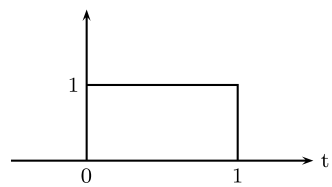
# Assignment 9

1. A random process is defined by

$$Y(t) = G(t - T)$$

where $g(t)$ is the rectangular pulse of following figure, and T is a uniformly distributed random variable in the interval (0,1).




   (a) Find the pmf of $Y(t)$.

   (b) Find $m_Y(t)$ and $C_Y(t_1, t_2)$.

2. The random process $H(t)$ is defined as the "hard-limited" version of $X(t)$:

$$H(t) = \begin{cases} +1 & \text{if } X(t) \geq 0 \\ -1 & \text{if } X(t) < 0. \end{cases}$$

   (a) Find the pdf, mean, and autocovariance of $H(t)$ if $X(t)$ is the sinusoid with a random amplitude presented in Example 9.2 in the text book.

   (b) Find the pdf, mean,and autocovariance of $H(t)$ if $X(t)$ is the sinusoid with random phase presented in Example 9.9 in the text book.

   (c) Find a general expression for the mean of $H(t)$ in terms of the cdf of $X(t)$

3. Let $X_n$ consist of an iid sequence of Poisson random variables with mean $\alpha$.

   (a) Find the pmf of the sum process $S_n$.

   (b) Find the joint pmf of $S_n$ and $S_{n+k}$.

4. Customers deposit \$1 in a vending machine according to a Poisson process with rate $\lambda$. The machine issues an item with probability $p$. Find the pmf for the number of items dispensed in time t.

5. Packets arrive at a multiplexer at two ports according to independent Poisson processes of rates $\lambda_1 = 1$ and $\lambda_2 = 2$ packets/second, respectively.

   (a) Find the probability that a message arrives first on line 2.

   (b) Find the pdf for the time until a message arrives on the either line.

   (c) Find the pmf for $N(t)$, the total number of messages that arrive in an interval of length t.

   (d) Generalize the result of part c for the "merging" of $k$ independent Poisson processes of rates $\lambda_1, \ldots, \lambda_k$, respectively:

$$N(t) = N_1(t) + \cdots + N_k(t).$$

6. Let $X(t)$ be a zero-mean Gaussian random process with autocovariance function given by

$$C_X(t_1, t_2) = 4e^{-2|t_1 - t_2|}.$$

7. Find the joint pdf of $X(t)$ and $X(t + s)$.