

X Lecture 7, May 24, 2011

Frequency shift keying (FSK)

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_i t) & 0 \leq t \leq T_s \\ 0 & \text{elsewhere} \end{cases}$$

for having continuous phase we require

$$f_c = \frac{n_c + 1}{T_s}$$

That is frequencies should be apart

by multiples of $\frac{1}{T_s}$. For orthogonality the min. spacing is $\frac{1}{2T_s}$

These signals are orthogonal also. The

basis functions are:

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_s}} \cos(2\pi f_i t) & 0 \leq t \leq T_s \\ 0 & \text{elsewhere} \end{cases}$$

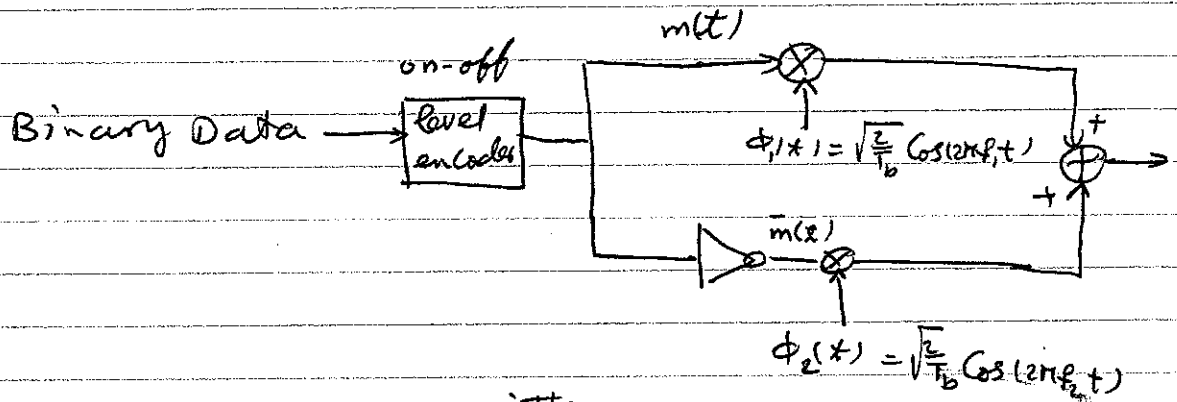
$$\underline{s}_1 = \begin{bmatrix} \sqrt{E_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \underline{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_s} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \dots \quad \underline{s}_m = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \sqrt{E_s} \end{bmatrix}$$

For the BFSK

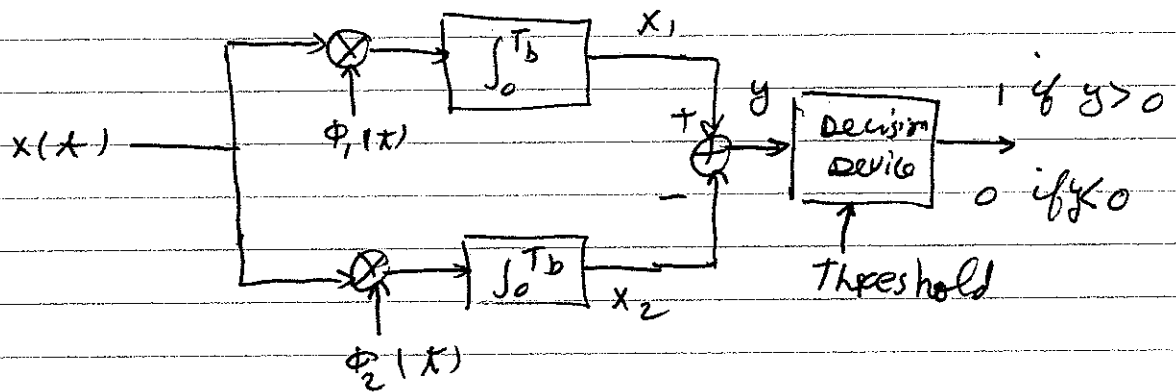
$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases}$$

$i=1, 2$.

Transmitter and Receiver for (Coherent) BPSK:



Transmitter



Receiver

Probability of error

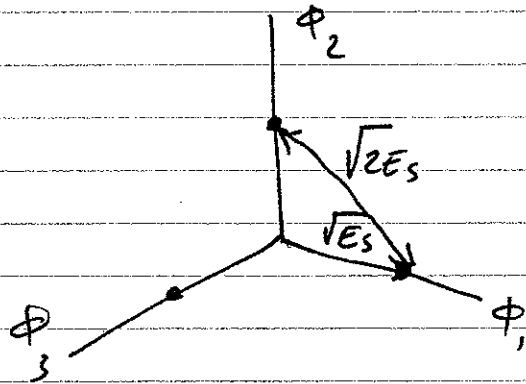
$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \text{ for BPSK}$$

Comparison with BPSK

7-3

M-ary FSK

Probability of error:



$$P_e \leq (M-1) Q\left(\frac{d}{\sqrt{2N_0}}\right) = (M-1) Q\left(\frac{\sqrt{2E_s}}{\sqrt{2N_0}}\right) = (M-1) Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

$$\text{But } E_s = E_b \log_2^M$$

So:

$$P_e \leq (M-1) Q\left(\sqrt{\frac{E_b \log_2^M}{N_0}}\right)$$

This is the probability of symbol error.

To find the probability of bit error, note that the probability of bit error given a symbol error is $\frac{M/2}{M-1}$ so

$$P_B \leq \frac{M}{2} Q\left(\sqrt{\frac{E_b \log_2^M}{N_0}}\right)$$

BER improves with increase in M .

