MECH 344/M

Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 2 Contd

Contents of today's lecture

- Stress Concentration
- Residual Stresses
- Thermal Stresses
- Failure Theories
  - Maximum Normal Theory
  - Maximum Shear Stress Theory
  - Maxium Energy Distortion Theory
  - Coulumb Mohr Theory

4.14 Residual Stresses Caused by Yielding—Axial Loading

- When a part is yielded nonuniformly throughout a CS, residual stresses remain in this CS after the external load is removed.
- For eg, the 4 levels of loading of the notched tensile bar shown in f.
- This same bar and the 4 levels of loading are represented in the left column
- Note that only slight yielding is involved—not major yielding such as often occurs in processing.
- The middle column shows the change in stress when the load is removed.

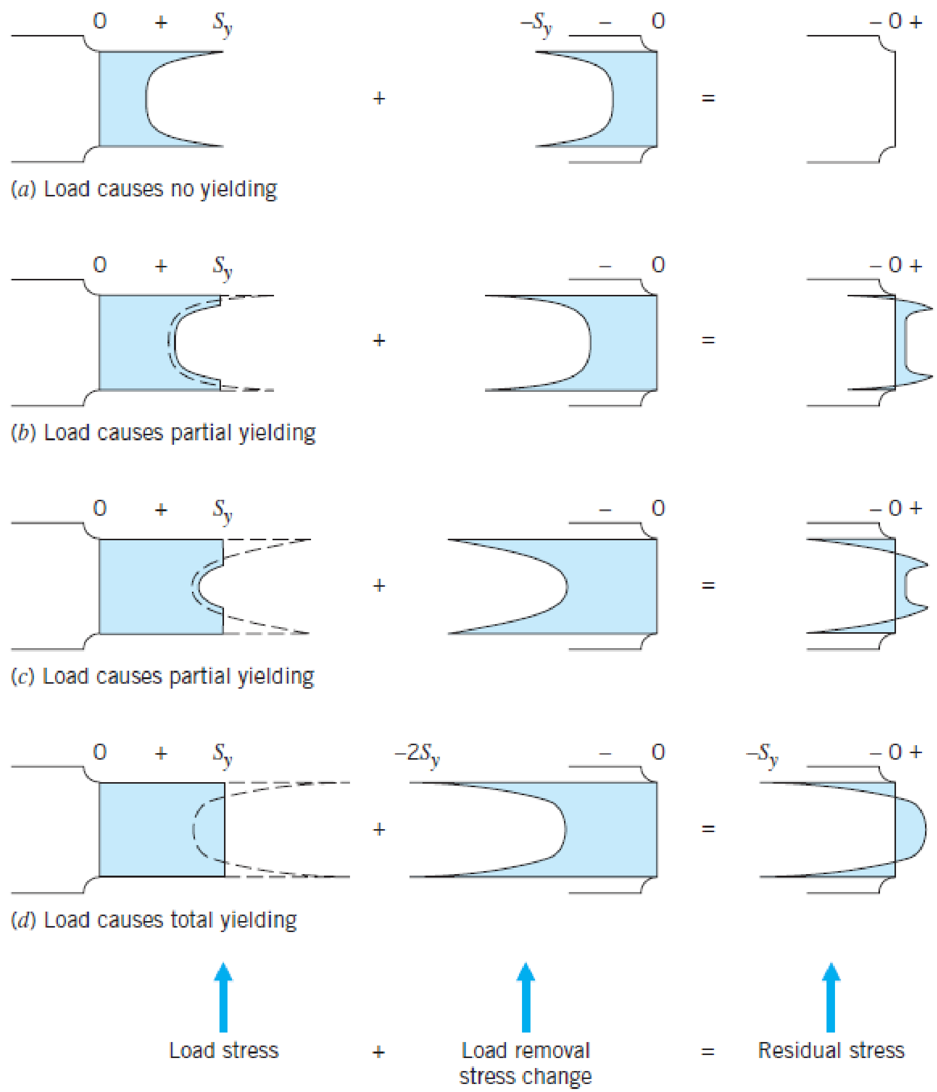


FIGURE 4.43
Residual stresses caused by yielding of a notched tensile bar of $K_t = 2$ for stress gradients a to d in Figure 4.42f.

4.14 Residual Stresses Caused by Yielding—Axial Loading

- Except for a, where the did not cause yielding at the notch root, the stress change when the load is removed does not exactly cancel the stresses by applying the load.
- Hence, residual stresses remain after the load is removed. These are shown in the right column
- In each case, the stress change caused by removing the load is elastic.
- It must be remembered, too, that this development of residual stress curves was based on assuming that the material conforms to the idealized stress–strain curve
- So the residual stress curves can only be good approximations.

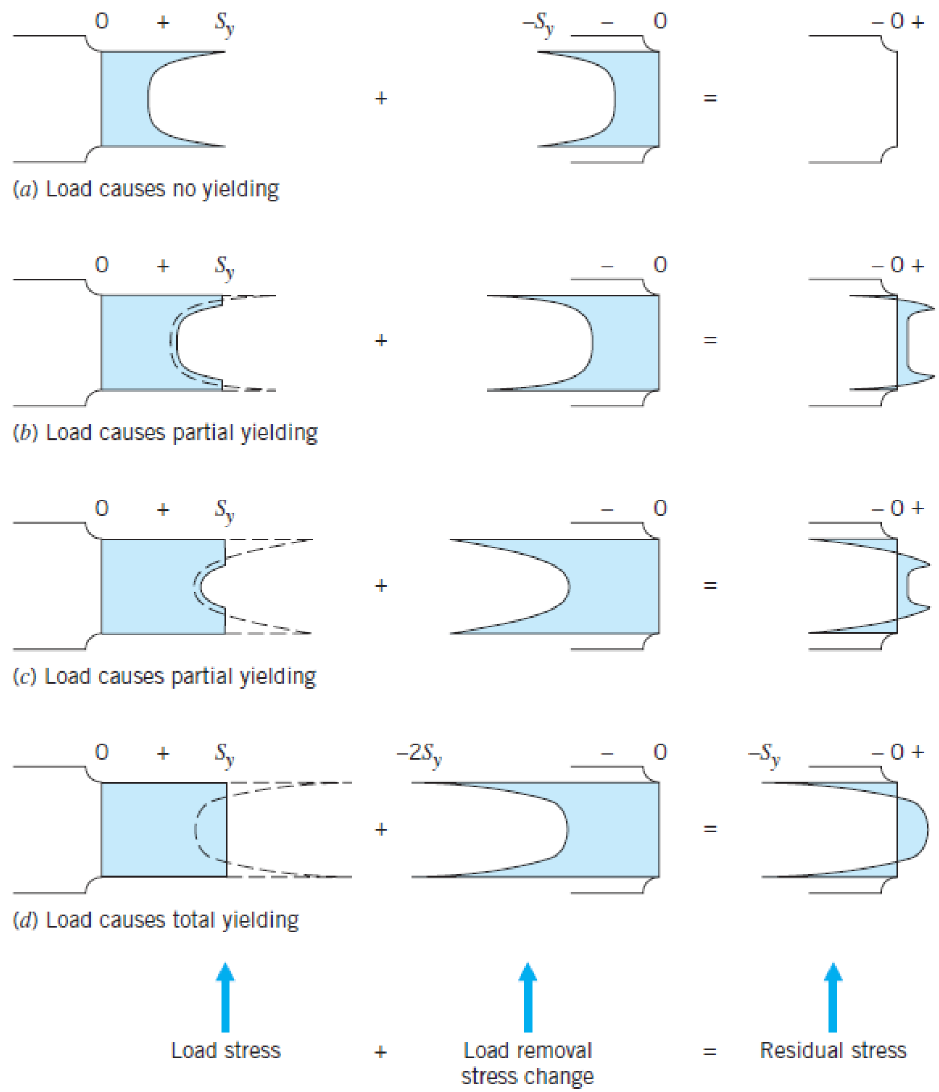
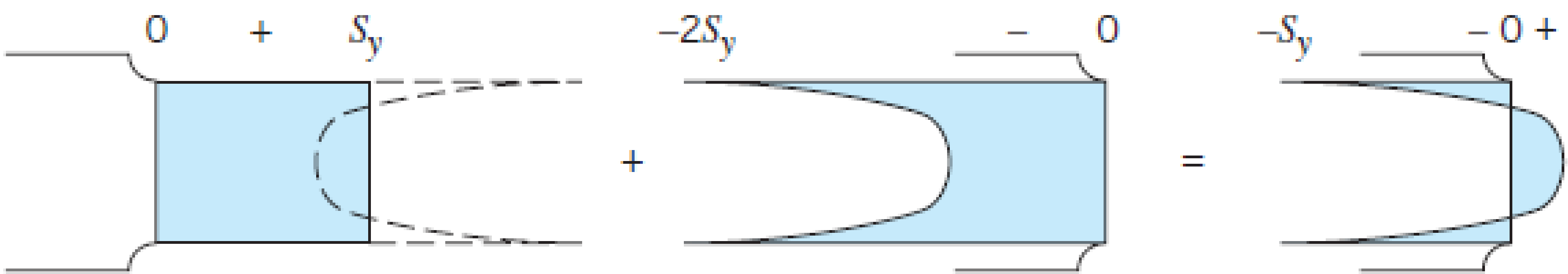


FIGURE 4.43
Residual stresses caused by yielding of a notched tensile bar of $K_t = 2$ for stress gradients a to d in Figure 4.42f.

4.14 Residual Stresses Caused by Yielding—Axial Loading



(d) Load causes total yielding

4.15 Residual Stresses Caused by Yielding—Bending and Torsional Loading

- Figure illustrates residual stresses caused by the bending of an unnotched 25 * 50-mm rectangular beam.
- made of steel having an idealized stress–strain curve with Sy = 300 MPa.
- Unknown moment M_1 produces the stress distribution shown in Figure, with yielding to a depth of 10 mm. Let us first determine the magnitude of moment M1
- If the distributed stress pattern is replaced with concentrated forces F1 and F2 at the centroids of the rectangular and triangular portions of the pattern, respectively,
- M1 is equal to the sum of the couples produced by F1 and F2. The magnitude of F1 is equal to the product of the average stress (300 MPa) times the area over which it acts (10 mm * 25 mm).

