MECH 344/M
Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 10

Contents of today's lecture

15 Spur Gears

15.1 Introduction and History

- Simplest method of transferring rotary motion from one shaft to another is by rolling cylinders, very old technique (4th century BC)
- In case of sufficient friction, this will work well, without slip (efficiency 98%)
- Low torque and slipping are a common problem in reality
- Gears are usually more costly than chains and belts.
- Gear manufacturing costs increase sharply with increased precision as required for the combination of high speeds and heavy loads, and for low noise levels.
- Standard tolerances for various degrees of manufacturing precision have been established by AGMA, American Gear Manufacturers Association.

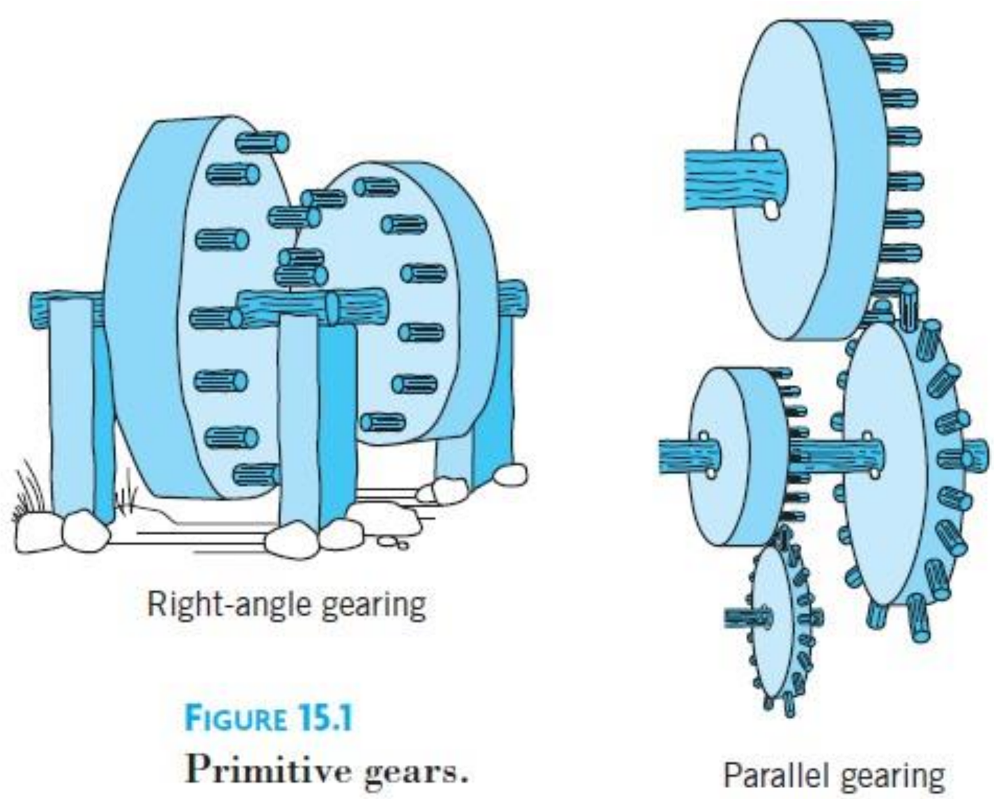


FIGURE 15.1
Primitive gears.

15.1 Introduction and History

- Spur gears are the simplest and most common type of gears. They are used to transfer motion between parallel shafts, and they have teeth that are parallel to the shaft axes.
- We will be concerned with gear geometry and nomenclature, gear force analysis, geartooth bending strength, and gear-tooth surface durability.
- Gear Designers should consult the pertinent standards of the AGMA, as well as other contemporary gear literature.

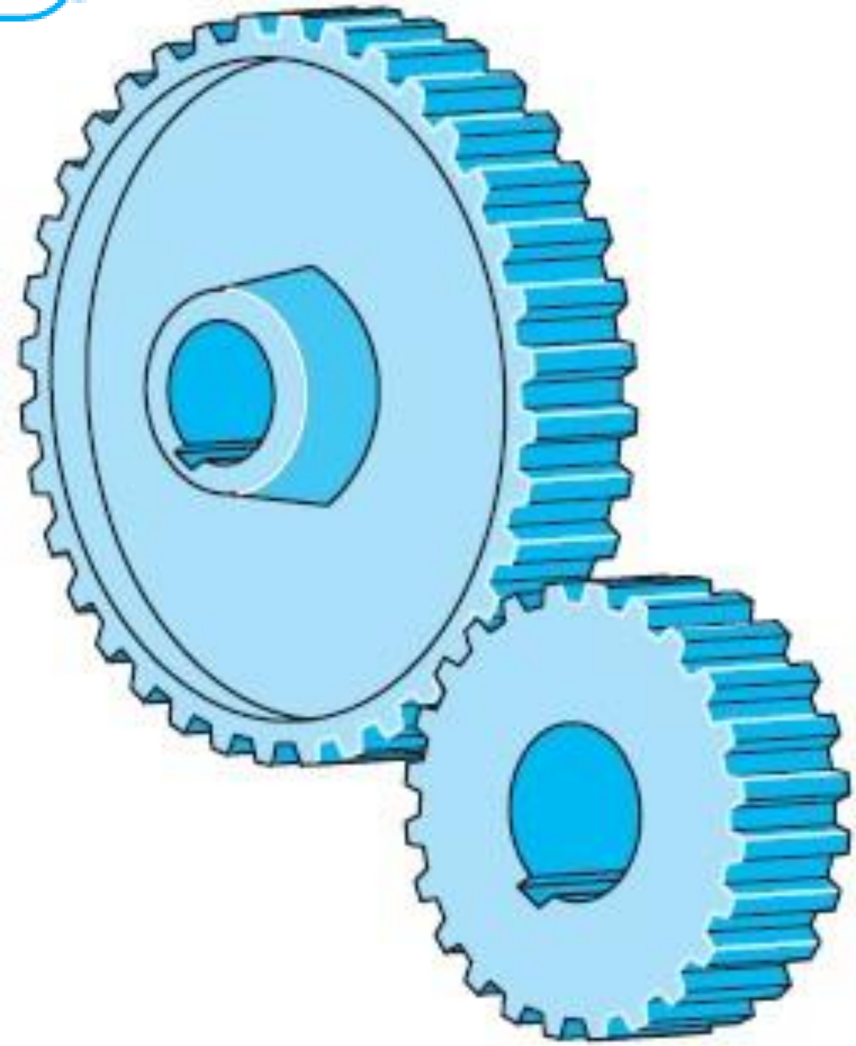
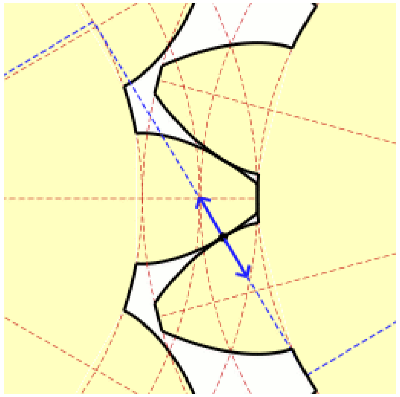


FIGURE 15.2
Spur gears.

15.2 Geometry and Nomenclature



- The basic requirement of gear-tooth geometry is that the angular velocity ratios are constant.
- For eg, the angular-velocity ratio between a 20-tooth and a 40-tooth gear must be precisely 2 in every position. Not 1.99 as teeth come into mesh and then 2.01 as they go out of mesh.
- Manufacturing inaccuracies may cause slight deviations, but acceptable tooth profiles are based on theoretical curves that meet this criterion.
- The action of a pair of gear teeth satisfying this requirement is termed conjugate gear-tooth action. The basic law of conjugate geartooth action states that

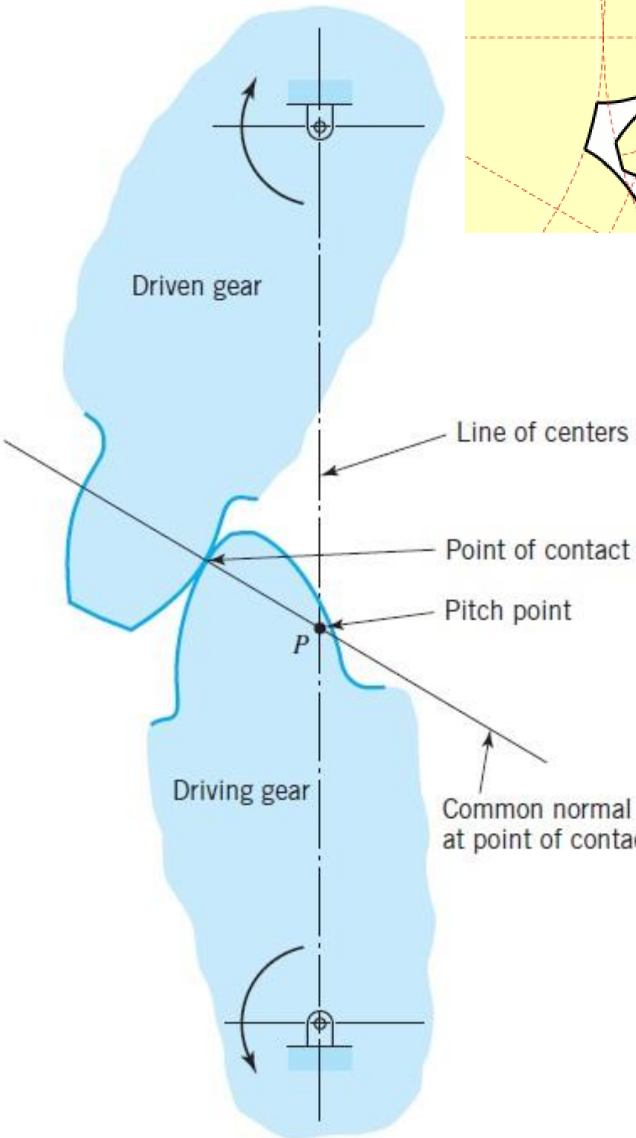


FIGURE 15.3
Conjugate gear-tooth action.

15.2 Geometry and Nomenclature

- The basic law of conjugate geartooth action states that

As the gears rotate, the common normal to the surfaces at the point of contact must always intersect the line of centers at the same point P, called the pitch point.

- The law of conjugate gear-tooth action can be satisfied by various tooth shapes, but involute is most important
- Involute is the curve generated by a point on a thread as it unwinds from the base circle.
- The generation of two involutes (Figure) - the dotted lines show how these could correspond to the outer portions of the right sides of adjacent gear teeth. Involutes done by unwinding in CW direction can complete the other side of the teeth
- At every point, the involute is perpendicular to the taut thread.
- An involute can be developed as far as desired outside the base circle, but an involute cannot exist inside its base circle.

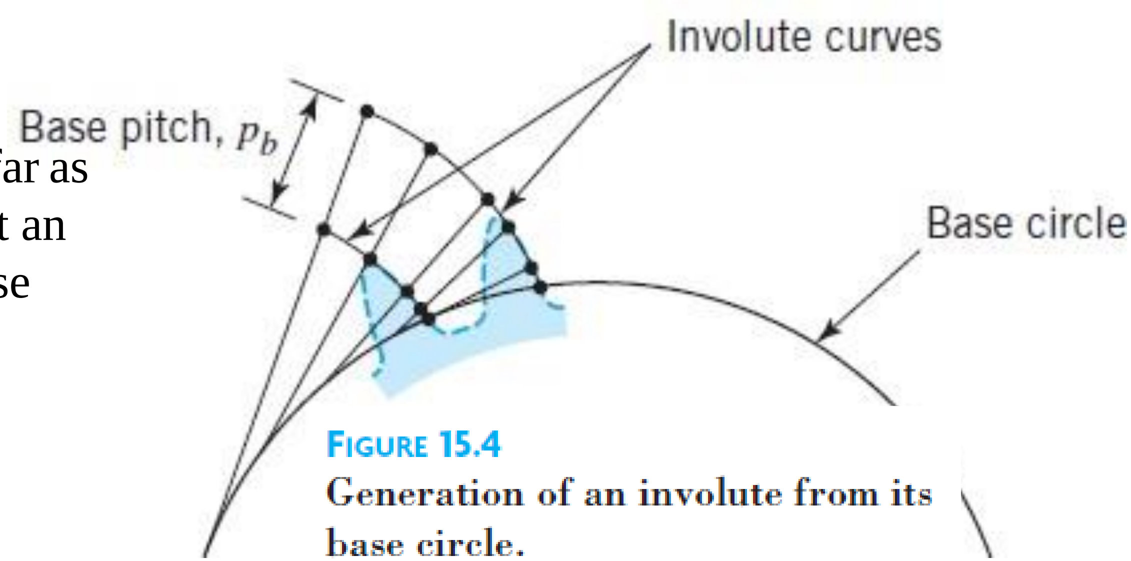


FIGURE 15.4
Generation of an involute from its base circle.

Involute Tooth Form

- The string is always tangent to the base circle.
- The centre of curvature of the involute is always at the point of tangency of the string with the base circle
- A tangent to the involute is always normal to the string, which is instantaneous radius of curvature of the involute curve

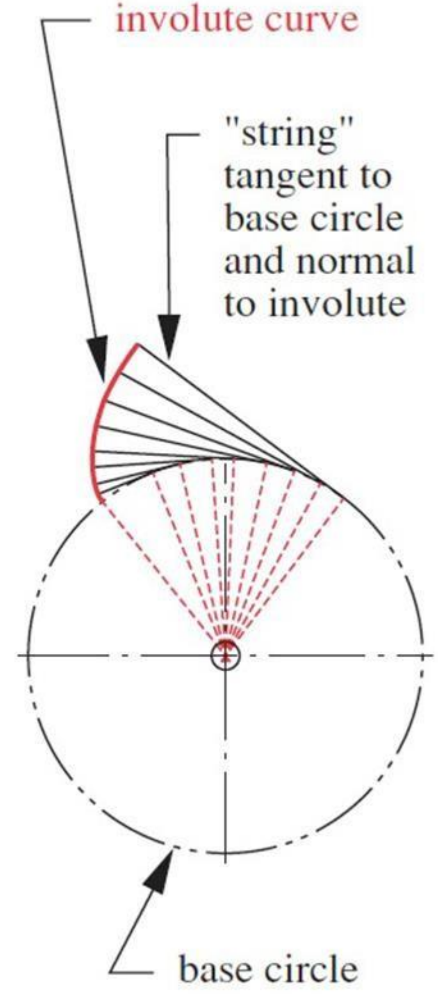


FIGURE 12-3

Development of the Involute of a Circle

15.2 Geometry and Nomenclature

- Figure shows two pitch circles as representing two cylinders pressed together.
- If no slippage, rotation of one cylinder will cause rotation of the other at an angular-velocity ratio inversely proportional to dia.
- In any pair of mating gears, the smaller is called pinion and the larger the gear.

$$\omega_p/\omega_g = -d_g/d_p \qquad \textbf{(15.1)}$$

- ω is the angular velocity, d is pitch dia, and - sign indicates opposite rotation direction
- The center distance is

$$c = (d_p + d_g)/2 = r_p + r_g \qquad \textbf{(15.1a)}$$

- r is the pitch circle radius

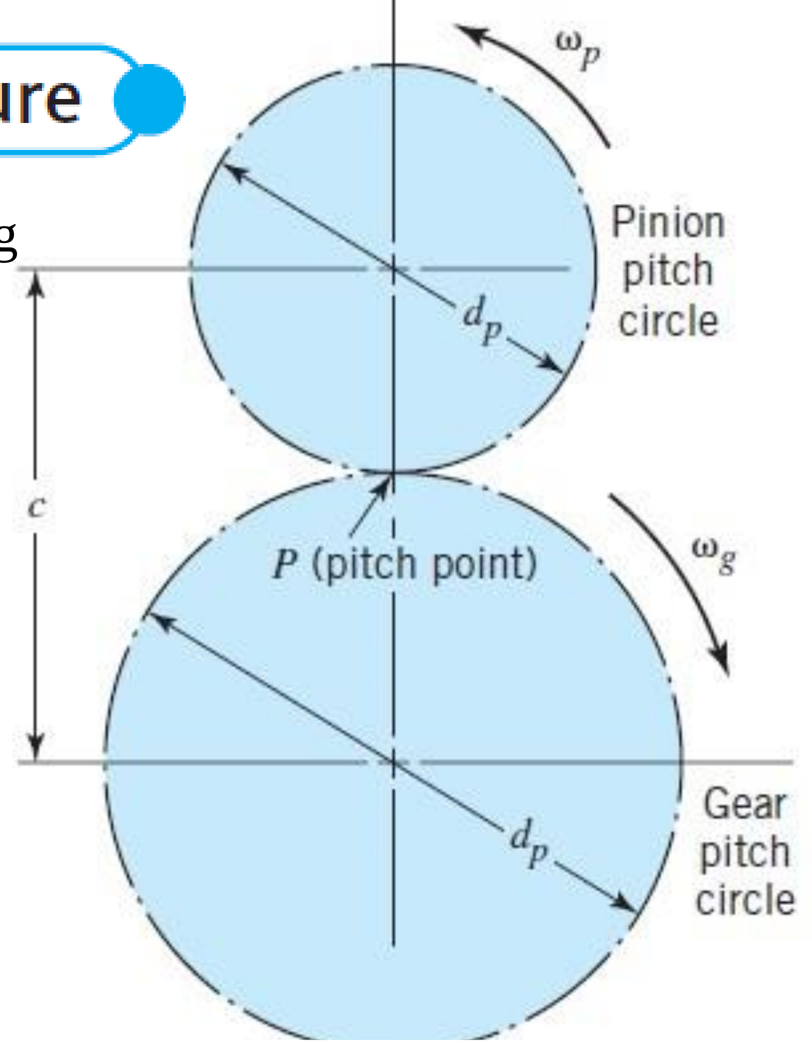


FIGURE 15.5
Friction gears of diameter d rotating at angular velocity ω.

15.2 Geometry and Nomenclature

- To transmit more torque than by friction gears alone, add a belt drive running between pulleys representing the base circles (figure).
- If the pinion is turned CCW, the belt will cause the gear to rotate in accordance with Eq. 15.1.

$$\omega_p/\omega_g = -d_g/d_p \qquad (15.1)$$

- In gear parlance, angle ϕ is called the pressure angle.
- From similar triangles the base circles have the same ratio as the pitch circles; thus, the velocity ratios provided by the friction and belt drives are the same.

