1.6 MOBILITY

One of the first concerns in either the design or the analysis of a mechanism is the number of degrees of freedom, also called the *mobility* of the device. The mobility* of a mechanism is the number of input parameters (usually pair variables) that must be controlled independently in order to bring the device into a particular position. Ignoring for the moment certain exceptions to be mentioned later, it is possible to determine the mobility of a mechanism directly from a count of the number of links and the number and types of joints that it includes.

To develop this relationship, consider that before they are connected together, each link of a planar mechanism has three degrees of freedom when moving relative to the fixed link. Not counting the fixed link, therefore, an n-link planar mechanism has $3(n - 1)$ degrees of freedom before any of the joints are connected. Connecting a joint that has one degree of freedom, such as a revolute pair, has the effect of providing two constraints between the connected links. If a two-degree-of-freedom pair is connected, it provides one

*The German literature distinguishes between *movability* and *mobility*. Movability includes the six degrees of freedom of the device as a whole, as though the ground link were not fixed, and thus applies to a kinematic chain. Mobility neglects these and considers only the internal relative motions, thus applying to a mechanism. The English literature seldom recognizes this distinction, and the terms are used somewhat interchangeably.

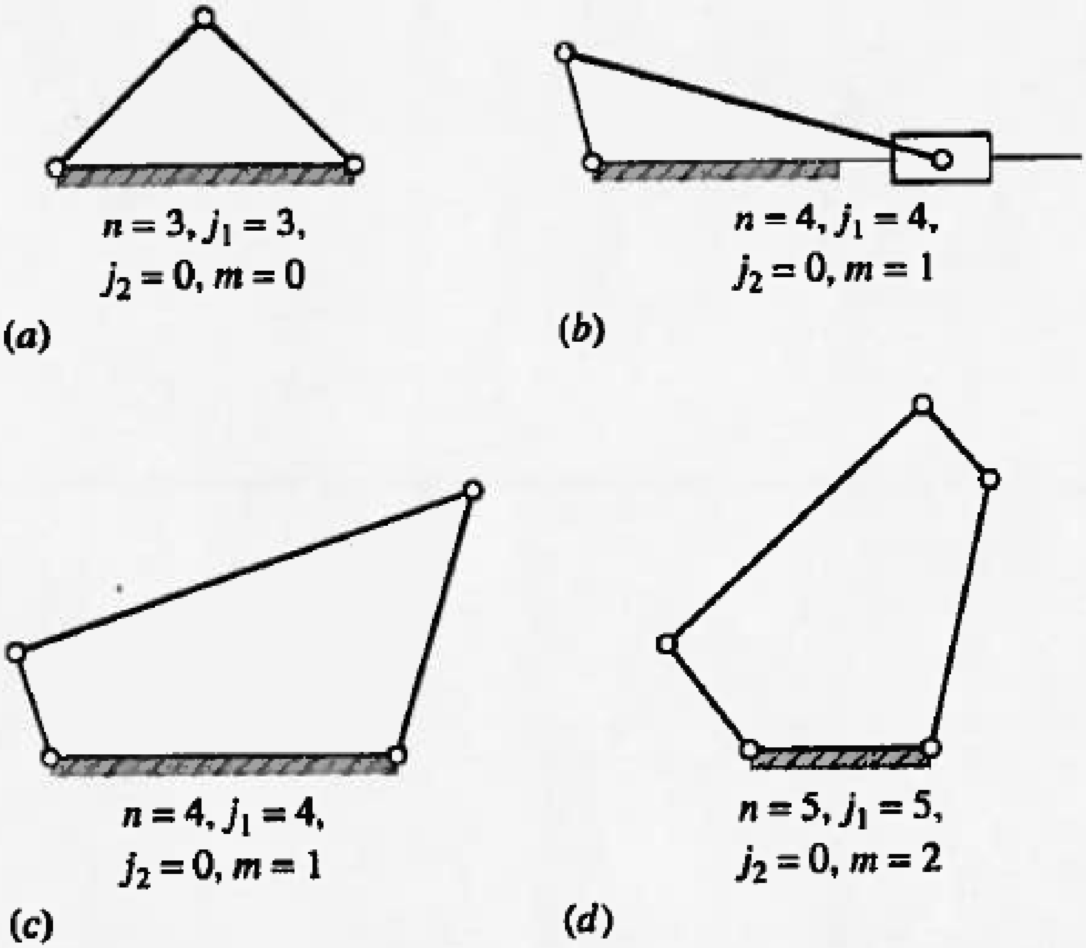


Figure 1.3 Applications of the Kutzbach mobility criterion.

