

***ELEC 691X/498X – Broadcast Signal Transmission
Fall 2015***

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Office Hours: Wednesday, Thursday, 14:00 – 15:00

Time: Tuesday, 2:45 to 5:30

Room: H 411

Lecture 9: Link Budget

In this lecture we cover the following topics:

- Noise Figure and Noise Temperature of system.
- Link Budget Calculation for Terrestrial Systems.
- Link Budget Calculation for Satellite Systems

Lecture 9: Link Budget Calculation

In the last lecture, we learnt how to trace the power from the output of the transmitter to the HPA to the input of the receiver. That is, we discussed about how to add to the transmitted power all the gains and subtract all the losses in order to find the received power.

$$P_r = P_t + G_t + G_r - L_s.$$

Where all quantities have been translated into dB. The linear formula (non dB) will be,

$$P_r = \frac{P_t G_t G_r}{L_s L_o}$$

Knowing P_r we can find the $E_b = \frac{P_r}{R_b}$.

The Path Loss, or space loss, L_s is given as,

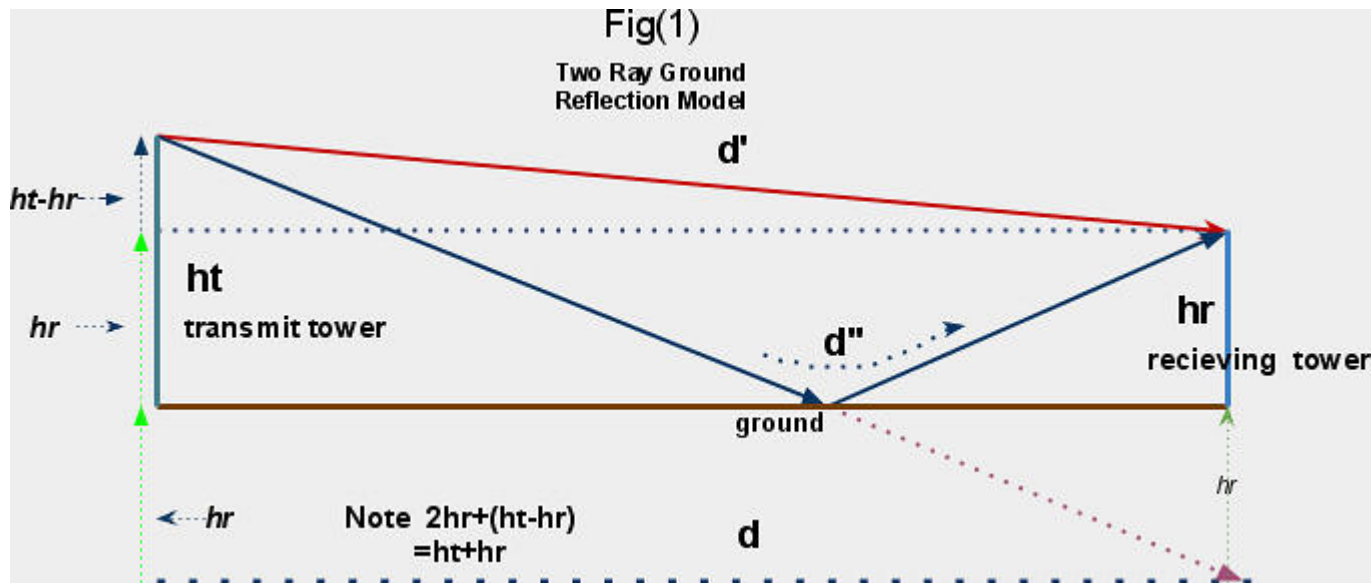
$$L_s = \left(\frac{4\pi d}{\lambda} \right)^2.$$

This means that path loss is increased by 20 dB per decade.

This formula is valid for the cases where electromagnetic wave travels through a single path from the transmitter antenna to the receiver antenna. This is the case

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when the antennas are located far above the ground or in the case of satellite links. This is called the Line of Sight (LOS) propagation model. In terrestrial broadcasting or communications, e.g., mobile telephony, the antennas, particularly, that of the receivers, are not well above the ground surface. In these cases, in addition to the direct signal, there will be waves reflected from the ground. A method commonly used to model this situation is the two-ray model.



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In this model, in addition to the direct ray, another wave reflected from the ground reaches the receiver antenna. For this model,

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}.$$

Note that here the loss is -40 dB per decade.