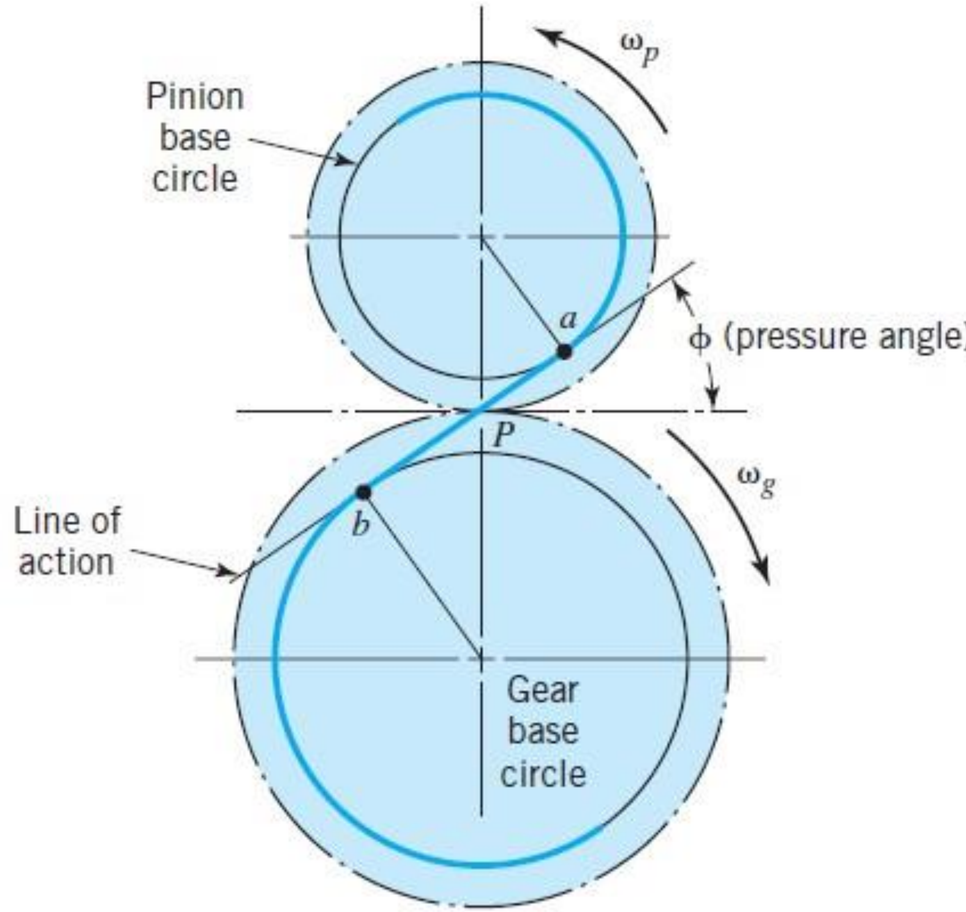
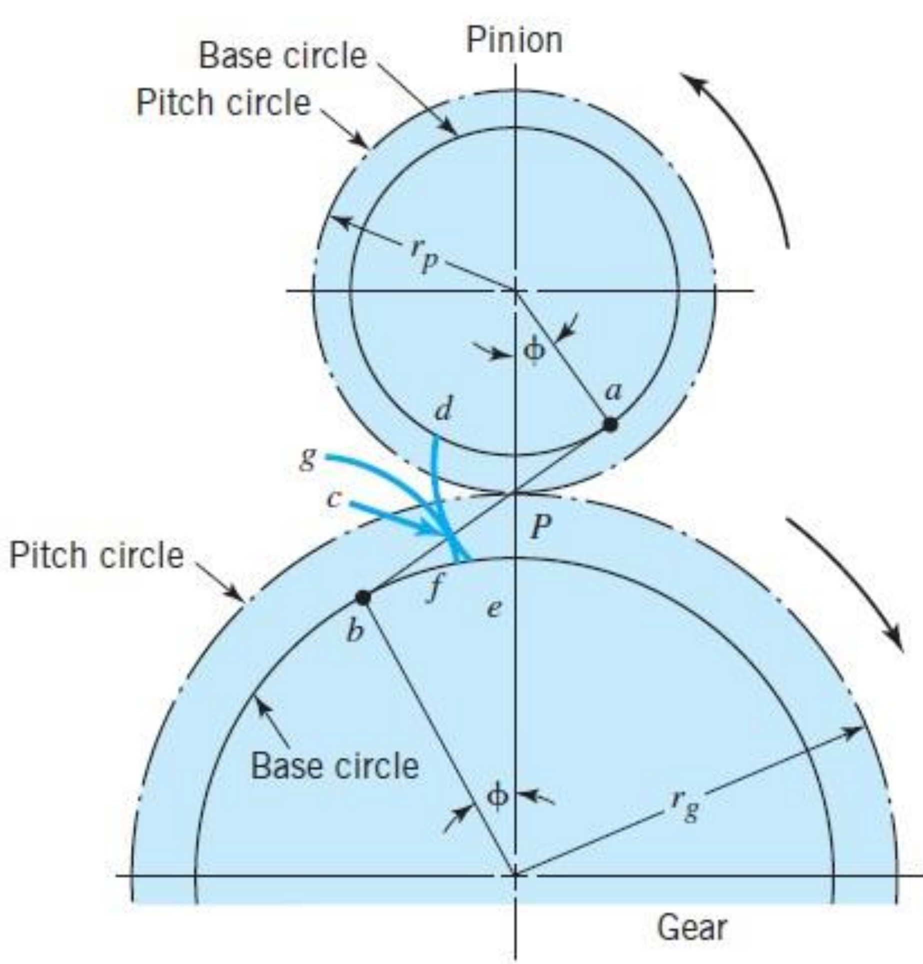


FIGURE 15.6
Belt drive added to friction gears.

15.2 Geometry and Nomenclature

- The belt is cut at point c, and the 2 ends are used to generate involutes de and fg
- Neglecting sliding friction, the force of one involute tooth pushing against the other is always at an angle equal to the pressure angle.



- Involute is the only geometric profile satisfying this law that maintains a constant pressure angle as the gears rotate.
- Note especially that conjugate involute action can take place only outside of both base circles.
- In Figure the conjugate involute profiles could be drawn only by "cutting the belt" at a point between a and b.

FIGURE 15.7

Belt cut at *c* to generate conjugate involute profiles.

15.2 Geometry and Nomenclature

- Figure shows the continued development of the gear teeth.
- The involute profiles are extended outward beyond the pitch circle by a distance called the addendum.
- The outer circle is the addendum circle.
- The tooth profiles are extended inward from the pitch circle by a distance called the dedendum.
- The involute portion can extend inward only to the base circle.

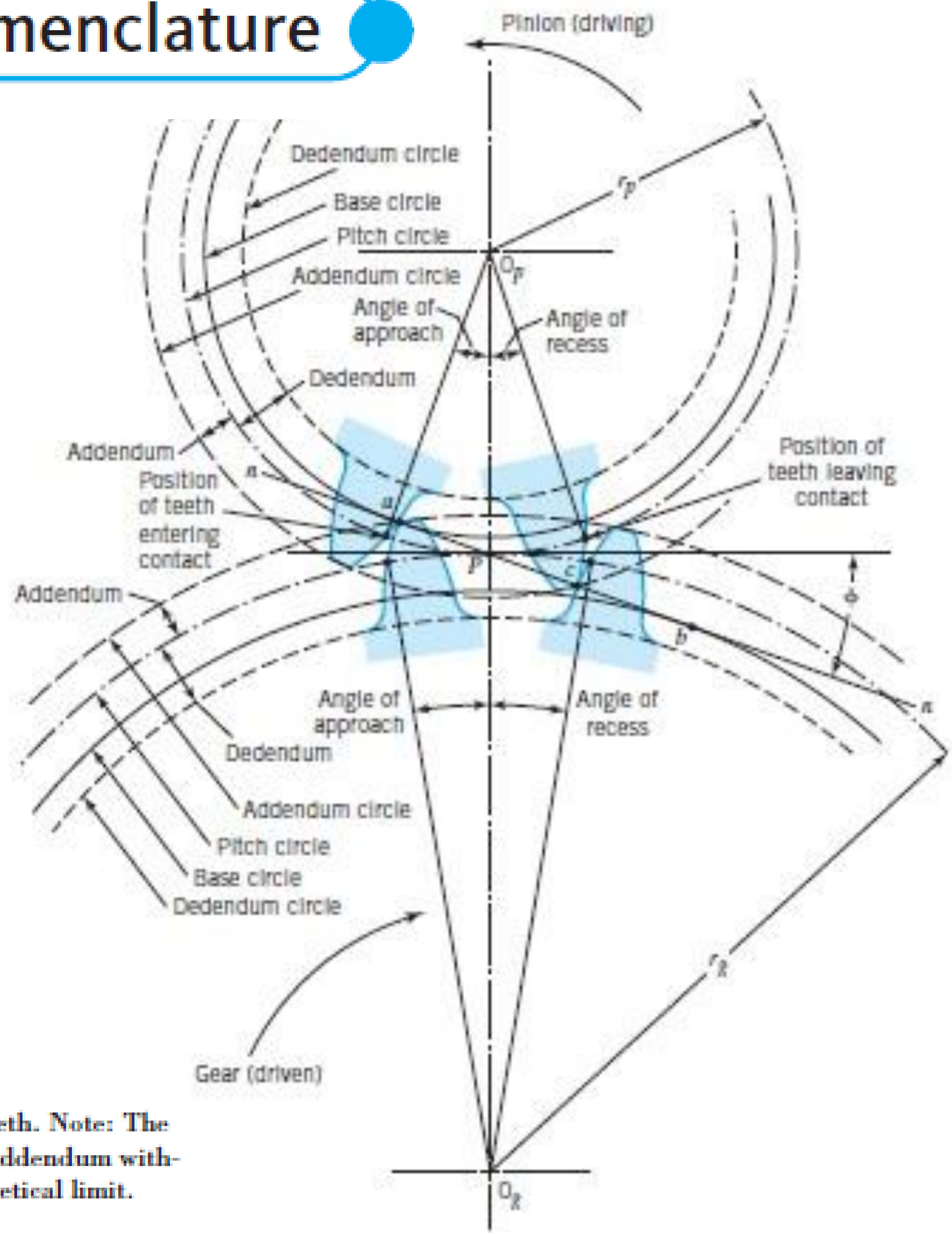
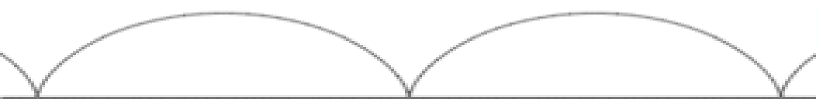


Figure 15.8
Further development and nomenclature of involute gear teeth. Note: The diagram shows the special case of maximum possible gear addendum without interference; pinion addendum is far short of the theoretical limit.

15.2 Geometry and Nomenclature

- The portion of the profile between the base and dedendum (root) circles cannot participate in the conjugate action - clear the tip of a mating tooth as the gears rotate.
- This portion is drawn as a radial line, but its actual shape is trochoidal.



- A line traced by a point on a circle that is travelling in straight line
- A fillet at the base of the tooth blends the profile into the dedendum circle - reduce bending stress concentration.

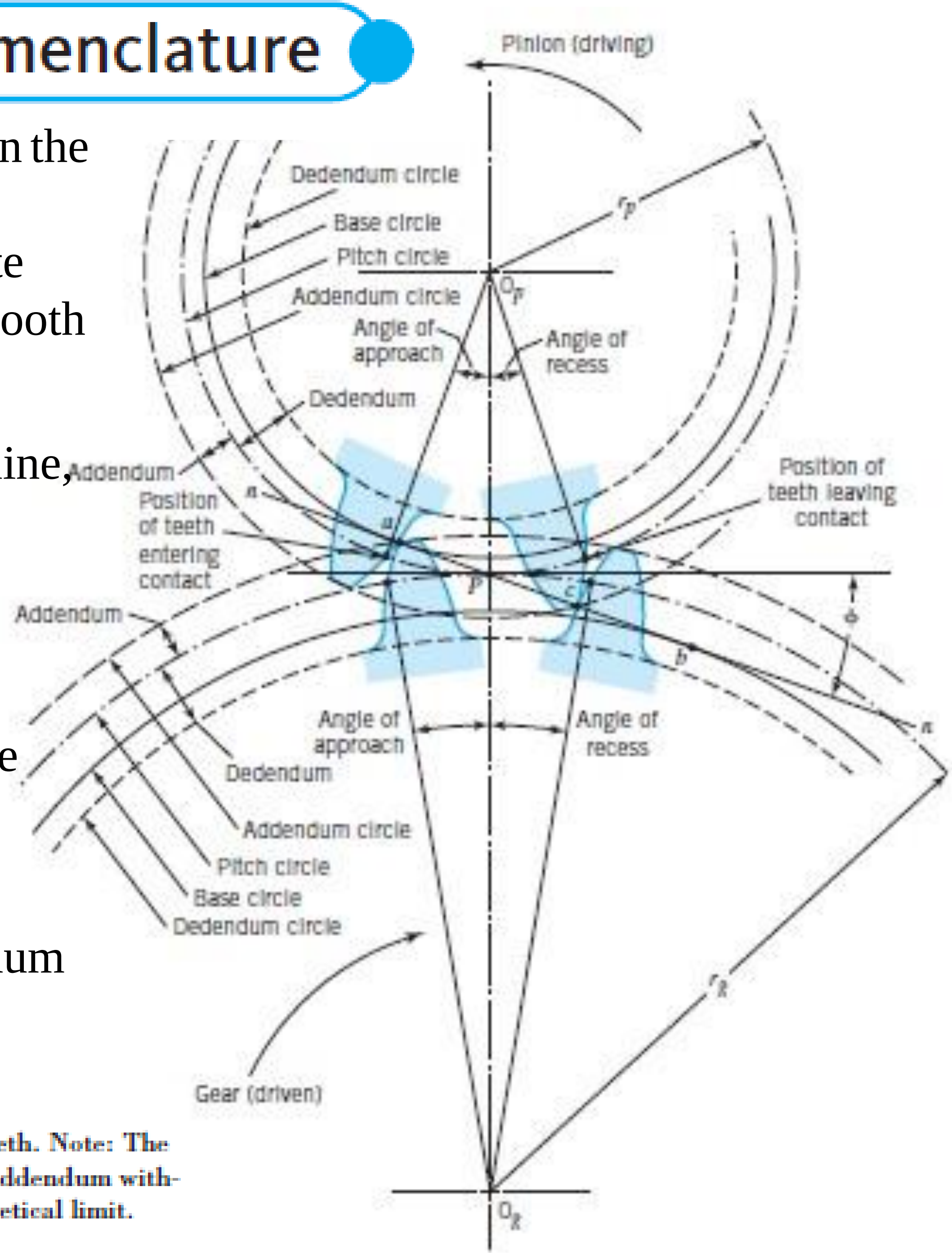


FIGURE 15.8
Further development and nomenclature of involute gear teeth. Note: The diagram shows the special case of maximum possible gear addendum without interference; pinion addendum is far short of the theoretical limit.

15.2 Geometry and Nomenclature

- The “dia” of a gear always refers to its pitch dia. If other dia (base, root, etc.) are intended, they are always specified.
- Similarly, d, without subscripts, refers to pitch diameter.
- The pitch dia of pinion and gear are distinguished by d_p and d_g.
- Figure shows the gear addendum extended exactly to point of tangency a.
- (The pinion addendum extends to arbitrary point c, which is short of tangency point b.)

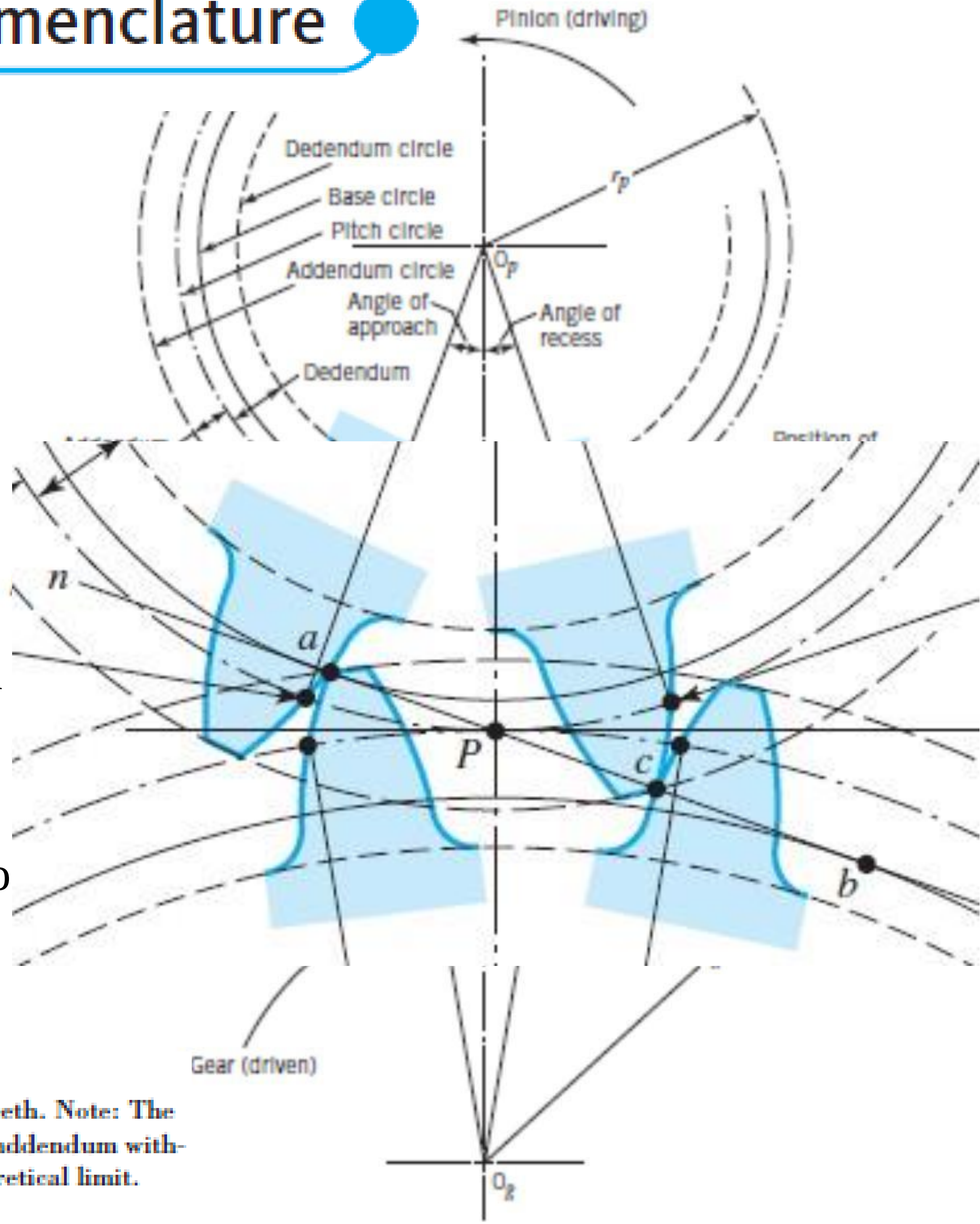
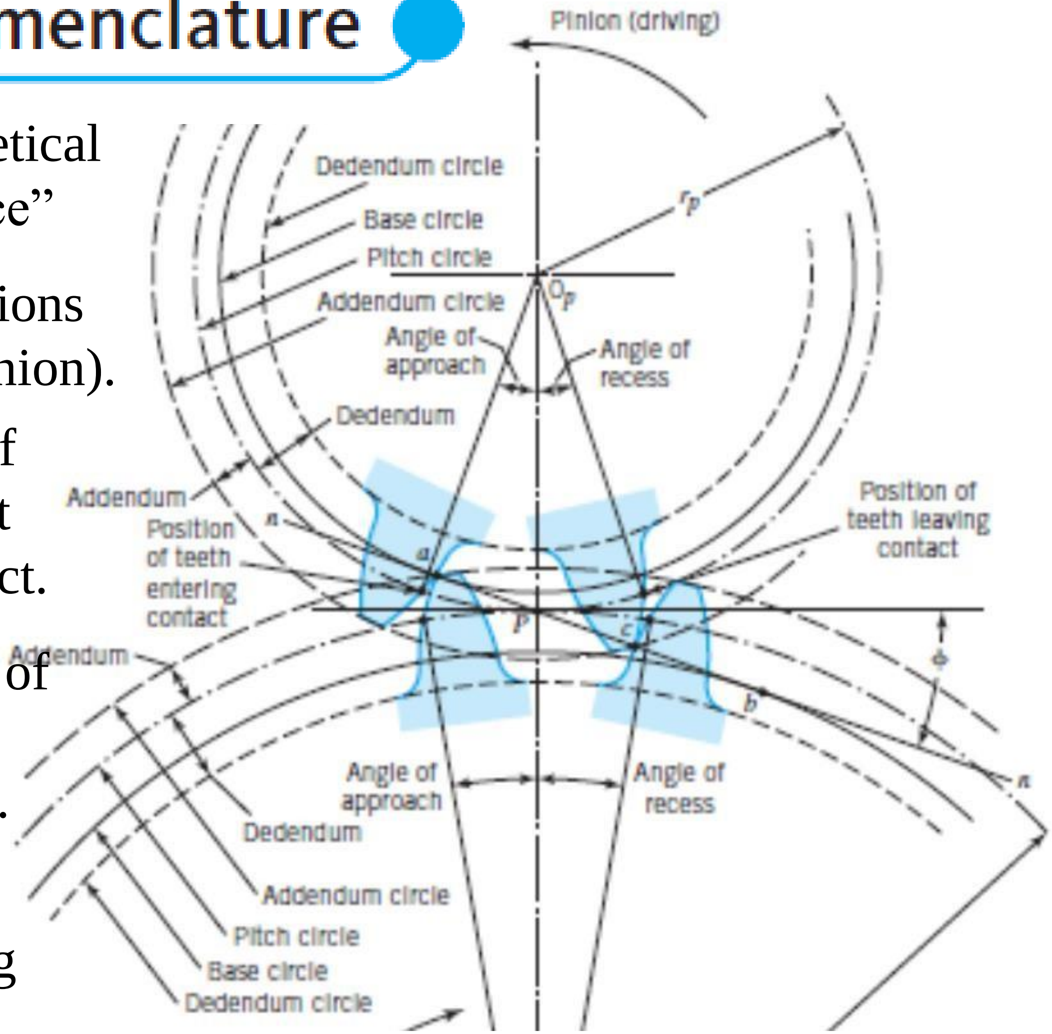


FIGURE 15.8
Further development and nomenclature of involute gear teeth. Note: The diagram shows the special case of maximum possible gear addendum without interference; pinion addendum is far short of the theoretical limit.

