ENCS6161: Probability and Stochastic Processes
Fall 2005
Final Exam

1) The random variable X has the *pdf*,

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x|}.$$

a) Find the *pdf* of the random variable $Y = |X|$ (6 Marks).

b) Find the expected value and variance of Y (4 Marks).

2) A fair coin is tossed 900 times. Find the probability that the number of heads is between 420 and 465 (6 Marks).

3) A point (X, Y) is selected at random inside a unit circle. Find the marginal *pdf* of X (6 Marks)

4) Independent random variables X and Y have the following probability density functions:

$$f_X(x) = \frac{1}{\sqrt{10\pi}} e^{\frac{-(x-2)^2}{10}}$$

and

$$f_Y(y) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(y-1)^2}{8}}.$$

Define random variable $Z = X + Y$ and $W = X - 2Y$. Find:

a) Joint probability density function of Z and W (8 Marks).
b) Marginal probability density functions of Z and W (4 Marks).

5) A random process is defined as,

$$X(t) = At + B,$$

where A and B are Gaussian random variables with means m_1, m_2, variance σ_1^2, σ_2^2, and correlation coefficient ρ.

a) Find the mean and autocovariance of $X(t)$ (5 Marks).
b) Find the *pdf* of $X(t)$ (3 Marks).

6) A linear time invariant system with input $X(t)$ is described by the equation,

$$\frac{d}{dt}Y(t) + aY(t) = X(t).$$

a) Find the power spectral density of $Y(t)$ if $X(t)$ is a white process with power spectral density $N_0/2$ (6 Marks).
b) Find the autocorrelation function of $Y(t)$ (4 Marks).

7) In a queue, arrivals are Poisson with a rate λ. There are two servers each with an exponential service time with a mean $\frac{1}{\mu}$. The size of the buffer is 5.

a) Draw the state transition diagram for the queue (2 Marks).

b) Find the steady state probabilities of the system (6 Marks).