

ENCS6161: Probability and Stochastic Processes
Fall 2005
Midterm Exam

1) In an urn there are 20 red balls, 30 green balls and 50 white balls. Ten balls are taken from the urn at random. What is the probability that there are 2 red and 4 green balls among the ten balls taken? (5 Marks).

2) A number X is selected in the interval $[-1, 1]$. Let A be the event $\{|X - 1/2| < 1\}$ and let B be the event $\{X > 3/4\}$. Find $P[A | B]$ and $P[B | A]$ (3 Marks).

3) A binary communication system transmits bits 0 and 1 with probability p and $1 - p$, respectively. A bit 0 is represented as -5 Volts and a bit 1 is represented as $+5$ Volts. A Gaussian with mean zero and variance σ^2 is added to the signal.

a) Design an optimum detector for this system, i.e., find the decision rule in terms of σ and p (4 Marks).

b) Find the probability of error if $\sigma = 1$ and $p = 0.5$ (3 Marks).

4) A random variable has the characteristic function given as,

$$\Phi_X(\omega) = \frac{a}{a - j\omega}.$$

a) Find the probability density function of this random variable (3 Marks).

b) Find mean and variance of X by applying the moment theorem (3 Marks).

5) Find the mean and variance of the random variable X with,

$$f_X(x) = e^{-2|x|}.$$

(3 Marks).

6) One of the two coins is selected at random (with equal probability) and tossed once. The probability of Head for the coin A is 0.1 and for coin B is 0.2. Find the entropy of the outcome (3 Marks).

7) Let X be a random variable uniform on the interval $[0, a]$ and $Y = 3X$. Find the differential entropy of X (1 Mark) and Y (2 Marks).