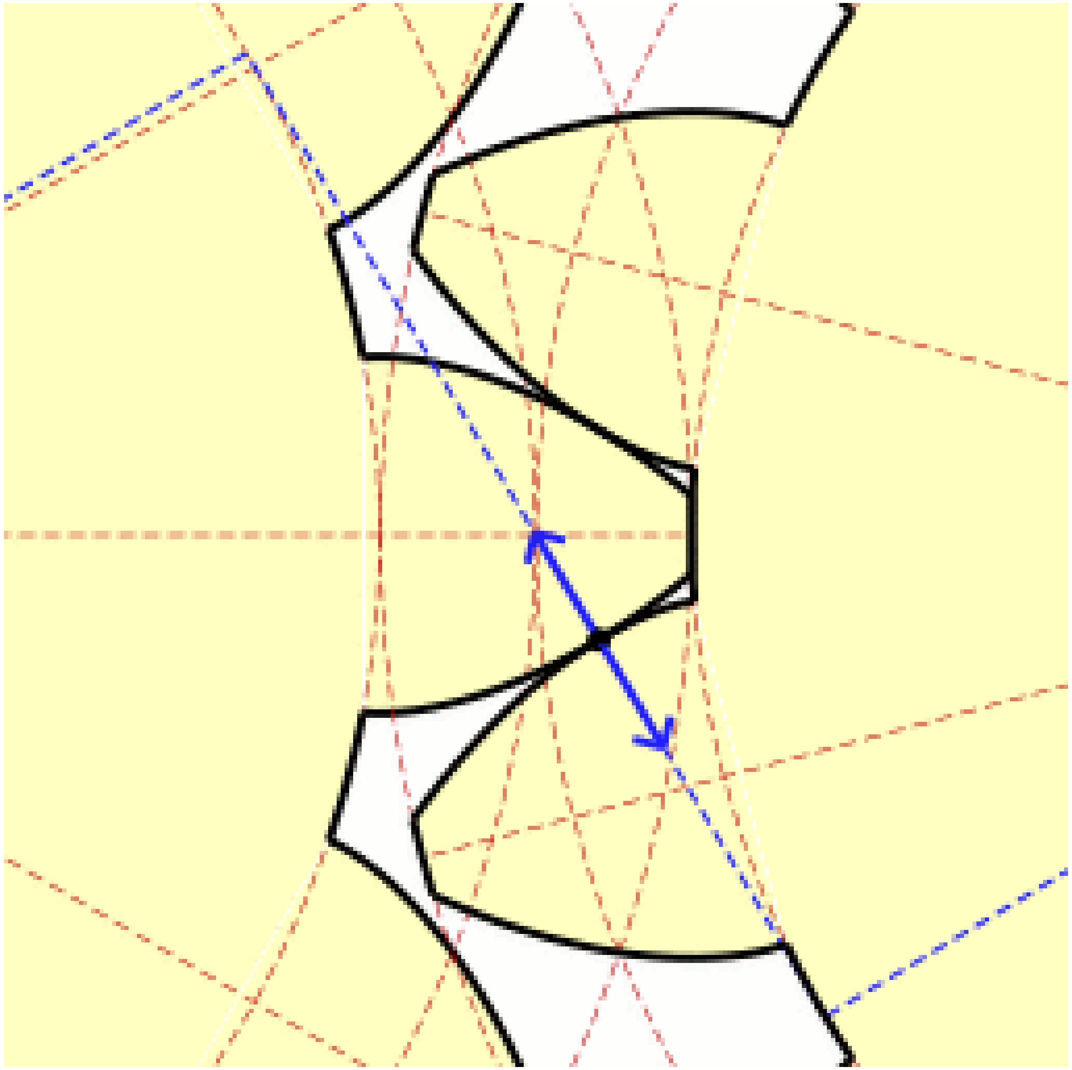
15.2 Geometry and Nomenclature

- Gear addendum represents theoretical max without causing "interference"
- Mating gears of standard proportions have shorter addenda (like the pinion).
- Figure shows position of a pair of mating teeth as they enter contact and again as they go out of contact.
- The angle of approach and angle of recess for both pinion and gear (measured to points on the pitch).
- Line nn is the LOA (neglecting friction, the force between mating teeth always acts along this line).



- The path of contact (locus of all points of tooth contact) is a segment of this line. In Figure the path of contact is the line segment ac.

Line of Action



- the contact point marching along the line of action
- the contact path bounded by two addenda
- LOA is the common normal (is a tangent to both base circles)
- LOA always meets at pitch point (*point with same linear velocity in both G and P)*, giving a constant m_v

15.2 Geometry and Nomenclature

- Further nomenclature relating to the complete gear tooth is shown in Figure. Various terms associated with gears and assemblies are given in Appendix J.

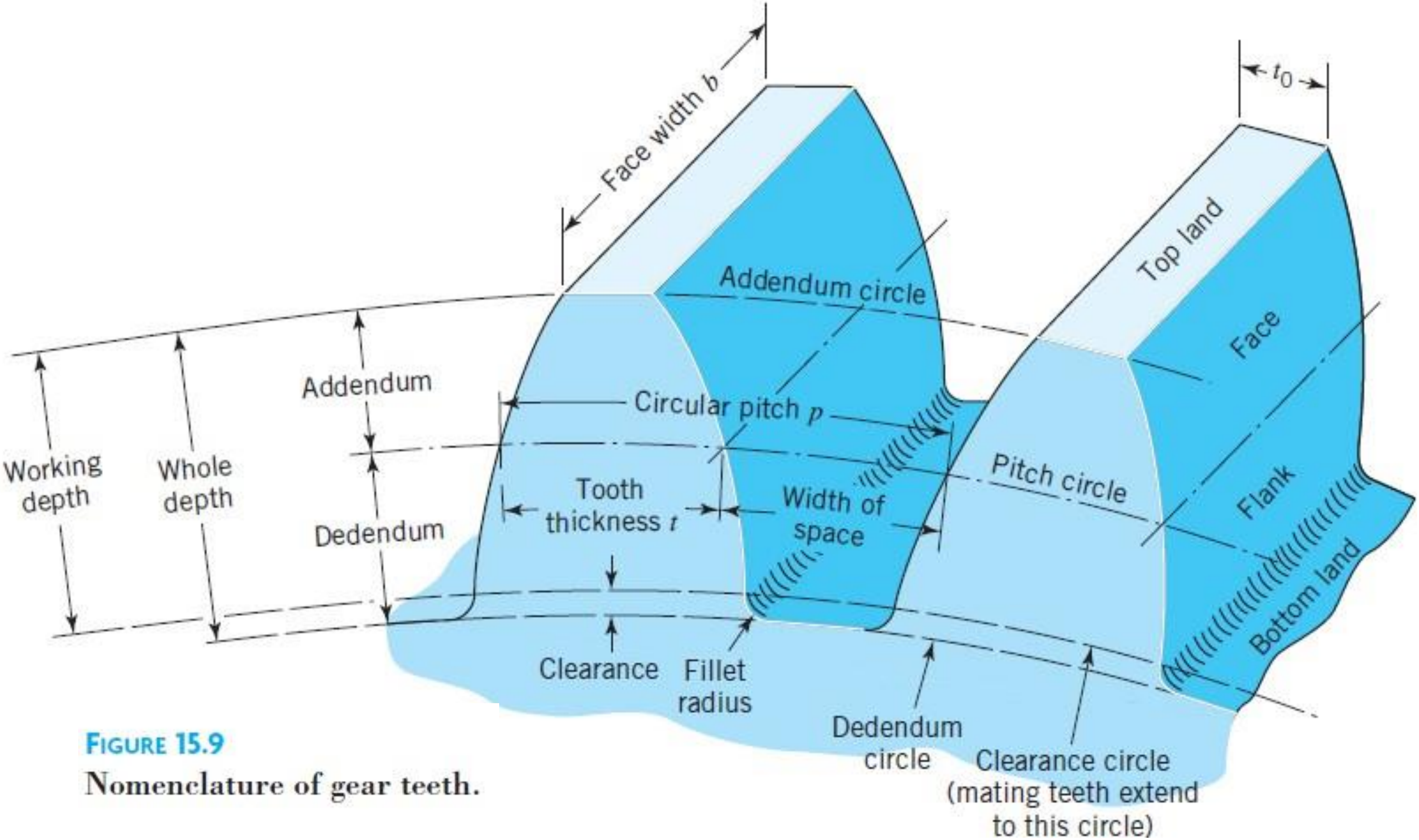


FIGURE 15.9
Nomenclature of gear teeth.

15.2 Geometry and Nomenclature

- Face and flank portions of the tooth surface are divided by the pitch cylinder. The circular pitch, p, and measured in inches or millimeters.
- If N is the number of teeth in the gear (or pinion), and d the pitch diameter, then

$$p = \frac{\pi d}{N}, \qquad p = \frac{\pi d_p}{N_p}, \qquad p = \frac{\pi d_g}{N_g} \qquad (p \text{ in inches}) \qquad \textbf{(15.2)}$$

- More commonly used indices diametral pitch P, and module m
- Diametral pitch is defined as the number of teeth per inch of pitch diameter:

$$P = \frac{N}{d}, \qquad P = \frac{N_p}{d_p}, \qquad P = \frac{N_g}{d_g} \qquad (P \text{ in teeth per inch}) \qquad \textbf{(15.3)}$$

- Module m, the reciprocal of P, is defined as the pitch dia in mm / # of teeth

$$m = \frac{d}{N}, \qquad m = \frac{d_p}{N_p}, \qquad m = \frac{d_g}{N_g} \qquad (m \text{ in millimeters per tooth}) \qquad \textbf{(15.4)}$$

- It is therefore

$$pP = \pi \qquad (p \text{ in inches; } P \text{ in teeth per inch})$$

15.2 Geometry and Nomenclature

- Face and flank portions of the tooth surface are divided by the pitch cylinder. The circular pitch, p, and measured in inches or millimeters.

$$p/m = \pi \qquad (p \text{ in millimeters; } m \text{ in millimeters per tooth}) \qquad \textbf{(15.6)}$$

$$m = 25.4/P \qquad \textbf{(15.7)}$$

- In English units “pitch,” means DP (“12-pitch gear” is 12 teeth per inch of pitch dia)
- In SI units “pitch” is circular pitch (“gear of pitch = 3.14 mm” is with cp = 3.14 mm).
- Figure shows the actual size of gear teeth of standard dp’s. Standard modules are
- 0.2 to 1.0 by increments of 0.1,
  1.0 to 4.0 by increments of 0.25
  4.0 to 5.0 by increments of 0.5
- Standard Pressure Angle ϕ is 20°
  14.5° extinct, 25° used only in the US

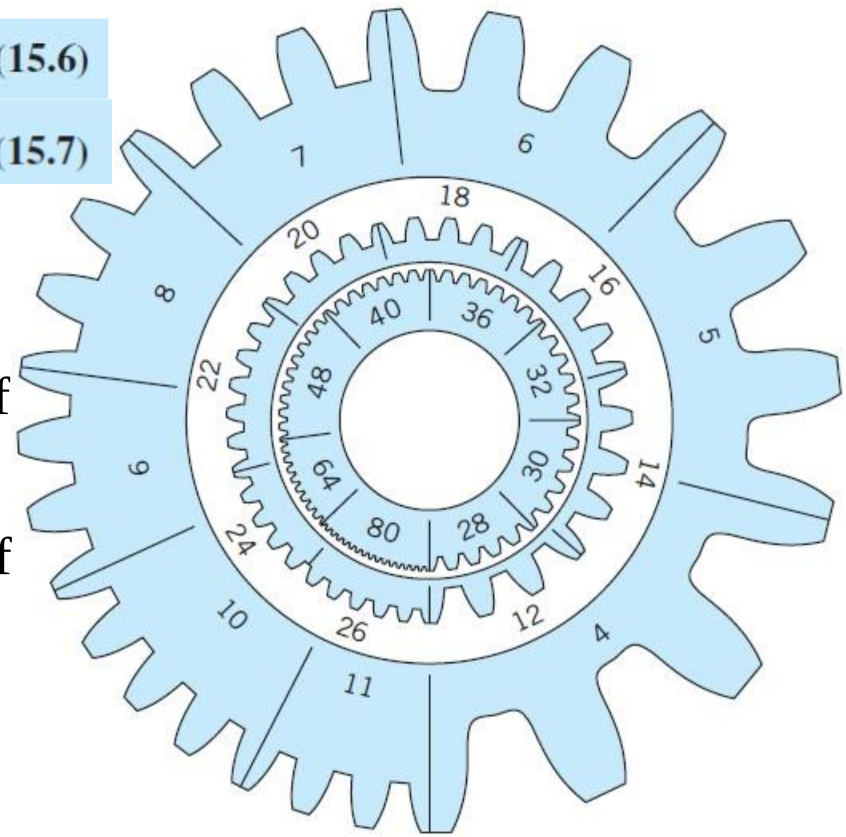
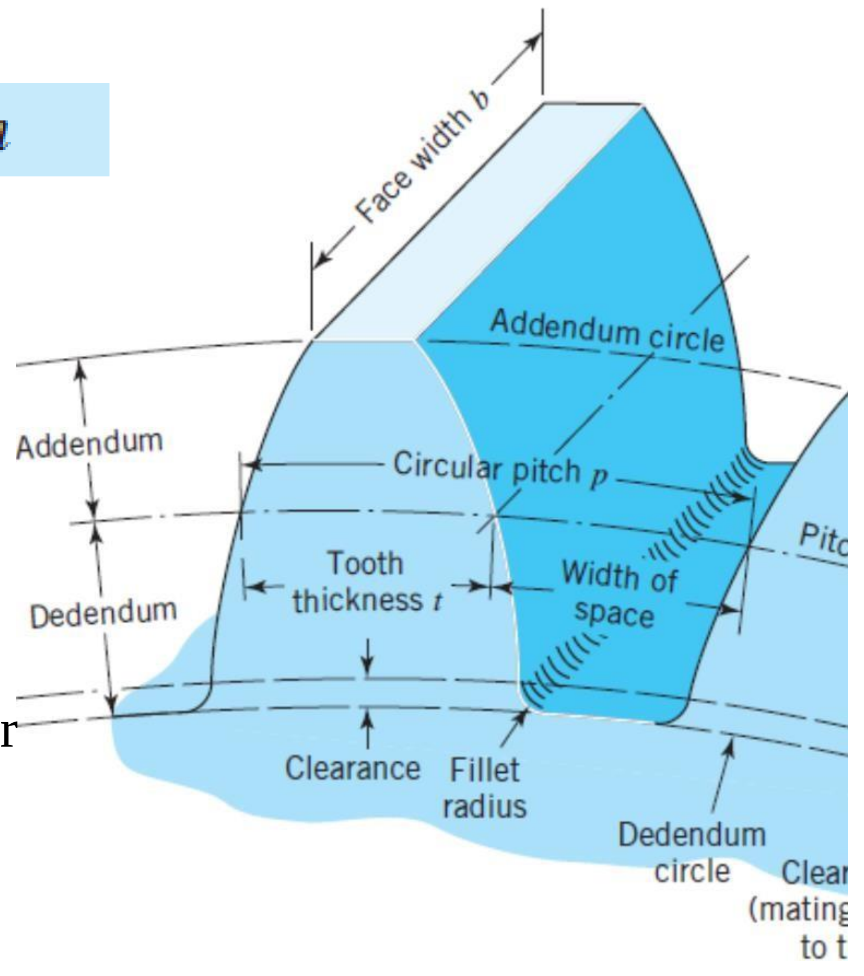


FIGURE 15.10
Actual sizes of gear teeth of various diametral pitches. Note: In general, fine-pitch gears have $P \geq 20$; coarse-pitch gears have $P < 20$. (Courtesy Bourn & Koch Machine Tool Company.)