Hellos

Bandwidth efficiency of M -ary FSK
orthogonal
for ~~continuous~~ phase FSK:

$$BW = \frac{M}{2T_s}$$

(for non-coherent $BW = \frac{M}{T_s}$)

$$T_s = T_b \log_2^M$$

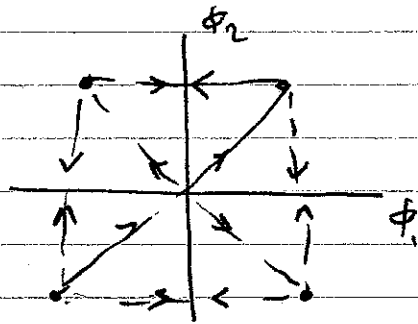
So

$$BW = \frac{M}{2T_b \log_2^M} = \frac{M R_b}{2 \log_2^M}$$

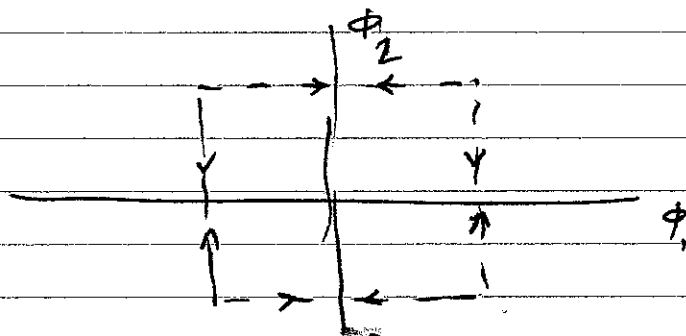
$$\eta = \frac{R_b}{BW} = \left(\frac{M}{2 \log_2^M} \right)^{-1}$$

the Bandwidth efficiency deteriorates as M increases.

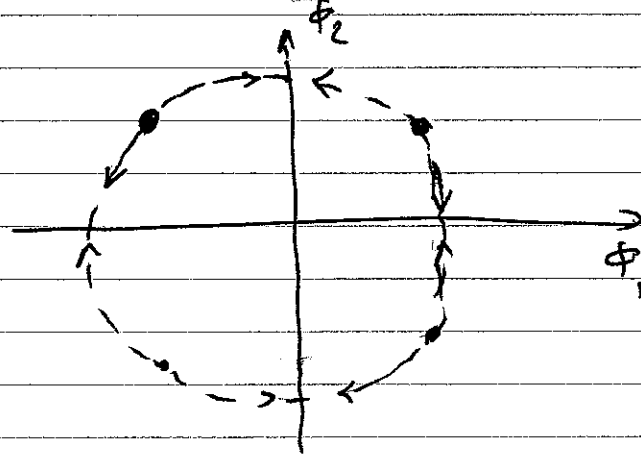
Minimum (Frequency) Shift Keying (MSK)



QPSK
phase-trajectory



O-QPSK
phase Trajectory



MSK
phase-trajectory

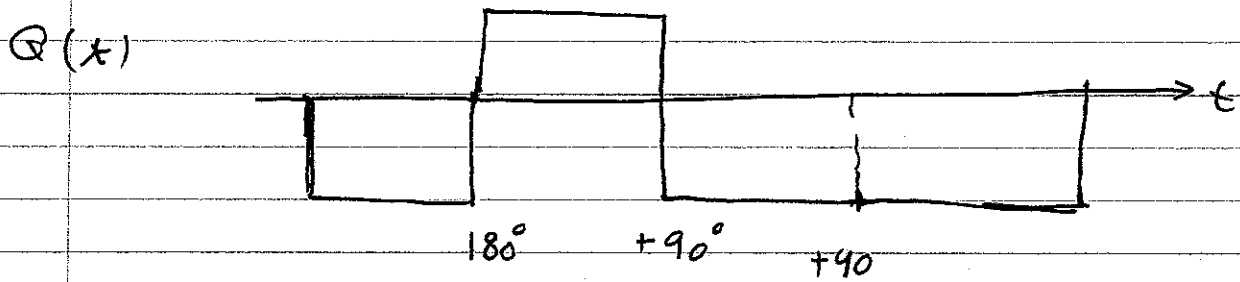
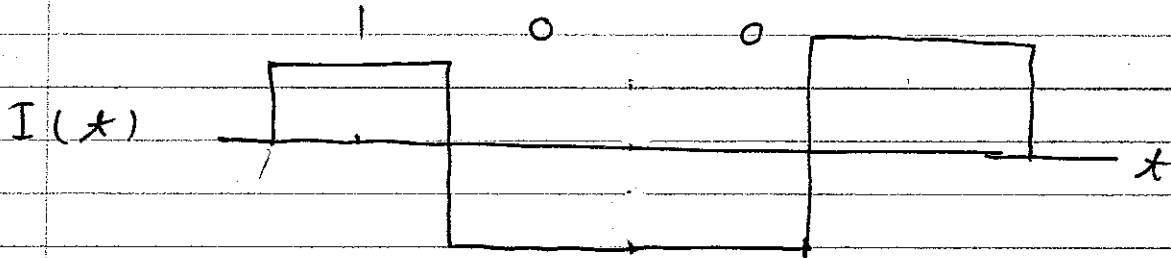
example

