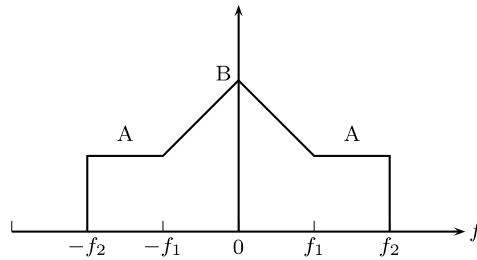


# Assignment 11

1. (a) Find the autocorrelation function corresponding to the power spectral density shown in the following figure.
- (b) Find the total average power.
- (c) Plot the power in the range  $|f| > f_0$  as a function of  $f_0 > 0$ .



2. Show that
  - (a)  $R_{X,Y}(\tau) = R_{Y,X}(-\tau)$ .
  - (b)  $S_{X,Y}(f) = S_{Y,X}^*(f)$ .
3. Let  $R_X(k) = 4(\alpha)^{|k|} + 16(\beta)^{|k|}, \alpha < 1, \beta < 1$ .
  - (a) Find  $S_X(f)$ .
  - (b) Plot  $S_X(f)$  for  $\alpha = \beta = 0.5$  and  $\alpha = 0.75 = 3\beta$  and comment on the effect of value of  $\alpha/\beta$ .
4. Let  $D_n = X_n - X_{n-d}$ , where  $d$  is an integer constant and  $X_n$  is a zero-mean, WSS random process.
  - (a) Find  $R_D(k)$  and  $S_D(f)$  in terms of  $R_X(k)$  and  $S_X(f)$ . What is the impact of  $d$ ?
  - (b) Find  $E[D_n^2]$ .

5. Let  $X(t)$  be a differentiable WSS random process, and define

$$Y(t) = \frac{d}{dt}X(t)$$

Find an expression for  $S_Y(f)$  and  $R_Y(\tau)$ . *Hint:* For this system,  $H(f) = j2\pi f$ .

6. Let  $Y(t)$  be a short-term integration of  $X(t)$ :

$$Y(t) = \frac{1}{T} \int_{t-T}^t X(t') dt'.$$

- (a) Find the impulse response  $h(t)$  and the transfer function  $H(f)$ .  
 (b) Find  $S_Y(f)$  in terms of  $S_X(f)$ .
7. In problem 6, Let  $R_X(\tau) = (1 - |\tau|/T)$  for  $|\tau| < T$  and zero elsewhere.
- (a) Find  $S_Y(f)$ .  
 (b) Find  $R_Y(\tau)$ .  
 (c) Find  $E[Y^2(t)]$ .
8. The input into a filter is zero-mean white noise with noise power density  $N_0/2$ . The filter has transfer function
- $$H(f) = \frac{1}{1 + j2\pi f}$$
- (a) Find  $S_{Y,X}(f)$  and  $R_{Y,X}(\tau)$ .  
 (b) Find  $S_Y(f)$  and  $R_Y(\tau)$ .  
 (c) What is the average power of the output?
9. (a) A WSS Gaussian random process  $X(t)$  is applied to two linear systems as shown in the following figure. Find an expression for the joint pdf of  $Y(t_1)$  and  $W(t_2)$ .  
 (b) Evaluate part a if  $X(t)$  is white Gaussian noise.