15.2 Geometry and Nomenclature

- standard addendum is 1/P" or m mm, and standard dedendum is 1.25 * addendum.
- The addendum is shortened to to 0.8/P. The fillet radius is 0.35/P" or m/3
- Face width, b, is not standardized, but generally,
- $\frac{9}{P} < b < \frac{14}{P}$ or $9m < b < 14m$
- The wider the face width, the more difficult it is to manufacture and mount the gears so that contact is uniform across the full face width.
- Gears made to standard systems are interchangeable and available in stock.
- Mass-produced gears for specific applications deviate from these standards for optimal performance under specific application (automobile transmission gears)



15.2 Geometry and Nomenclature

- Figure shows a pinion in contact with a rack, a segment of a gear - infinite dia.
- Next one shows a pinion in contact with an internal gear or annulus, or ring gear,
- Commonly used in the planetary gear trains of automotive automatic transmissions
- Diameters of internal gears are considered negative; hence, Eq. 15.1 indicates that a pinion and internal gear rotate in the same direction.

$$\omega_p/\omega_g = -d_g/d_p \qquad (15.1)$$

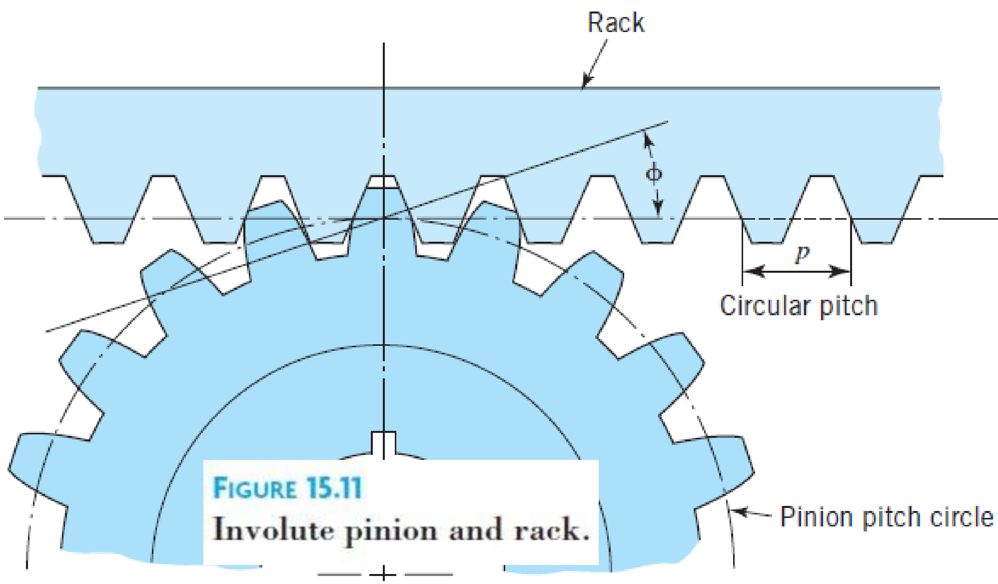


FIGURE 15.11
Involute pinion and rack.

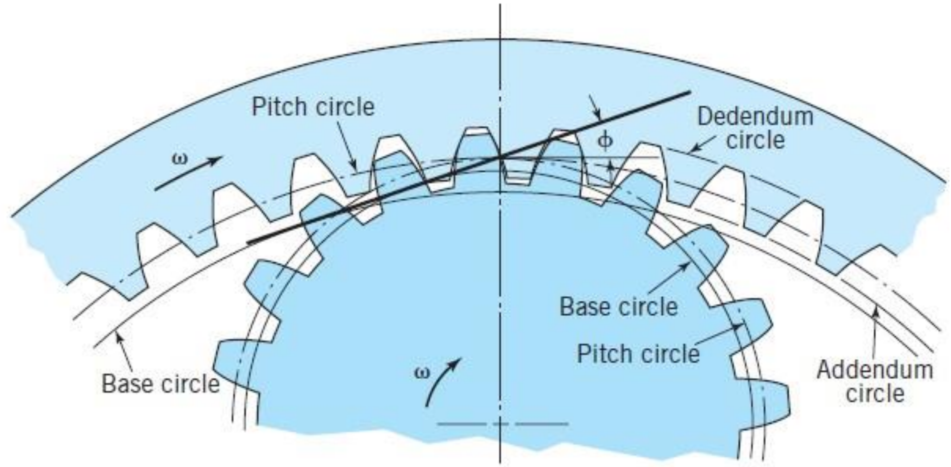
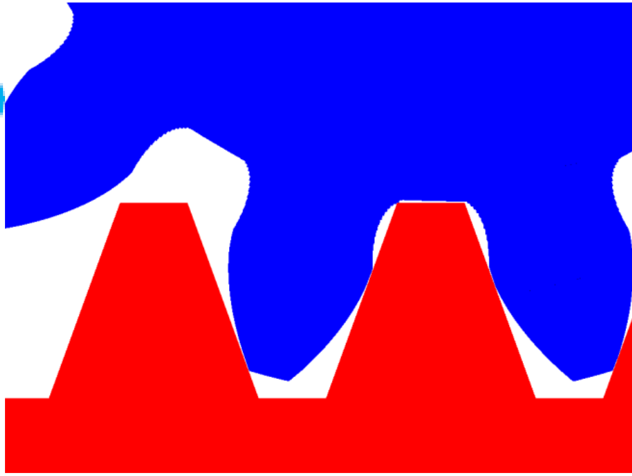


FIGURE 15.12
Involute pinion and internal gear. Note that both rotate in the same direction.

15.2 Geometry and Nomenclature



- Advantage of the involute is that it provides theoretically perfect conjugate action even when the shaft center distances are not exactly correct.
- If the shafts are separated, proper action continues with an increased ϕ. The backlash increases with increase in separation. Sometimes necessary for lubricating oil film between mating teeth, or to counter thermal expansion.
- Another advantage of the involute system is that the profile for the basic rack is a straight line. This facilitates cutter manufacture and gear-tooth generation.
- Gears are manufactured by gear hobber or shaper
- Or they are done by rack cutter

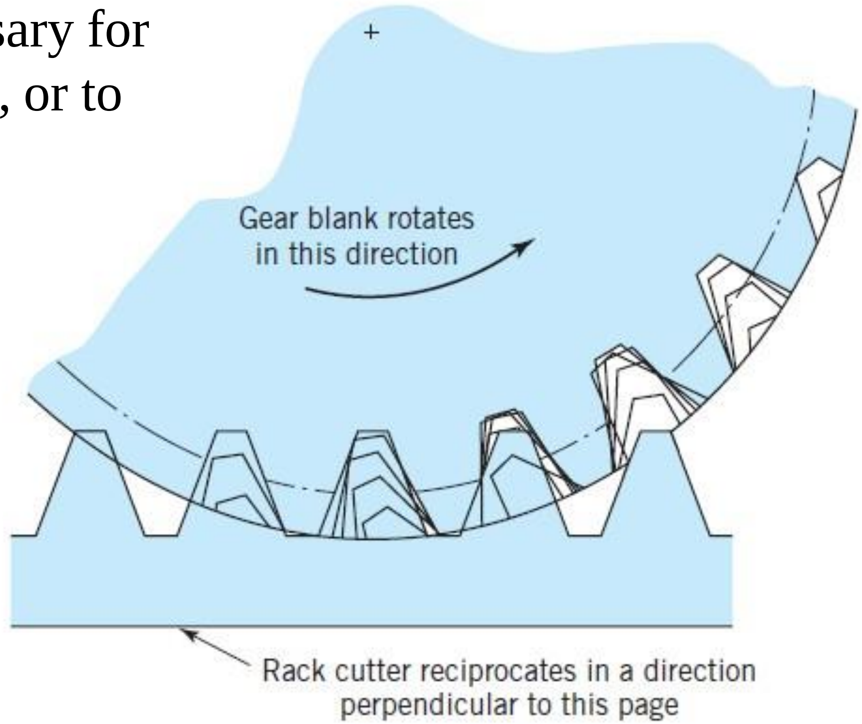
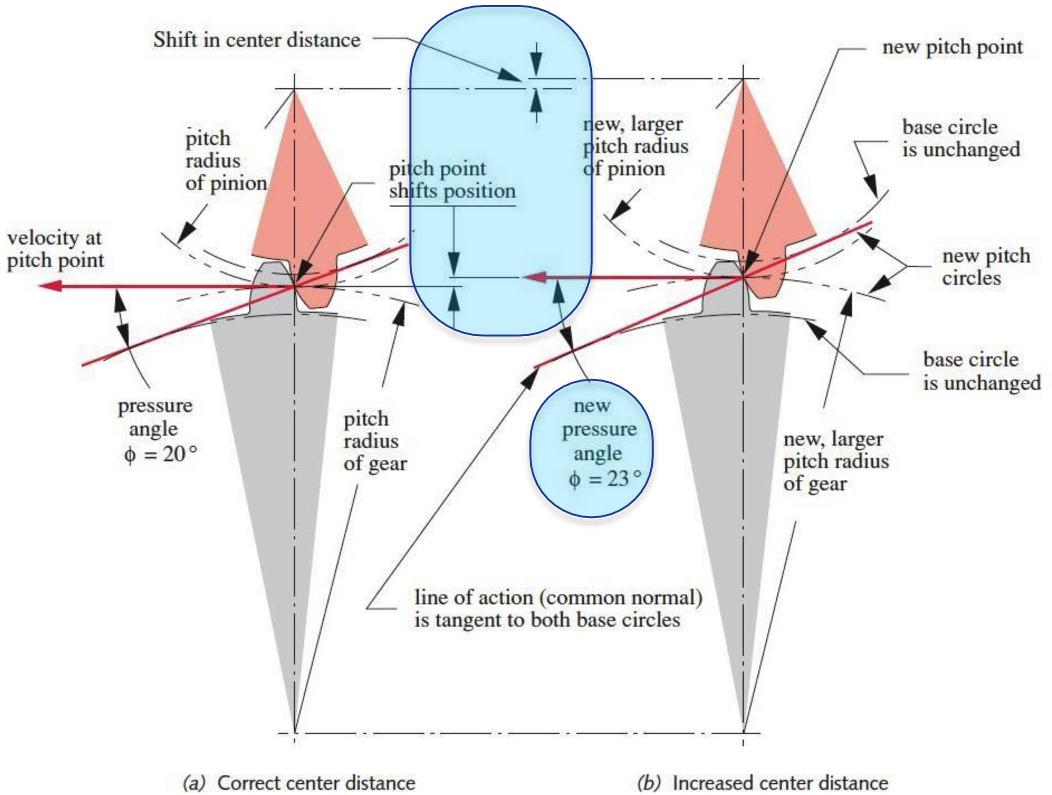


FIGURE 15.14
Shaping teeth with a rack cutter.

Why Involute

- The fundamental law of gearing: the common normal of the tooth profiles, As the pitch point shifts in proportion with the change in the center distance in involute profiles, the velocity ratio remains constant.
- Only the pressure angle increases when the center distance grows.



ꟷURE 12-7

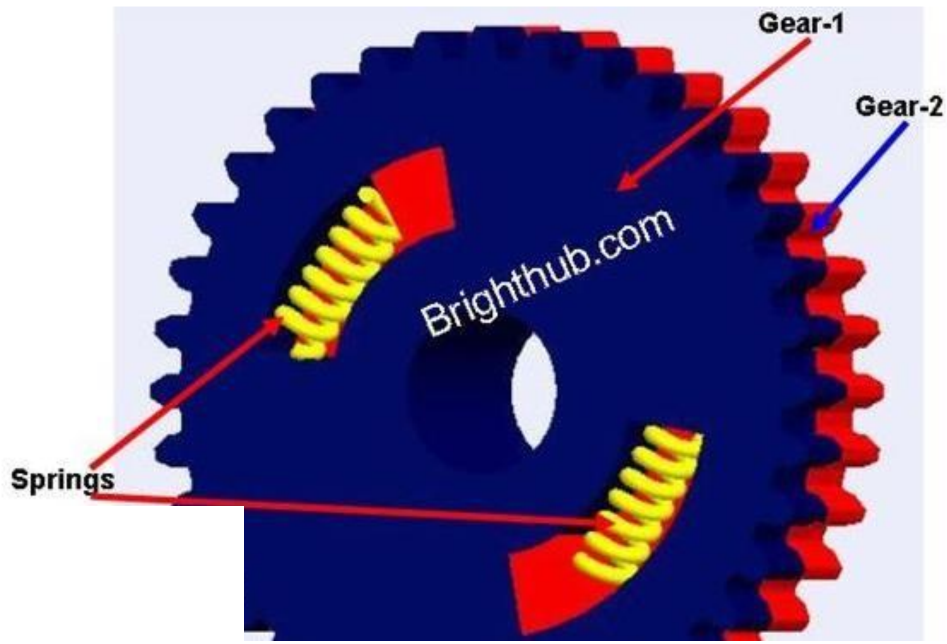
nging Center Distance of Involute Gears Changes Only the Pressure Angle and Pitch Diameters

Why Involute

- With an involute tooth form, the centre-distance errors do not violate the velocity ratio *which is fixed to pitch dia that does not change*

Backlash

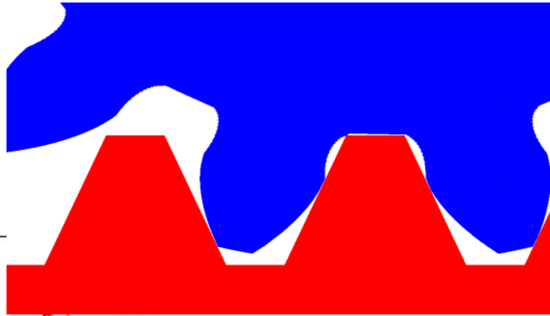
- Backlash is defined as the gap between mating teeth measured along the circumference of the pitch circle
- Increasing the centre-to-centre distance C to $\bar{C}$ will increase the backlash



Two gears mounted with spring to prevent backlash for critical applications

Backlash (measured on pitch circle)	$\bar{B}$	in.	$\bar{B} = 2(\bar{c} - c)\tan\bar{\phi}$

15.3 Interference and Contact Ratio



- Interference occurs, preventing rotation of the mating gears, if addendum extends beyond tangent points a and b.
- In Figure both addendum circles extend beyond the interference points;
- These gears will not operate without modification.
- The preferred correction is to remove the interfering tooth tips, shown shaded.
- Alternatively, the tooth flanks of the mating gear can be undercut in order to clear the offending tips, but this weakens the teeth.

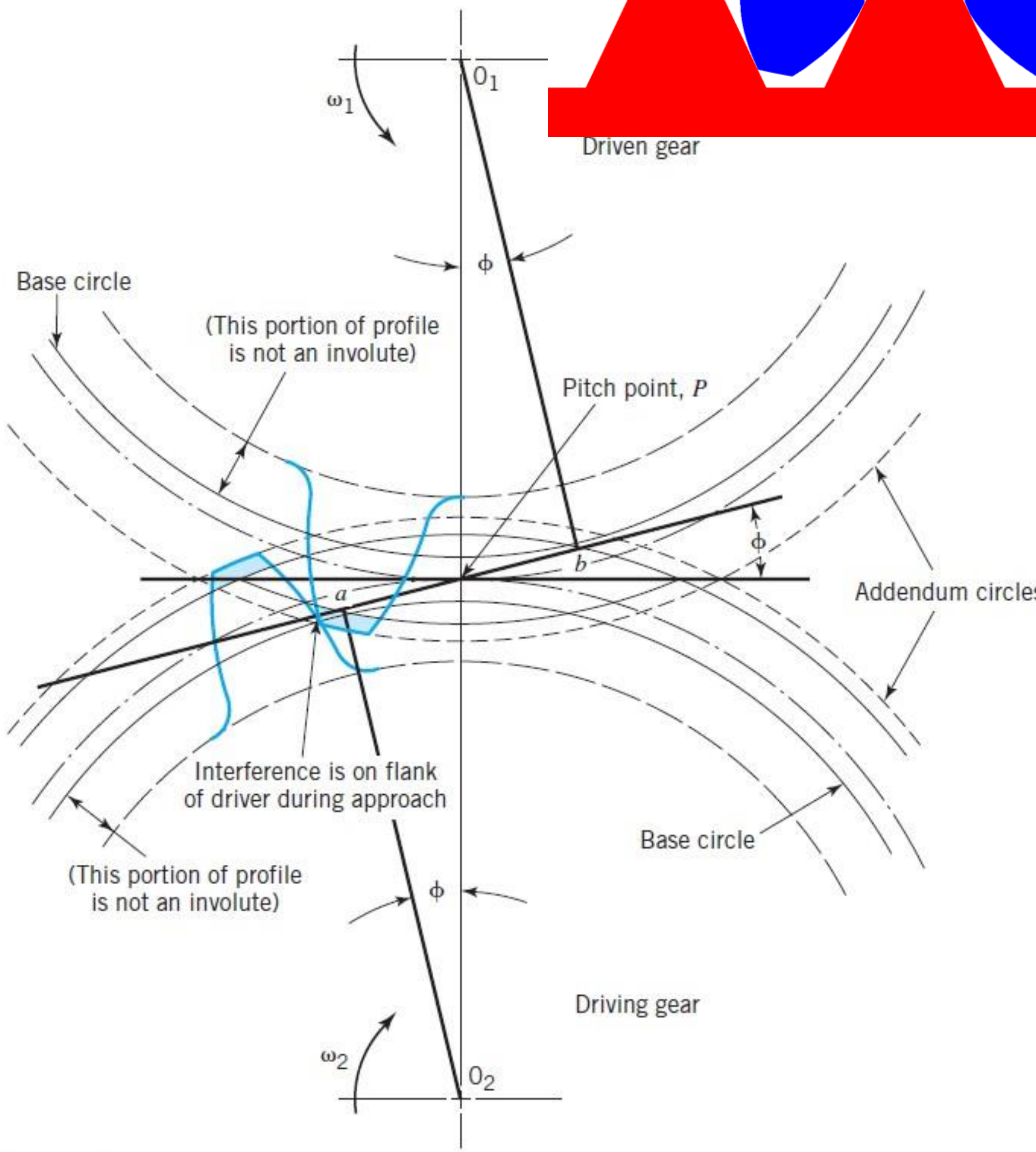


FIGURE 15.15
Interference of spur gears (eliminated by removing the shaded tooth tips).

15.3 Interference and Contact Ratio

- It is impossible to have useful contact of the shaded tips, as conjugate involute action is not possible beyond these points.
- When teeth are generated with a rack cutter, the teeth are automatically undercut if they would interfere with a rack.
- This undercutting takes place with 20° pinions < 18 teeth, and 25° pinions < 12 teeth.
- For this reason pinions with fewer than these numbers of teeth are not normally used with standard tooth proportions.

