Assignment 4

1. Eight number are selected at random from the unit interval.
   (a) Find the probability that the first four numbers are less than 0.25 and the last four are greater than 0.25.
   (b) Find the probability that four numbers are less than 0.25 and four are greater than 0.25.
   (c) Find the probability that the first three numbers are less than 0.25, the next two are between 0.25 and 0.75, and the last three are greater than 0.75.
   (d) Find the probability that three numbers are less than 0.25, two are between 0.25 and 0.75, and three are greater than 0.75.
   (e) Find the probability that the first four numbers are less than 0.25 and the last four are greater than 0.75.
   (f) Find the probability that four numbers are less than 0.25 and four are greater than 0.75.

2. The number of orders waiting to be processed is given by a Poisson random variable with parameter $\alpha = \lambda/n\mu$, where λ is the average number of orders that arrive in a day, μ is the number of order that can be processed by an employee per day, and n is the number of employees. Let $\lambda = 5$ and $\mu = 1$. Find the number of employees required so the probability that more than four orders are waiting is less than 90%. What is the probability that there are no orders waiting?

3. The number X of photons counted by a receiver in an optical communication system is a Poisson random variable with rate λ_1 when a signal is present and a Poison random variable with rate $\lambda_0 < \lambda_1$ when a signal is absent. Suppose that a signal is present with probability p.
   (a) Find P[signal present $\mid X = k$] and P[signal absent $\mid X = k$].
   (b) The receiver uses the following decision rule:
   If P[signal present $\mid X = k$] $>$ P[signal absent $\mid X = k$], decide signal present; otherwise, decide signal absent.
   Show that this decision rule leads to the following threshold rule:
   If $X > T$, decide signal present; otherwise, decide signal absent.
   (c) What is the probability of error for the above decision rule?

4. Find the variance of the exponential random variable.

5. Explain why the mean of the Cauchy random variable does not exist.

6. Two chips are being considered for use in a certain system. The lifetime of chip 1 is modeled by a Gaussian random variable with mean 20,000 hours and standard deviation 5000 hours. (The probability of negative lifetime is negligible) The lifetime of chip 2 is also a Gaussian random variable with mean 22,000 hours and standard deviation 1000 hours. Which chips is preferred if the target lifetime of the system is 20,000 hours? 24,000 hours?

7. Let X be a Gaussian random variable with mean 2 and variance 4. The reward in a system is given by $Y = (X)^+$. Find the pdf of Y.

8. Let $Y = e^X$.

   (a) Find the cdf and pdf of Y in the terms of the cdf and pdf of X.

   (b) Find the pdf of Y when X is a Gaussian random variable. In this case Y is said to be a lognormal random variable. Plot the pdf and cdf of Y when X is zero-mean with variance 1/8; repeat with variance 8.