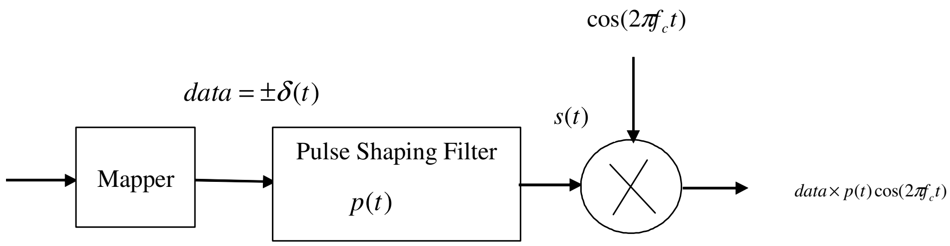
Problem BPSK: (Binary Phase-shift keying)

Consider a Binary antipodal system which produces data of "$-\delta(t)$" or "$+\delta(t)$" for binary "0" and "1" respectively. This data is passed to pulse shaping filter and the output of the pulse shaping filter is multiplied by $\cos(2\pi f_c t)$ to have a pass-band spectrum in the channel (up-conversion).



a) Evaluate time domain and frequency domain representation of transmitted signal.
Why pass-band communication is required? What is the signal bandwidth at the transmitter output if Square-Root-Raised Cosine (SQRC) filter is used as pulse shaping filter?

b) What is the optimum receiver structure? What is the transfer function of matched filter? Discuss the probability of error for the pass-band system.

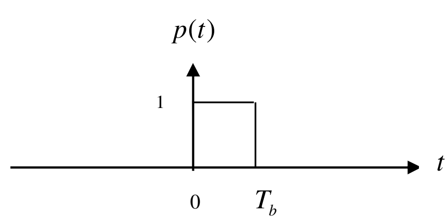
c) If the phase of the carrier in the transmitter and the receiver are not the same, what happens? Can we recover the data in this case?

Solution:

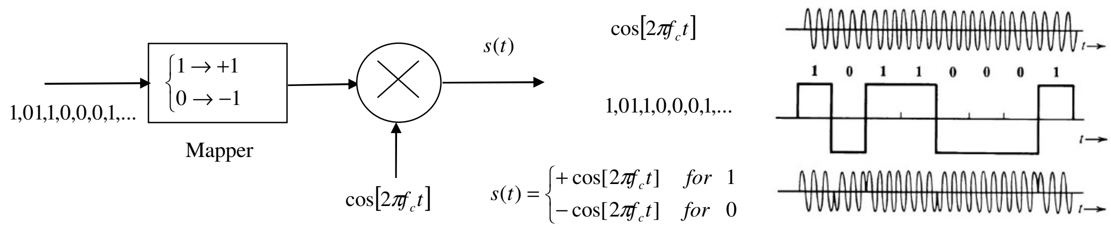
Part a)

The transmitter is transmitting either $p(t)\cos(2\pi f_c t)$ or $-p(t)\cos(2\pi f_c t)$ or in other words it is transmitting either $p(t)\cos(2\pi f_c t)$ or $p(t)\cos(2\pi f_c t + \pi)$. This is why we call this system "binary phase shift keying" (BPSK). We either send the cosine with phase of "0" or with a phase of π.

Consider a pulse shaping filter of $p(t)$ as shown below:



Then, the transmitter structure and output of transmitter in time domain is as shown below:



The signal at the output of the transmitter in the frequency domain is:
$0.5P(f + f_c) + 0.5P(f - f_c)$

There are two reasons for having passband communication:

- To divide the available bandpass bandwidth between several users.
- To have small size antennas due to having high frequencies.