We also saw that $N_0 = k_B T$, where T is the noise spectral density at the receiver. We also showed that the overall noise temperature of a cascade of circuits with gains G_1, G_2, G_3, \dots and noise temperatures T_1, T_2, T_3, \dots is,

$$T_{eq} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots$$

Where the first term T_1 is the noise temperature of the first component in the cascade referred to the input.

In communications (including broadcasting) systems, the receiver antenna is connected to the first component in the receiver chain and the noise temperature of the antenna T_{ant} is given at its output. So, T_{ant} and T_1 are both at the input of the first stage of the cascade.

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Therefore, we have,

$$T_{eq} = T_{ant} + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots$$

or

$$T_{eq} = T_{ant} + T_{sys}$$

where,

$$T_{sys} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots$$

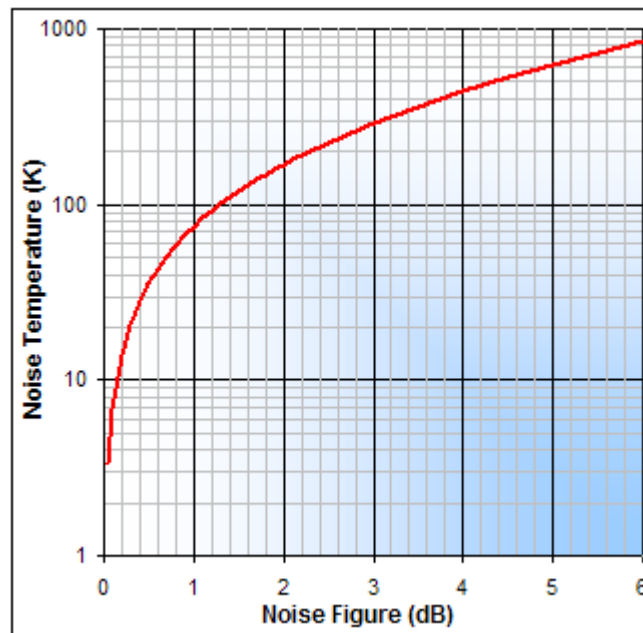
Another quantity we discussed was the noise factor F and we saw that the relationship between the noise factor F and the noise temperature T is, $F = 1 + \frac{T}{T_{in}}$ or $T = (F - 1)T_{in}$ where T_{in} is a reference noise temperature at the input of the amplifier. It is usually assumed to be 290 degrees Kelvin.

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Noise factor when represented in dB is called the Noise Figure (NF). That is, $NF = 10\log(F)$. So,

$$T = 290(F - 1) = 290\left(10^{\frac{NF}{10}} - 1\right).$$

$$NF = 10\log\left(\frac{T}{290} + 1\right)$$



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Explaining noise temperature, noise factor or noise figure is easy for the active components, i.e., those that amplify the signal. But for passive components the subject is sometimes puzzling.

Take a lossy line such as a coaxial cable. It attenuates the signal by a factor L . We say that the loss of the line is L . This can be considered as a gain less than unity $G=1/L$.

The signal at the input of the line will be attenuated by L and be $S_{out} = \frac{S_{in}}{L}$ at the end of the line. But the noise remains unchanged, $N_{out} = N_{in}$. So,

$$SNR_{out} = \frac{S_{in}/L}{N_{in}}$$

Therefore, $F = L$ or $T = 290(L - 1)$.

Example: A low noise amplifier with equivalent noise temperature of 30° K and power gain of 20 dB is connected with a microwave receiver with a noise figure of 25 dB. What is the overall noise temperature if the ambient temperature is 27° C.

Solution: $T_e = T_1 + \frac{T_2}{G_1}$ where $T_1 = 30^\circ$ C and $G_1 = 20$ dB = 100.

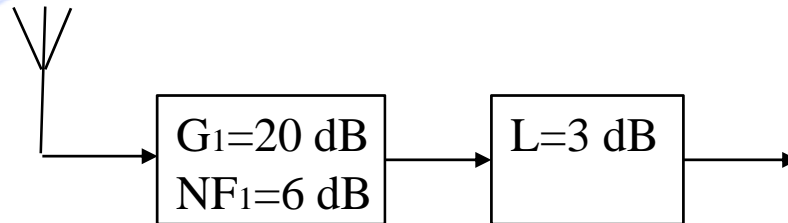
$NF_2 = 25$ dB. So, $F_2 = 10^{2.5} = 316.23$.