constraint. When the constraints for all joints are subtracted from the total freedoms of the unconnected links, we find the resulting mobility of the connected mechanism. When we use j_1 to denote to number of single-degree-of-freedom pairs and j_2 for the number of two-degree-of-freedom pairs, the resulting mobility m of a planar n-link mechanism is given by

$$m = 3(n - 1) - 2j_1 - j_2 \tag{1.1}$$

$m = 1$, the mechanism can be driven by a single input motion. If $m = 2$, then two separate input motions are necessary to produce constrained motion for the mechanism; such a case is shown in Fig. 1.3*d*.

If the Kutzbach criterion yields $m = 0$, as in Fig. 1.3*a*, motion is impossible and the mechanism forms a structure. If the criterion gives $m = -1$ or less, then there are redundant constraints in the chain and it forms a statically indeterminate structure. Examples are shown in Fig. 1.4. Note in these examples that when three links are joined by a single pin, two joints must be counted; such a connection is treated as two separate but concentric pairs.

Figure 1.5 shows examples of Kutzbach's criterion applied to mechanisms with two-degree-of-freedom joints. Particular attention should be paid to the contact (pair) between the wheel and the fixed link in Fig. 1.5*b*. Here it is assumed that slipping is possible

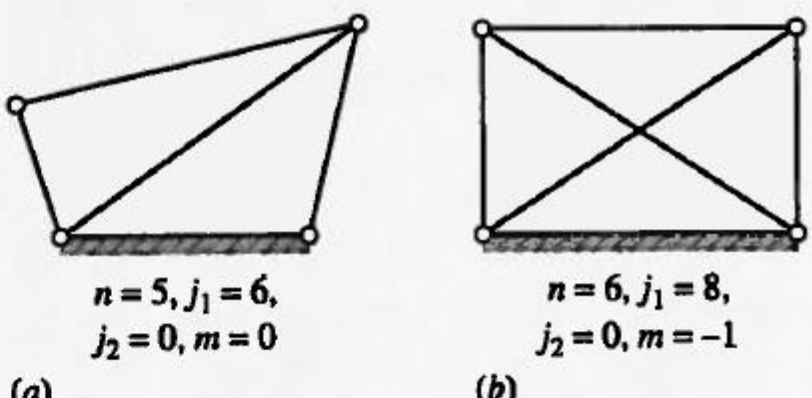
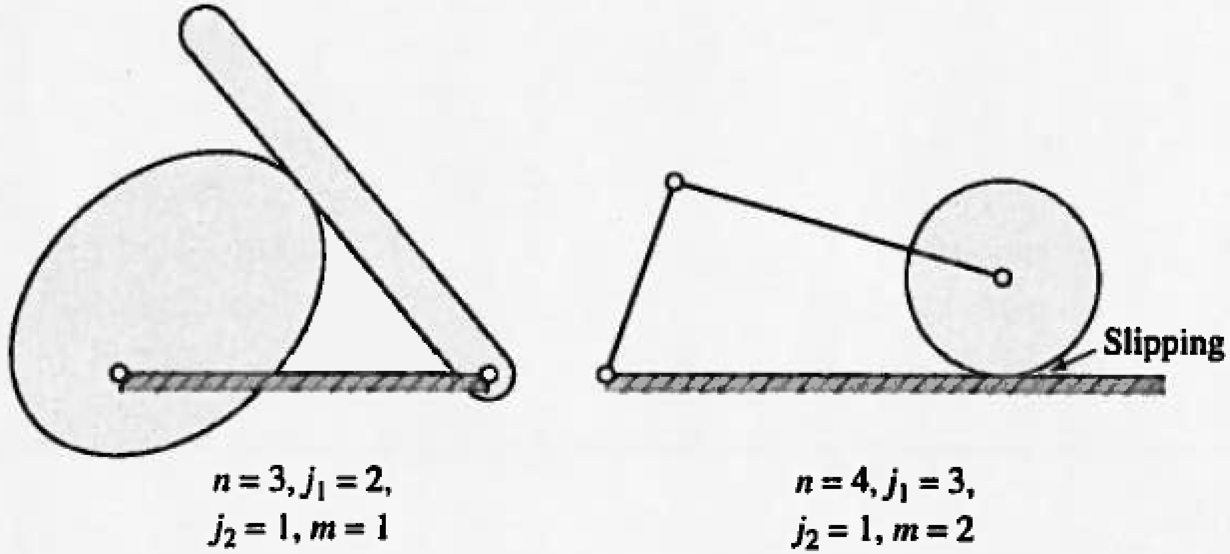