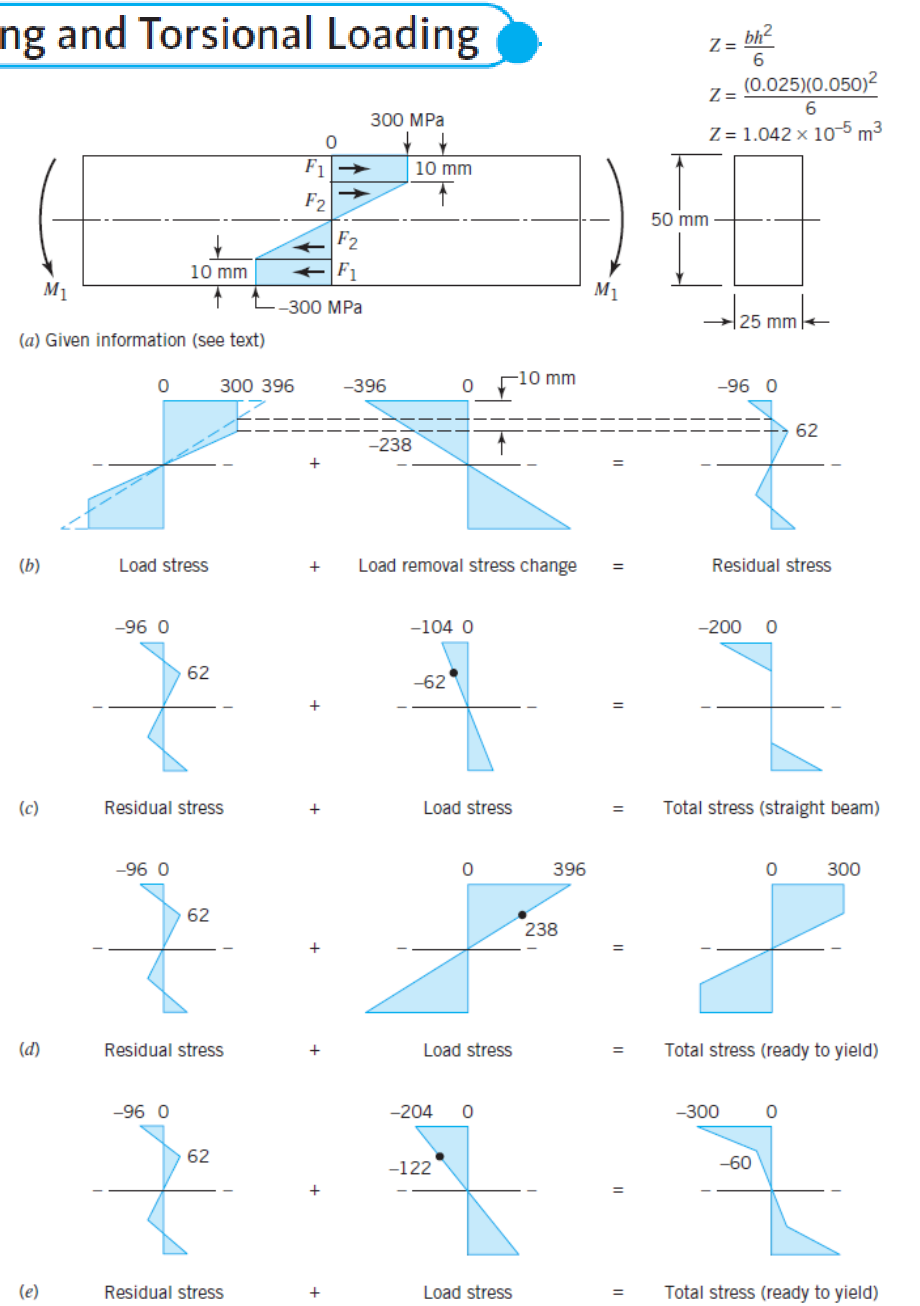


FIGURE 4.44
Residual stresses in an unnotched rectangular beam.

4.15 Residual Stresses Caused by Yielding—Bending and Torsional Loading

- Similarly, F2 is equal to an average stress of 150 MPa times an area of 15 mm * 25 mm. The moment arms of the couples are 40 mm and 20 mm, respectively

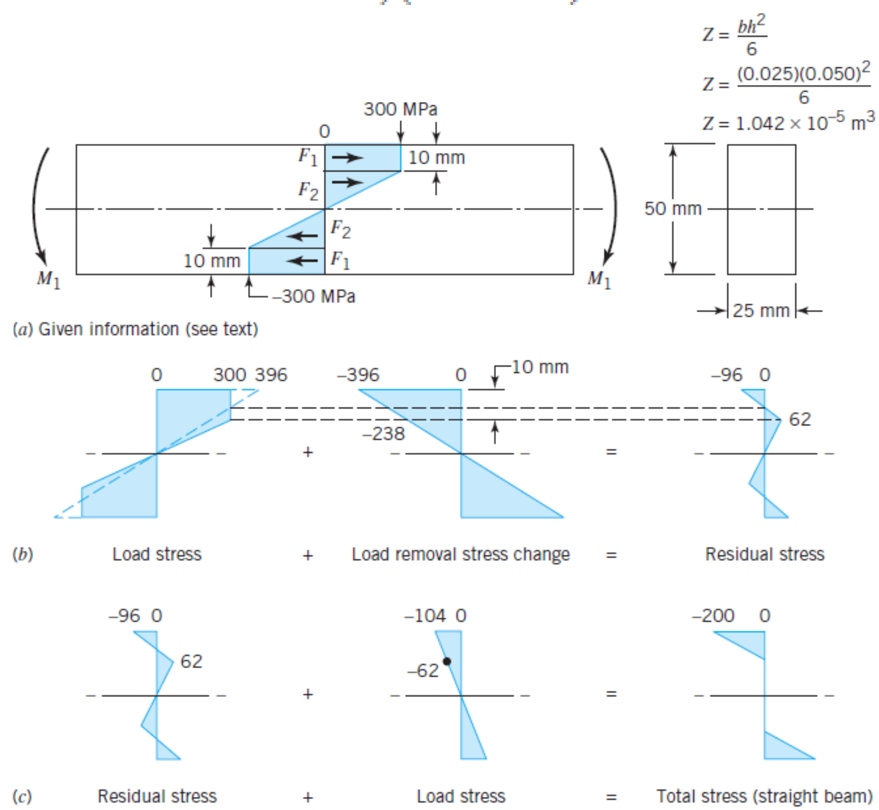
$$M_1 = (300\ \text{MPa} \times 250\ \text{mm}^2)(0.040\ \text{m}) + (150\ \text{MPa} \times 375\ \text{mm}^2)(0.020\ \text{m})$$
$$= 4125\ \text{N}\cdot\text{m}$$

- After M_1 is removed

$$\sigma = M/Z = 4125\ \text{N}\cdot\text{m}/(1.042 \times 10^{-5}\ \text{m}^3)$$
$$= 3.96 \times 10^8\ \text{Pa} = 396\ \text{MPa}$$

- Note that at this point the beam is slightly bent

- Figure 4.44c shows that the desired center portion stress-free condition requires superimposing a load that develops a compressive stress of 62 MPa, 10 mm below the surface



4.15 Residual Stresses Caused by Yielding—Bending and Torsional Loading

- With this load in place, total stresses are as shown.
- Since center portion stresses are zero, the beam is indeed straight.
- Stress 396 MPa is due to moment of 4125 Nm.
- By simple proportion, a stress of 104 MPa requires a moment of 1083 N # m.
- Let us now determine the elastic bending moment capacity of the beam after the residual stresses have been established.
- A moment in the same direction as M_1 can be added that superimposes a surface stress of +396 MPa without yielding. It is of 4125 Nm.
- The release of original moment M_1 = 4125 Nm caused no yielding; so, it can be reapplied.

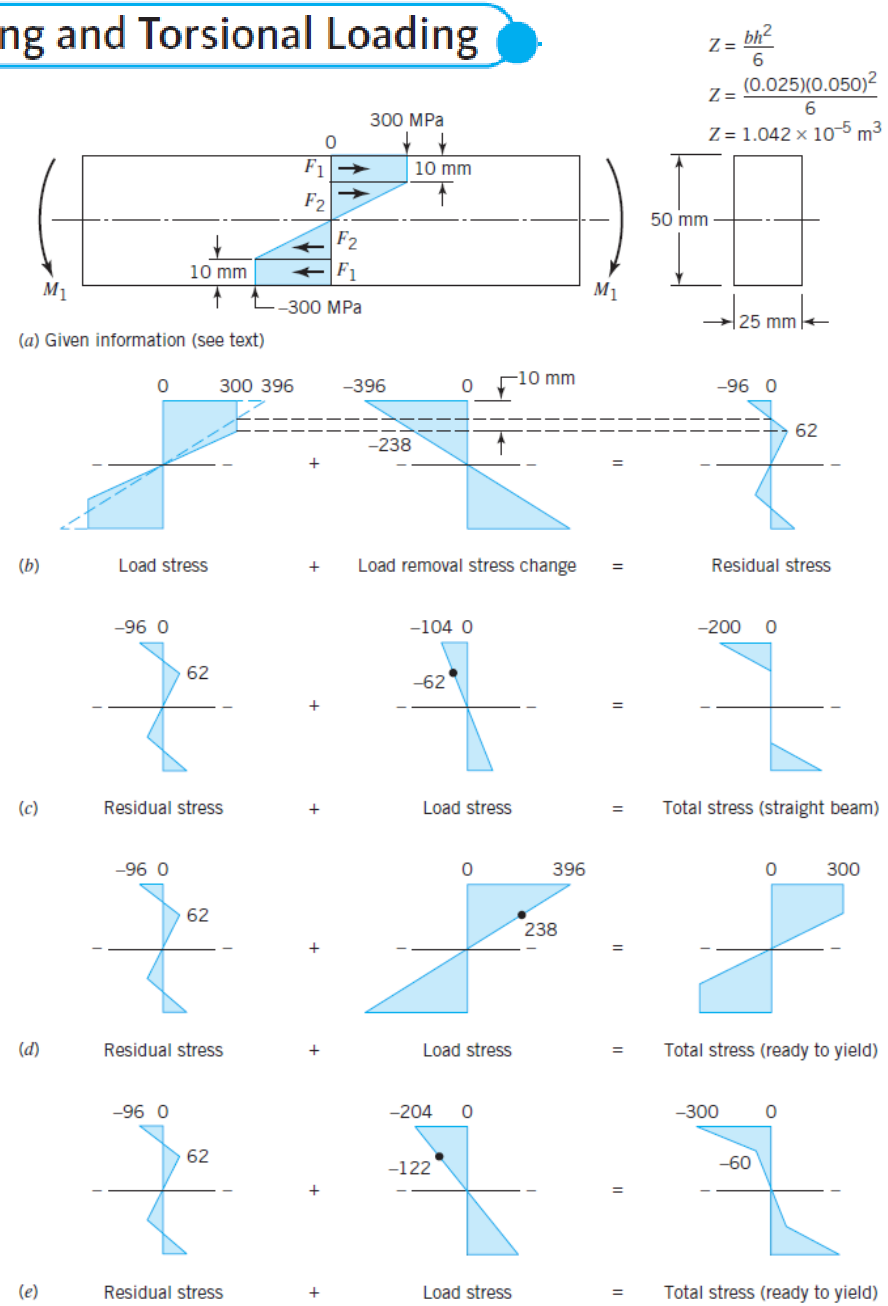


FIGURE 4.44
Residual stresses in an unnotched rectangular beam.