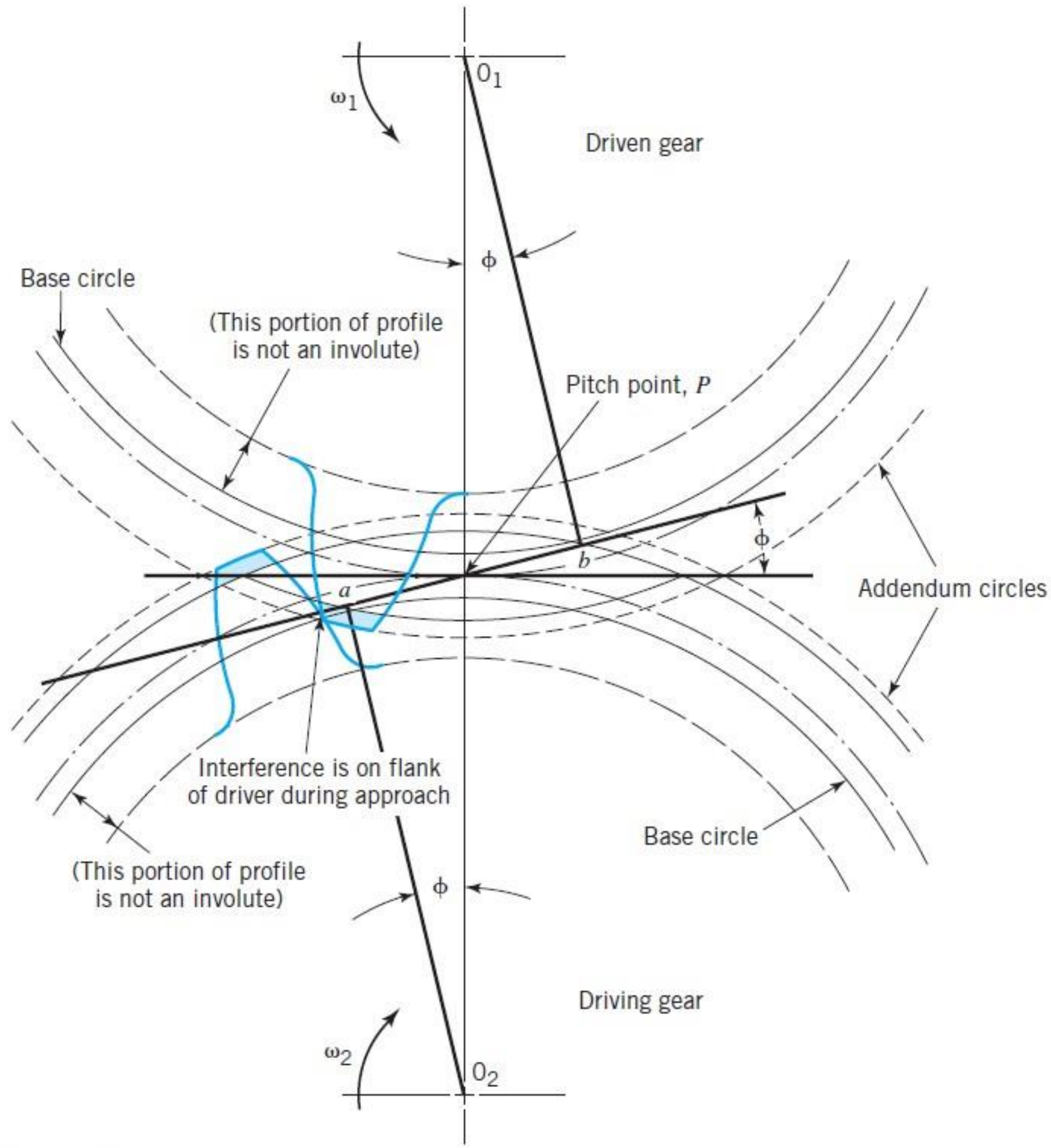


Figure 15.15
Interference of spur gears (eliminated by removing the shaded tooth tips).

15.3 Interference and Contact Ratio

- $r_a = r + a$, Where r_a is radius of addendum circle, r is pitch circle radius and a is addendum
- So max r_a without interference will be

$$r_{a(\max)} = \sqrt{r_b^2 + c^2 \sin^2 \phi}$$

- Where r_b is base circle radius of the same member, c is center distance, and ϕ is actual not nominal PA
- *i*nterference is more likely to involve the tips of the gear teeth than the tips of the pinion teeth
- Interference is promoted by having a small number of pinion teeth, a large number of gear teeth, and a small pressure angle.

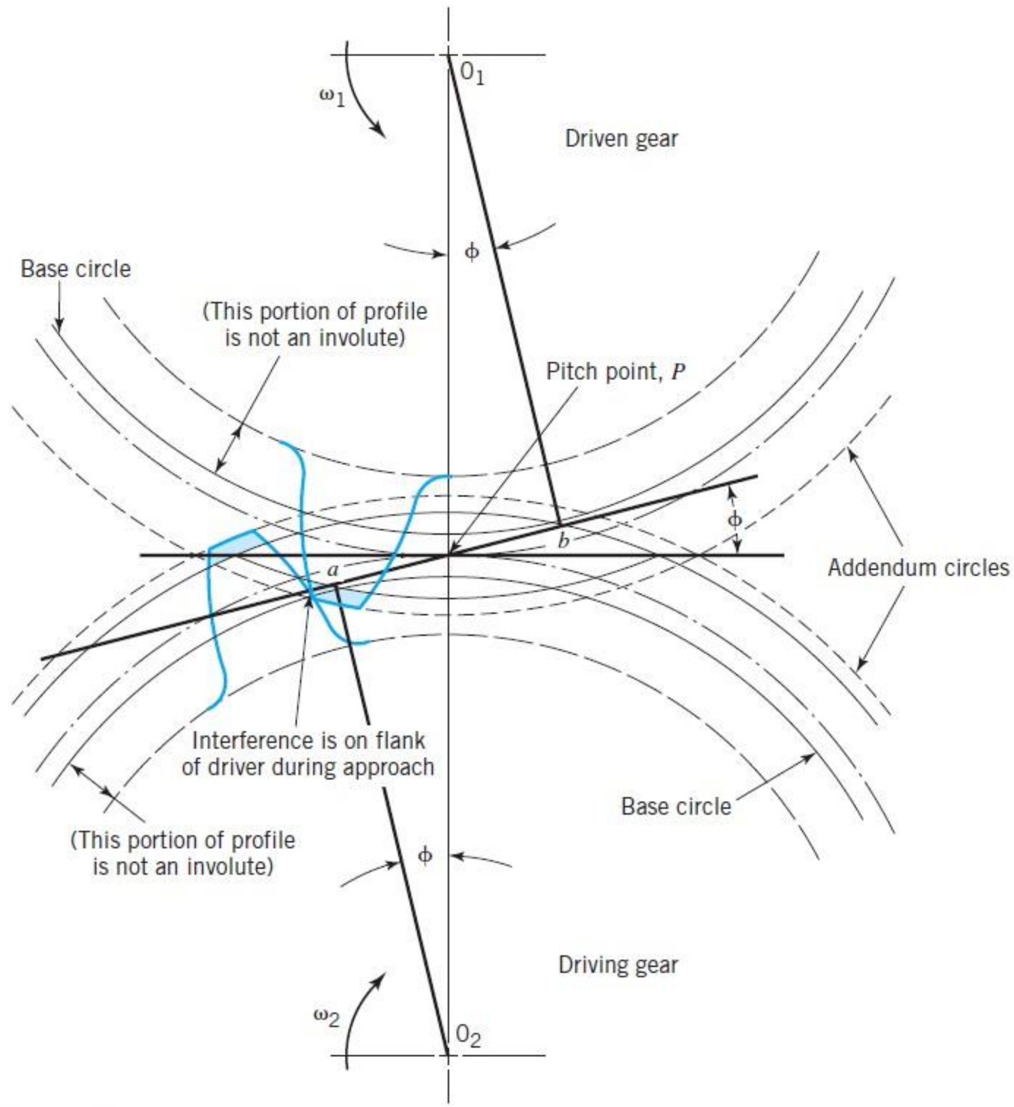
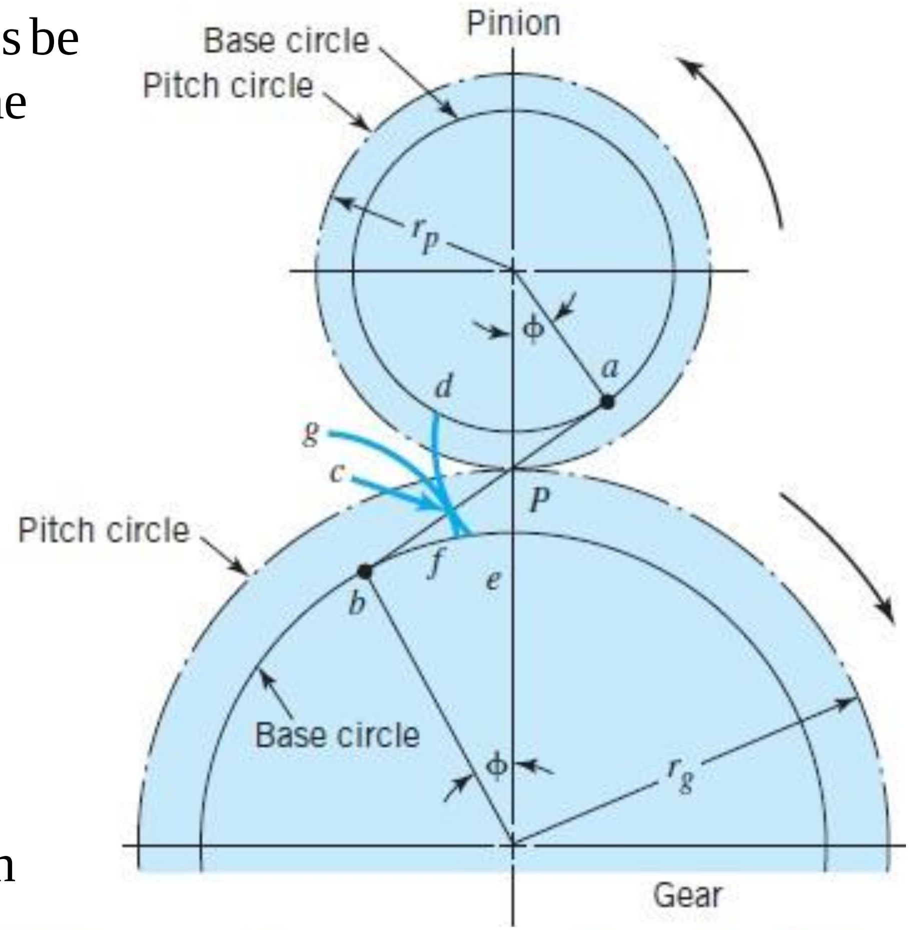


Figure 15.15
Interference of spur gears (eliminated by removing the shaded tooth tips).

15.3 Interference and Contact Ratio

- It is important to proportion tooth profiles be so that a second pair of mating teeth come into contact before the first pair is out of contact. The average number of teeth in contact as the gears rotate together is the contact ratio (CR),



$$\mathrm{CR} = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin\phi}{p_b} \tag{15.9}$$

where

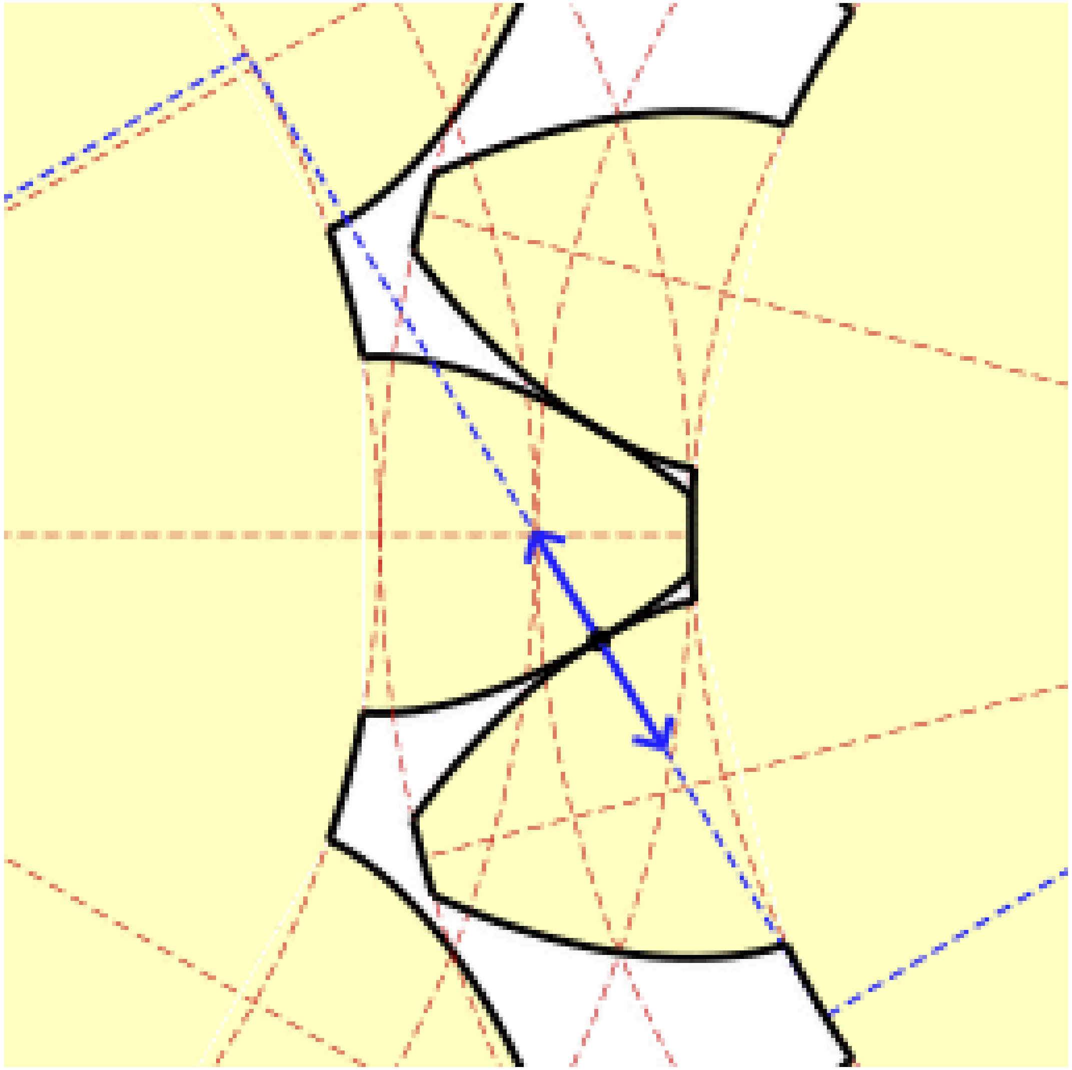
r_{ap}, r_{ag} = addendum radii of the mating pinion and gear

r_{bp}, r_{bg} = base circle radii of the mating pinion and gear

- Base pitch $P_b = \pi d_b/n$ where
- d_b is the dia of base circle and n # of teeth
- From figure $d_b = d\cos\phi, \quad r_b = r\cos\phi, \quad \text{and} \quad p_b = p\cos\phi \quad (15.11)$
- The base pitch is like the circular pitch except is an arc in base circle
- > the CR, the smoother the gear. CR>2 implies that atleast 2 pairs of teeth are theoretically in contact at all times.

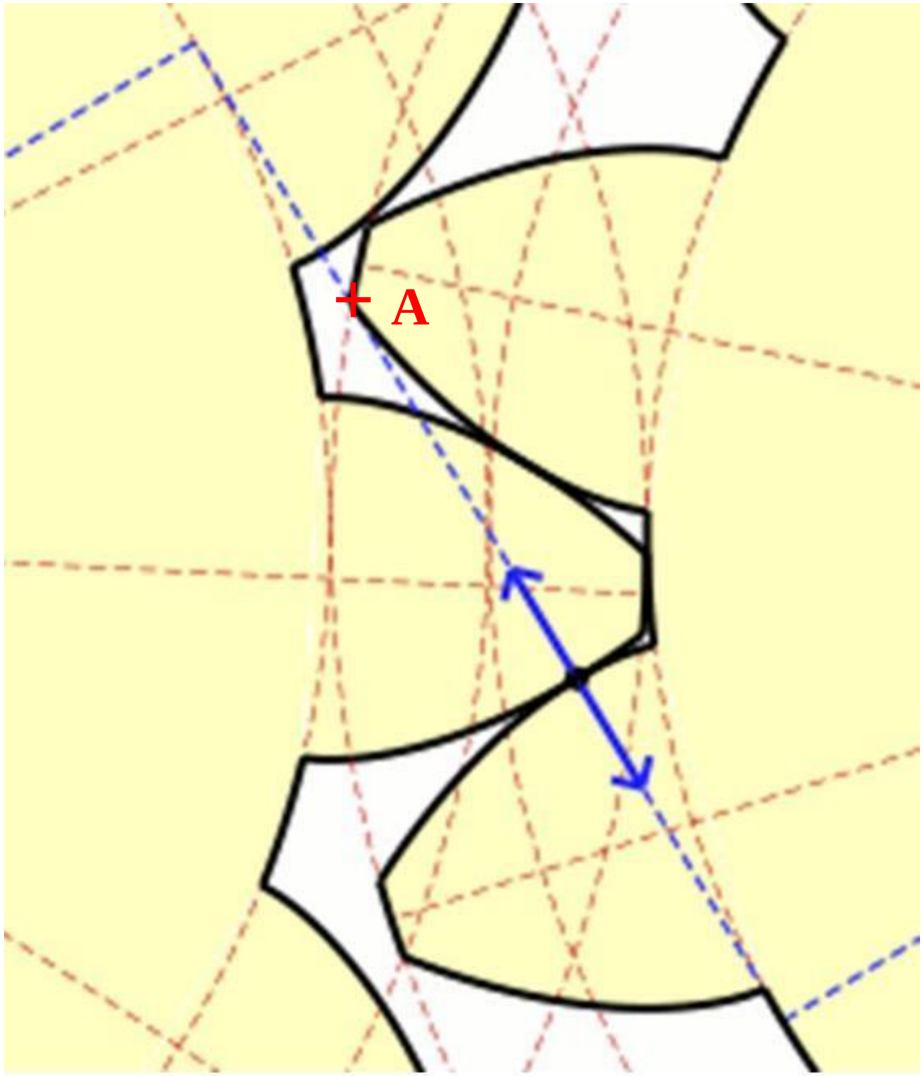
15.3 Interference and Contact Ratio

- the contact point marching along the line of action
- the contact path bounded by two addenda
- LOA is the common normal (is a tangent to both base circles)
- LOA always meets at pitch point(*point with same linear velocity in both G and P)*, giving a constant ω ratio



15.3 Interference and Contact Ratio

- Figure shows three phase positions of contacting involute gear teeth
- Teeth comes in contact at A at first where addendum of driven cuts the line of action
- Contact follows the line of action to point P and then to B where addendum of driver cuts the with line of action
- Line AB (bound by the two addenda) is the path of point of contact and its length is the Length of the path of contact or **Length of Action**



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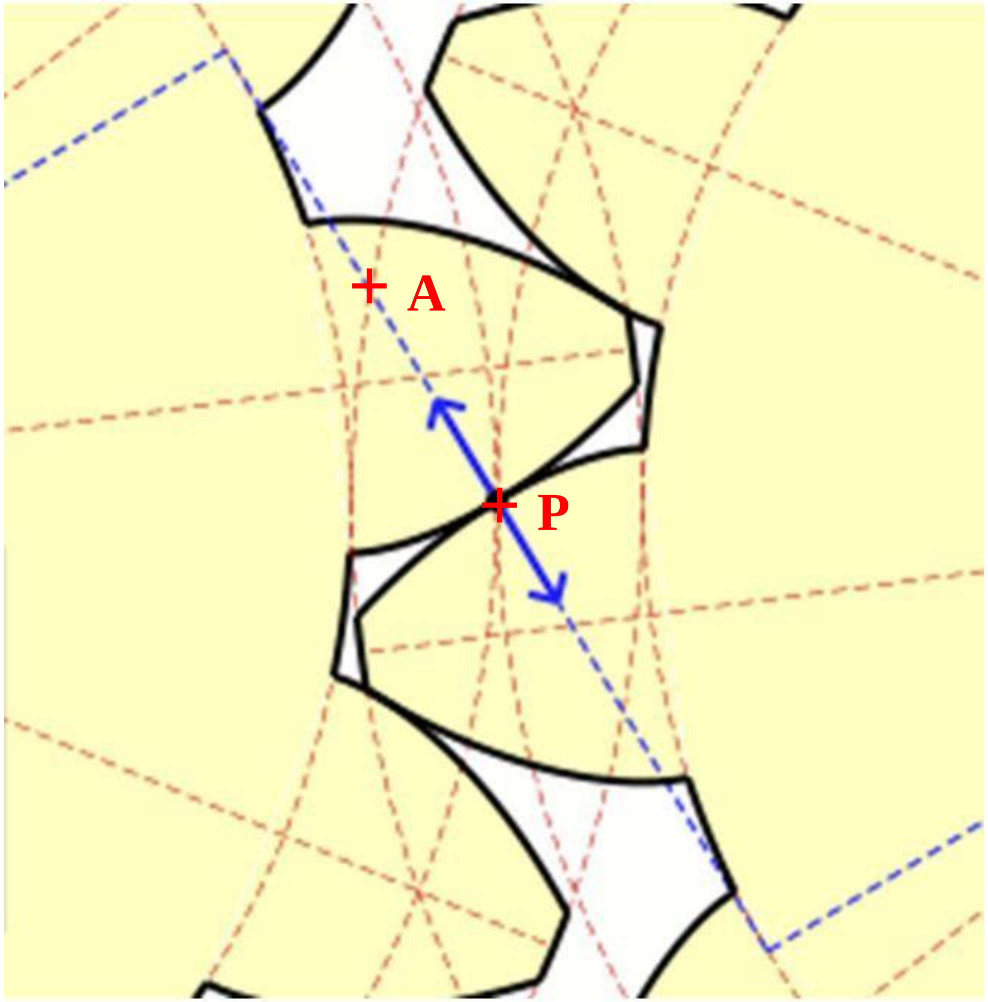