Figure 1.4 Applications of the Kutzbach criterion to structures.



(*a*) (*b*)

Figure 1.5

between the links. If this contact included gear teeth or if friction was high enough to prevent slipping, the joint would be counted as a one-degree-of-freedom pair, because only one relative motion would be possible between the links.

Sometimes the Kutzbach criterion gives an incorrect result. Notice that Fig. 1.6*a* represents a structure and that the criterion properly predicts $m = 0$. However, if link 5 is arranged as in Fig. 1.6*b*, the result is a double-parallelogram linkage with a mobility of 1 even though Eq. (1.1) indicates that it is a structure. The actual mobility of 1 results only if the parallelogram geometry is achieved. Because in the development of the Kutzbach criterion no consideration was given to the lengths of the links or other dimensional properties, it is not surprising that exceptions to the criterion are found for particular cases with equal link lengths, parallel links, or other special geometric features.

Even though the criterion has exceptions, it remains useful because it is so easily applied. To avoid exceptions, it would be necessary to include all the dimensional properties of the mechanism. The resulting criterion would be very complex and would be useless at the early stages of design when dimensions may not be known.

An earlier mobility criterion named after Grübler applies to mechanisms with only single-degree-of-freedom joints where the overall mobility of the mechanism is unity. Putting $j_2 = 0$ and $m = 1$ into Eq. (1.1), we find Grübler's criterion for planar mechanisms with constrained motion:

$$3n - 2j_1 - 4 = 0 \tag{1.2}$$

From this we can see, for example, that a planar mechanism with a mobility of 1 and only single-degree-of-freedom joints cannot have an odd number of links. Also, we can find the simplest possible mechanism of this type; by assuming all binary links, we find

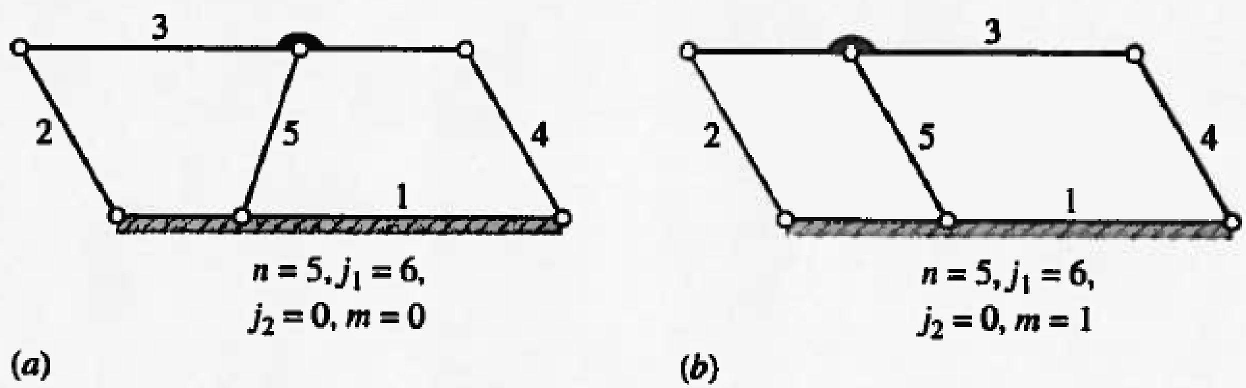


Figure 1.6

$n = j_1 = 4$. This shows why the four-bar linkage (Fig. 1.3*c*) and the slider-crank mechanism (Fig. 1.3*b*) are so common in application.