4.15 Residual Stresses Caused by Yielding—Bending and Torsional Loading

- Figure e shows that in the direction opposite the original moment M_1, a moment giving a surface stress of 204 MPa is all that can be elastically withstood.
- This corresponds to moment of 2125Nm.
- *An overload causing yielding produces residual stresses that are favorable to future loads in the same direction and unfavorable to future loads in the opposite direction.*
- Furthermore, on the basis of the idealized stress–strain curve, the increase in load capacity in one direction is exactly equal to the decrease in load capacity in the opposite direction.

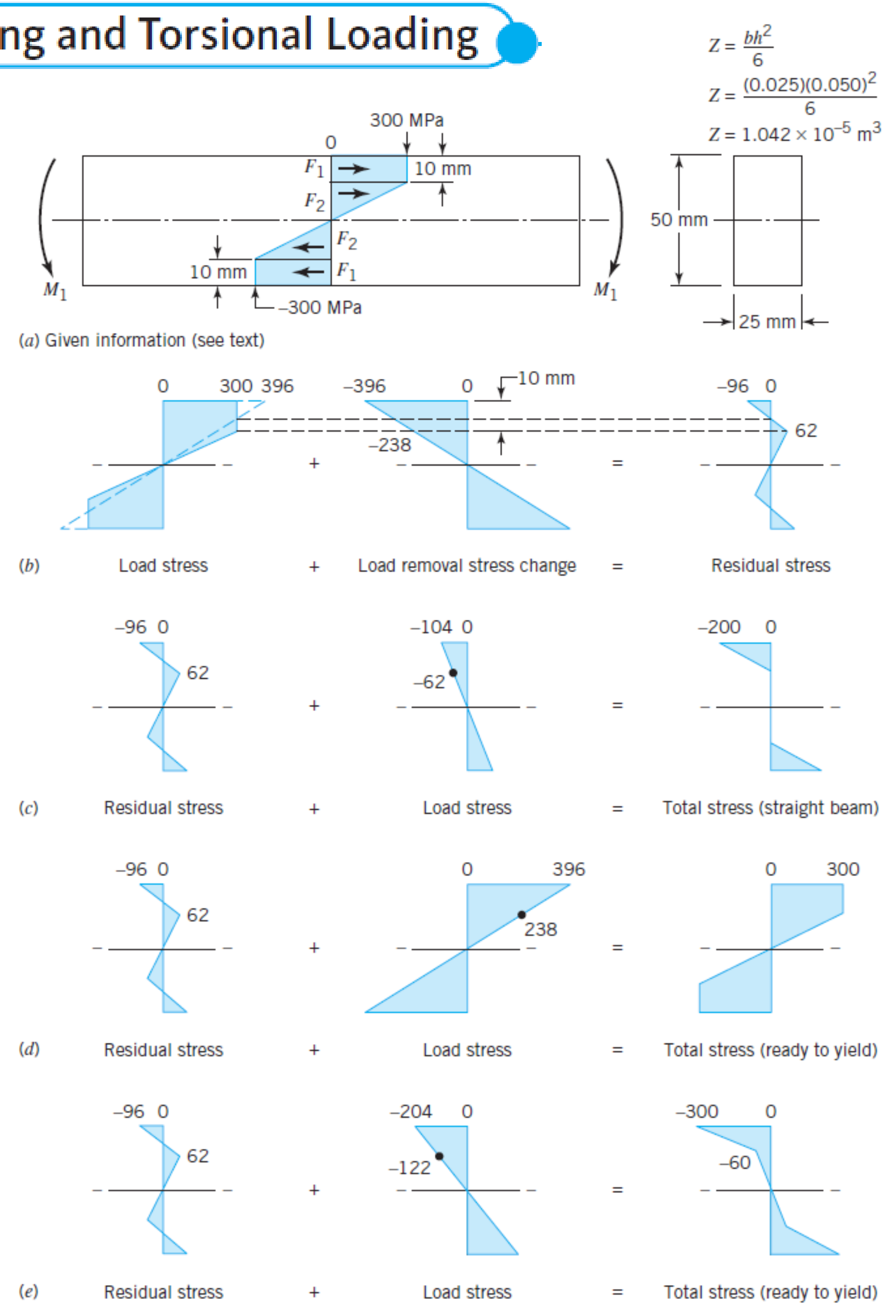


FIGURE 4.44
Residual stresses in an unnotched rectangular beam.

4.16 Thermal Stresses

- We ve seen stresses caused by external loads. Stresses can also be caused by expansion and contraction due either to temperature changes or to a material phase change.
- It is important to become familiar with the basic principles. When the temperature of an unrestrained homogeneous, isotropic body is uniformly changed, it expands (or contracts) uniformly in all directions, according to the relationship

$$\epsilon = \alpha \Delta T$$

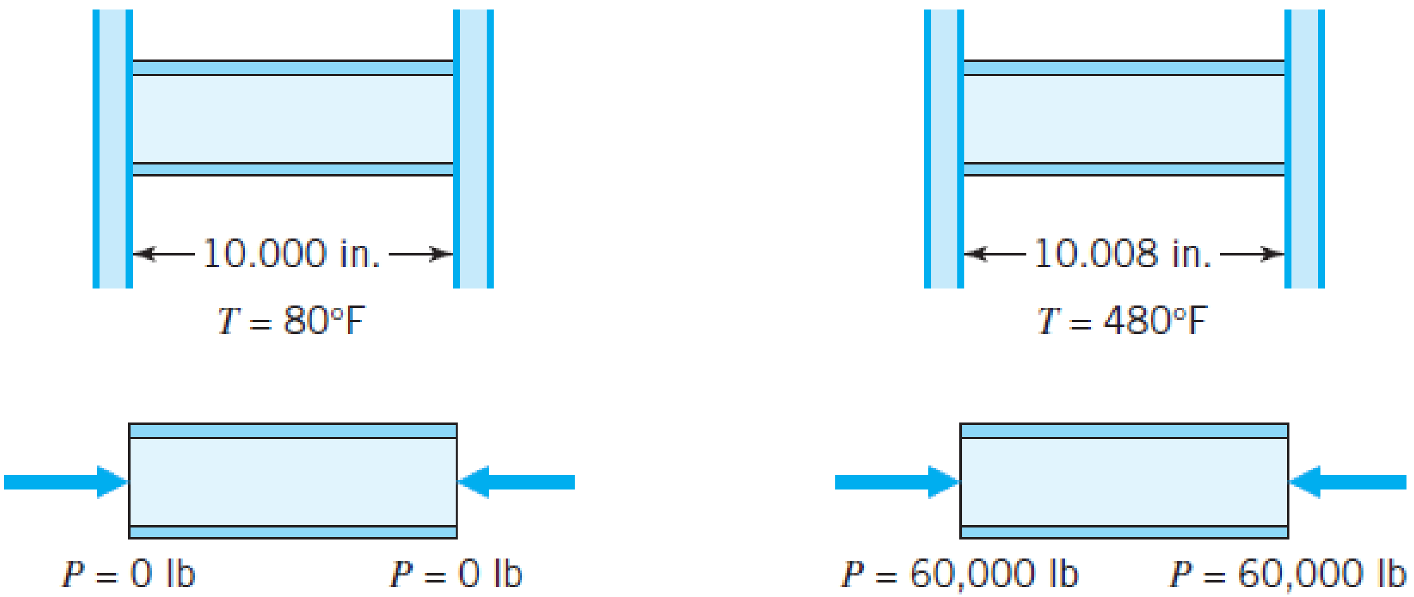
- where ε is the strain, α is the thermal expansion coef and ΔT is the temperature change. Values of α for several common metals are given in Appendix C-1.
- If restraints are placed on the member during the temperature change, the resulting stresses can be determined by
- (1) computing the dimensional changes that would take place in the absence of constraints,
- (2) determining the restraining loads necessary to enforce the restrained dimensional changes, and
- (3) computing the stresses associated with these restraining loads.

SAMPLE PROBLEM 4.5 Thermal Stresses in a Tube

- We ve seen stresses caused by external loads. Stresses can also be caused by expansion and contraction due either to temperature changes or to a material phase change. A 10-in. L steel tube ($E = 30 \times 10^6$ psi and $\alpha = 7 \times 10^{-6}$ per ° F) having a CSA of 1 in^2 is installed with “fixed” ends so that it is stress-free at 80° F. In operation, the tube is heated throughout to a uniform 480° F. Careful measurements indicate that the fixed ends separate by 0.008 in. What loads are exerted on the ends of the tube, and what are the resultant stresses?

- **Known**: A given length of steel tubing with a known CSA expands 0.008 in. from a stress-free condition at 80° F when the tube is heated to a uniform 480° F

Schematic and Given Data:



Assumptions:

1. The tube material is homogeneous and isotropic.
2. The material stresses remain within the elastic range.

Analysis:

1. For the unrestrained tube

$$\epsilon = \alpha \Delta T = (7 \times 10^{-6})(400) = 2.8 \times 10^{-3}$$
$$\Delta L = L\epsilon = 10 \text{ in.} (2.8 \times 10^{-3}) = 0.028 \text{ in.}$$

2. Since the measured expansion was only 0.008 in., the constraints must apply forces sufficient to produce a deflection of 0.020 in. From the relationship

$$\delta = \frac{PL}{AE}$$

which is from elementary elastic theory, and reviewed in Chapter 5,

$$0.020 = \frac{P(10)}{(1)(30 \times 10^6)}, \qquad \text{or} \quad P = 60{,}000\ lb$$

3. Because the area is unity, $\sigma = 60\ ksi$.

Comment: Since these answers are based on elastic relationships, they are valid only if the material has a yield strength of at least 60 ksi at $480°F$.

