ENCS 6161 Midterm, Oct 2015
30 points total
Time allotted: 2 hours

1. A fair die with numbers 1 to 6 on its faces is rolled n times. Let X be the number of times number 6 shows up. What is the average value of X^2 and $aX + bX^2$?(2 points)

2. For a random variable X with PDF $f(x) = \lambda e^{-\lambda x}$ (for X>0), find $E[x^2]$ (you must show how you do the integral to get the grade). (2 points)

3. A binary transmission system transmits a signal X (-1 to send "0" bit and 1 to send "1" bit). Assume 0 bits are 2 times as likely as 1 bits. For X=-1, the received signal is Y=X+2N, and for X=1, the received signal is Y=X+N, where noise N has zero-mean Gaussian distribution with variance σ^2.

a) Find the conditional PDF of Y given X. That is, $f_Y(y \mid X=-1)$ and $f_Y(y \mid X=1)$. (2 points)

b) Receiver decides a 0 bit was transmitted if the observed value of y satisfies

$$f_Y(y \mid X=-1)\, P(X=-1) > f_Y(y \mid X=1)\, P(X=1)$$

Find the range that receiver decides 0 bit was transmitted. Explain the results. (8 point)

4. Let Y = X^3.

a) Find the pdf of Y in terms of pdf of X. (2 points)

b) Assume X has uniform distribution, $f(x) = 1$ for $0 \le x \le 1$. Find f(y). (2 points)

c) In part b, what is f(y=0)? (2 points)

5) A communication channel accepts inputs X = 0, 1, 2, 3, and outputs Z = X+Y, where Y is a random variable taking values 0 and 1 with equal probability. Assume all values of input X have equal probabilities, and X & Y are independent.

a) Calculate the entropy of Z. (2 points)

b) Find the entropy of X given that Y=1. (2 points)

c) Find the entropy of X given that Z=1. (2 points)

6) For a random variable X with PDF $f(x) = \lambda e^{-\lambda x}$ (for X>0), find P(X>8 | X>2). (2 points)

7) Assume current measurements I in a wire are positive, have a mean of 12 mA and standard deviation of 1mA. Approximate both Markov and Chebychev upper bounds for prob. of I>15mA. Make necessary assumptions and state them. (2 points)

In case you need to solve $ax^2+bx+c=0$, the solution is $x=(-b \pm \sqrt{(b^2-4ac)}) / 2a$