15.3 Interference and Contact Ratio

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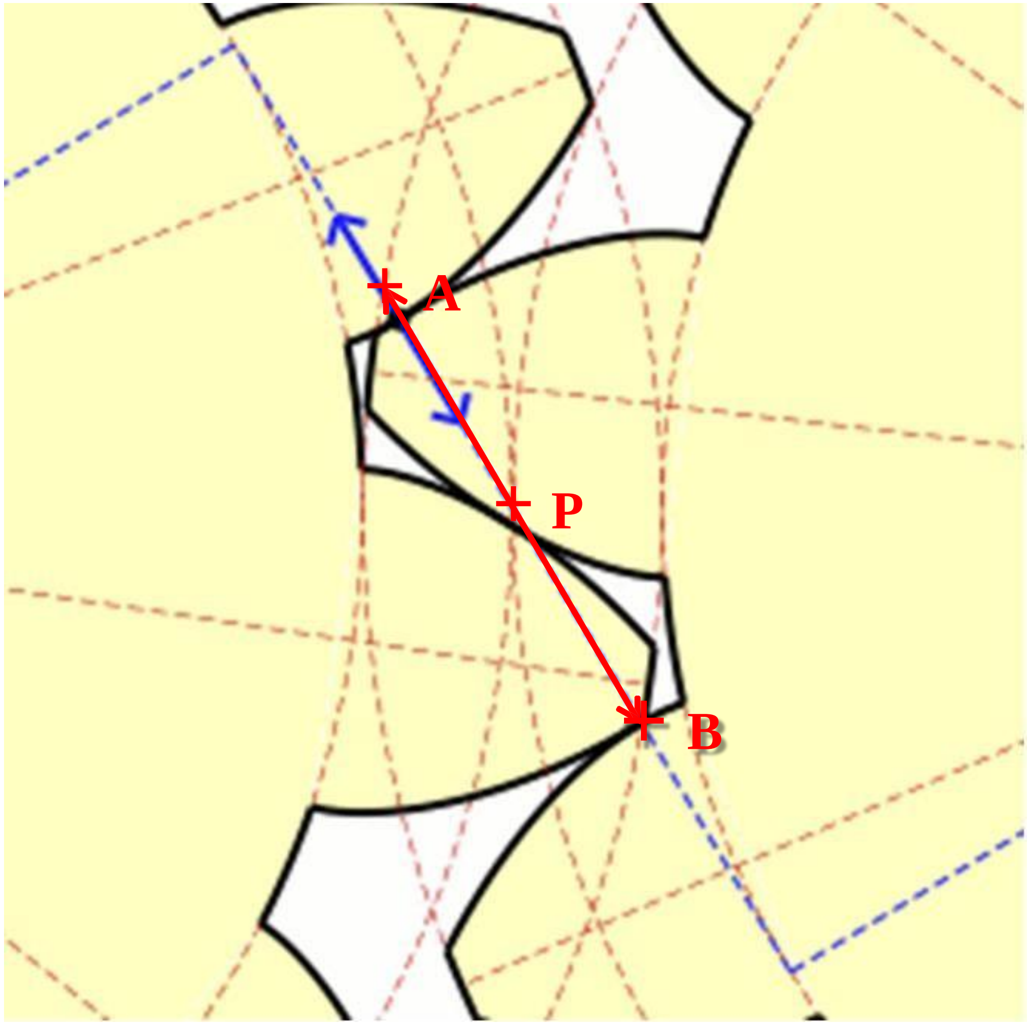
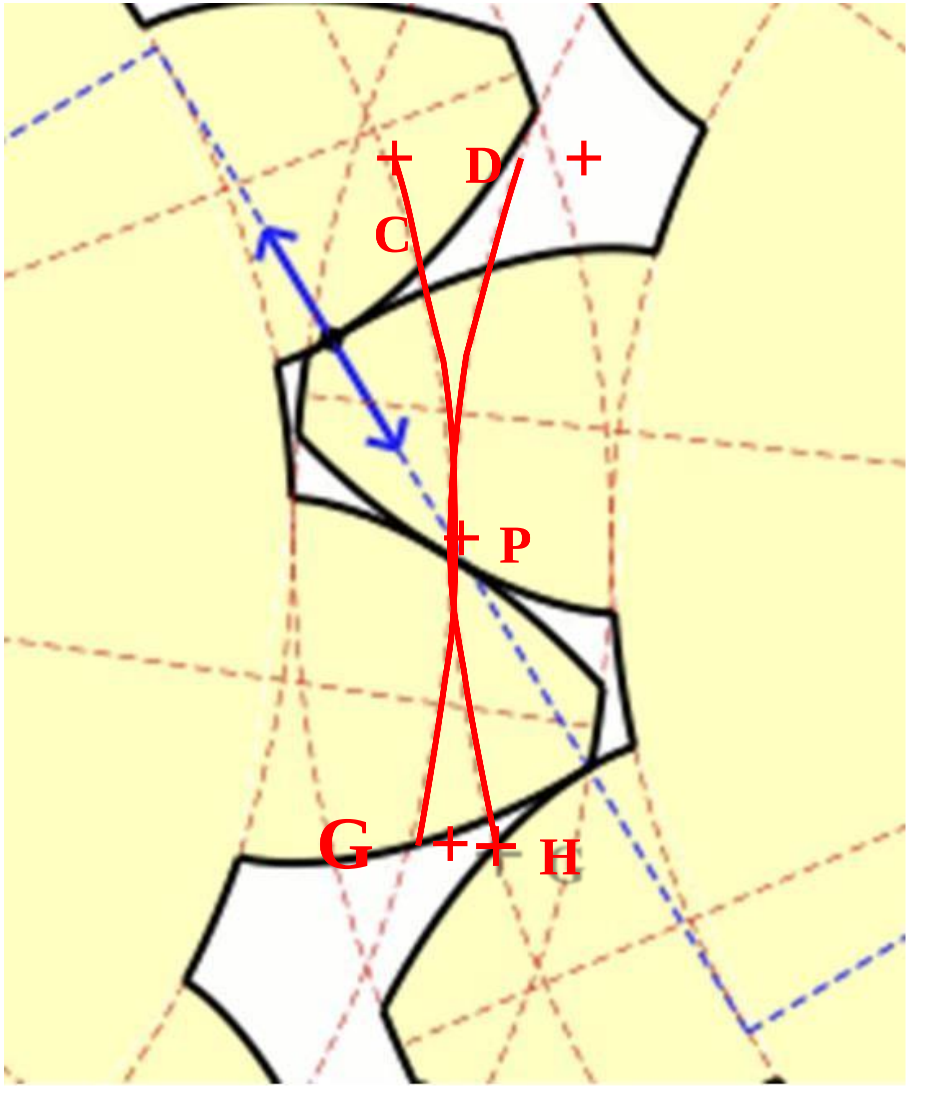


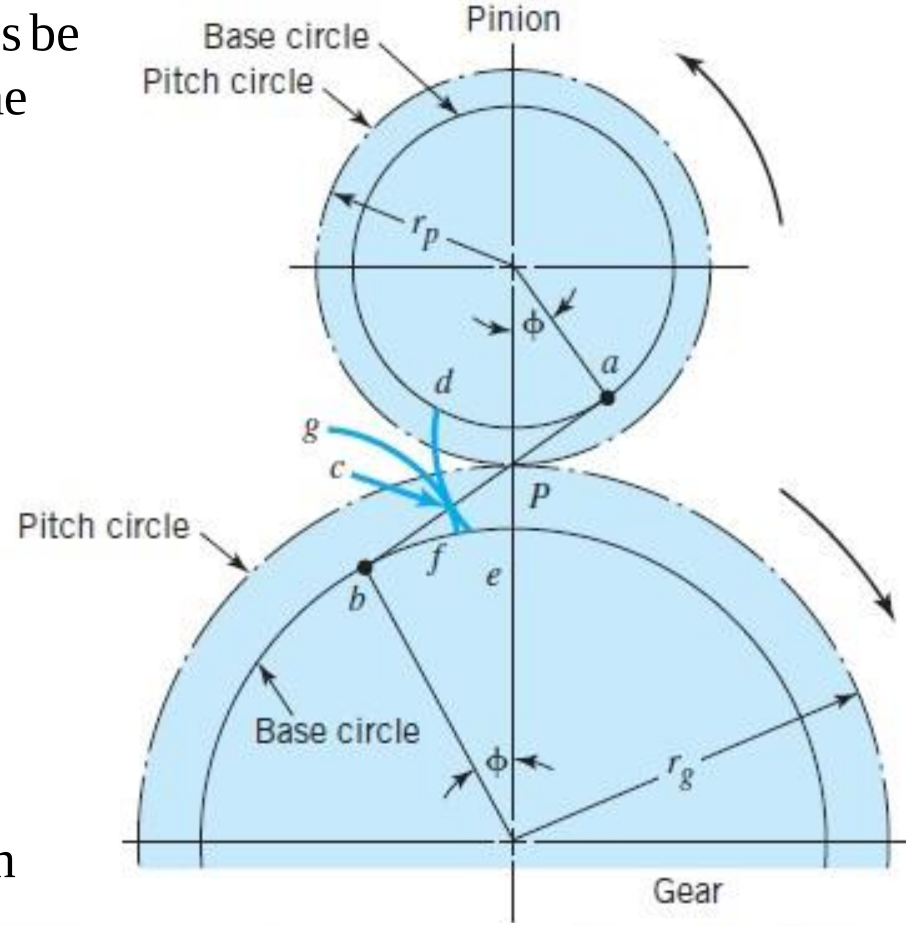
15.3 Interference and Contact Ratio

- Point C is the intersection of tooth profile on gear 2 with pitch circle at initial contact and G is the same point on gear 2 at end of contact
- Similarly points D and H for gear 3. The arcs CPG and DPH are the arcs of action
- These arcs are equal as pitch circles roll on one another Arc
- Contact ratio is average number of Pairs of teeth which are in contact
- Is equal to number of base pitch in the length of point of contact
- $m_c = \frac{lenght\ of\ path\ of\ contact}{base\ pitch} = \frac{AB}{p_b}$
- AB can be found graphically are from the following



15.3 Interference and Contact Ratio

- It is important to proportion tooth profiles be so that a second pair of mating teeth come into contact before the first pair is out of contact. The average number of teeth in contact as the gears rotate together is the contact ratio (CR),



$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin\phi}{p_b} \qquad (15.9)$$

where

r_{ap}, r_{ag} = addendum radii of the mating pinion and gear
r_{bp}, r_{bg} = base circle radii of the mating pinion and gear

- Base pitch $P_b = \pi d_b/n$ where
- d_b is the dia of base circle and n # of teeth
- From figure $d_b = d\cos\phi$, $r_b = r\cos\phi$, and $p_b = p\cos\phi$ (15.11)
- The base pitch is like the circular pitch except is an arc in base circle
- > the CR, the smoother the gear. CR>2 implies that atleast 2 pairs of teeth are theoretically in contact at all times.

SAMPLE PROBLEM 15.1D Meshing Spur Gear and Pinion

Two parallel shafts with 4-in. center distance are to be connected by 6-pitch, 20° spur gears providing a velocity ratio of −3.0. (a) Determine the pitch diameters and numbers of teeth in the pinion and gear. (b) Determine whether there will be interference when standard full-depth teeth are used. (c) Determine the contact ratio. (See Figure 15.16.)

SOLUTION

Known: Spur gears of known pitch size, pressure angle, and center distance mesh to provide a known velocity ratio.

Find:

a. Determine pitch diameters (d_p, d_g) and the numbers of teeth (N_p, N_g).

b. Determine the possibility of interference with standard full-depth teeth.

c. Calculate the contact ratio (CR).

Schematic and Given Data:

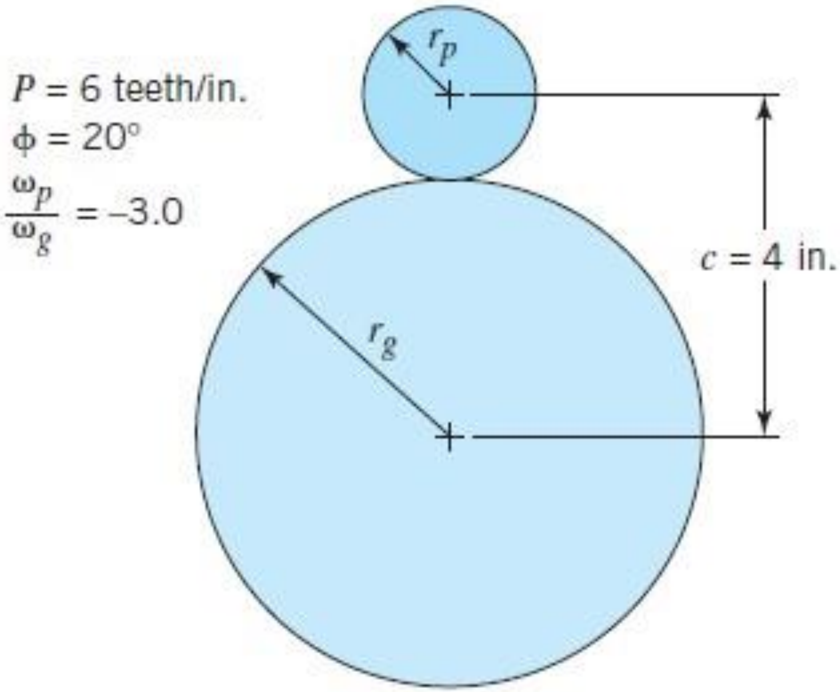


FIGURE 15.16

Spur gears for Sample Problem 15.1D.

Decisions and Assumptions:

1. If interference results from the use of standard full-depth gear teeth, unequal addenda gears will be selected.
2. The gear teeth will have standard involute tooth profiles.
3. The two gears will be located at their theoretical center distance, $c = (d_p + d_g)/2$ where $d_p = N_p/P$, $d_g = N_g/P$; that is, the gears will mesh at their pitch circles.

Design Analysis:

1. We have $r_p + r_g = c = 4$ in.; $r_g/r_p = -$velocity ratio $= 3$; hence, $r_p = 1$ in., $r_g = 3$ in., or $d_p = 2$ in., $d_g = 6$ in.
2. The term "6-pitch gears" means that $P = 6$ teeth per inch of pitch diameter; hence, $N_p = 12$, $N_g = 36$.
3. In order to use Eq. 15.8 to check for interference, we first determine the base circle radii of pinion and gear. From Eq. 15.11, $r_{\text{bp}} = 1$ in.(cos 20°), and $r_{\text{bg}} = 3$ in.(cos 20°). Substitution in Eq. 15.8 gives $r_{a(\max)} = 1.660$ in. for the pinion and 3.133 in. for the gear.

$$d_b = d\cos\phi, \qquad r_b = r\cos\phi, \qquad \text{and} \quad p_b = p\cos\phi \qquad \textbf{(15.11)}$$

$$r_{a(\max)} = \sqrt{r_b^2 + c^2\sin^2\phi}$$

4. The limiting outer gear radius is equivalent to an addendum of only 0.133 in., whereas a standard full-depth tooth has an addendum of $1/P = 0.167$ in. Clearly, the use of standard teeth would cause interference.
5. Let us use unequal addenda gears (nonstandard), with somewhat arbitrarily chosen addenda of $a_g = 0.060$ in. for the gear and $a_p = 0.290$ in. for the pinion. (The reasoning is to select maximum addenda for greatest contact ratio, while at the same time limiting the gear addendum to stay safely away from interference, and limiting the pinion addendum to maintain adequate width of top land. The latter is shown as t_0 in Figure 15.9, and its minimum acceptable value is sometimes taken as $0.25/P$.)

Name	Symbol	Units	Relationship	Example
Number of teeth	N_p N_g			$N_p = 12$ $N_g = 36$
Nominal diametral pitch	P	per in.		$P = 6$
Nominal pressure angle	ϕ	degrees		$\phi = 20°$
Gear ratio	η		$\eta = N_g/N_p$	$\eta = 3$
Base width	b	in.	$9/P < b < 14/P$	$1.5 < b < 2.33$
Nominal pitch radius	r_p r_g	in.	$r_p = N_p/(2P)$ $r_g = N_g/(2P)$	$r_p = 1$ $r_g = 3$
Nominal base radius	r_{bp} r_{bg}	in.	$r_{bp} = r_p \cos \phi$ $r_{bg} = r_g \cos \phi$	$r_{bp} = 0.939$ $r_{bg} = 2.82$
Nominal center distance	c	in.	$c = \frac{(N_p + N_g)}{(2P)}$	$c = 4$
Maximum addendum radius to avoid interference	r_{ap}^{max} r_{ag}^{max}	in.	$r_{ap}^{max} = \sqrt{r_{bp}^2 + c^2 \sin^2 \varphi}$ $r_{ag}^{max} = \sqrt{r_{bg}^2 + c^2 \sin^2 \varphi}$	$r_{ap}^{max} = 1.66$ $r_{ag}^{max} = 3.13$

Name	Symbol	Units	Relationship	Example
Standard addendum radius	r_{ap} r_{ag}	in.	$r_{ap} = (N_p + 2)/(2P)$ $r_{ag} = (N_g + 2)/(2P)$	$r_{ap} = 1.17$ $r_{ag} = 3.17$ *(Interference)*
Addendum radius	r_{ap} r_{ag}	in.	*(Non-standard)*	$r_{ap} = 1.29$ $r_{ag} = 3.06$
Standard dedendum radius	r_{dp} r_{dg}	in.	$r_{dp} = (N_p - 2.5)/(2P)$ $r_{dg} = (N_g - 2.5)/(2P)$ *(Standard)*	$r_{dp} = 0.792$ $r_{dg} = 2.792$
Nominal module	m	mm	$m = 25.4/P$	$m = 4.23$
Nominal circular pitch	p	in.	$p = \pi /P$	$p = 0.523$
Nominal tooth thickness	t	in.	$t = \pi /(2P)$	$t = 0.262$
Nominal contact ratio	CR		$\Delta_p = \sqrt{r_{ap}^2 - r_{bp}^2}$ $\Delta_g = \sqrt{r_{ag}^2 - r_{bg}^2}$ $CR = \dfrac{\Delta_p + \Delta_p - c \sin \phi}{p \cos \phi}$	$CR = 1.43$

6. Substitution in Eq. 15.11 gives $p_b = (\pi/6) \cos 20° = 0.492$ in. Substitution in Eq. 15.9 [with $r_{ap} = 1.290$ in., $r_{bp} = 1$ in.(cos 20°), $r_{ag} = 3.060$ in., $r_{bg} = 3$ in.(cos 20°)] gives CR = 1.43, which should be a suitable value.