- If stresses caused by temperature change are undesirably large, the best solution is often to reduce the constraint. - using expansion joints, loops, or telescopic joints
- Thermal stresses also result due to *temperature gradients* - if a thick metal plate is heated in the center of one face with a torch, the hot surface is restrained from expanding by the cooler surrounding material; it is in a state of compression.
- Then the remote cooler metal is forced to expand, causing tensile stresses.
- If the forces and moments do not balance for the original geometry, it will distort or warp to bring about internal equilibrium.
- If stresses are within the elastic limit, the part will revert to its original geometry when the initial temperature conditions are restored.
- If some portion of the part yields, this portion will not tend to revert to the initial geometry, and there will be warpage and internal (residual) stresses when initial temperature conditions are restored. This must be taken into account in the design

- Residual stresses are added to any subsequent load stresses in order to obtain the total stresses.
- If a part with residual stresses is machined, the removal of residually stressed material causes the part to warp or distort. As this upsets the internal equilibrium.
- A common (destructive) method for determining the residual stress in a particular zone of a part is to remove very carefully material from the zone and then to make a precision measurement of the resulting change in geometry. (Hole drilling method)
- Residual stresses are often removed by annealing. The unrestrained part is uniformly heated (to a sufficiently high temperature and for a sufficiently long period of time) to cause virtually complete relief of the internal stresses by localized yielding.
- The subsequent slow cooling operation introduces no yielding. Hence, the part reaches room temperature in a virtually stress-free state.

4.17 Importance of Residual Stresses

- In general, residual stresses are important in situations in which stress concentration is important.
- These include brittle materials involving all loading types, and the fatigue and impact loading of ductile as well as brittle materials.
- For the static loading of ductile materials, harmless local yielding can usually occur to relieve local high stresses resulting from either (or both) stress concentration or superimposed residual stress.
- It is easy to overlook residual stresses because they involve nothing that ordinarily brings them to the attention of the senses. When one holds an unloaded machine part, for example, there is normally no way of knowing whether the stresses are all zero or whether large residual stresses are present.
- a reasonable qualitative estimate can often be made by considering the thermal and mechanical loading history of the part.
- An interesting example shows that residual stresses remain in a part as long as heat or external loading does not remove them by yielding.

4.17 Importance of Residual Stresses

- The Liberty Bell, cast in 1753, has residual tensile stresses in the outer surface because the casting cooled most rapidly from the inside surface.
- After 75 years of satisfactory service, the bell cracked, probably as a result of fatigue from superimposed vibratory stresses caused by ringing the bell.
- Holes were drilled at the ends to keep the crack from growing, but the crack subsequently extended itself.
- Almen and Black cite this as proof that residual stresses are still present in the bell.

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Lecture 3

6

Failure Theories, Safety Factors, and Reliability

6.1 Introduction

- Previously we have dealt with the determination of loads (Chapter 2), with the stresses and deflections caused by those loads (Chapters 4.
- Here we will be looking at
- (1) predicting the capability of materials to withstand the infinite combination of nonstandard loads to which they are subjected as fabricated machine and structural components and
- (2) the selection of appropriate safety factors to provide the required safety and reliability.
- The primary concern is with static loads.
- Reliability is a matter of extreme importance in the engineering of a product, and this point is becoming increasingly recognized.
- On the other hand, it is important that components not be overdesigned to the point of making them unnecessarily costly, heavy, bulky, or wasteful of resources.

6.2 Types of Failure

- predicting the capability of materials to withstand the infinite combination of nonstandard loads to which they are subjected as fabricated machine and structural components and
- the selection of appropriate safety factors to provide the required safety and reliability.
- The primary concern is with static loads.
- Reliability is a matter of extreme importance in the engineering of a product, and this point is becoming increasingly recognized.
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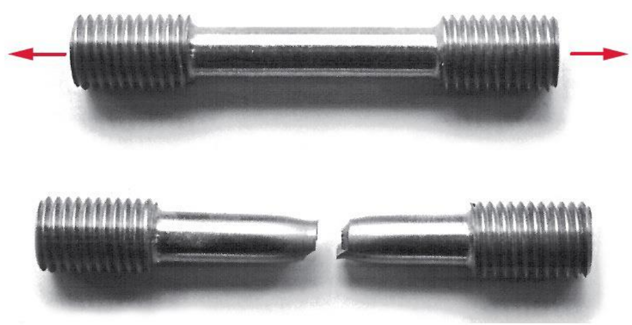


FIGURE 2-3

A Tensile Test Specimen of Mild, Ductile Steel Before and After Fracture

Failure along principal shear stress plane

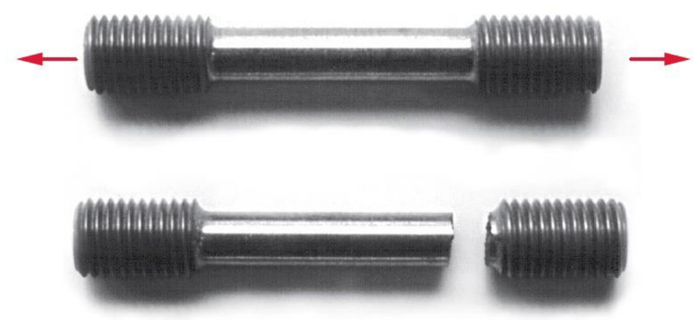
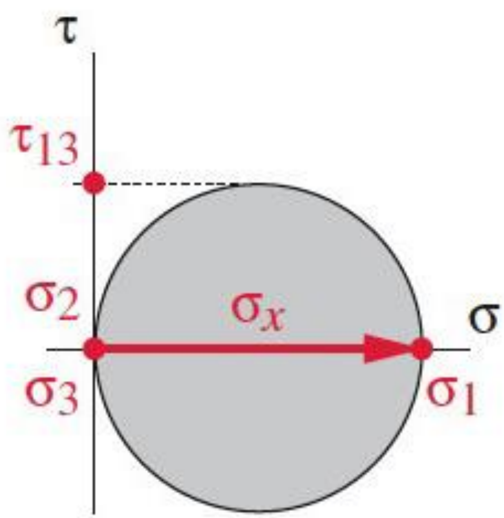


FIGURE 2-5

A Tensile Test Specimen of Brittle Cast Iron Before and After Fracture

Failure along principal normal stress plane



Cast iron has C between 2.1% to 4% and Si between 1% and 3%
C contents less than 2.1% are steels.

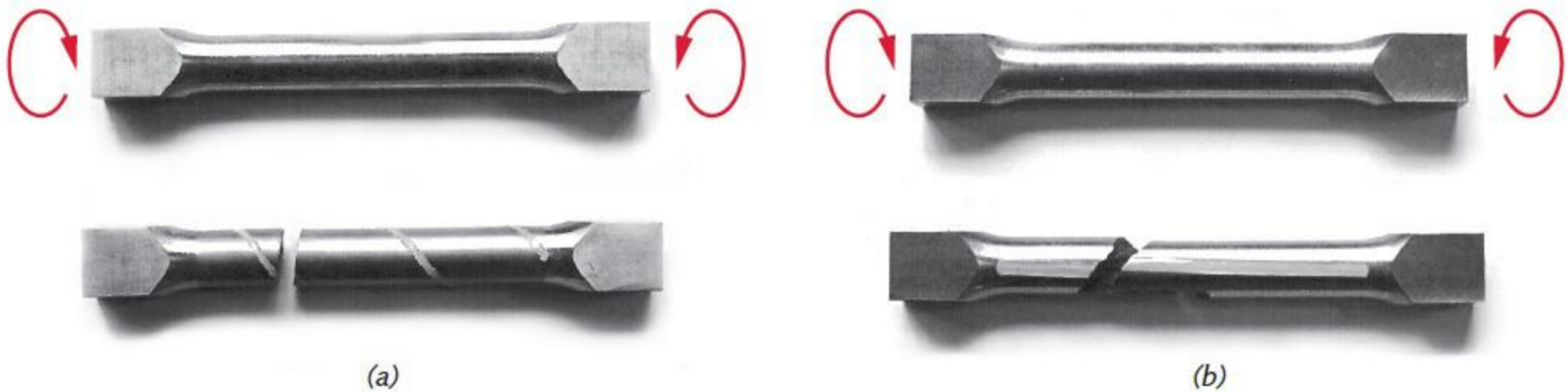
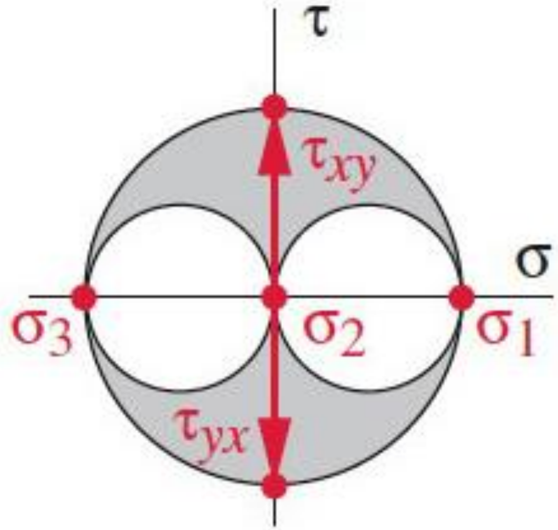


FIGURE 2-8

Torsion Test Specimens Before and After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

Failure along principal shear stress plane

Failure along principal normal stress plane



(*a*)

FIGURE 5-1

Mohr's Circles for Unidirectional Tensile Stress (*a*) and Pure Torsion (*b*)

(*b*)

- In general, ductile, isotropic materials are limited by their shear strengths.
- Brittle materials are limited by their tensile strengths.
- If cracks are present in a ductile material, it can suddenly fracture at nominal stress levels well below its yield strength, even under static loads.

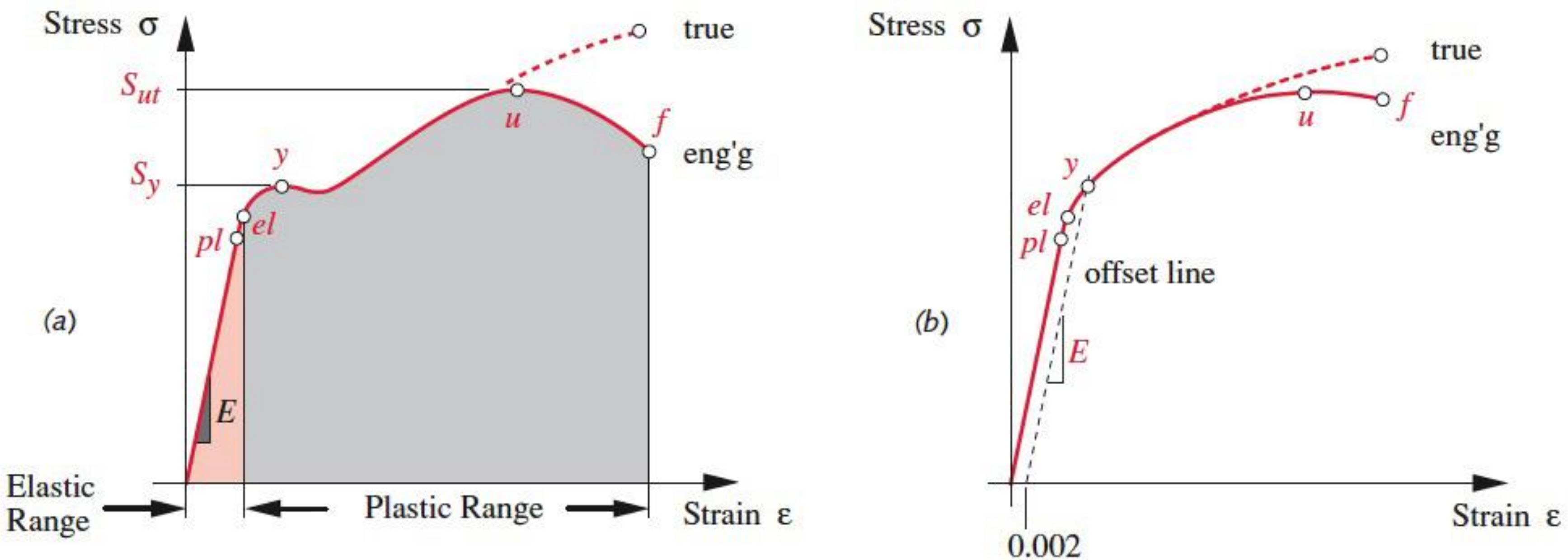


FIGURE 2-2

Engineering and True Stress-Strain Curves for Ductile Materials: (*a*) Low-Carbon Steel (*b*) Annealed High-Carbon Steel

- Static loads are slowly applied and remain constant with time.
- Dynamic loads are suddenly applied (impact), or repeatedly varied with time (fatigue), or both.

