ASSIGNMENT 1

(1) In a specified 6-AM-to-6-AM 24-hour period, a student wakes up at time t_1 and goes to sleep at some later time t_2.
   (a) Find the sample space and sketch in on the $x - y$ plane if the outcome of this experiment consists of the pair(t_1, t_2).
   (b) Specify the set A and sketch the region on the plane corresponding to the event "student is asleep at noon."
   (c) Specify the set B and sketch the region on the plane corresponding to the event "student sleeps through breakfast (7-9 AM)."
   (d) Sketch the region corresponding to $A \cap B$ and describe the corresponding event in words.

(2) A random experiment has sample space $S = a, b, c, d$. Suppose that $P[c, d] = 3/8$, $P[b, c] = 6/8$, and $P[d] = 1/8$. Use the axioms of probability to find the probabilities of the elementary events.

(3) Find the probabilities of the following events in terms of $P[A]$, $P[B]$, $P[A \cap B]$:
   (a) A accurs and B does not occur; B accurs and A does not occur.
   (b) Exactly one of A or B occurs.
   (c) Nither A nor B occur.

(4) Use Corollary 7 to prove the following:
$$(a) P[A \cup B \cup C] \leq P[A] + P[B] + P[C].$$
$$(b) P\left[\bigcup_{k=1}^{n} A_k\right] \leq \sum_{k=1}^{n} P[A_k]$$
$$(c) P[\bigcap_{k=1}^{n} A_k] \geq 1 - \sum_{k=1}^{n} P[A_k^c]$$

The second expression is called the **union bound**.

(5) Let p be the probability that a single character appears incorrectly in this book. Use the union bound for the probability of there being any errors in a page with n characters.

(6) A park has N raccoons of which eight were previously captured and tagged. Suppose that 20 raccoons are captured. Find the probability that four of these are found to be tagged. Denote this probability, which depends on N, by $p(N)$. Find the value of N that maximizes this probability. *Hint:* Compare the ratio $p(N)/p(N-1)$ to unity.