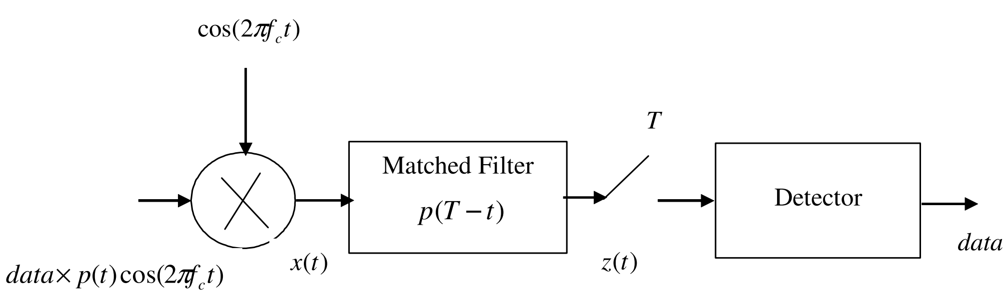
The bandwidth of the signal at the output of the passband transmitter is:

$$BW_{signal} = (1+\beta)\frac{1}{T}$$

which is twice the baseband bandwidth.

Part b)

Optimum receiver is as follows:



Receiver

Therefore, the above is the optimum bandpass receiver.

$$x(t) = data \times p(t)\cos^2(2\pi f_c t) = data \times p(t)\left\{\frac{1+\cos(4\pi f_c t)}{2}\right\} = \frac{data \times p(t)}{2} + \frac{data \times p(t)\cos(4\pi f_c t)}{2}$$

Since matched filter is a law pass filter we have:

$$z(t) = \frac{data}{2} \times p(t) * p(T-t)$$

Therefore, pass band system is equivalent to baseband system.

The matched filter has the same transfer function as pulse shaping filter and therefore it is SQRC.

The probability of error of the passband system is the same as baseband system and therefore the BER would be as calculated before:

$$BER(BPSK) = Q\sqrt{\frac{2E_b}{N_0}}$$

Part c)

Assume a phase difference of φ,

$$x(t) = data \times p(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \varphi)$$

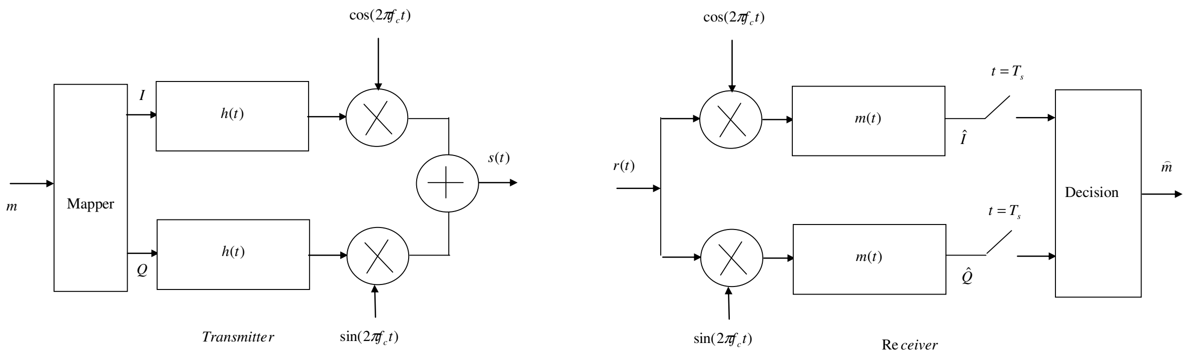
$$= data \times p(t)\left\{\frac{\cos(4\pi f_c t + \varphi) + \cos(\varphi)}{2}\right\} = \frac{data \times p(t)\cos(\varphi)}{2} + \frac{data \times p(t)\cos(4\pi f_c t + \varphi)}{2}$$

Since matched filter is a law pass filter we have:

$$z(t) = \frac{data}{2} \times p(t) * p(T-t) \times \cos(\varphi)$$

Since φ is a random number between 0 and 2π, we cannot make a decision using maximum likelihood detector. To be able to recover the data, we should first find φ.