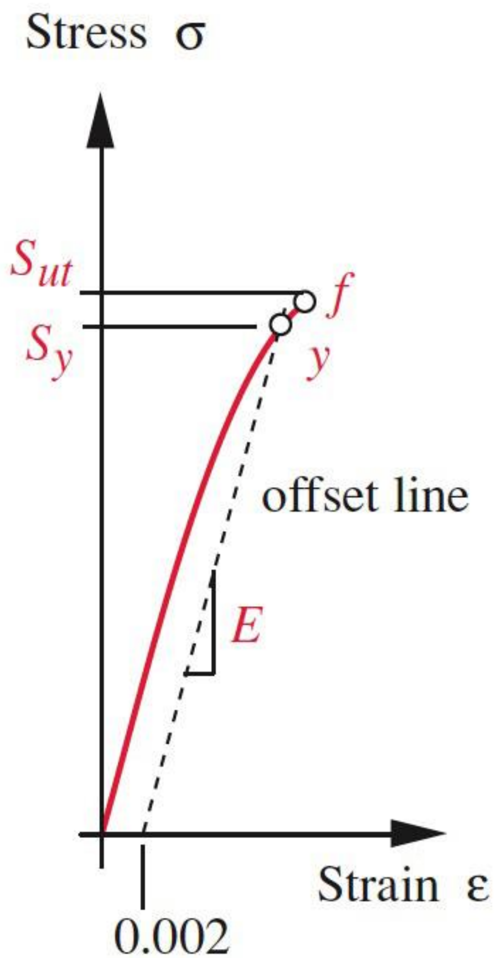
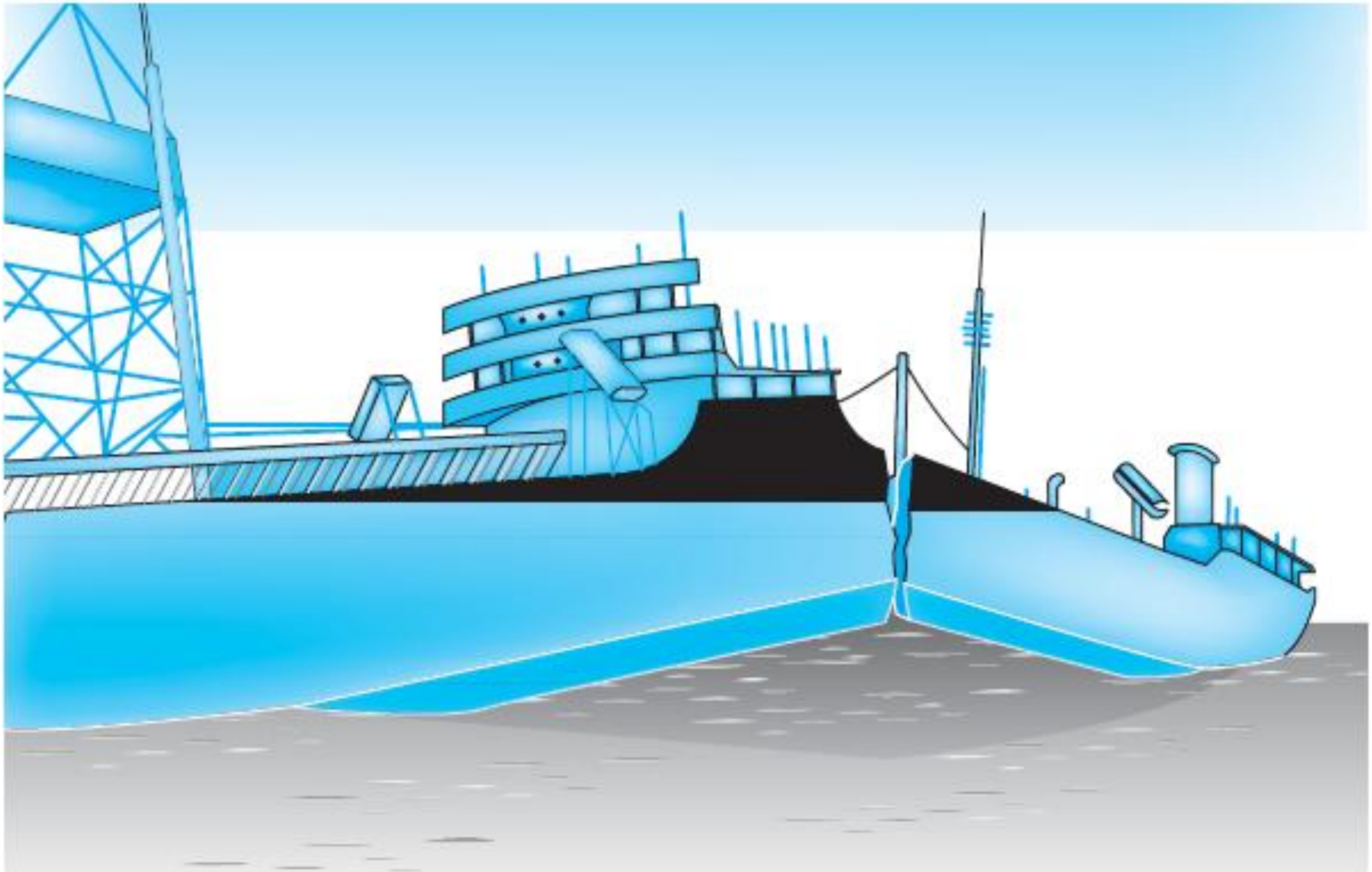


FIGURE 2-4

Stress-Strain Curve of a Brittle Material

- In dynamic loading, the distinction between failure mechanisms of ductile and brittle materials blurs.

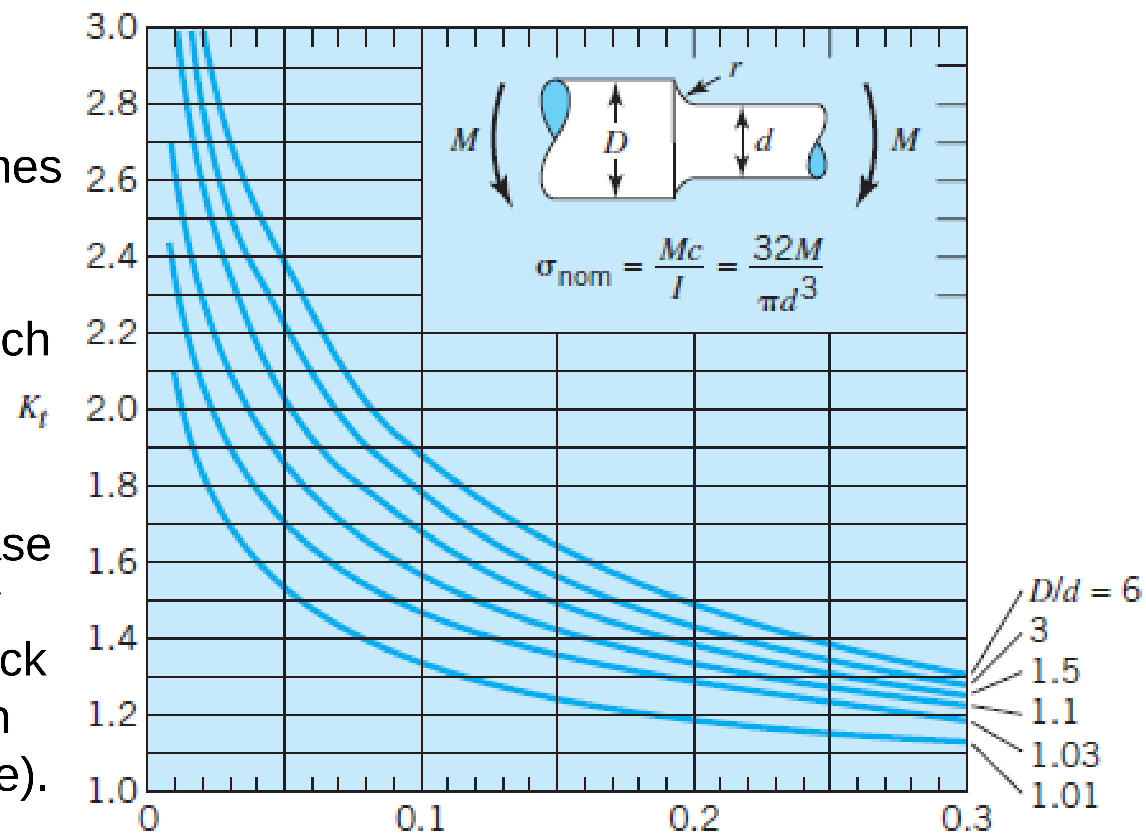
- Ductile materials often fail like brittle materials in dynamic loading.



- Brittle fracture of WWII tanker though made with ductile steel.
- The mechanisms of brittle fracture are the concern of a relatively new discipline, fracture mechanics..

6.3 Fracture Mechanics—Basic Concepts

- The fracture mechanics approach begins with the assumption that all real materials contain cracks of some size - even if only submicroscopic.
- If brittle fracture occurs, it is because the conditions of loading and environment (primarily temperature) are such that they cause an almost instantaneous propagation to failure of one or more of the original cracks.
- If there is fatigue loading, the initial cracks may grow very slowly until one of them reaches a critical size (for the loading, geometry, material, and environment involved), at which time total fracture occurs.
- Theoretically, the stress concentration factor at the base of a crack approaches infinity because the radius at the crack root approaches zero (as with r/d approaching zero in Figure).