1 0 0 1 0 0 1 0

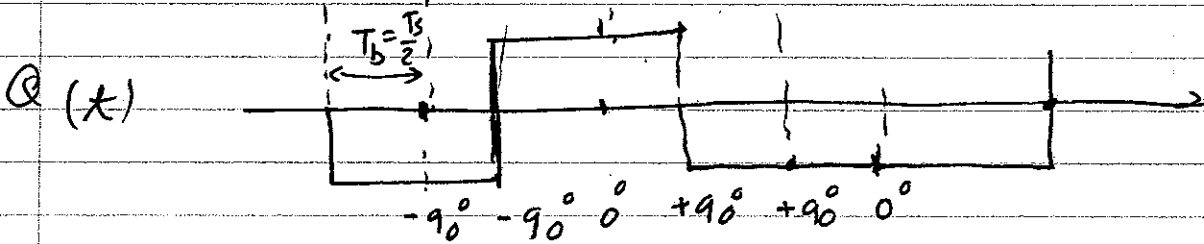
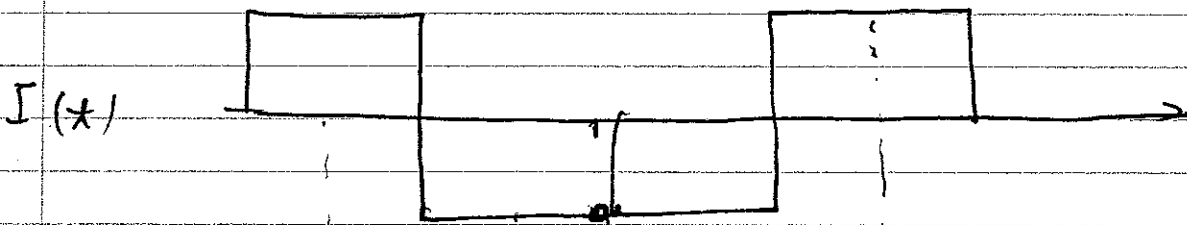
I 1 0 0 1

Q 0 1 0 0

For QPSK



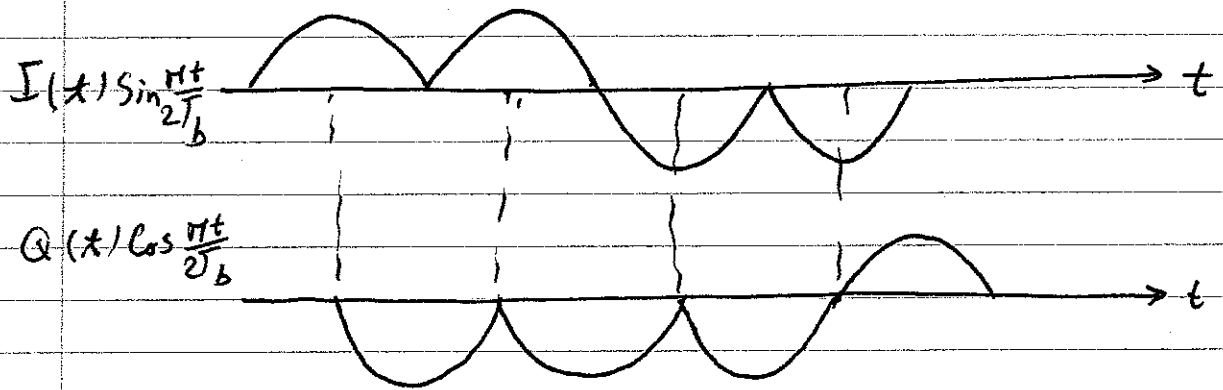
For OQPSK



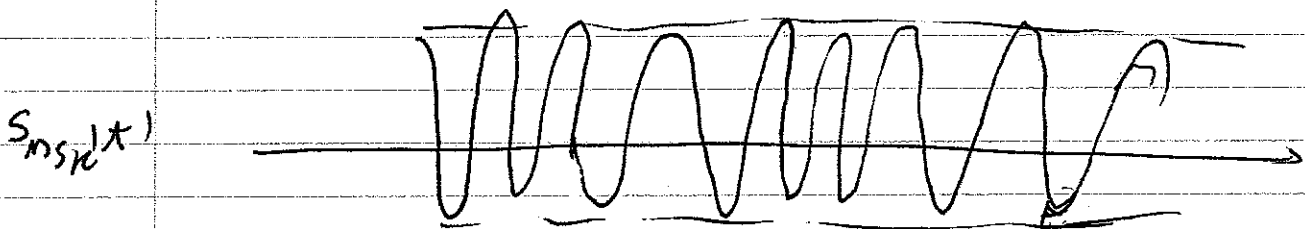
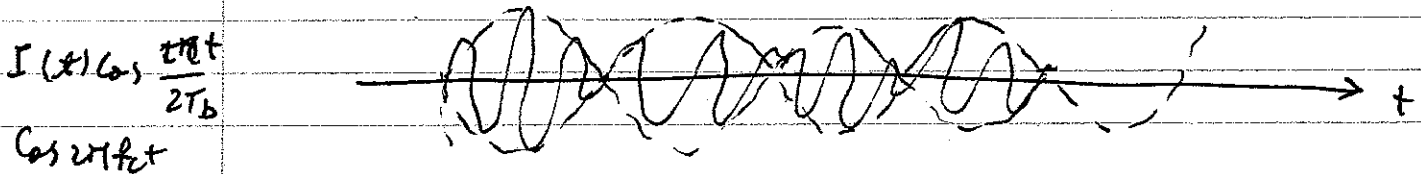
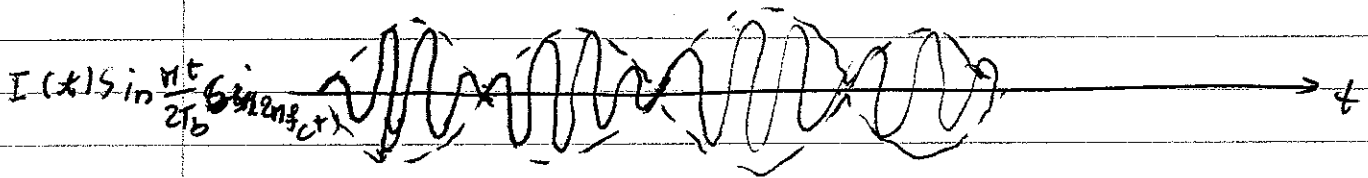
For MSK

of QPSK

We multiply $I(t)$ by $\sin(\frac{\pi t}{2T_b}) = \sin(\frac{\pi t}{T_s})$
and multiply $Q(t)$ by $\cos(\frac{\pi t}{2T_b})$



then modulate these



$P_e = Q(\sqrt{\frac{2E_b}{N_0}})$ the same as BPSK or QPSK.

