

Concordia University
ENCS6161 – Probability and Stochastic Processes

Instructor: Dr. D. Qiu

Midterm Exam

Winter 2006

- 1) a) A random experiment has sample space $S=\{a,b,c\}$. Suppose that $P[\{a,c\}]=3/8$ and $P[\{b,c\}]=7/8$. Find the probabilities of the elementary events. **(2 Marks)**
b) Two numbers are selected at random from the interval $[0,1]$. Find the probability that they differ by less than $1/2$. **(2 Marks)**
c) An urn contains four black balls and three white balls. Two balls are selected at random from the urn without replacement. Find the probability that both are black. **(2Marks)**
d) How many odd three-digit numbers can be formed from the digits 0, 1, 2, 3, 4, and 5, if each digit can be used only once? **(2 Marks)**
- 2) How many words can be formed with the letters of the word 'OMEGA' when:
a) 'E' being always in the middle? **(2 Marks)**
b) Vowels occupying odd-places? **(2 Marks)**
c) 3 Vowels being not together? (note: 2 vowels being together is allowed) **(2 Marks)**
- 3) Consider a coin with $P(\text{Head}) = 0.4$ and $P(\text{Tail}) = 0.6$. The coin is flipped 100 times. Let X be the number of heads obtained. Using the Chebyshev inequality, find a lower bound for $P(30 < X < 50)$. **(3 Marks)**
- 4) a) Let $Y=aX+b$ where a and b are constants. Find the characteristic function of Y in terms of the characteristic function of X . **(2 Marks)**
b) If a random variable X has the characteristic function $\Phi(\omega) = e^{-|\omega|}$, find the PDF of X . **(3 Marks)**
- 5) A communication channel accepts the input $X \in \{0,1,2,3\}$ and outputs $Y=X+Z$ where Z is a binary random variable taking values -1 and $+1$ with equal probability. Assume X and Z are independent and all values of the input X have equal probability.
a) Find the entropy of Y . **(2 Marks)**
b) Find the entropy of X given that $Y=1$. **(2 Marks)**
- 6) In a data network, packets containing 64 bits are transmitted over a channel with a bit error probability of $\varepsilon = 10^{-2}$. If a packet is received in error, i.e., containing one or more errors, the receiver asks for retransmission.
a) What is the probability that a packet is successfully transmitted in the first attempt? **(2 Marks)**
b) What is the average number of transmissions per packet? **(2 Marks)**