assignment 2

1. Show that

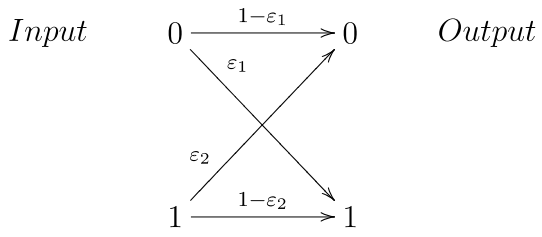
$$\binom{n}{k} = \binom{n}{n-k}$$

2. Show that if $P[A \cap B \cap C] = P[A \mid B \cap C]P[B \mid C]P[C]$.

3. A nonsymmetric binary communications channel is shown in the figure below. Assume the input is "0" with probability p and "1" with probability $1 - p$.

   (a) Find the probability that the out put is 0.

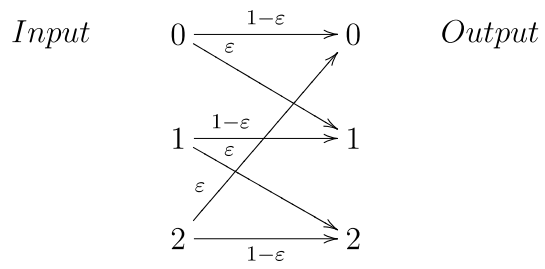
   (b) Find the probability that the input was 0 given that the output is 1. Find the probability that the input is 1 given that the output is 1. Which input is more probable?



4. A ternary communication system is shown in the figure below. Suppose that input symbols 0, 1 and 2 occur with probability 1/3 respectively.

   (a) Find the probabilities of the output symbols.

   (b) Suppose that a 1 is observed at the output. What is the probability that the input was 0? 1? 2?



5. In the binary communication system in attached example, find the value of ε for which the input of the channel is independent of the output of the channel. Can such a channel be used to transmit information?

   Attached Example: Binary Communication System
   Many communication systems can be modeled in the following way. First, the user inputs a 0 or a 1 into the system, and a corresponding signal is transmitted. Second, the receiver makes a decision about what was the input to the system, based on the signal it received. Suppose that the user sends 0s with probability $1-p$ and 1s with probability p, and suppose that the receiver makes random decision errors with probability ε. For $i = 0, 1$, let A_i be the event "input was i," and let B_i be the event "receiver decision was i." Find the probabilities $P[A_i \cap B_j]$ for $i = 0, 1$ and $j = 0, 1$.
   Then we have

$$\begin{aligned} P[A_0 \cap B_0] &= (1-p)(1-\varepsilon) \\ P[A_0 \cap B_1] &= (1-p)\varepsilon \\ P[A_1 \cap B_0] &= p\varepsilon \\ P[A_1 \cap B_1] &= p(1-\varepsilon) \end{aligned}$$