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$$T_2 = T_{ref}(F_2 - 1) = 300(316.23 - 1) \approx 94568.$$

$$\text{So } T_{eq} = 30 + \frac{94568}{100} = 975.68^\circ K.$$

Example: Assume that a preamplifier with a gain of 20 dB and noise figure of 6 dB is connected to a receiver with a cable with a 3 dB loss.



- Find the system equivalent noise temperature.
- Find the system equivalent noise temperature if the antenna is connected to the amplifier via the cable.

Solution: a) $T_{eq} = T_1 + \frac{T_2}{G_1}$ where $T_1 = 290(10^{0.6} - 1) \approx 870^\circ K$, $G_1 = 10^2 = 100$

and $T_2 = 290(L - 1) = 290(2 - 1) = 290$. So, $T_{eq} = 870 + \frac{290}{100} = 872.9^\circ K$.

b) $T_{eq} = T_2 + \frac{T_1}{1/L} = T_2 + T_1 L = 290 + 870 \times 2 = 2030^\circ K$.

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After finding the noise temperature T for the system, we can use the formula $N_0 = k_B T$ to find the noise spectral density. Having the E_b and N_0 , now we can have the performance of the system. That is given the type of modulation, we use the $\frac{E_b}{N_0}$ to find the bit error rate (BER).

A more common practice is to approach the problem from two different ways and see whether they match or not:

First: We start from the requirement, i.e., the BER required for a given application and for a given available bandwidth and bit rate, find the $\left(\frac{E_b}{N_0}\right)_{req}$. This is the $\frac{E_b}{N_0}$ required to achieve the desired performance.

Second: We start from the system components and find the actual $\frac{E_b}{N_0}$ as described in the previous slides. If this available $\frac{E_b}{N_0}$ exceeds the $\left(\frac{E_b}{N_0}\right)_{req}$ we are safe. That is the link performs as we wished. Else, we need to add something to increase the $\frac{E_b}{N_0}$. This may include increasing the transmitter power, size of the antennas, reducing the rate, etc. The difference between the $\frac{E_b}{N_0}$ and $\left(\frac{E_b}{N_0}\right)_{req}$ is called the link margin,

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Always remember that subtraction of this sort can only be used when the quantities are in dB and not ratio.

$$M = \frac{E_b}{N_0} - \left(\frac{E_b}{N_0} \right)_{req}$$

Example: Assume that a Television station is transmitting an HDTV signal with a bit rate of 15 Mbps in a 6 MHz. bandwidth using MPSK with roll-off factor 0.2. The requirement is that no more than one packet of TS be dropped in an hour. Assume that a packet is dropped if there is any error in it. The power of the transmitter is 30 W and the transmitter antenna gain is 15 dBi. The receiver antenna has a gain of 5 dBi and noise temperature 2500° K. The receiver has an LNB with NF=6 dB and $G_{LNA} = 20$ dB connected with a cable with 3 dB loss to an amplifier with 30 dB gain and NF=9 dB. The receiver, including the receiver front end and the demodulator, has a noise figure of 10 dB. Assuming a pointing loss of 0.5 dB and implementation loss of 1.5 dB. If the transmission frequency is 700 MHz., find the maximum distance the station can cover with the required quality: a) without FEC 2) with RS coding.

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Solution:

$$W = R_s(1 + \beta)$$

So,

$$6 = R_s(1 + 0.2) \text{ or } R_s = 5 \text{ M} \frac{\text{symbols}}{\text{sec}}$$

and

$$\log_2 M = \frac{R_b}{R_s} = 3 \text{ and } M = 8.$$

For 8PSK, we have,

$$BER \approx \frac{2}{3} Q \left(\sqrt{6 \frac{E_b}{N_0} \sin \frac{\pi}{8}} \right)$$

Let's now find the required BER:

The rate of the video is 15 Mbps. The length of each TS packet is $188 \times 8 = 1504$ bits. So, there are approximately 9973 packets per second. Requiring that we get one lost packet per hour means that the packet error probability should be less than 2.79×10^{-8} .

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If the BER is p then we need to have

$$1 - (1 - p)^{1504} \approx 1504p \leq 2.79 \times 10^{-8}$$

or $p \leq 1.85 \times 10^{-11}$.

From:

$$1.85 \times 10^{-11} \approx \frac{2}{3} Q \left(\sqrt{6 \frac{E_b}{N_0} \sin \frac{\pi}{8}} \right)$$

we get $\frac{E_b}{N_0} = 49 = 16.9 \text{ dB}$.