Two dimensional modulation schemes:

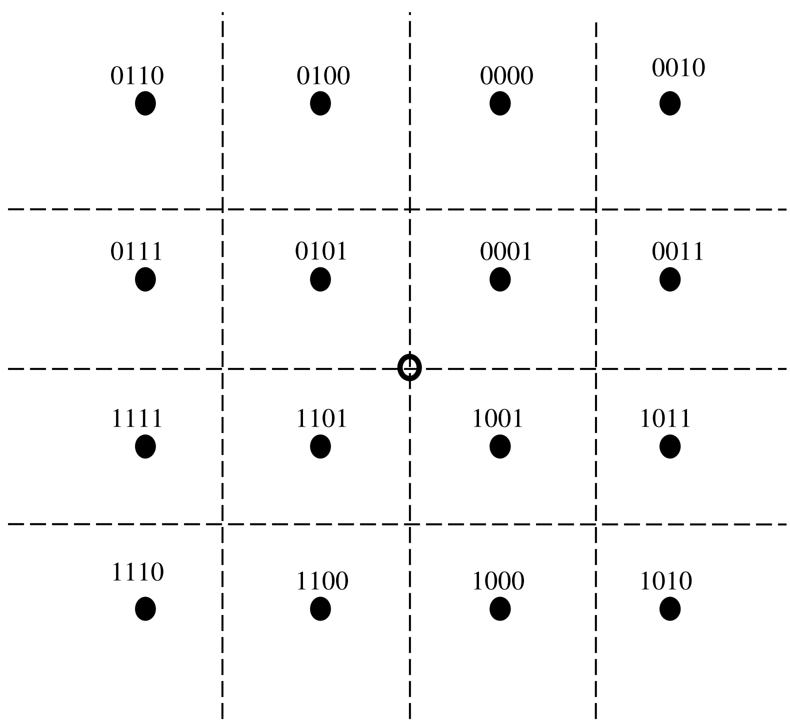


Bandwidth:

BW= $(1+\beta)\frac{1}{T_s} = (1+\beta)R_s$

The constellation diagram of 16QAM is as follows:

Let's consider the 16QAM system below, where adjacent points (horizontally or vertically) have equal distance from each other. Decision boundaries for maximum likelihood detection are shown by dashed lines.



$$P_E(16QAM) = 4\left(\frac{\sqrt{16}-1}{\sqrt{16}}\right)Q\left(\sqrt{\left(\frac{3}{16-1}\right)\frac{E_s}{N_0}}\right) = 3Q\left(\sqrt{\frac{E_s}{5N_0}}\right)$$

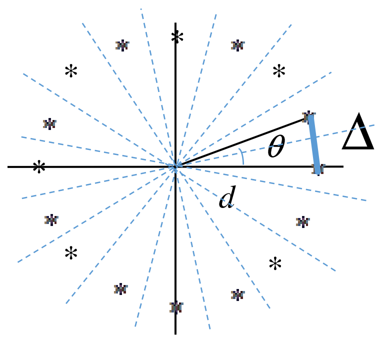
$$P_B(16QAM) = \frac{P_E(16QAM)}{\log_2 16} = \frac{3}{4}Q\left(\sqrt{\frac{E_s}{5N_0}}\right)$$

	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
I	1	1	3	3	-3	-3	-1									
Q	3	1	1	3	3	1	1									

The decision block works based on decision regions.

The constellation diagram of 16PSK is as follows:

Let's consider the 16PSK system below, where adjacent points have equal angular distance. Decision boundaries for maximum likelihood detection are shown by dashed lines.



$\theta = \pi/8$

For 16PSK, we can use Gray coding as shown in the constellation diagram and therefore:

$$P_b = \frac{P_E}{\log_2 16} = \frac{1}{2}Q\left(\sqrt{\frac{2E_s}{N_0}}\sin(\pi/16)\right) = \frac{1}{2}Q\left(\sqrt{\frac{2\times 4E_b}{N_0}}\sin(\pi/16)\right) = \frac{1}{2}Q\left(\sqrt{\frac{2\times 4P_r}{N_0 R_b}}\sin(\pi/16)\right)$$

Answer: $P_b = \frac{1}{2}Q\left(\sqrt{\frac{8P_r}{N_0 R_b}}\sin(\pi/16\right)$

Gray coding is obvious. If $s_1, s_2, s_3, \ldots, s_{16}$ are points in the constellation diagram in clock-wise order, then Gray coding can be shown in the mapper as follows:

	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
I	1	x	0.7	y	0	-y	-0.7	-x	-1	-x	-0.7	-y	0	y	0.7	x
Q	0	y	0.7	x	1	x	0.7	y	0	-y	-0.7	-x	-1	-x	-0.7	-y

$x = \cos(\pi/8)$ $y = \sin(\pi/8)$

The decision block first evaluates the phase of the received vector $\varphi_r = \tan^{-1}\left(\frac{Q_r}{I_r}\right)$ and then decisions are made as follows:

If $\frac{(i-1)\pi}{16} < \varphi_r < \frac{(i+1)\pi}{16}$, decide that s_i has been transmitted. (for $i = 1,2,3,..,16$)

Problem: Design of Communication Systems

A stream of digital data with data rate of 12 Mb/s is to be transmitted in an additive white Gaussian noise channel with power spectral density N_0 of 2×10^{-10} Watts/Hz where the available bandwidth is 4 MHz. The required system performance is a bit error rate of 10^{-6}.

Consider MQAM or MPSK and design the best passband transmitter and receiver by determining all the parameters of the design (including the received power) and drawing a system block diagram.

Solution:

assume $r = 0.3 \;\Rightarrow\; M_{DSB} = (1+r)R_s = (1+0.3)\dfrac{R_b}{\log_2 M} = 1.3\times\dfrac{12\times10^6}{\log_2 M} \le 4\times10^6\,Hz$

$\log_2 M \ge 3.9$, so we can choose $M = 16$ and corresponding schemes are 16-QAM or 16-PSK.

$$P_E(16QAM) = 4\left(\frac{\sqrt{16}-1}{\sqrt{16}}\right)Q\left(\sqrt{\left(\frac{3}{16-1}\right)\frac{E_s}{N_0}}\right) = 3Q\left(\sqrt{\frac{E_s}{5N_0}}\right)$$

$$P_B(16QAM) = \frac{P_E(16QAM)}{\log_2 16} = \frac{3}{4}Q\left(\sqrt{\frac{E_s}{5N_0}}\right)$$

$$P_B(16QAM) = 10^{-6} = \frac{3}{4}Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)$$

$$1.33\times10^{-6} = Q\left(\sqrt{\frac{2E_b}{5\times10^{-10}}}\right)$$

From Q-function table $x = \sqrt{\dfrac{2E_b}{5\times10^{-10}}} = 4.7$ and therefore:

$$\Rightarrow E_b = 5.5225\times10^{-9}$$
$$\Rightarrow P_r = E_b\cdot R_b = 5.5225\times10^{-9}\times12\times10^6 = 66.27mW$$

$P_E(16PSK) = P_E(M) \approx 2Q\left(\sqrt{\dfrac{2E_s}{N_0}}\sin\dfrac{\pi}{M}\right)$ and $P_B = \dfrac{P_E}{\log_2 M}$ (Gray coded)

$$\Rightarrow P_B = \frac{P_E}{\log_2 16} = \frac{1}{2}Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{16}\right) = 10^{-6}$$

From Q-function table:

$$\Rightarrow x = \sqrt{\frac{2E_s}{N_0}}\cdot\sin\frac{\pi}{16} = 4.6 \;\Rightarrow\; \frac{E_s}{N_0} = 277.98 \;\Rightarrow\; E_s = 277.98\times2\times10^{-10} = 5.56\times10^{-8}$$

$$\Rightarrow P_r = E_bR_b = \frac{E_s}{\log_2 M}R_b = \frac{5.56\times10^{-8}}{\log_2 16}\times12\times10^6 = 166.8mW$$