Non-Coherent Communication

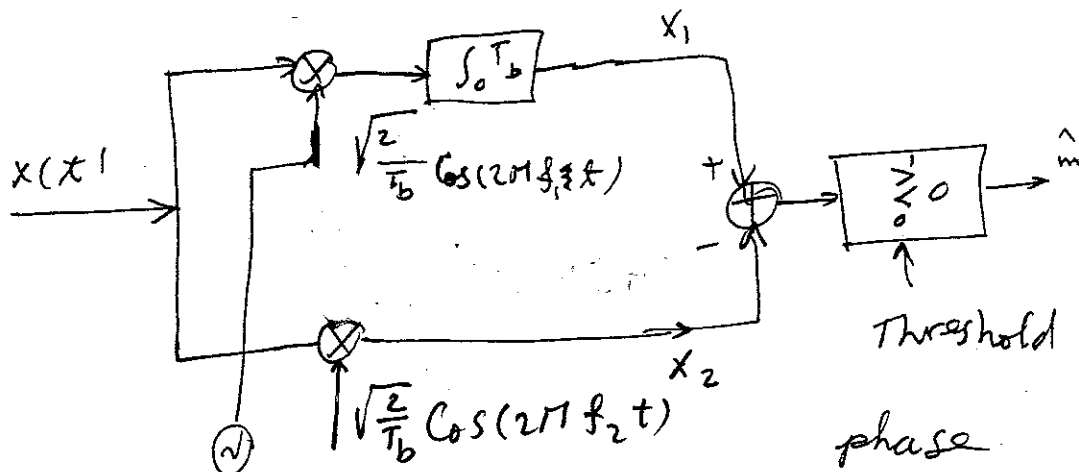
Non-Coherent BPSK

We have

$$S_i(x) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leq x \leq T_b \\ 0 & \text{elsewhere} \end{cases}$$

for $i=1, 2$

For coherent receiver



Say, the receiver reference is off by ϕ

then

$$x_1 = \frac{2}{T_b} \sqrt{E_b} \int_0^{T_b} \cos(2\pi f_1 t) \cos(2\pi f_1 t + \phi) dt$$

$$= \frac{2}{2T_b} \sqrt{E_b} \int_0^{T_b} \cos(\phi) dt + \frac{2}{2T_b} \sqrt{E_b} \int_0^{T_b} \cos(2\pi(2f_1)t) dt$$

$$= \sqrt{E_b} \cos \phi$$

and

$$x_2 = \int_0^{T_b} \frac{2}{T_b} \sqrt{E_b} \cos(2\pi f_2 t + \phi) \cos(2\pi f_1 t) dt$$

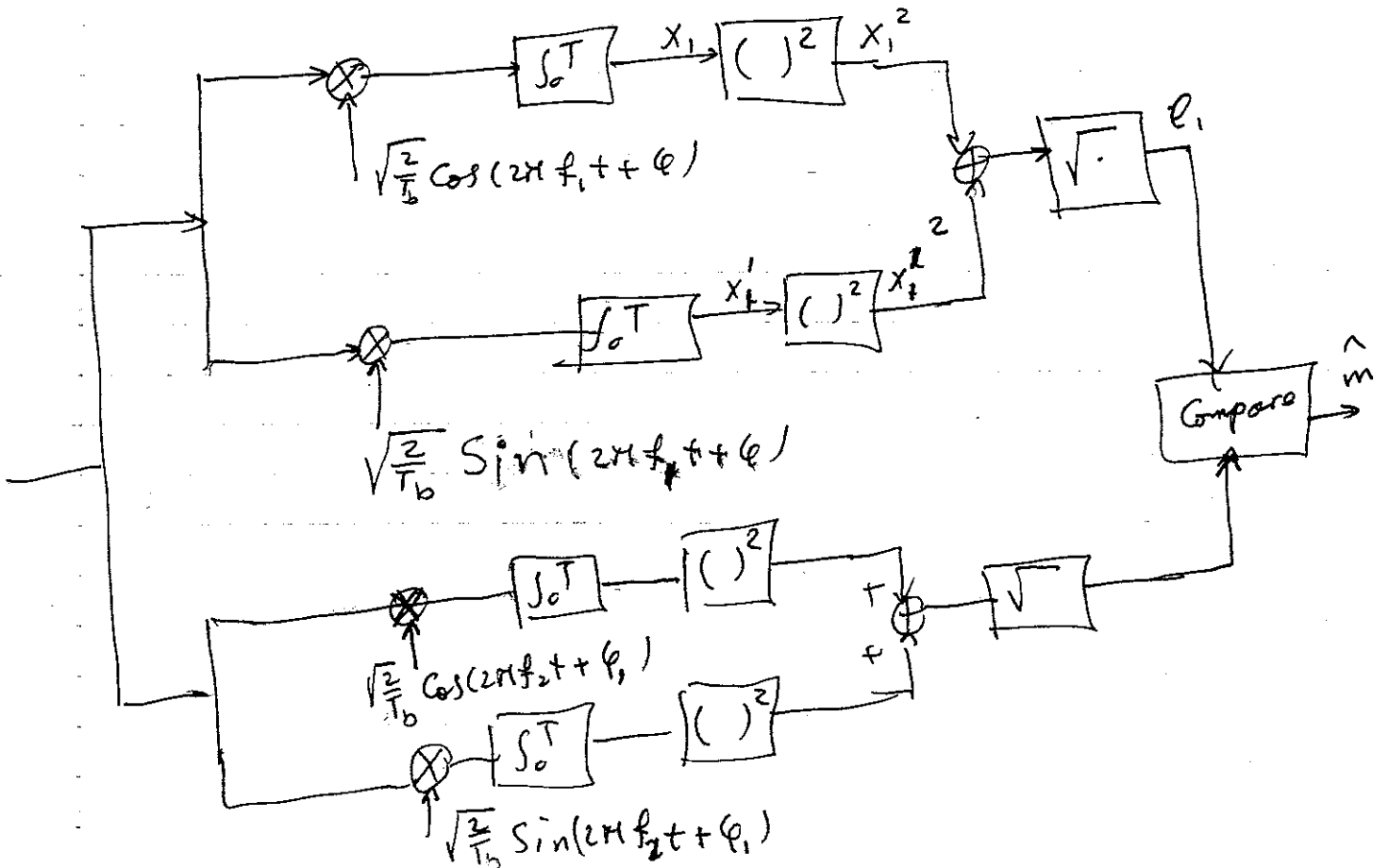
$$x_2 = \frac{\sqrt{E_b}}{T_b} \int_0^{T_b} \cos(2\pi (f_2 - f_1) t + \phi) dt + \frac{\sqrt{E_b}}{T_b} \int_0^{T_b} \cos(2\pi (f_2 + f_1) t + \phi) dt$$

