Assignment 10

1. Let $Y(t) = X(t+d) - X(t)$, where $X(t)$ is a Gaussian random process.

   (a) Find the mean and autocovariance of $Y(t)$.

   (b) Find the pdf of $Y(t)$.

   (c) Find the joint pdf of $Y(t)$ and $Y(t+s)$.

   (d) Show that $Y(t)$ is a Gaussian random process.

2. Let $X(t)$ be a zero-mean Gaussian random process with autocovariance function given by $C_X(t_1, t_2)$. If $X(t)$ is the input to a "square law detector," then the output is

   $$Y(t) = X(t)^2$$

   Find the mean and autocovariance of the output $Y(t)$.

3. Let $Y(t) = X^2(t)$, where $X(t)$ is the Wiener process.

   (a) Find the pdf of $Y(t)$.

   (b) Find the conditional pdf of $Y(t_2)$ given $Y(t_1)$.

4. Let $Z(t) = X(t) - aX(t-s)$, where $X(t)$ is the Wiener process.

   (a) Find the pdf of $Z(t)$.

   (b) Find $m_Z(t)$ and $C_Z(t_1, t_2)$.

5. Let $X(t)$ be defined by

   $$X(t) = A\cos\omega t + B\sin\omega t,$$

   Where A and B are iid random variables.

   (a) Under what conditions is $X(t)$ wide-sense stationary?

   (b) Show that $X(t)$ is not stationary. *hint*: Consider $E[X^3(t)]$.

6. Let $X(t)$ and $Y(t)$ be independent, wide-sense stationary random processes with zero means and the same covariance function $C_X(\tau)$. Let $Z(t)$ be defined by

$$Z(t) = 3X(t) - 5Y(t)$$

(a) Determine whether $Z(t)$ is also wide-sense stationary.

(b) Determine the pdf of $Z(t)$ if $X(t)$ and $Y(t)$ are also jointly Gaussian zero-mean random processes with $C_X(\tau) = 4e^{-|\tau|}$.

(c) Find the joint pdf of $Z(t_1)$ and $Z(t_2)$ in part b.

(d) Find the cross-covariance between $Z(t)$ and $X(t)$. Are $Z(t)$ and $X(t)$ jointly stationary random processes?

(e) Find the joint pdf of $Z(t_1)$ and $X(t_2)$ in part b. *Hint*: Use auxilliary variables.