

**Concordia University**  
**Department of Electrical and Computer Engineering**  
**ELEC6831/ELEC462: Digital Transmission I**  
**Final Exam**  
**April 12, 2012**

1) The output of a 256 level PCM system is transmitted using 16 QAM modulation with a raised cosine filter having a rolloff factor of 0.25 over a channel with a bandwidth of 2 MHz. What is the maximum message bandwidth for which the system works satisfactorily (7 Marks).

2) Find the output of a differential encoder if the input is 110011001010 (3 Marks).

3) A digital communication system uses QPSK to transmit data at a rate of 250 kbps. The bit error probability is  $10^{-8}$ . What would be the error probability if the bit rate is increased to 1 Mbps? (5 Marks) How much (how many dB's) should the power be increased if we would like to keep the same error probability of  $10^{-8}$ ? (3Marks)

4) Five audio channels each with a bandwidth of 20 kHz. are sampled at a 10% higher than the Nyquist rate and quantized using a 16 bit quantizer. These five channels are then time multiplexed.

- a. Find the overall bit rate of the output of multiplexer (4 Marks) and
- b. Find the minimum bandwidth required to transmit the output of the multiplexer when using 16-QAM with a raised cosine filter with a rolloff factor of 0.25 (2 Marks).

5) A (15, 5) cyclic code has the following generator polynomial:

$$g(X) = 1 + X + X^2 + X^5 + X^8 + X^{10}$$

a) Draw the diagram of the encoder for this code (3 Marks).

b) Find the codeword polynomial (in systematic form) for the message

$$m(X) = 1 + X^2 + X^4 \text{ (3 Marks).}$$

c) Is  $r(X) = 1 + X^4 + X^6 + X^8 + X^{14}$  a legitimate code polynomial for this code? Justify your answer (3 Marks).

6) A convolutional code with constraint length  $K = 3$  has generator polynomials

$$g_1(D) = 1, g_2(D) = D^2 + 1 \text{ and } g_3(D) = D^2 + D + 1.$$

a) Draw the encoder of this code (2 Marks).

b) Is the code systematic? Explain (1 Mark).

c) Draw the state diagram (1 Mark) and trellis of the code (2 Marks).

d) Find the output of the encoder if the input is 0010100 (2 Marks).

- 7) A convolutional code with constraint length  $K = 2$  has generator polynomials  $g_1(D) = 1$  and  $g_2(D) = D + 1$ .
- Draw the state diagram of the code (1 Marks).
  - Find the transfer function of the code and determine the minimum free distance of the code (3 Marks).
- 8) For  $m = 8$ , determine the Reed Solomon code that can correct 16 errors (3 Mark). What is the rate of the code? (1 Mark). What is the length of each codeword in bits (1 Mark).
- 9) A communication system uses QPSK using raised cosine filter with roll-off 0.5 to transmit a stream at the rate of 4 Mbps (Mega bits per second).
- What is the required bandwidth? (2 Marks)
  - What is the  $E_b / N_0$  required to achieve a bit error probability of  $10^{-6}$ ? (2 Marks)
  - If one uses a (127, 64) BCH code with  $t=10$ , what would be the required  $E_b / N_0$  to achieve the same probability of error of  $10^{-6}$ ? (3 Marks)
  - What is the coding gain at  $10^{-6}$  in dB? (2 Marks)
  - What is the required bandwidth for the coded system? (1 Mark)