong \frac{1}{\log_2 M}\, Q\left(\sqrt{\frac{2E_b\log_2 M}{N_0}}\sin\frac{\pi}{M}\right)$$

for large M

$$P_B \cong \frac{1}{\log_2 M}\, Q\left(\sqrt{\frac{2E_b\log_2 M}{N_0}}\,\frac{\pi}{M}\right)$$

while the exponent for M-QAM decreases as $\sqrt{\frac{\log_2 M}{M-1}}$ for M-PSK the decrease is as $\frac{\sqrt{\log_2 M}}{M}$ which is faster. So, the performance QAM is better.