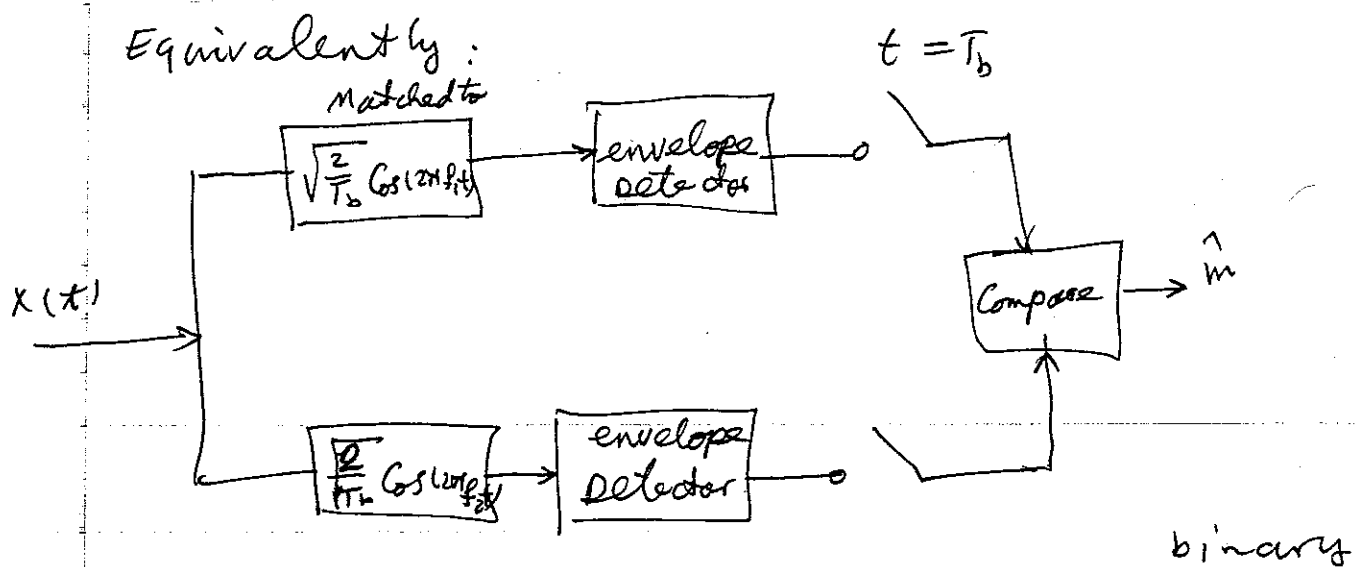
$$x_2 = \sqrt{E_b} \sin \phi \quad (\text{using the fact that } f_2 - f_1 = \frac{1}{T_b})$$

If ϕ is large x_1 can become smaller than x_2 resulting in erroneous decision.

Solution

use $\sqrt{x_1^2 + x_2^2}$ as the decision parameter





Bit error rate for non-coherent FSK

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$$

For coherent case, we had: ~~much worse than coherent~~

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \approx \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{E_b}{2N_0}\right)$$

Differential Phase Shift Keying (DPSK)

Instead of comparing against a reference phase, we compare with the signal in the previous signalling interval:

Assume the previous bit was "1" so:

$$s(t) = \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b$$

Now if the present bit is "1", we have

$$s^n(t) = \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) \quad T_b \leq t \leq 2T_b$$