Both the Kutzbach criterion, Eq. (1.1), and the Grübler criterion, Eq. (1.2), were derived for the case of planar mechanisms. If similar criteria are developed for spatial mechanisms, we must recall that each unconnected link has six degrees of freedom; and each revolute pair, for example, provides five constraints. Similar arguments then lead to the three-dimensional form of the Kutzbach criterion,

$$m = 6(n - 1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5 \tag{1.3}$$

and the Grübler criterion,

$$6n - 5j_1 - 7 = 0 \tag{1.4}$$

The simplest form of a spatial mechanism,* with all single-freedom pairs and a mobility of 1, is therefore $n = j_1 = 7$.

1.9 GRASHOF'S LAW

A very important consideration when designing a mechanism to be driven by a motor, obviously, is to ensure that the input crank can make a complete revolution. Mechanisms in which no link makes a complete revolution would not be useful in such applications. For the four-bar linkage, there is a very simple test of whether this is the case.

Grashof's law states that *for a planar four-bar linkage, the sum of the shortest and longest link lengths cannot be greater than the sum of the remaining two link lengths if there is to be continuous relative rotation between two members.* This is illustrated in Fig. 1.23, where the longest link has length l, the shortest link has length s, and the other two links have lengths p and q. In this notation, Grashof's law states that one of the links, in particular the shortest link, will rotate continuously relative to the other three links if and only if

$$s + l \leq p + q \tag{1.6}$$

If this inequality is not satisfied, no link will make a complete revolution relative to another.