6.3 Fracture Mechanics—Basic Concepts

- This means that if the material has any ductility, yielding will occur within some small volume of material at the crack tip, and the stress will be redistributed.
- Thus, the effective stress concentration factor is considerably less than infinity, and furthermore it varies with the intensity of the applied nominal stress.
- In the fracture mechanics approach, one does not attempt to evaluate an effective stress concentration per se; rather, a stress intensity factor, K, is evaluated.
- This can be thought of as a measure of the effective local stress at the crack root. Once evaluated, K is then compared with a limiting value of K that is necessary for crack propagation in that material.
- This limiting value is a characteristic of the material, called fracture toughness, or critical stress intensity factor K_c, which is determined from standard tests.
- Failure is defined as whenever the stress intensity factor, K, exceeds the critical stress intensity factor, K_c .
- Thus, a safety factor, SF, for failure by fracture can be defined as K_c/K.

6.3 Fracture Mechanics—Basic Concepts

- Most available values of K and K_c are for tensile loading - mode I. K_I & K_{IC}
- Modes II and III pertain to shear loading.
- Set of fracture toughness values (K_{Ic}) are presented in Table 6.1 & in Appendix F
- Available values of K_{Ic} are for relatively thick members, such that the material at the crack root is approximately in a state of plane strain. That is, material that surrounds the crack and is under low stress resists "Poisson's ratio" contraction at the crack root, thereby enforcing $\epsilon_3 \approx 0$ in the thickness direction.

TABLE 6.1 Strength Properties of 1-in.-Thick Plates—Values of K_{Ic}, Critical Stress Intensity Factor

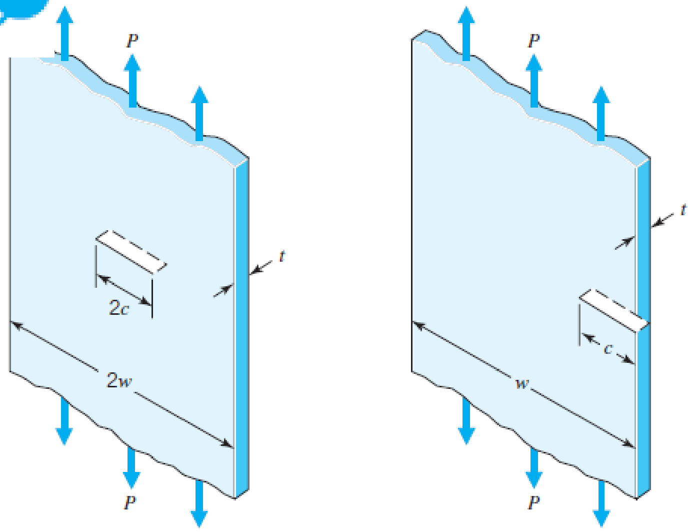
Material	Temperature	S_u (ksi)	S_y (ksi)	K_{Ic} (ksi $\sqrt{\text{in.}}$)
7075-T651 Aluminum	Room	78	70	27
Ti-6Al-4V (annealed)	Room	130	120	65
D6AC Steel	Room	220	190	70
D6AC Steel	−40°F	227	197	45
4340 Steel	Room	260	217	52

Source: A. Gomza, Grumman Aerospace Corporation.

6.3 Fracture Mechanics—Basic Concepts

- Crack root material in sufficiently thin members is free to contract in the thickness direction, giving $\sigma_3 \approx 0$ or a condition of plane stress.
- The plane strain tensile loading, with σ_3 being tensile, offers less opportunity for redistributing high crack root stresses by shear yielding.
- (This is evident by considering three-dimensional Mohr stress circles for $\sigma_3 = 0$ and for $\sigma_3 =$ a positive value.)
- Because of this, values of K_{Ic} for plane strain are substantially lower than those for plane stress.
- Thus, the more readily available plane strain values of K_{Ic} are often used for conservative calculations when the value of K_{Ic} for the actual thickness is not known.

6.4 Fracture Mechanics—Applications



(a) Center crack (b) Edge crack

- Figure a shows a "thin" plate with a central crack of length 2c extending through the full thickness.
- If the crack length is a small fraction of the plate width, and if the P/A stress figured on the basis of the net area, t(2w–2c), is less than the yield strength, then the stress intensity factor at the edges of the crack is approximately

$$K_I \cong K_o = \sigma\sqrt{\pi c} = (1.8\sqrt{c})\sigma_g \quad (6.1)$$

- $\sigma = \sigma_g$ is the gross-section tensile stress, P/2wt and K_o is the stress intensity factor for a short central crack of length 2c in a flat infinite sheet of thickness t subjected to the uniform tensile stress σ_g.
- Rapid fracture occurs when K_I becomes equal to K_{Ic}. In this case of a thin plate, the plane stress value of K_{Ic} would be preferred. Thus, failure occurs when 3 variables reach the approximate relationship:

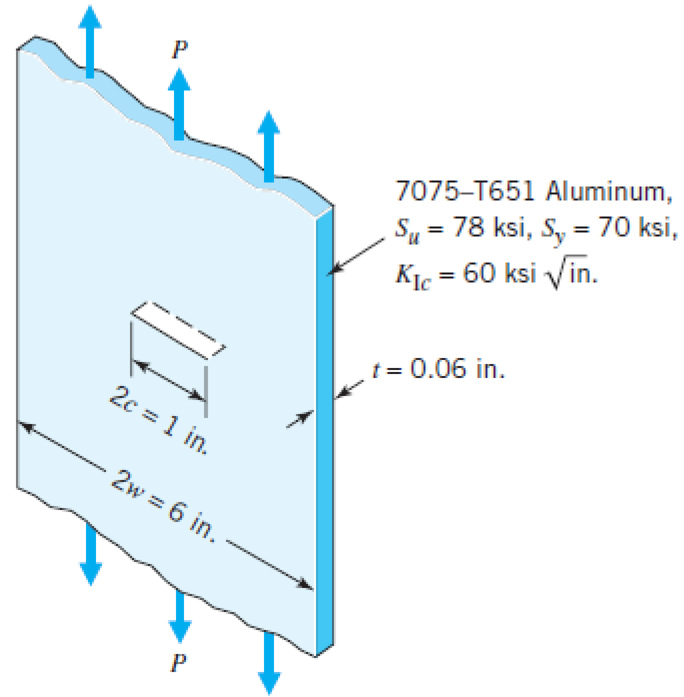
$$K_{Ic} = (1.8\sqrt{c})\sigma_g \quad (6.2)$$

- For geometries that differ from that of central crack, as in edge crack, configuration factor, Y, is introduced that accounts for the particular geometry and loading
- Thus changing the relationship approximately to

$$K_{Ic} = K_I = YK_o = \sigma Y\sqrt{\pi c} = (2.0\sqrt{c})\sigma_g$$

SAMPLE PROBLEM 6.1 Determine the Critical Load for a "Thin" Plate with a Central Crack

- A plate of 2w = 6 in. and t = 0.06 in. is made of aluminum (S_u = 78 ksi, S_y = 70 ksi). It has a plane stress K_{Ic} = 60 ksi
- It is used in an aircraft component which will be inspected periodically for cracks. Estimate the highest load, P that can be applied without causing fracture when central crack grows to length, 2c =1 in.



Assumptions:

1. Yielding has occurred within some small volume of material at the crack tip.
2. Crack propagation to total fracture occurs instantaneously when the limiting value of the stress intensity factor K_I equals or exceeds the fracture toughness K_{Ic} for the material.
3. The crack is a small fraction of the plate width.
4. The tensile stress based on the net area (minus the area of the crack) is less than the yield strength.

SAMPLE PROBLEM 6.1 Determine the Critical Load for a "Thin" Plate with a Central Crack

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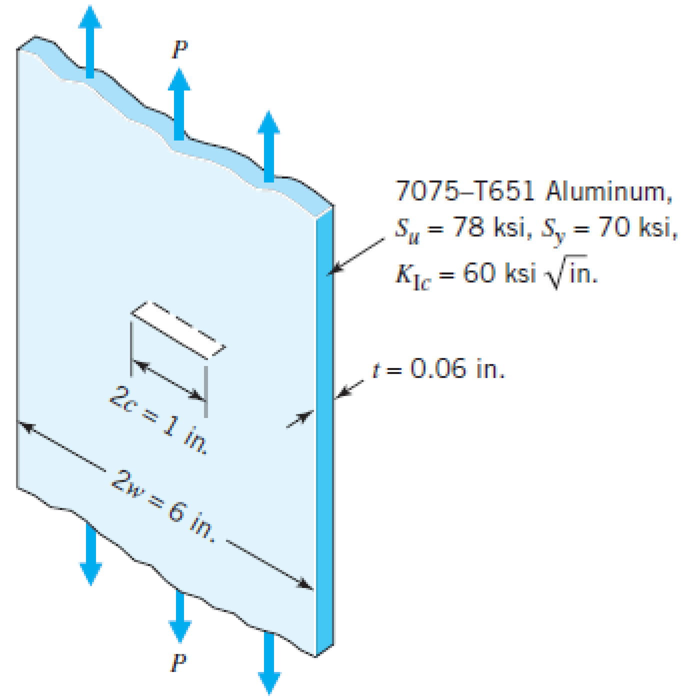


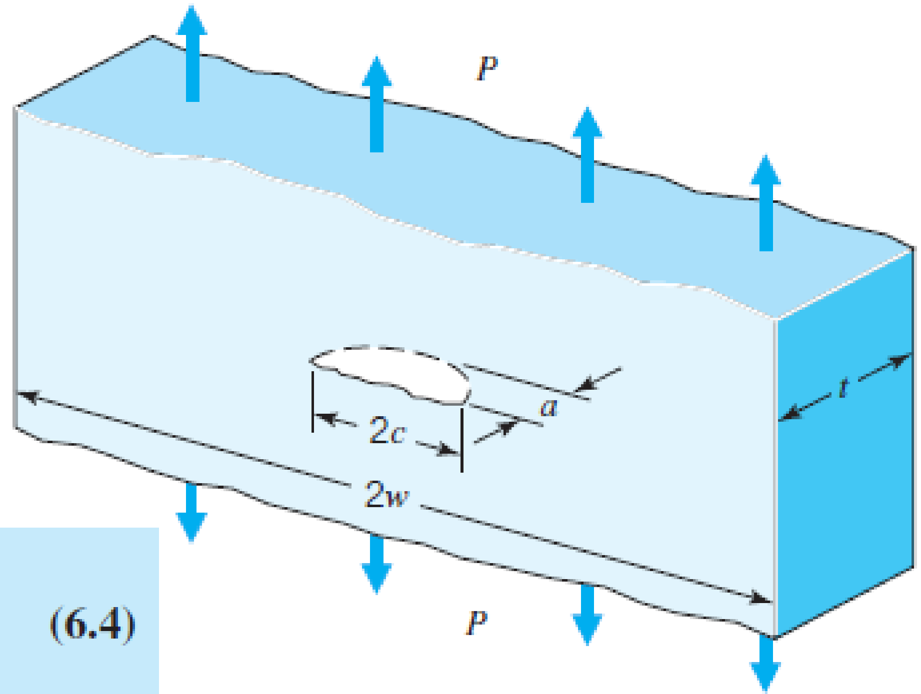
Analysis: From Eq. 6.2,

$$\sigma_g = \frac{K_{Ic}}{1.8\sqrt{c}} = \frac{60}{1.8\sqrt{0.5}} = 47.14 \text{ ksi}$$

$$P = \sigma_g(2wt) = 47{,}140 \text{ psi } (6 \text{ in.} \times 0.06 \text{ in.}) = 16{,}970 \text{ lb}$$

Comment: The P/A stress based on the net area, $t(2w-2c)$, is 56,567 psi. This value is less than the yield strength ($S_y = 70$ ksi).