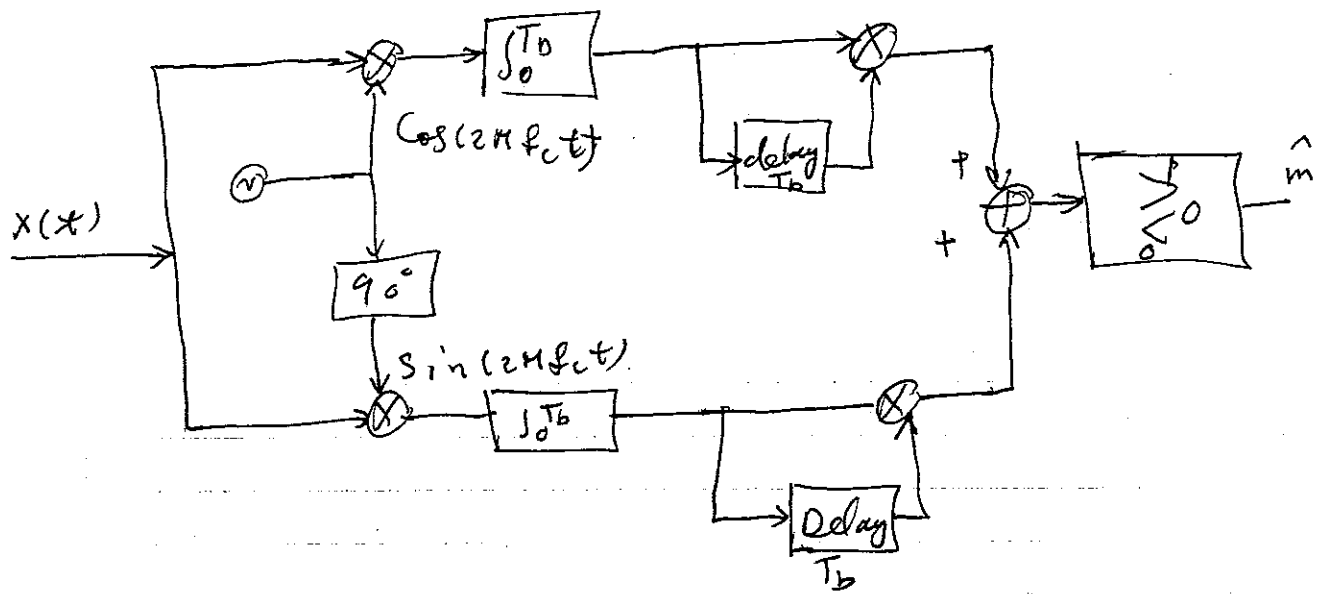
and if the present bit is "0", we have

$$s^n(t) = \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t)$$

So if, we correlate $s^n(t)$ (or noisy version of it $x^n(t)$) with the previously received signal, we get:

$$\frac{E_b}{2T_b} \int_0^{T_b} \cos^2(2\pi f_c t) dt = \begin{cases} E_b & \text{if "1"} \\ -E_b & \text{if "0"} \end{cases}$$

Alternatively, we can find the phase at time n w.r.t. a reference ~~phase~~ carrier and compare with that of the previous epoch.



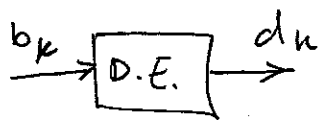
For this ~~mod~~ demodulation scheme

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$$

~~But~~ for coherent PSK, we had

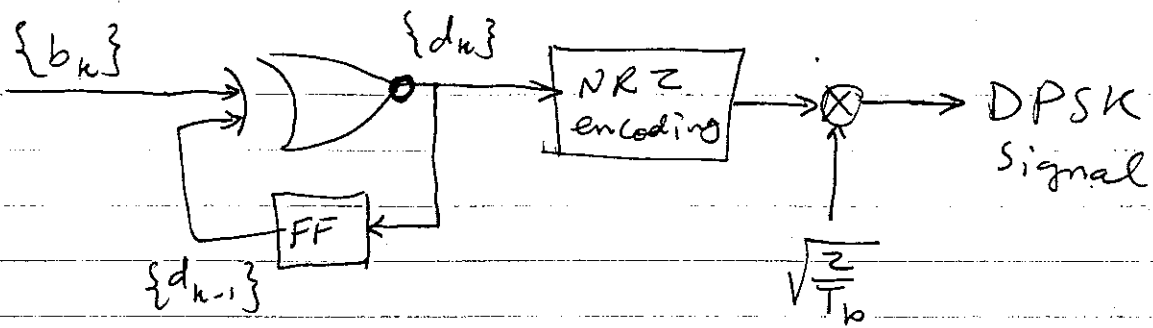
$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \approx \frac{1}{\sqrt{\pi}} \exp\left(-\frac{E_b}{N_0}\right)$$

Differential encoding:



if b_k is 1 d_k remains unchanged
 if b_k is 0 d_k is inverted

$$d_k = b_k \oplus d_{k-1}$$

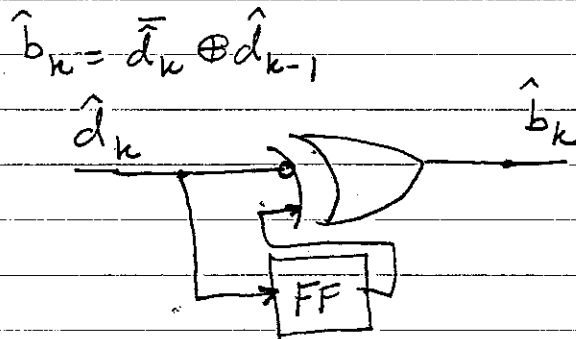


$\{b_k\}$ 1 0 0 1 0 0 1 1

$\{d_k\}$ 1 1 0 1 0 1 1 1

$\varphi_n = \text{phase}$ 0 0 π 0 0 π 0 0

At the receiver (for Coherent DPSK).



