Concordia University
ENCS6161 – Probability and Stochastic Processes

Instructor: Dr. D. Qiu | Final Exam | Winter 2006

1) Show that the Q-function for the Gaussian random variable satisfies (**5 marks**)
$$Q(-x) = 1 - Q(x)$$

2) Let X_1 be uniformly distributed in the interval [0,1], X_2 be uniformly distributed in [0, X_1], X_3 be uniformly distributed in [0, X_2].
a) Find the joint pdf of (X_1, X_2, X_3). (**5 marks**)
b) Find the marginal pdf of X_3 .(**10 marks**)

3) Let $X_1, X_2, \cdots$ be a sequence of iid random variables, let N be a positive integer-valued random variable independent of X_j 's, and let $S = \sum_{k=1}^{N} X_k$. Find the mean and variance of S in terms of the mean and variance of X and N. (**15 marks**)

4) Let $X(t) = A\cos\omega t + B\sin\omega t$, where A and B are iid Gaussian random variables with zero mean and variance σ^2 .
a) Find the mean and autocovariance of $X(t)$. (**8 marks**)
b) Find the joint pdf of $X(t)$ and $X(t+s)$. (**7 marks**)

5) Let Z_n be the random process defined by:
$$Z_n = \frac{1}{2} Z_{n-1} + X_n \qquad Z_0 = 0,$$
where X_n is a zero-mean, unit-variance iid process.
a) Find the autocovariance of Z_n and determine whether Z_n is wide-sense stationary. (**10 marks**)
b) If X_n is an iid sequence of zero-mean, unit-variance Gaussian random variables, find the pdf of Z_n as $n \to \infty$ (**5 marks**)

6) The input into a filter is zero-mean white noise with noise power density $N_0 / 2$. The filter has transfer function
$$H(f) = \frac{1}{1 + j2\pi f}$$
a) Find $S_Y(f)$ and $R_Y(\tau)$. (**5 marks**)
b) What is the average power of the output? (**5 marks**)

(See next page for more questions)

7) A die is rolled repeatedly. Which of the following are Markov chains? For those that are, find the transition probability matrix.
a) The largest number X_n shown up to the *n*th roll. (**7 marks**)
b) The number N_n of sixes in *n* rolls. (**8 marks**)

8) Two types of customers come to a queue independently with Poisson arrival rates of λ_1 and λ_2 respectively. Both types of customers require exponentially distributed services time of rate μ. Type 1 customers are always accepted while type 2 customers are rejected when the total number of customers in the system exceeds K.
a) Let *X(t)* be the total arrival process. Prove that *X(t)* is a Poisson process with rate $\lambda_1 + \lambda_2$. (**5 marks**)
b) Draw the transition diagram for *N(t)*, the number of customers in the system. (**5 marks**)