6.4.2 Thick Plates



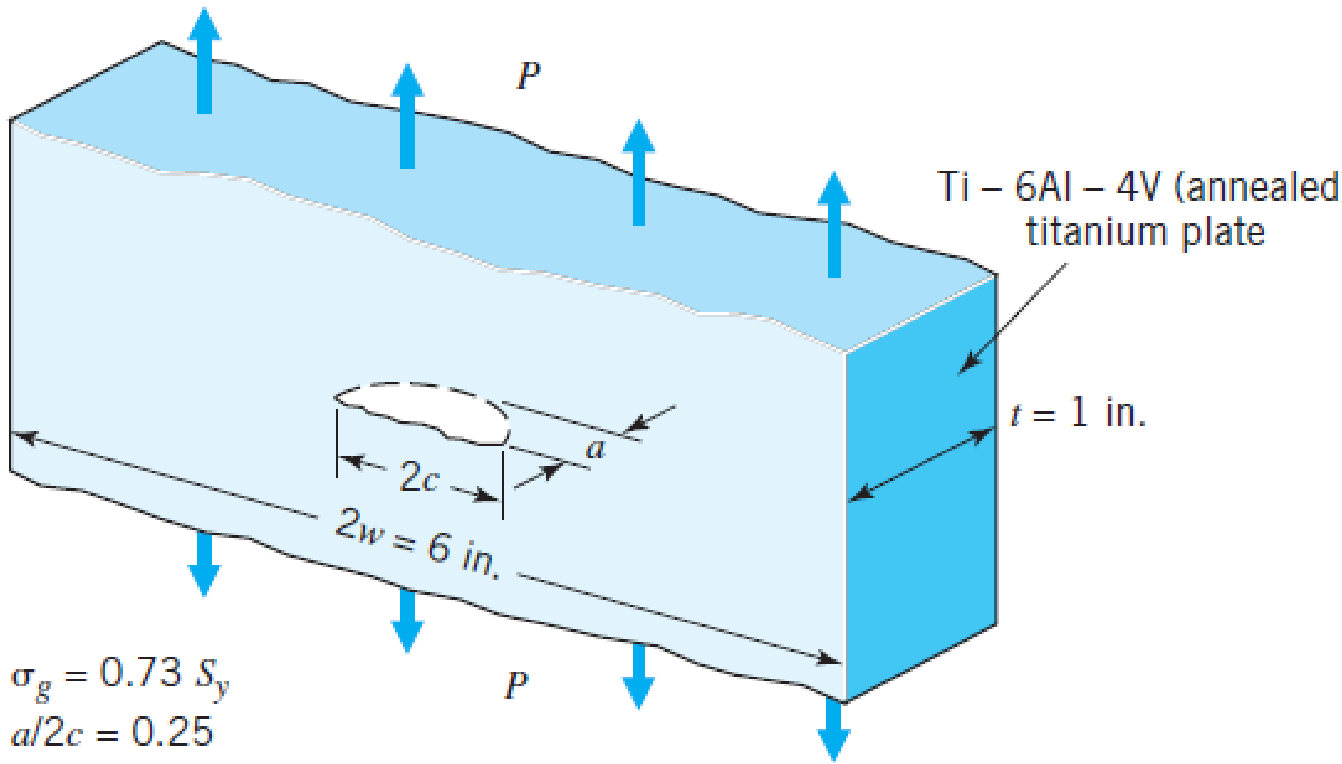
- Cracks in thick plates generally begin at the surface, taking a somewhat elliptical form, as shown in Figure 6.4a.
- If $2w/t > 6$, $a/2c$ = about 0.25, $w/c > 3$, $a/t < 0.5$, and $\sigma_y/\sigma_g < 0.8$, the stress intensity factor at the edges of the crack is approximately

$$K_I = K = \frac{\sigma_g \sqrt{a}}{\sqrt{0.39 - 0.053(\sigma_g/S_y)^2}} \tag{6.4}$$

- Fracture would be predicted for values of K exceeding K_{ic}
- Table 6.1 gives typical mechanical properties of 1-in.-thick plates made of various structural materials. Note particularly
- (1) the relatively high fracture toughness of the titanium alloy in comparison to its S_u
- (2) the room temperature comparison of K_{Ic} for the two steels of nearly equal S_u
- (3) the reduction in K_{Ic} with temperature for the high-toughness D6AC steel.

SAMPLE PROBLEM 6.2 Determine the Critical Crack Depth for a Thick Plate

- A Ti-6Al-4V (annealed) titanium plate is loaded as in Figure to a gross-area stress σ_g of $0.73S_y$.
- t = 1 in., 2w = 6 in., a/2c = 0.25
- Estimate the critical crack depth, a_{cr} , at which rapid fracture will occur.
- **Assumptions**: @ room temp 70° F
- Fracture occurs if (SIF) $K > K_{Ic}$.



Sample Problem 6.2 Determine the Critical Crack Depth for a Thick Plate

Analysis:

1. From Table 6.1 for Ti-6Al-4V (annealed) at room temperature, we have $S_y = 120$ ksi and $K_{Ic} = 65 \text{ ksi}\sqrt{\text{in.}}$
2. From Eq. 6.4, and setting $K = K_{Ic}$ ($a = a_{cr}$),

$$a_{cr} = \left(\frac{K_{Ic}\sqrt{0.39 - 0.053(\sigma_g/S_y)^2}}{\sigma_g}\right)^2$$

$$= \left(\frac{65\sqrt{0.39 - 0.053(0.73)^2}}{(0.73)(120)}\right)^2 = 0.20 \text{ in.}$$

Comments:

1. Equation 6.4 is appropriate if $2w/t > 6$, $a/2c$ = about 0.25, $w/c > 3$, $a/t < 0.5$, and $\sigma_g/S_y < 0.8$. For this problem, $2w/t = 6$, $a/2c = 0.25$, $w/c \geq 7.5$, $a_{cr}/t = 0.20$, and $\sigma_g = 0.73S_y$.
2. An important design requirement of internally pressurized members is that a crack be able to propagate through the full wall thickness (thereby causing a leak that can be readily detected) without becoming unstable and leading to total fracture.

6.4.3 Stress-Intensity Factors[1]

- Evaluate stress intensity factors associated with geometry and loadings so that the maximum stress intensity factor existing in a part can be determined.
- In order to handle other than very simple cases, experimental and analytical methods for determining stress intensity factors were developed and used. The results of many of these studies are available in the form of graphs 6.5a to 6.5h.
- For geometries that differ from that of a central crack in a small fraction of the plate width (central crack in an infinite sheet), a configuration factor, Y, is introduced
- The configuration factor, $Y = K_I/K_o$, is plotted versus dimensionless ratios, indicating that only the loading and the shape (for relatively large sizes) of the part influences the configuration factor involved.
- The figures give values of the SIF, K_I , at the crack tip (based on linear, elastic, homogenous, and isotropic material).
- The value of K_o is the SIF for a short central crack of length 2c in an infinite sheet subjected to a uniform uniaxial tensile stress, σ, where $K_o = \sigma\sqrt{\pi c} = (1.8\sqrt{c})\sigma_g$.
- The stress intensity factor K_I will reflect the particular geometry and loading, and will thus differ from K_o except for a plate with a small central crack.
- As mentioned before, the value of K_I is compared to the value of K_{Ic} to determine if failure occurs.

FIGURE 6.5a

Rectangular sheet with through-the-thickness central crack subjected to a uniform uniaxial tensile load [10].

- Figure 6.5a shows a rectangular sheet of width 2w and height 2h, with a central crack of length 2c.
- A uniform tensile stress acts over the ends of the sheet and is perpendicular to the direction of the crack. Figure 6.5a presents curves, Y versus c/w for different h/w.
- K_o is the SIF for a central crack in an infinite sheet (h = w = ∞) and given by

$K_o = \sigma\sqrt{\pi c}$. Here, $K_I = YK_o$.

