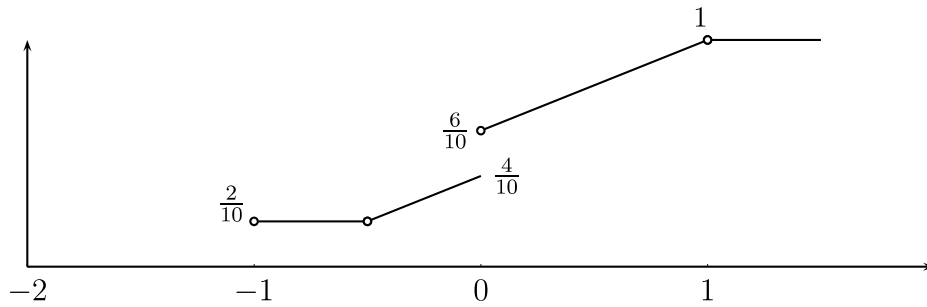


assignment 3

- Let X be the maximum of the number of heads obtained when Carlos and Michael each flip a fair coin twice.
 - Describe the underlying space S of this random experiment and specify the probabilities of its elementary events.
 - Show the mapping from S to S_X , the range of X .
 - Find the probabilities for the various values of X .
- A dart is equally likely to land at any point inside a circular target of radius 2. Let R be the distance of the landing point from the origin.
 - Find the sample space S and the sample space of R , S_R .
 - Show the mapping from S to S_R .
 - The “bull’s eye” is the central disk in the target of radius 0.25. Find the event A in S_R corresponding to “dart hits the bull’s eye.” Find the equivalent event in S and $P[A]$.
 - Find and plot cdf of R .
- The cdf of the random variable X is given by:

$$F_X(x) = \begin{cases} 0 & x < -1 \\ 0.5 & -1 \leq x \leq 0 \\ (1+x)/2 & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

- Plot the cdf and identify the type of random variable.
 - Find $P[X \leq 2]$, $P[X = 0]$, $P[X < 0]$, $P[2 < X < 6]$, $P[X > 10]$.
- The random variable X has cdf shown in the figure below.
 - What type of random variable is X ?
 - Find the following probabilities: $P[X < -1]$, $P[X \leq -1]$, $P[-1 < X < -0.75]$, $P[-0.5 \leq X < 0]$, $P[-0.5 \leq X \leq 0.5]$, $P[|X - 0.5| < 0.5]$.



5. A random variable X has pdf:

$$f_X(x) = \begin{cases} c(1 - x^2) & -1 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- Find c and plot the pdf.
- Plot the cdf of X .
- Find $P[X = 0]$, $P[0 < X < 0.5]$, and $P[|X - 0.5| < 0.25]$.

6. Find the cdf of the Cauchy random variable which has pdf:

$$f_X(x) = \frac{\alpha/\pi}{x^2 + \alpha^2} \quad -\infty < x < \infty$$

7. The loose minute hand of a clock is spun hard. The coordinates (x, y) of the point where the tip of the hand comes to rest is noted. Z is defined as the sgn function of the product of x and y , where $\text{sgn}(t)$ is 1 if $t > 0$, 0 if $t = 0$, and -1 if $t < 0$.

- Find and plot the cdf of the random variable Z .
- Does the cdf change if the clock hand has a propensity to stop at 3, 6, 9, and 12 o'clock?