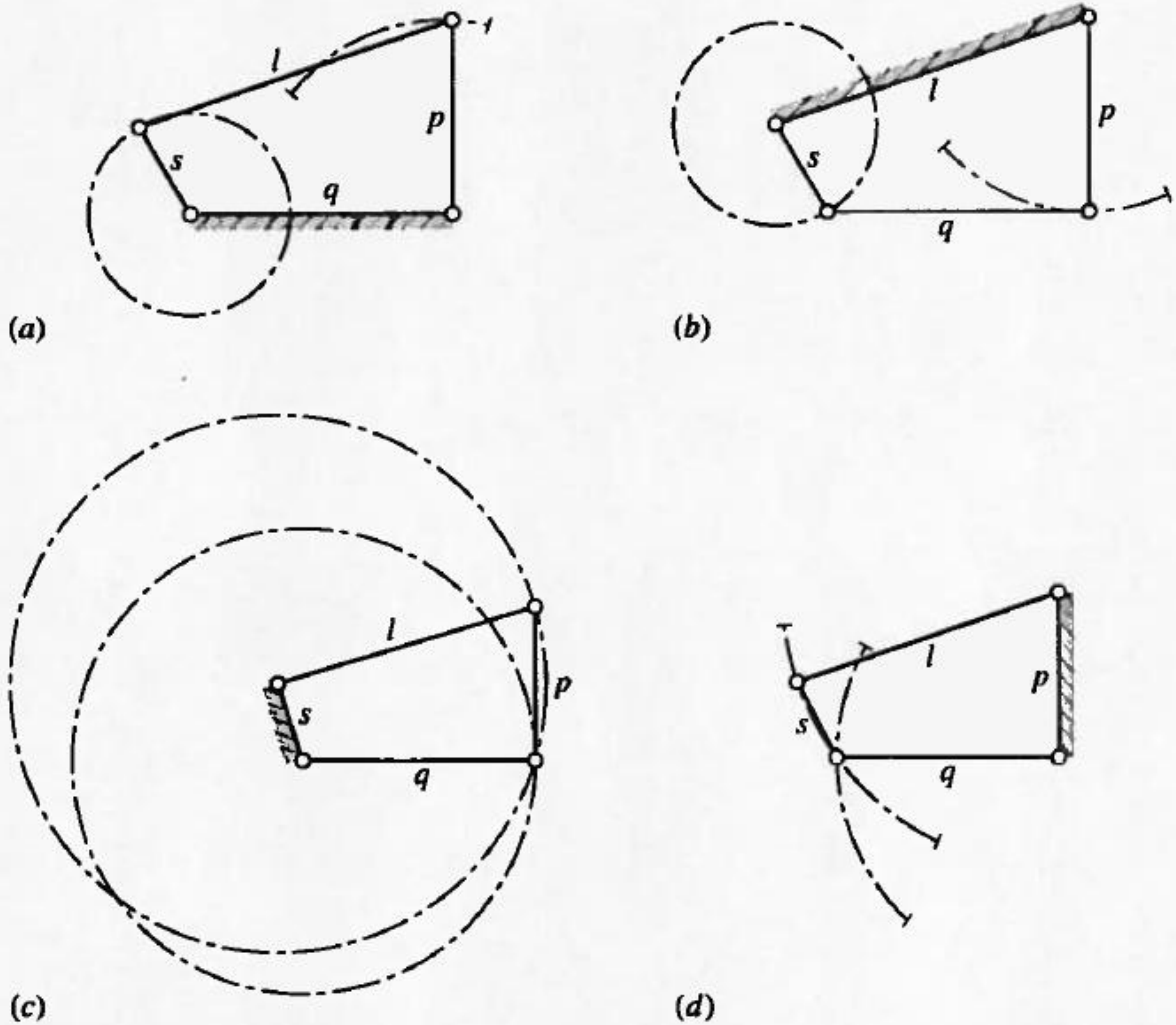


Figure 1.23 Four inversions of the Grashof chain: (*a*, *b*) crank-rocker mechanisms; (*c*) drag-link mechanism; and (*d*) double-rocker mechanism.

Attention is called to the fact that nothing in Grashof's law specifies the order in which the links are connected or which link of the four-bar chain is fixed. We are free, therefore, to fix any of the four links. When we do so, we create the four inversions of the four-bar linkage shown in Fig. 1.23. All of these fit Grashof's law, and in each the link s makes a complete revolution relative to the other links. The different inversions are distinguished by the location of the link s relative to the fixed link.

If the shortest link s is adjacent to the fixed link, as shown in Figs. 1.23*a* and 1.23*b*, we obtain what is called a *crank-rocker* linkage. Link s is, of course, the crank because it is able to rotate continuously; and link p, which can only oscillate between limits, is the rocker.

The *drag-link* mechanism, also called the *double-crank* linkage, is obtained by fixing the shortest link s as the frame. In this inversion, shown in Fig. 1.23*c*, both links adjacent to s can rotate continuously, and both are properly described as cranks; the shorter of the two is generally used as the input.

Although this is a very common mechanism, you will find it an interesting challenge to devise a practical working model that can operate through the full cycle.

By fixing the link opposite to s we obtain the fourth inversion, the *double-rocker* mechanism of Fig. 1.23*d*. Note that although link s is able to make a complete revolution, neither link adjacent to the frame can do so; both must oscillate between limits and are therefore rockers.

In each of these inversions, the shortest link s is adjacent to the longest link l. However, exactly the same types of linkage inversions will occur if the longest link l is opposite the shortest link s; you should demonstrate this to your own satisfaction.

Reuleaux approaches the problem somewhat differently but, of course, obtains the same results. In this approach, and using Fig. 1.23*a*, the links are named

