Assignment 12

1. Let M_n denote the sequence of sample means form an iid random process X_n:

$$M_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

   (a) Is M_n a Markov process?

   (b) If the answer to part a is yes, find the following state transition pdf: $f_{M_n}(X|M_{n-1} = y)$.

2. (a) Show that the following autoregressive process is a Markov process:

$$Y_n = rY_{n-1} + X_n \qquad\qquad Y_0 = 0$$

   where X_n is an iid process.

   (b) Find the transition pdf if X_n is an iid Gaussian sequence.

3. Let X_n be an iid random process. Show that X_n is a Markov process and give its one-step transition probability matrix.

4. A very popular barbershop is always full. The shop has two barbers and three chairs for waiting, and as soon as a customer completes his service and leaves the shop, another enters the shop. Assume the mean service time is m.

   (a) Use Little's formula to relate the arrival rate and the mean time spent in the shop.

   (b) Use Little's formula to relate the arrival rate and the mean time spent in service.

   (c) Use the above formulas to find an expression for the mean time spent in the system in terms of the mean service time.

5. A communication network receives messages from R sources with mean arrival rates $\lambda_1, \ldots, \lambda_R$. On the average there are $E[N_i]$ messages from source i in the network.

   (a) Use the Little's formula to find the average time $E[T_i]$ spent by type i customers in the network.

(b) Let λ denote the total arrival rate into the network. Use Little's formula to find an expression for the mean time $E[T]$ spent by customers (of all types) in the network in terms of the $E[N_i]$.

(c) Combine the results of part a and part b to obtain an expression for $E[T]$ in terms of $E[T_i]$. Derive the same expression using $A(t)$ the arrival processes for each type.

6. (a) Find $P[N \geq n]$ for an M/M/1 system.

   (b) What is the maximum allowable arrival rate in a system with service rate μ, if we require that $P[N \geq 10] = 10^{-3}$?

7. Consider an M/M/1 queueing system in which each customer arrival brings in a profit of \$5 but in which each unit time of delay costs the system \$1. Find the range of arrival rates for which the system makes a net profit.