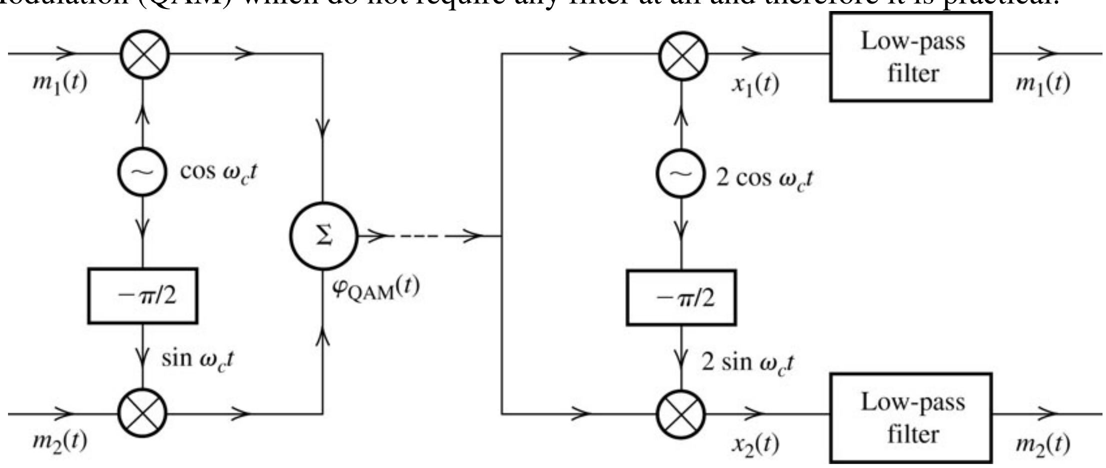
Therefore,16-QAM is better, since the received power is $1/3$ of 16-PSK.

Problem (Analog Communication)

The problem of the structure of problem B.2 is the use of ideal filters. We would like to design another transmitter which uses no filters. This kind of modulator is called Quadrature-Amplitude-Modulator (QAM). Draw the structure of this new transmitter to transmit the information about $m_1(t)$ and $m_2(t)$ such that the output signal $tx\_out(t)$ has the same center frequency f_c and the same bandwidth as of problem B.2. Show that your design works by showing the structure of the receiver and explaining the recovery of both signals $m_1(t)$ and $m_2(t)$. Draw the frequency spectrum of all the signals in the transmitter and the receiver as well.

Solution:

Single sideband signals are hard to generate since as shown in part "a" of this problem ideal filters are required. To transmit two signals $m_1(t)$ and $m_2(t)$, we can use Quadrature Amplitude Modulation (QAM) which do not require any filter at all and therefore it is practical.



The QAM transmitter output is $\varphi_{QAM}(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$ and the carrier frequency is $f_c = 545\ kHz$. To find the bandwidth and center frequency of $\varphi_{QAM}(t)$, we should examine two signals $m_1(t)\cos(2\pi\times545t)$ and $m_2(t)\sin(2\pi\times545t)$. The base-band bandwidth of $m_1(t)$ and $m_2(t)$ is 5kHz and as we know the bandwidth of each of the Double –Side-Band modulated signals $m_1(t)\cos(2\pi\times545t)$ and $m_2(t)\sin(2\pi\times545t)$ would be double of that which is 10kHz. The center frequency of each of these signals would be the carrier frequency of 545kHz.

Therefore, the transmitted signal $\varphi_{QAM}(t) = m_1(t)\cos(2\pi\times545t) + m_2(t)\sin(2\pi\times545t)$ will have center frequency of 545kHz with bandwidth of 10kHz.

To recover $m_1(t)$ and $m_2(t)$, coherent QAM receiver as shown above could be used. This can be proved by calculating $x_1(t)$ and $x_2(t)$ based on the structure of the receiver.