Summary (Comparison)

a) Coherent PSK $Q(\sqrt{\frac{2E_b}{N_0}})$
 " QPSK

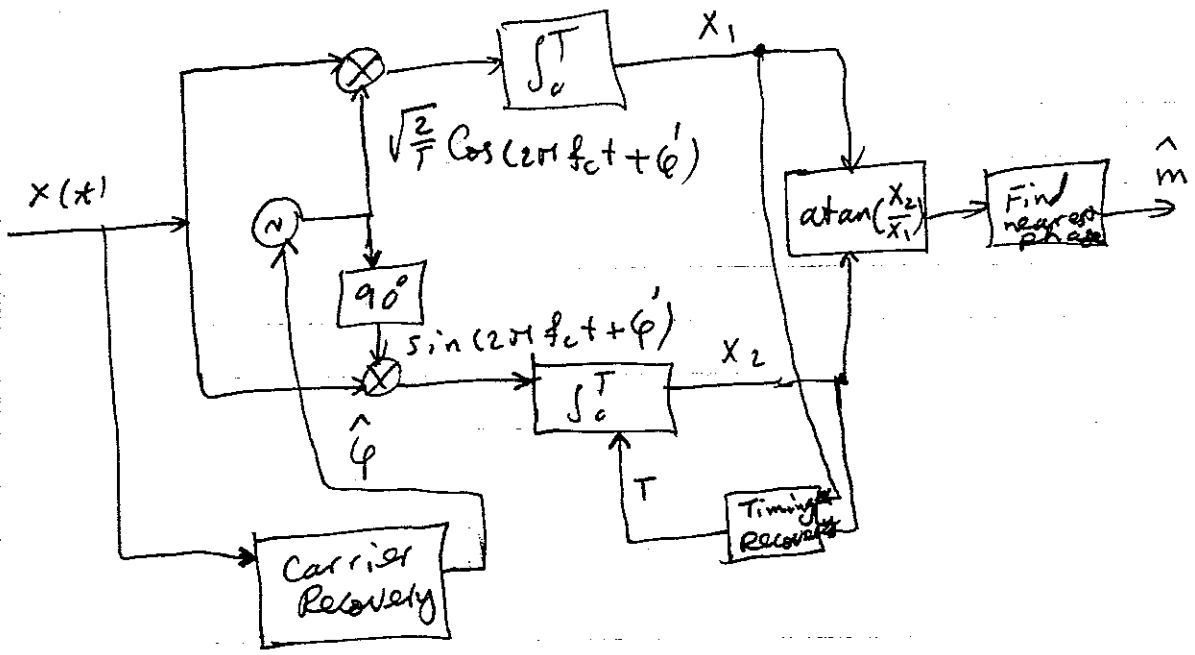
b) Coherent BFSK $Q(\sqrt{\frac{E_b}{N_0}})$

c) DPSK $\frac{1}{2} \exp(-\frac{2E_b}{N_0})$

d) Non-coherent BFSK $\frac{1}{2} \exp(-\frac{E_b}{N_0})$

7-14

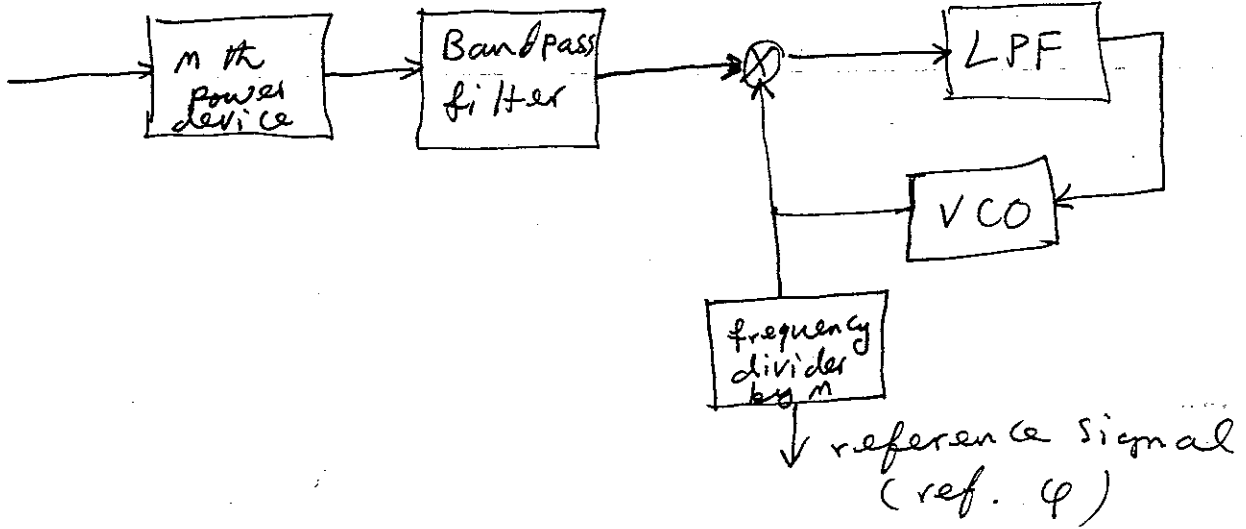
Synchronization:



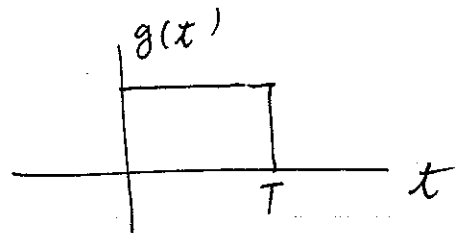
$$x(t) = A \cos(2\pi f_c t + \phi + \phi_{data}) + \text{noise}$$

$$\phi_{data} = 0, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \quad (\text{e.g., QPSK})$$