Comments:

1. If after the gears are mounted, the center distance is found to be slightly greater than the theoretical (calculated) center distance of 4.0 in., this would mean that the calculated diameters, d_p and d_g, are smaller than the actual gear and pinion pitch diameters and that the backlash is greater than initially calculated.
2. Had we wished to use standard tooth proportions in solving this sample problem, we could have (a) increased the diametral pitch (thereby giving more teeth on the pinion—and this outweighs the influence of giving more teeth to the gear) or (b) increased the pressure angle to 25° (which would be more than enough to eliminate interference).
3. This problem may also be solved using the worksheet in Appendix J.

15.4 Gear Force Analysis

- In Figures - line ab is normal to the contacting tooth surfaces, and that (neglecting sliding friction) it was the line of action of the forces between mating teeth.

- The force between mating teeth can be resolved at the pitch point (P) into two components.

1. Tangential component F_t, which, when multiplied by the pitch line velocity, accounts for the power transmitted.

2. Radial component F_r, which does no work but tends to push the gears apart.

- From figure the relationship between F_t & F_r is

$$F_r = F_t \tan \phi \qquad (15.12)$$

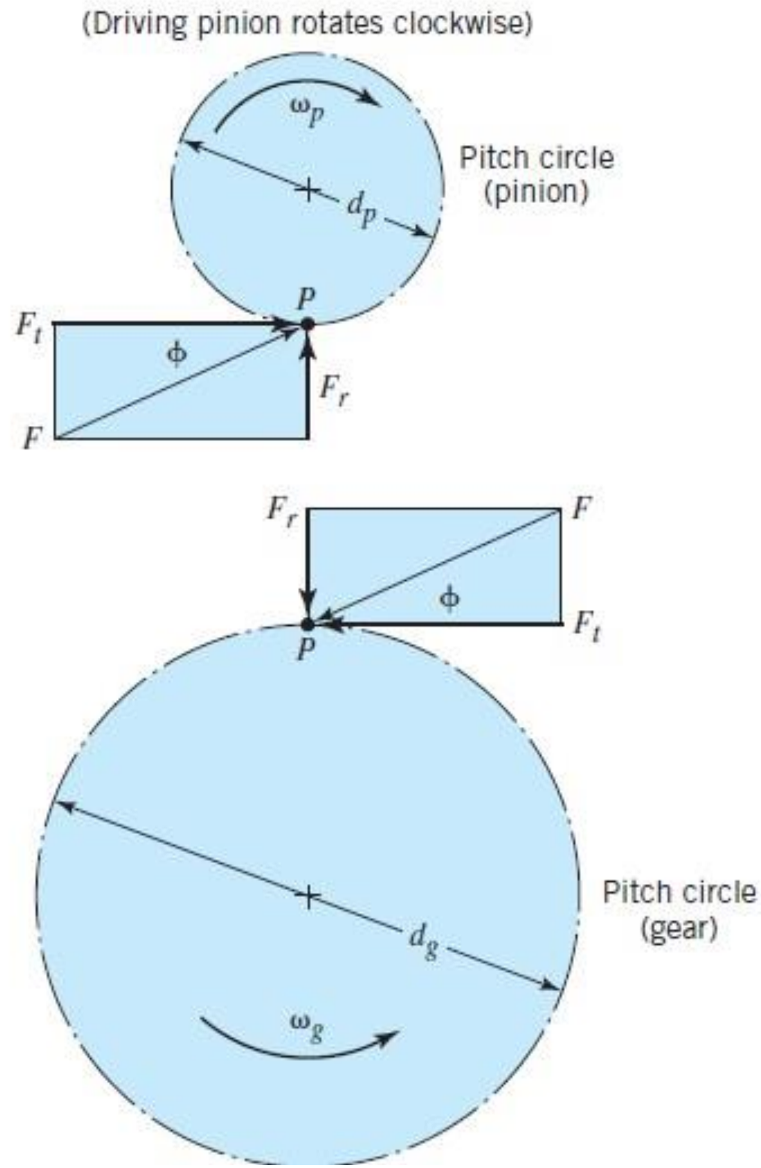


FIGURE 15.17
Gear-tooth force F, shown resolved at pitch point. The driving pinion and driven gear are shown separately.

15.4 Gear Force Analysis

- To analyze the relationships between the gear force components and the associated shaft power and rotating speed, the gear pitch line velocity V, in feet per minute, is

$$V = \pi dn/12 \tag{15.13}$$

- where d is the pitch diameter in inches of the gear rotating n rpm. The transmitted power in horsepower (hp) is

$$\dot{W} = F_t V/33{,}000 \tag{15.14}$$

- where F_t is in pounds and V in feet per minute.
- In SI units $V = \pi dn/60{,}000$ (15.13a)
- where d is the pitch diameter in mm of the gear rotating n rpm and V is in m/s. The transmitted power in watts (W) is

$$\dot{W} = F_t V \tag{15.14a}$$

- Where F_t is in newtons

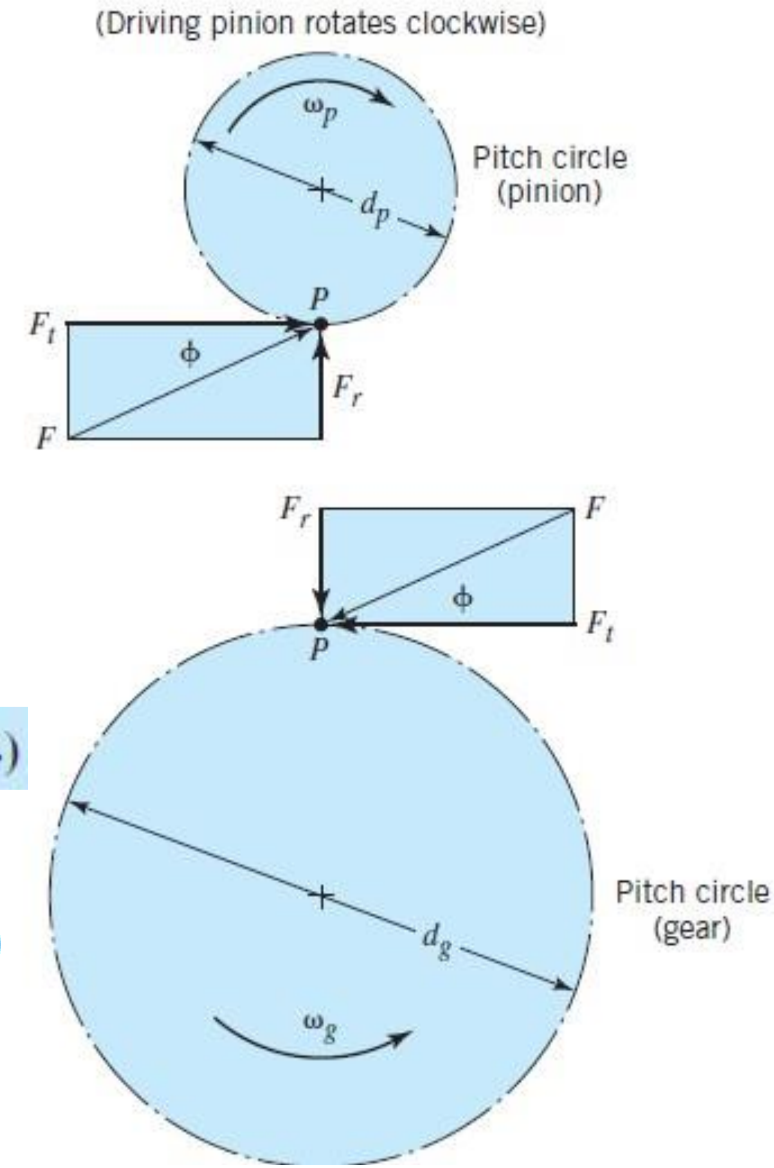


FIGURE 15.17
Gear-tooth force F, shown resolved at pitch point. The driving pinion and driven gear are shown separately.

SAMPLE PROBLEM 15.2 Forces on Spur Gears

Figure 15.18*a* shows three gears of $P = 3$, $\phi = 20°$. Gear *a* is the driving, or input, pinion. It rotates counterclockwise at 600 rpm and transmits 25 hp to idler gear *b*. Output gear *c* is attached to a shaft that drives a machine. Nothing is attached to the idler shaft, and friction losses in the bearings and gears can be neglected. Determine the resultant load applied by the idler to its shaft.

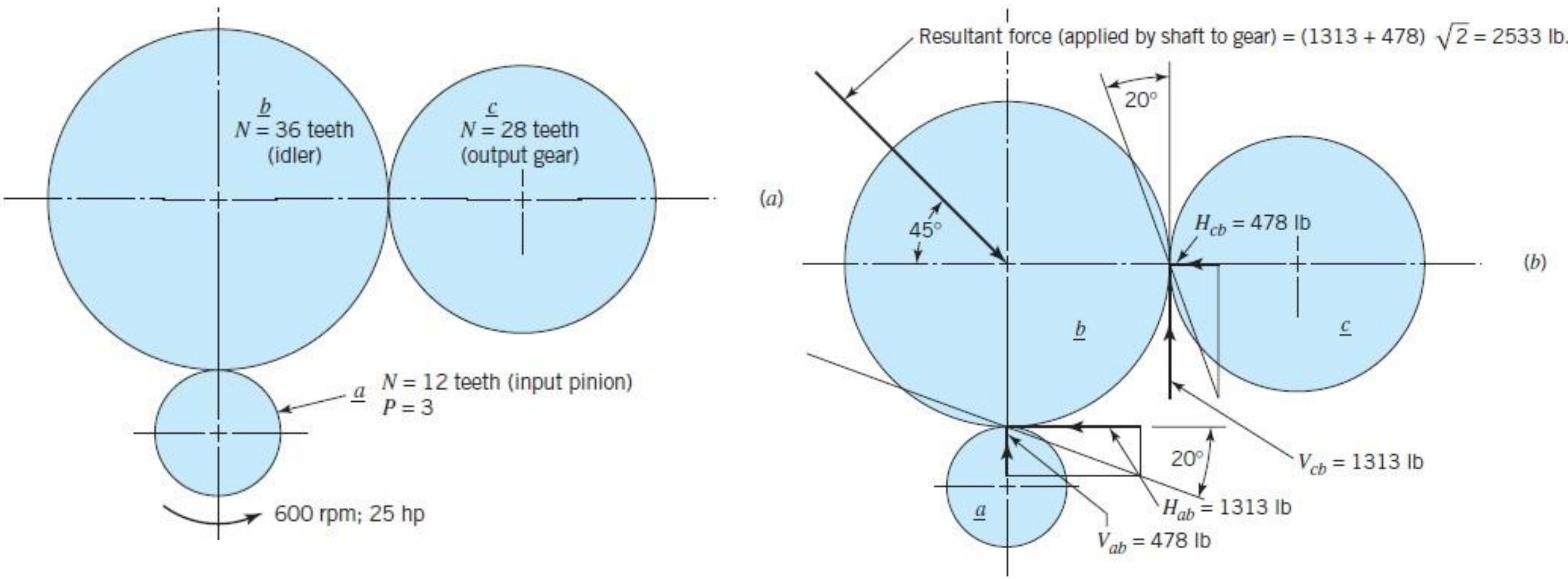


FIGURE 15.18
Gear forces in Sample Problem 15.2. (*a*) Gear layout. (*b*) Forces acting on idler *b*.

Solution

Known: Three spur gears of specified diametral pitch, numbers of teeth, and pressure angle mesh to transmit 25 hp from input gear to output gear through an idler gear. The input gear rotation speed and direction are given.

Find: Determine the resultant load of the idler gear on its shaft.

Schematic and Given Data: See Figure 15.18.

Assumptions:

1. The idler gear and shaft serve the function of transmitting power from the input gear to the output gear. No idler shaft torque is applied to the idler gear.
2. Friction losses in the bearings and gears are negligible.
3. The gears mesh at the pitch circles.
4. The gear teeth have standard involute tooth profiles.
5. The shafts for gears a, b, and c are parallel.

Analysis:

1. Applying Eq. 15.3 to gear *a* gives

$$d_a = N_a/P = (12 \text{ teeth})/(3 \text{ teeth per inch}) = 4 \text{ in.}$$

2. All three gears have the same pitch line velocity. Applying Eq. 15.13 to gear *a*, we have

$$V = \frac{\pi d_a n_a}{12} = \frac{\pi(4 \text{ in.})(600 \text{ rpm})}{12} = 628.28 \text{ ft/min}$$

3. Applying Eq. 15.14 to gear *a* and solving for F_t gives

$$F_t = \frac{33{,}000(25 \text{ hp})}{628.28 \text{ fpm}} = 1313 \text{ lb}$$

This is the horizontal force of gear *b* applied *to* gear *a*, directed to the right. Figure 15.18*b* shows the equal and opposite horizontal force of *a* applied to *b*, labeled H_{ab}, and acting to the left.

4. From Eq. 15.12, the corresponding radial gear-tooth force is $F_r = V_{ab} = (1313)(\tan 20°) = 478$ lb.
5. Forces H_{cb} and V_{cb} are shown in proper direction in Figure 15.18*b*. (Remember, these are forces applied *by c to b*.) Since the shaft supporting idler *b* carries no torque, equilibrium of moments about its axis of rotation requires that $V_{cb} = 1313$ lb. From Eq. 15.12, $H_{cb} = (1313)(\tan 20°)$, or 478 lb.
6. Total gear-tooth forces acting on *b* are $1313 + 478 = 1791$ lb both vertically and horizontally, for a vector sum of $1791\sqrt{2} = 2533$ lb acting at 45°. This is the resultant load applied *by* the idler *to* its shaft.

Comment: The equal and opposite force applied *by* the shaft *to* the idler gear is shown in Figure 15.18*b*, where the idler is shown as a free body in equilibrium.

15.5 Gear-Tooth Strength

- After gear geometry and force analysis, looking into how much power a gear pair will transmit without tooth failure.
- Figure shows photoelastic pattern of stresses in gear-tooth. Highest stresses ex i where the lines are closest together.

1. the point of contact with the mating gear, where force F is acting,
2. in the fillet at the base of the tooth.

- Designing gears will deal with bending fatigue at the base of the tooth and with surface durability
- As will be seen, the load capacity and failure mode of a pair of gears are affected by their rotating speed.

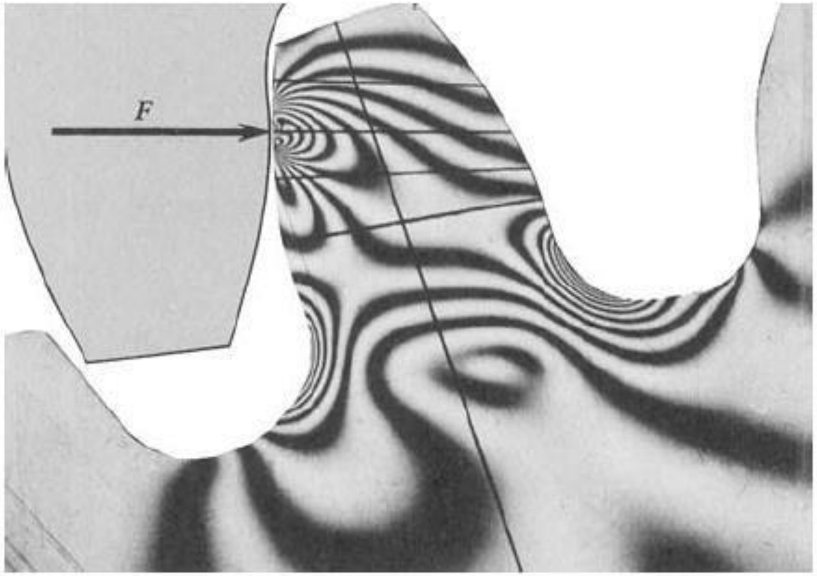


FIGURE 15.19
Photoelastic pattern of stresses in a spur gear tooth. (From T. J. Dolan and E. L. Broghammer, "A Study of Stresses in Gear Tooth Fillets," Proc. 14th Eastern Photoelasticity Conf., PE December 1941.)

15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- The first analysis of gear-tooth stresses was presented by Lewis in 1892 - still used for gear-tooth bending stress analysis.
- Figure shows a gear tooth loaded as a cantilever beam, with resultant force F applied to the tip. Mr. Lewis made the following simplifying assumptions.
- The full load is applied to the tip of a single tooth. This is obviously the most severe condition and is appropriate for gears of "ordinary" accuracy.
- For high precision gears, however, the full load is never applied to a single tooth tip.
- With a CR > 1, each new pair of teeth comes into contact while the previous pair is still engaged.

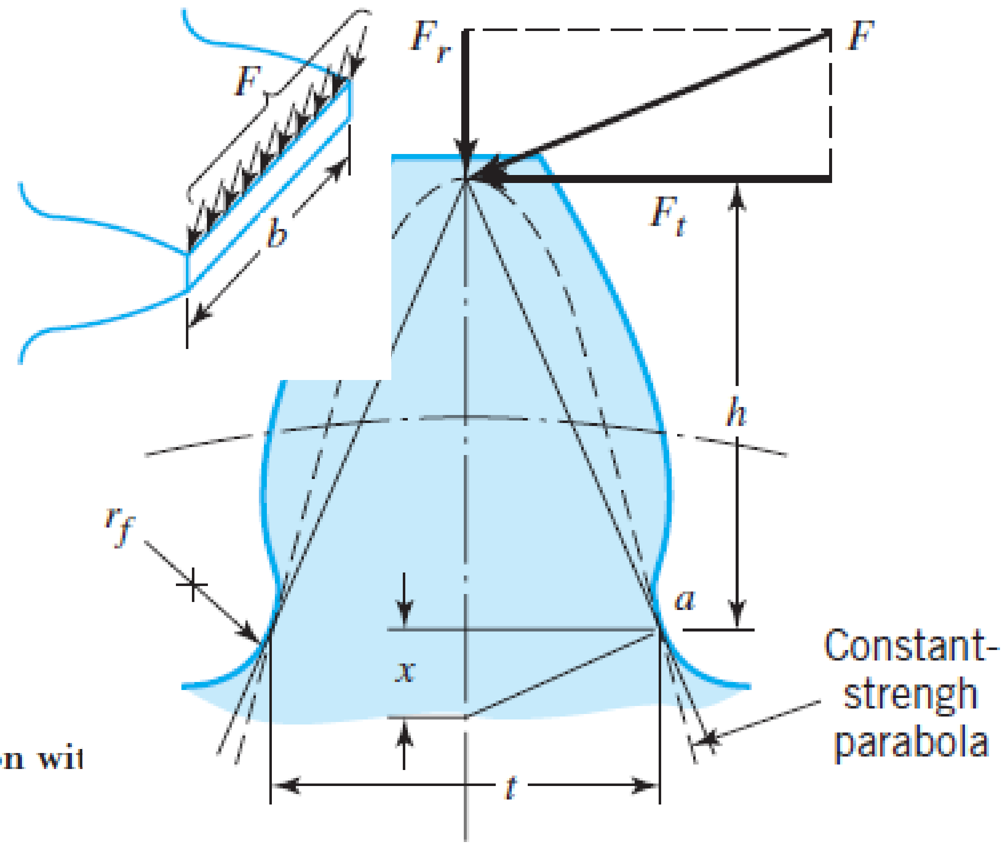


FIGURE 15.20
Bending stresses in a spur gear tooth (comparison wit constant-stress parabola).