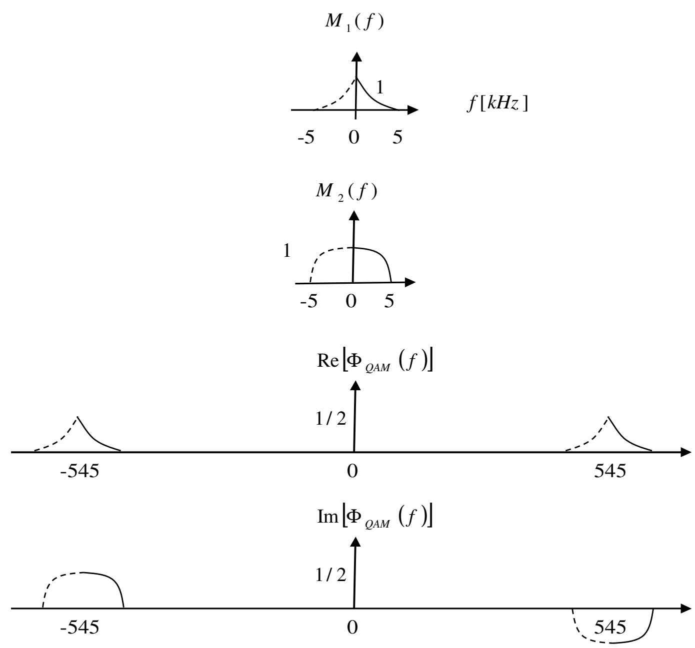
$$x_1(t) = 2\varphi_{QAM}(t)\cos(2\pi\times545t) = 2[m_1(t)\cos(2\pi\times545t) + m_2(t)\sin(2\pi\times545t)]\cos(2\pi\times545t)$$

$$= m_1(t) + m_1(t)\cos(2\pi\times1090t) + m_2(t)\sin(2\pi\times1090t)$$

$$x_2(t) = 2\varphi_{QAM}(t)\sin(2\pi\times545t) = 2[m_1(t)\cos(2\pi\times545t) + m_2(t)\sin(2\pi\times545t)]\sin(2\pi\times545t)$$

$$= m_2(t) - m_2(t)\cos(2\pi\times1090t) + m_1(t)\sin(2\pi\times1090t)$$

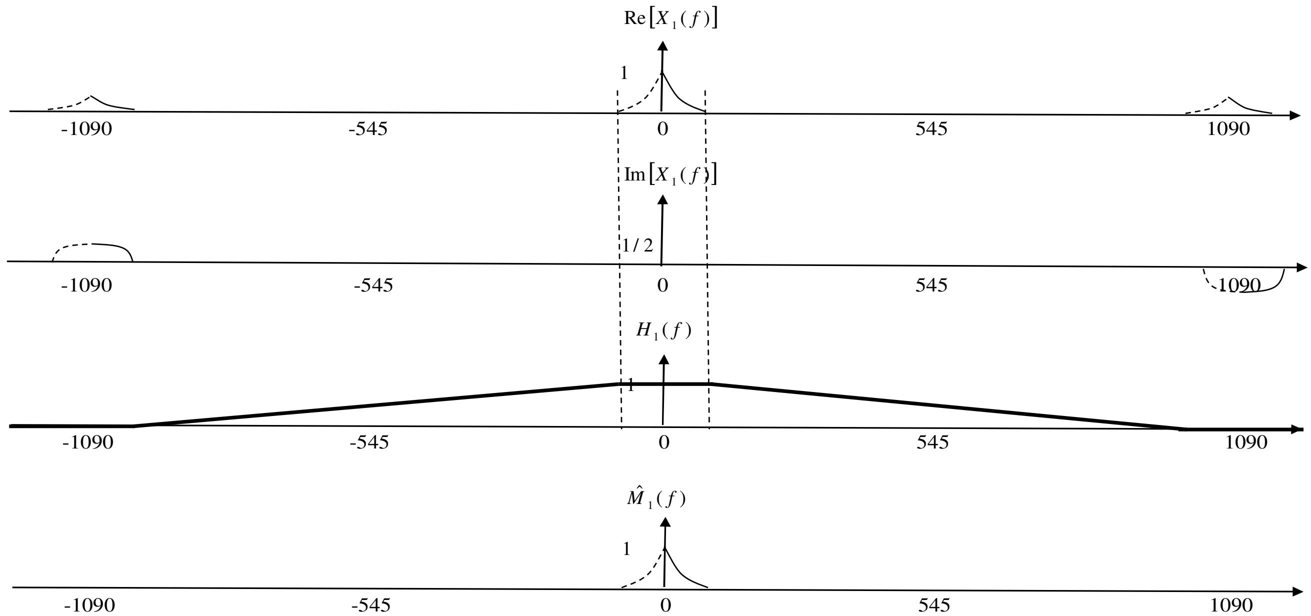
The last two terms in both equations are bandpass signals centered at $2f_c = 1090kHz$ which are suppressed by the lowpass filter and therefore, the output of these filters will recover $m_1(t)$ and $m_2(t)$.
Note that these low-pass filters are not ideal.
Frequency spectrum of all the signals are provided below:

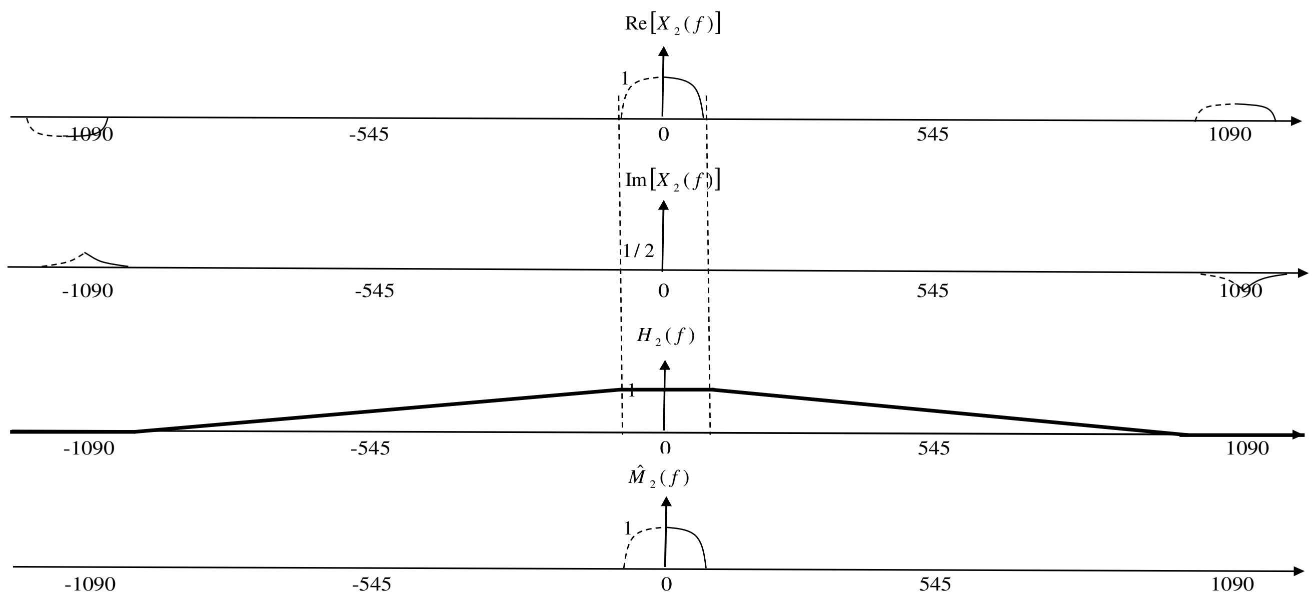


We note that the transmitted frequency spectrum is complex. Therefore,

$$\Phi_{QAM}(f) = \mathrm{Re}[\Phi_{QAM}(f)] + j\,\mathrm{Im}[\Phi_{QAM}(f)]$$

Considering that low pass filters in the receiver have transfer functions of $H_1(f)$ and $H_2(f)$, we can show the frequency spectrum of all the signals in the receiver and also come up with transfer functions of these two filters.





Based on the above two lowpass filters are identical with pass band of 0 to 5Kz, stop-band of larger than 1085 KHz and transition bands of 5KHz to 1085 KHz. The half amplitude cut-off frequency is 545 KHz.
Note that using these filters and assuming that there is no noise in the channel, we have:

$$\hat{M}_1(f) = M_1(f) \Rightarrow \hat{m}_1(t) = m_1(t)$$

$$\hat{M}_2(f) = M_2(f) \Rightarrow \hat{m}_2(t) = m_2(t)$$