s the crank	p the lever
l the coupler	q the frame

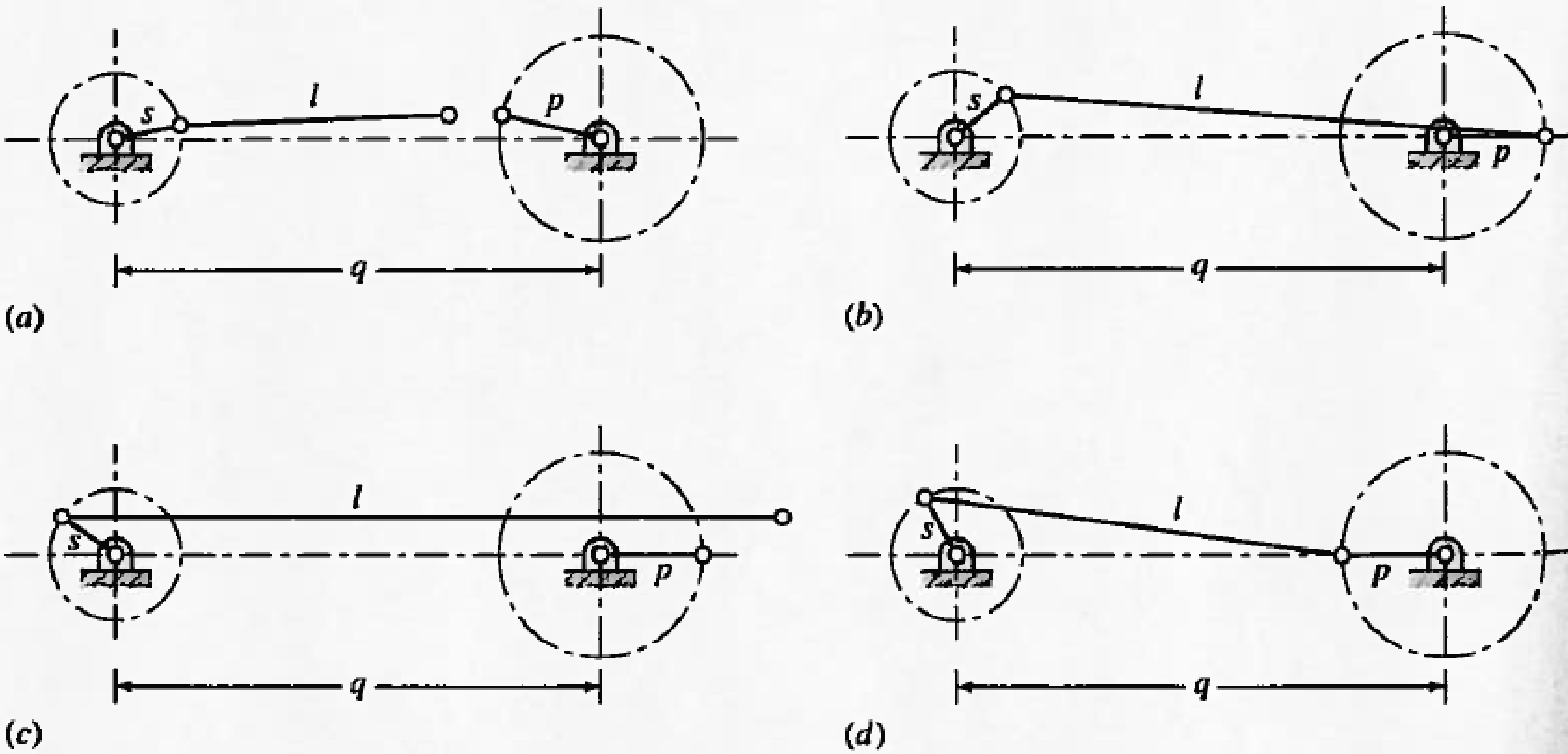


Figure 1.24 (*a*) Equation (1.7); $s + l + p < q$ and the links cannot be connected. (*b*) Equation (1.8); $s + l - p > q$ and s is incapable of rotation. (*c*) Equation (1.9); $s + q + p < l$ and the links cannot be connected. (*d*) Equation (1.10); $s + q - p < l$ and s is incapable of rotation.

where l need not be the longest link. Then the following conditions apply:

$$s + l + p \geq q \tag{1.7}$$

$$s + l - p \leq q \tag{1.8}$$

$$s + q + p \geq l \tag{1.9}$$

$$s + q - p \leq l \tag{1.10}$$

These four conditions are illustrated in Fig. 1.24 by showing what happens if the conditions are not met.