15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- After the contact point moves down some distance from the tip, the previous teeth go out of engagement and the new pair carries the full load (unless, CR>2).
- This is the situation depicted in stress Figure. Thus, with precision gears, the tooth should be regarded as carrying only part of the load at its tip, and the full load at a point on the tooth face where the bending moment arm is shorter.

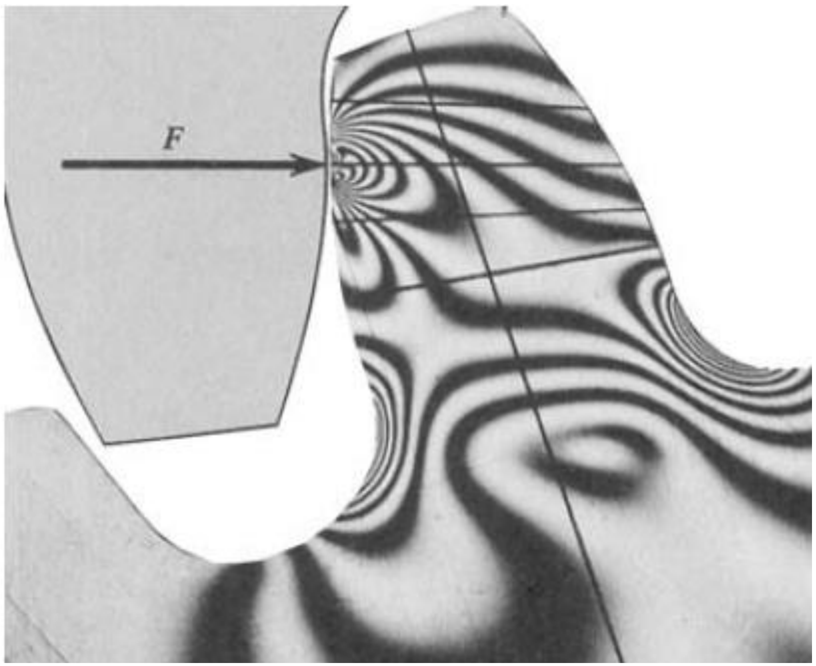
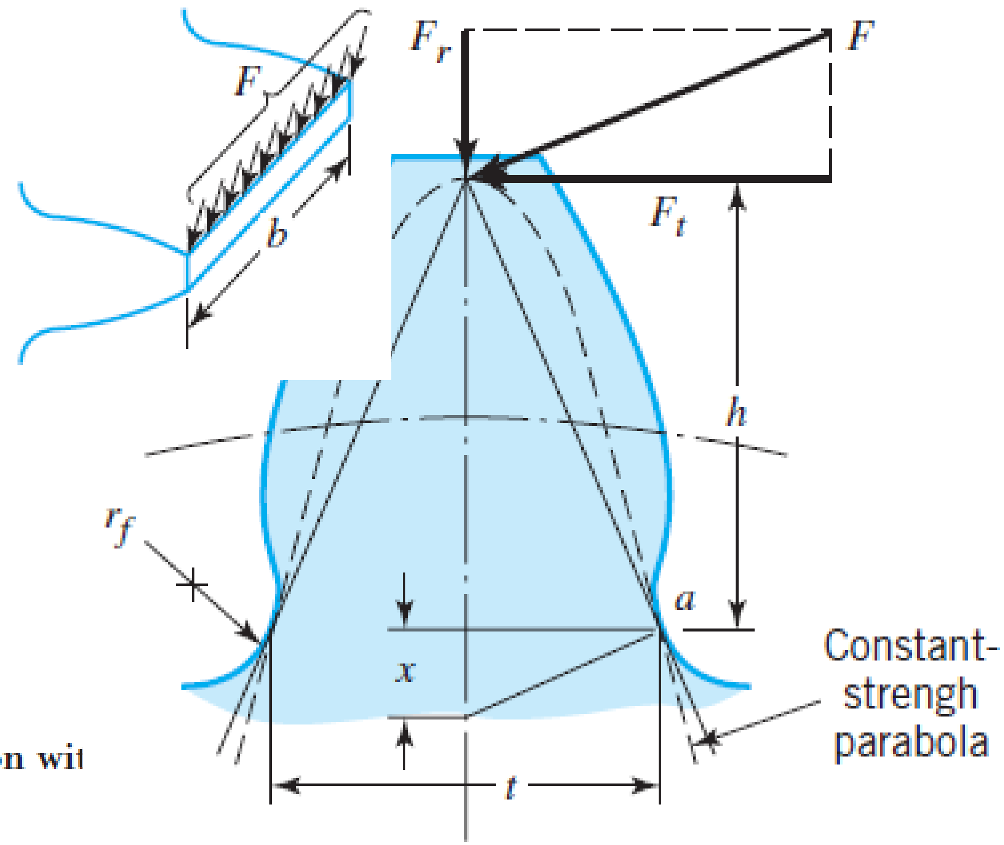
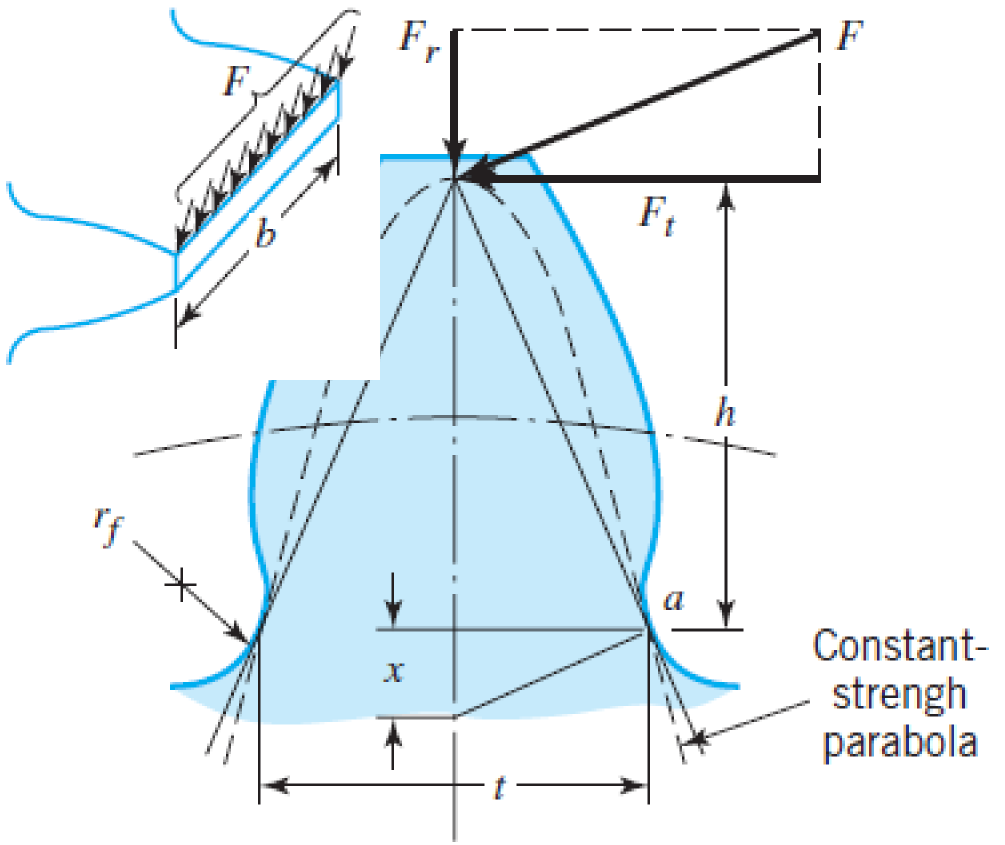


FIGURE 15.20
Bending stresses in a spur gear tooth (comparison wit constant-stress parabola).



15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- The radial component, F_r , is negligible. This is a conservative assumption, as F_r produces a compressive stress that subtracts from the bending tension at point a of Figure. (The fact that it adds to the bending compression in the opposite fillet is unimportant because fatigue failures always start on the tensile side.)
- The load is distributed uniformly across the full face width. This is a non-conservative assumption and can be instrumental in gear failures involving wide teeth and misaligned or deflecting shafts.
- Forces which are due to tooth sliding friction are negligible.
- Stress concentration in the tooth fillet is negligible. K factors were unknown in Mr. Lewis's time but are now known to be important.



15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- Proceeding with the development of the Lewis equation, from Figure, the gear tooth is everywhere stronger than the inscribed constant strength parabola, except for the section at a where the parabola and tooth profile are tangent. At point a

$$\sigma = \frac{Mc}{I} = \frac{6F_t h}{bt^2} \qquad \sigma = M/Z \text{ and } Z = bt^2/6 \qquad \text{(c)}$$

- For similar triangles

$$\frac{t/2}{x} = \frac{h}{t/2}, \qquad \text{or} \qquad \frac{t^2}{h} = 4x \qquad \text{(d)}$$

- Substituting d into c gives

$$\sigma = \frac{6F_t}{4bx} \qquad \text{(e)}$$

- if lewis factor y is 2x/3p then

$$\sigma = \frac{F_t}{bpy}$$

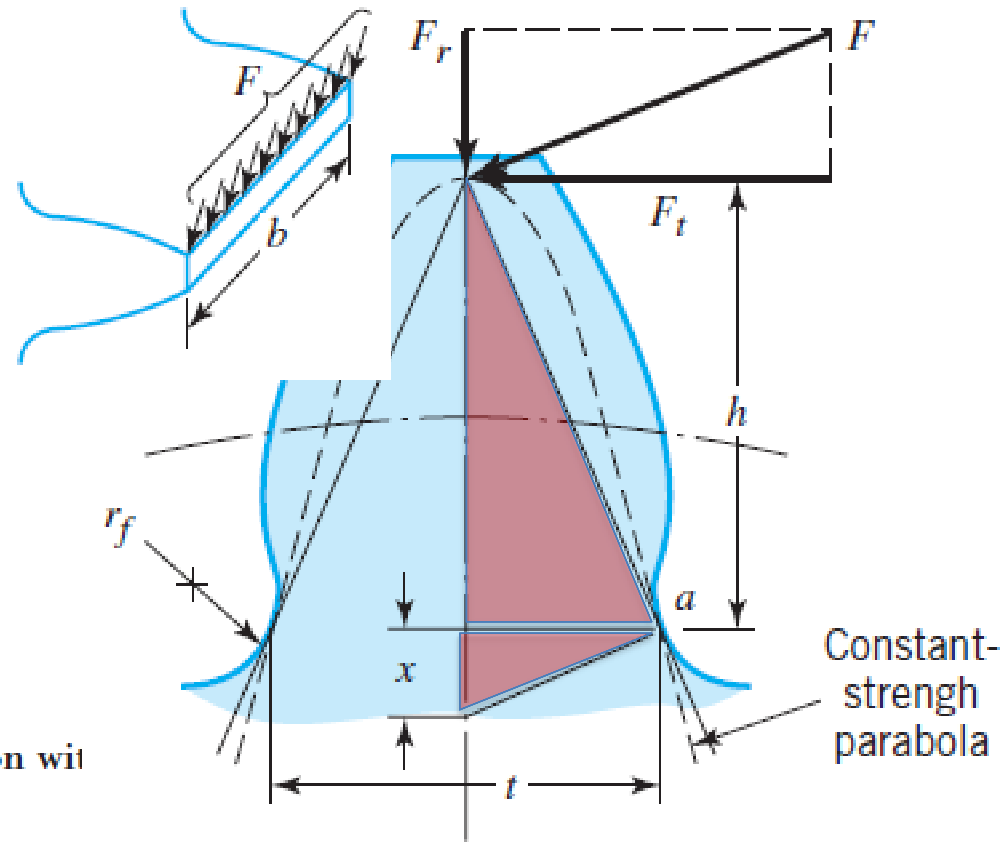


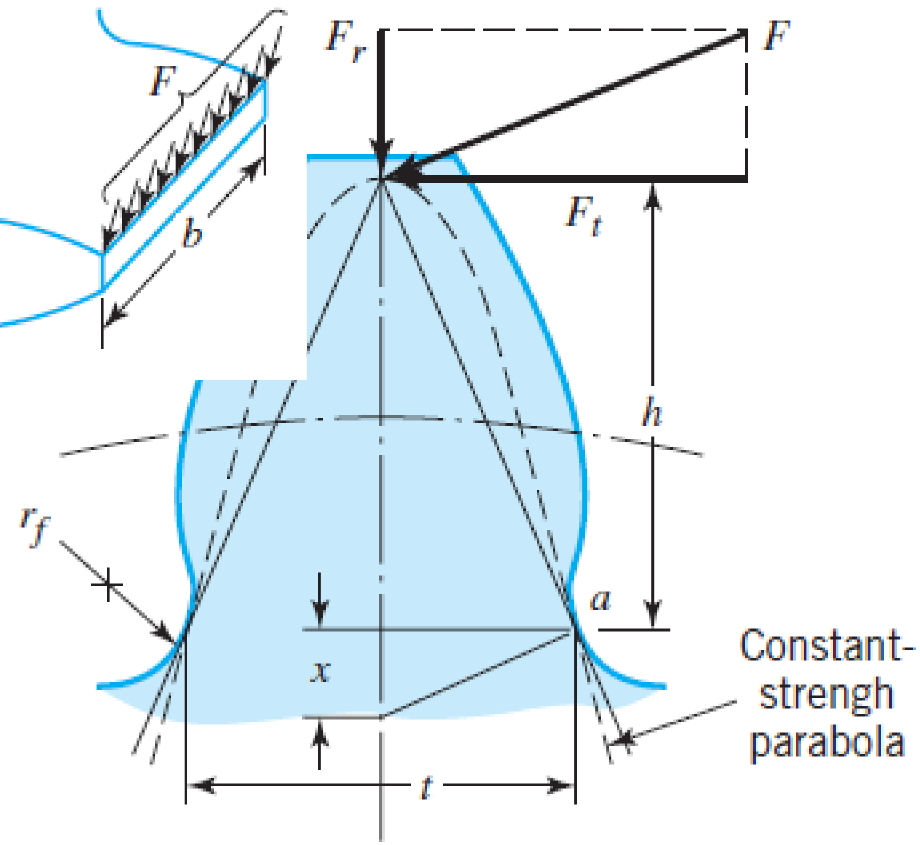
FIGURE 15.20
Bending stresses in a spur gear tooth (comparison wit constant-stress parabola).

15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- Gears are made to standard diametral pitch P, which is p = π/P and then y = Y/ π

$$\sigma = \frac{F_t P}{bY} \quad \text{or} \quad \sigma = \frac{F_t}{mbY} \qquad (15.16)$$

- where Y is the Lewis form factor based on DP, Por Module, m. Both Y & y are functions of tooth shape (not size) and therefore vary with the number of teeth in the gear.
- Values of Y for standard gear systems are given in Figure 15.21.
- For nonstandard gears, the factor can be obtained by graphical layout of the tooth or by digital computation.
- Lewis equation indicates that tooth-bending stresses vary (1) directly with load F_t, (2) inversely with tooth width b, (3) inversely with tooth size p, 1/P, or m and (4) inversely with tooth shape factor Y or y.



15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

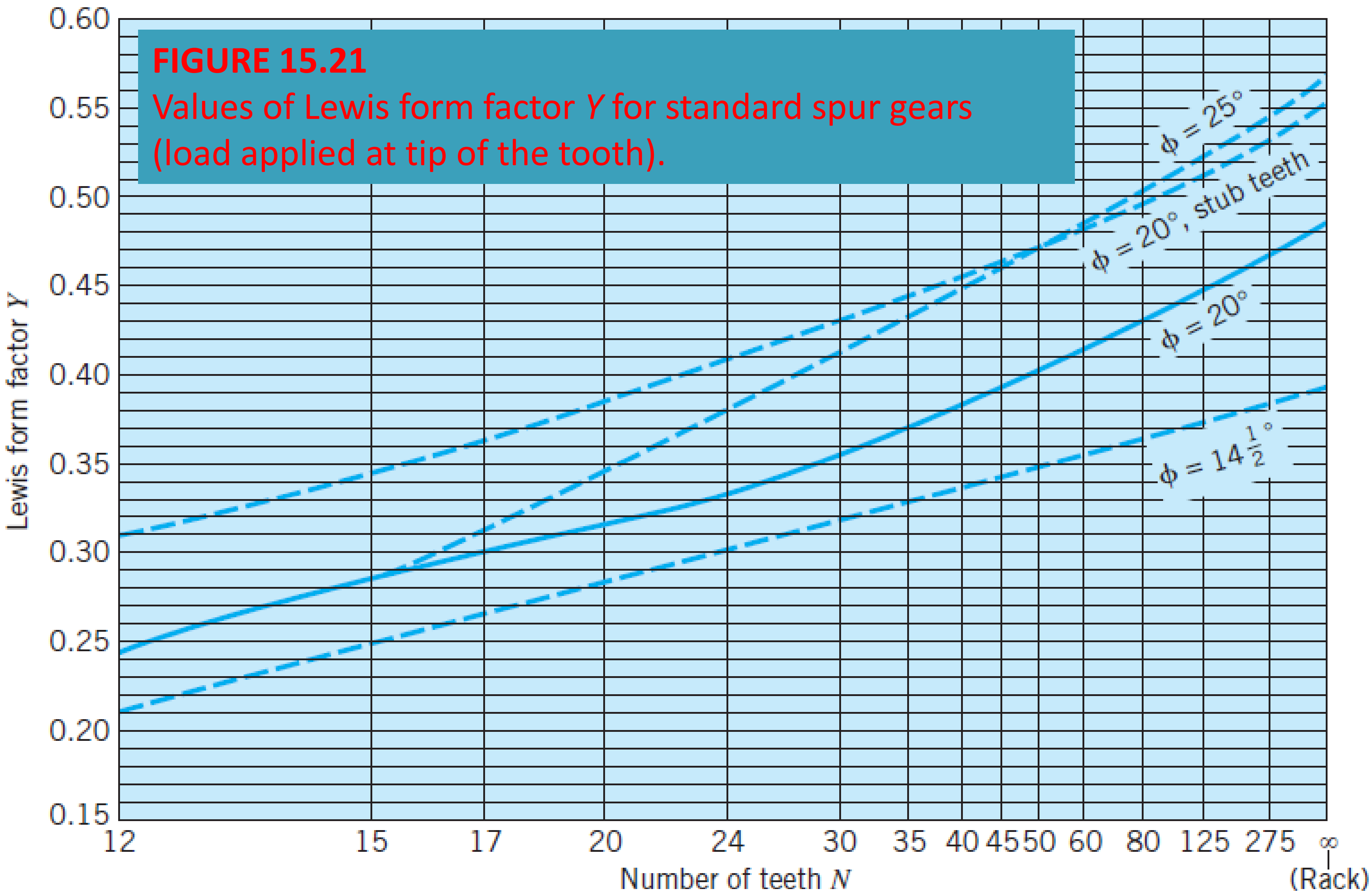


FIGURE 15.21
Values of Lewis form factor *Y* for standard spur gears (load applied at tip of the tooth).