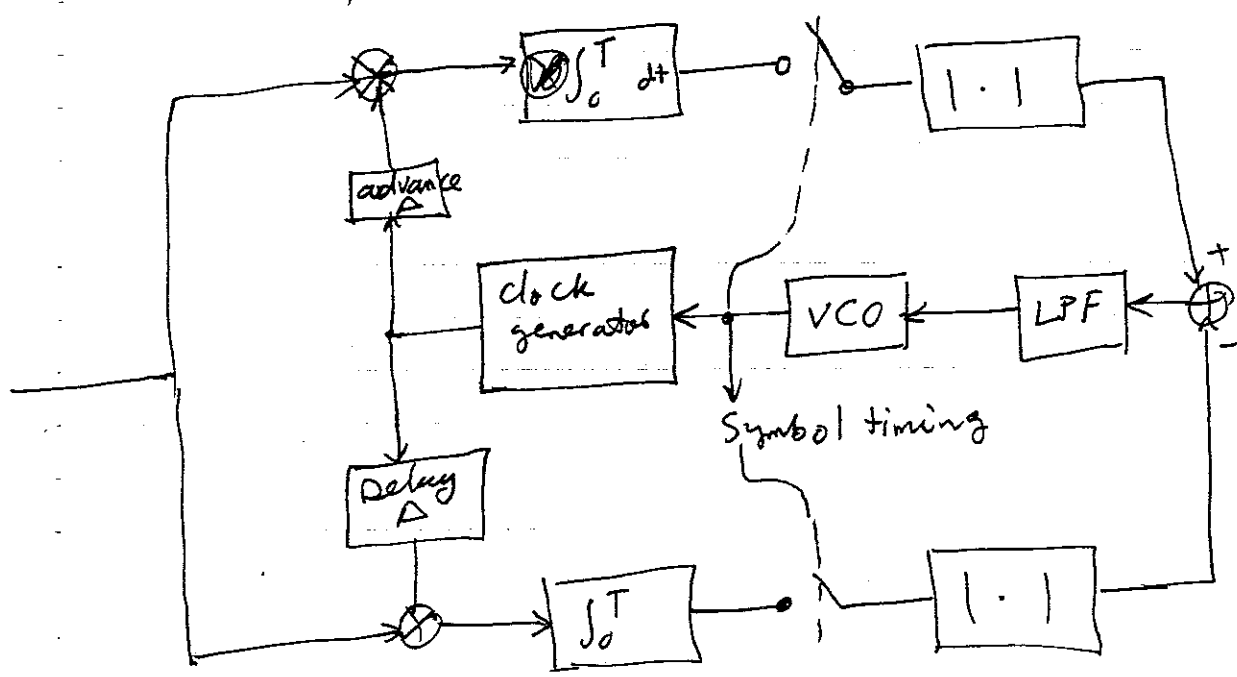
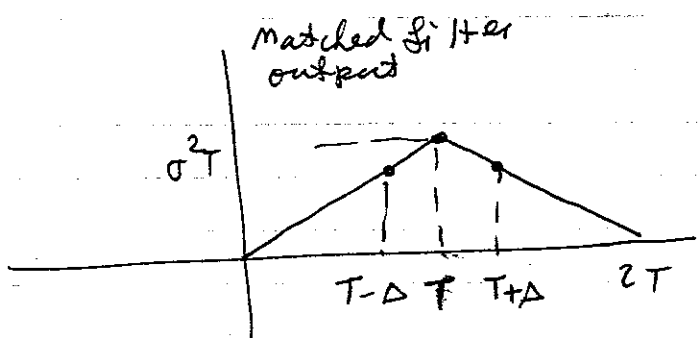
Carrier Recovery (Phase Estimation)



Timing Recovery (Bit Timing / Symbol Timing Estimation)



Matched filter
 $g(t) = h(T-t)$



Early - late gate

TABLE 9.1 Summary of formulas for the bit error probability P_e for different data transmission systems

	P_e
Coherent PSK	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
Coherent FSK (with 1-bit decoding)	$Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
MSK	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
QPSK	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
Noncoherent FSK	$\frac{1}{2} \exp\left(-\frac{E_b}{2N_0}\right)$
DPSK	$\frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$

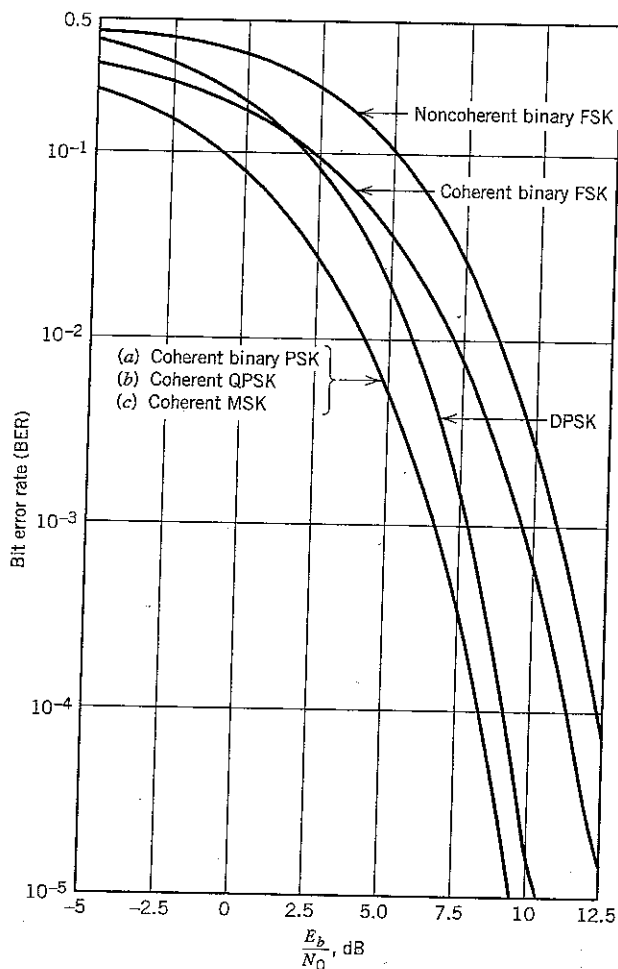


FIGURE 9.20 Comparison of the noise performances of different PSK and FSK systems.