Now, we try to find P_r :

$$P_r = P_t + G_t + G_r - L_s - L_o = 14.77 + 15 + 5 - L_s - 0.5 - 1.5$$

So, $P_r = 32.77 - L_s$ and,

$$E_b = \frac{10^{3.277 - \frac{L_s}{10}}}{15 \times 10^6}$$

To find the noise temperature:

$$T_{eq} = T_{ant} + T_{LNA} + \frac{T_C}{G_{LNA}} + \frac{T_{AMP}}{G_{LNA} G_C} + \frac{T_{rec}}{G_{LNA} G_C G_{AMP}}$$

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Where

$$T_{ant} = 2500^\circ \text{ K.}$$

$$T_{LNA} = 290(10^{0.6} - 1) \approx 864.5^\circ \text{ K and } G_{LNA} = 100.$$

$$T_C = 290(10^{0.3} - 1) \approx 290^\circ \text{ K and } G_C = 0.5.$$

$$T_{AMP} = 290(10^{0.9} - 1) \approx 2013.55^\circ \text{ K and } G_{AMP} = 1000.$$

$$T_{rec} = 290(10 - 1) = 2610^\circ \text{ K.}$$

So,

$$T_{eq} = 2500 + 864.5 + \frac{290}{100} + \frac{2013.55}{100 \times 0.5} + \frac{2610}{100 \times 1000 \times 0.5} = 3408.9^\circ \text{ K.}$$

$$\text{Therefore, } N_0 = k_B T_{eq} = 1.38 \times 10^{-23} \times 3408.9 \approx 4.7 \times 10^{-20}.$$

Letting $\frac{E_b}{N_0} = 49$, we get,

$$\frac{E_b}{N_0} = \frac{10^{3.277 - \frac{L_S}{10}}}{15 \times 10^6 \times 4.7 \times 10^{-20}} = 49.$$

Or $L_S = 137.08 \text{ dB}$

Remember that $L_S = 10 \log \left(\frac{4\pi d}{\lambda} \right)^2$ for the LOS where $\lambda = \frac{3 \times 10^8}{700 \times 10^6} = \frac{3}{7} \text{ m.}$

Solving for d we get $d \approx 244 \text{ km.}$

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Now, let's use the (204,188) Reed Solomon code. We can do that by either increasing the bandwidth by a ratio of $204/188=1.085$, i.e., by 9% increase in bandwidth cost or a reduction in the rate of encoding by $188/204=0.92$ from 15 Mbps to 13.8 Mbps, or using a higher order modulation. Let's choose the latter option.

Now the overall bit rate will be $R_c = 15 \times \frac{204}{188} = 16.2766$ Mbps. Keeping the bandwidth 6 MHz. as before, we have,

$$6 \geq \frac{16.2766}{\log_2 M} (1 + 0.2).$$

So, $\log_2 M \geq 3.25$. Therefore, we have $M=16$. So, we use 16PSK

With RS coding, we will have a bad (erroneous) packet only if there are more than 8 bytes of error in a packet.

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At the rate of 15 Mbps there are 9973 packets per second. So, in order to comply with the performance requirement, we are allowed to have no more than one error every $3600 \times 9973 = 35904255$ packets or the packet error rate should be less than 2.79×10^{-8} .

Probability of error is

$$P_E = \sum_{i=9}^{204} \binom{204}{i} p_c^i (1 - p_c)^{204-i}$$

Approximating this expression by taking only the first term (assuming that p_c is very small),

$$P_E \approx \binom{204}{9} p_c^9 (1 - p_c)^{195} = 2.79 \times 10^{-8}$$

Solving this, we get $p_c \approx 3 \times 10^{-3}$.

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So, the required BER (before decoding) is $\approx 3.75 \times 10^{-4}$
 Substituting this in the formula for BER of 16 PSK, we get,

$$3.375 \times 10^{-4} = \frac{1}{2} Q \left(\sqrt{8 \frac{E_b}{N_0}} \sin \left(\frac{\pi}{16} \right) \right)$$

Solving for $\frac{E_b}{N_0}$ we get $\frac{E_b}{N_0} = 33.03$

Letting

$$\frac{E_b}{N_0} = \frac{10^{3.277 - \frac{L_s}{10}}}{\left(\frac{204}{188}\right) \times 15 \times 10^6 \times 4.7 \times 10^{-20}} = 33.03.$$

We get $L_s \approx 138.8$ dB.

$$L_s = 10 \log \left(\frac{4\pi d}{\lambda} \right)^2 = 138.8.$$

Solving for d , we get $d \approx 296$ km.