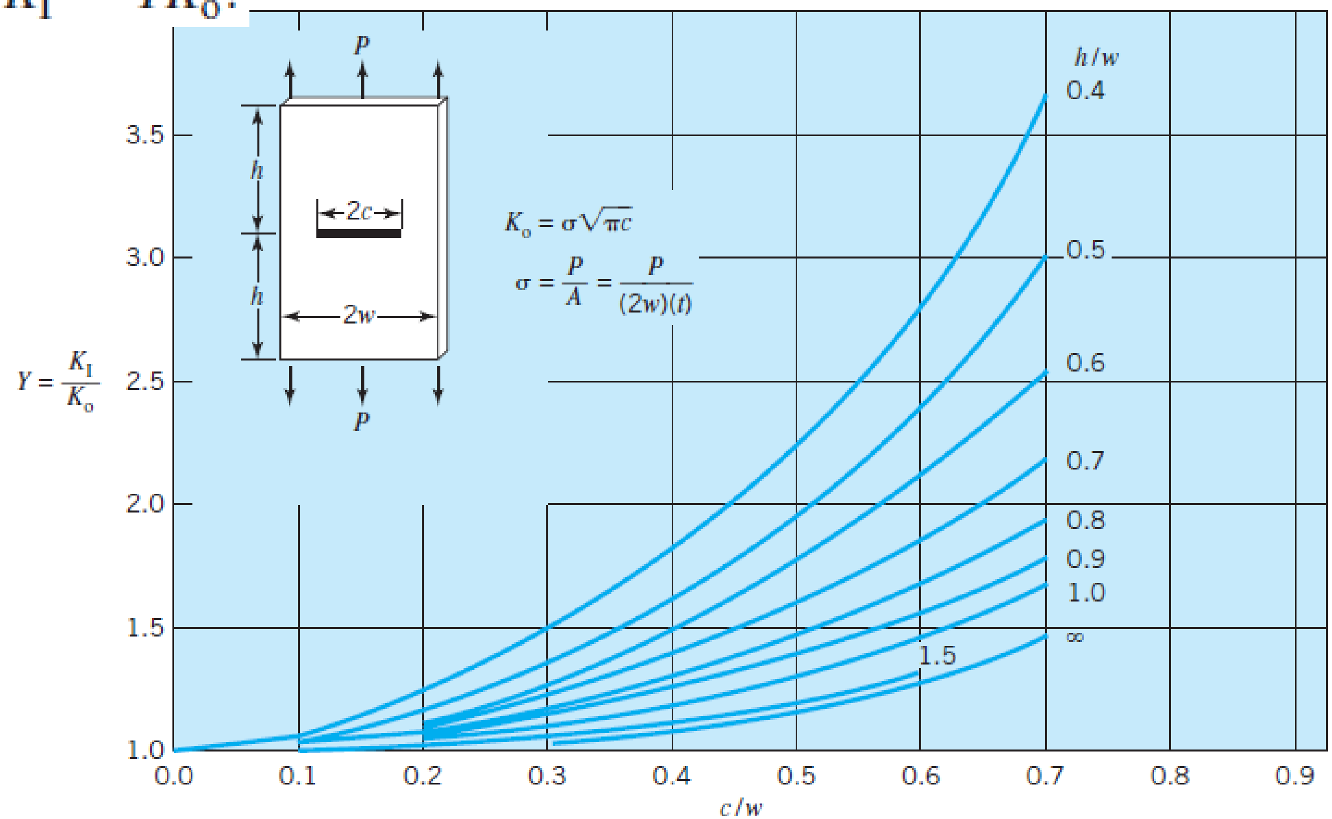
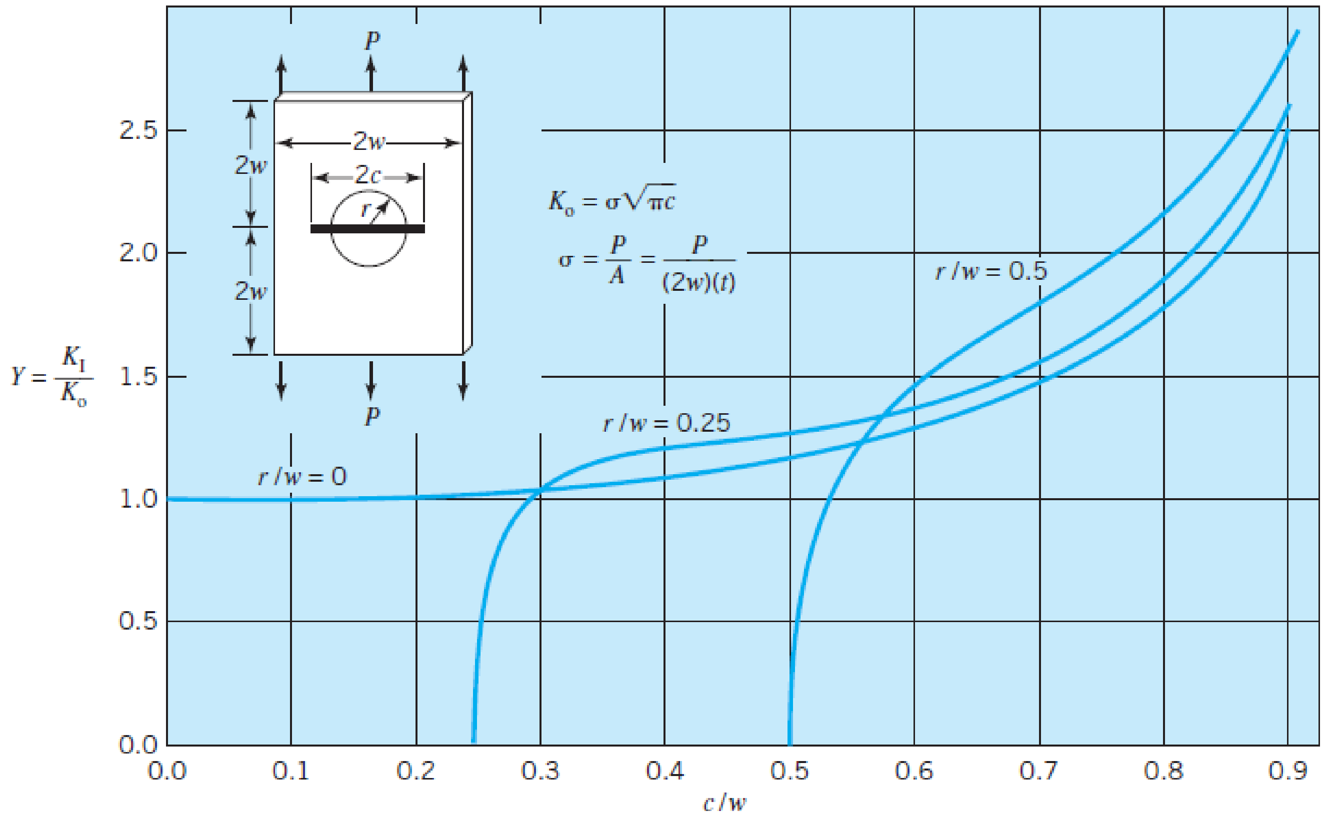


FIGURE 6.5b

Rectangular sheet with a central circular hole and two cracks subjected to a uniform uniaxial tensile load [10].

- Figure 6.5b shows a rectangular sheet of width 2w and height 4w, with two cracks each of equal length, (c-r), at a central hole of radius r. The cracks are diametrically opposite and perpendicular to load P direction or uniform σ. The crack tips are a distance 2c apart.
- Figure 6.5b presents curves of configuration factor, Y, versus c/w for different r/w. Height/width ratio is 2, and for the case r/w = 0, the results correspond and agree with those for a central crack in a rectangular sheet (see Figure 6.5a).



- Figure 6.5c shows a flat sheet with width w and height 2h. The sheet is loaded with a uniform tensile stress σ acting perpendicular to an edge crack of lgth c.
- Figure presents curves of Y, versus c/w for different h/w.
- Two cases are presented:
- (i) where the ends are free to rotate—bending unrestrained,
- (ii) where the ends are constrained from rotating—bending restrained.
- As in the previous figures,

$$K_I = YK_0 \text{ and } K_0 = \sigma\sqrt{\pi c}.$$

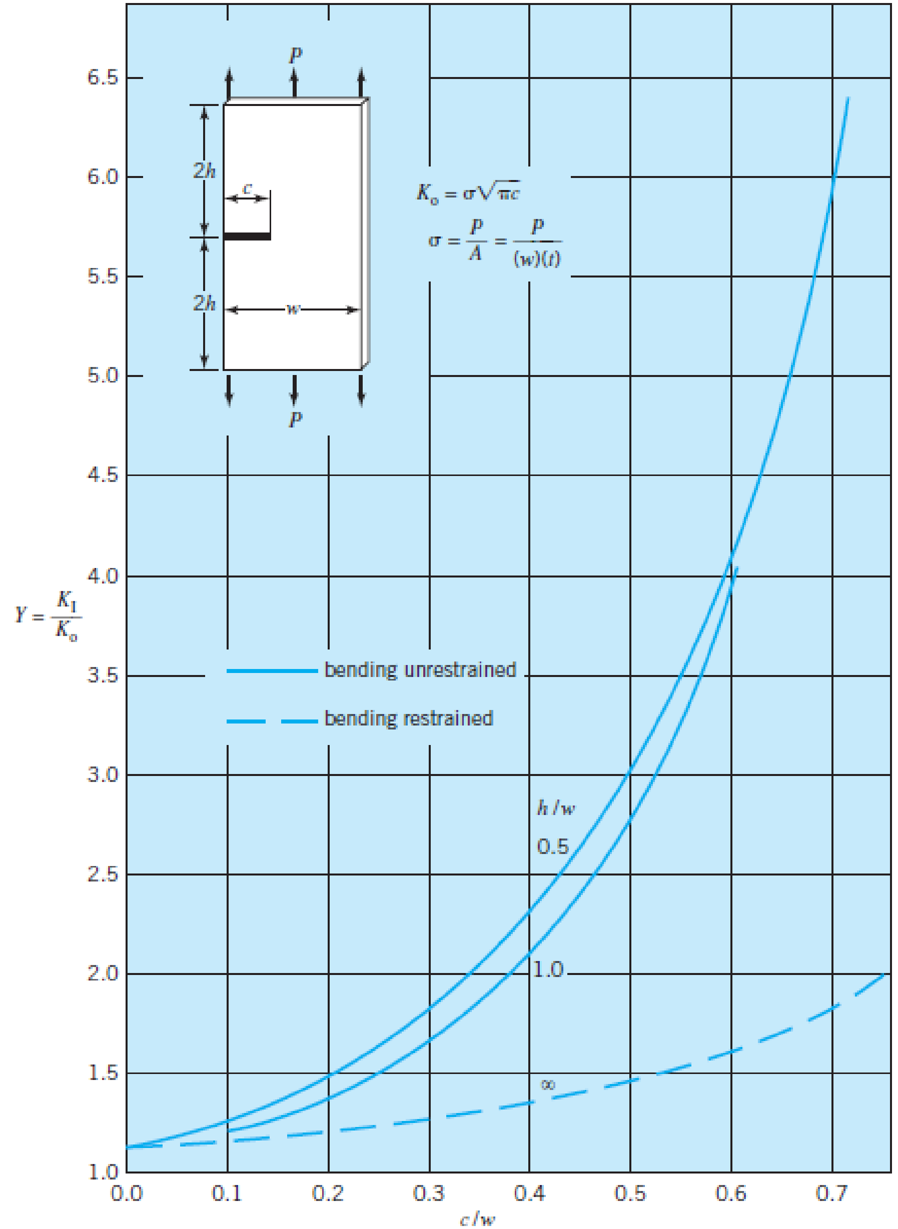


FIGURE 6.5c
Rectangular sheet with an edge crack subjected to a uniform uniaxial tensile load acting perpendicular to the direction of the crack with and without bending constraints [10].

- Figure 6.5d shows a flat sheet of width w and height 2h. The sheet contains an edge crack in the middle of and perpendicular to one side. A splitting force, P, acts symmetrically along the side containing the crack of length c.
- Figure 6.5d presents curves of configuration factor, Y, versus c/w for various values of c/w.
- Here, $K_0 = P\sqrt{w}/(ht)$.

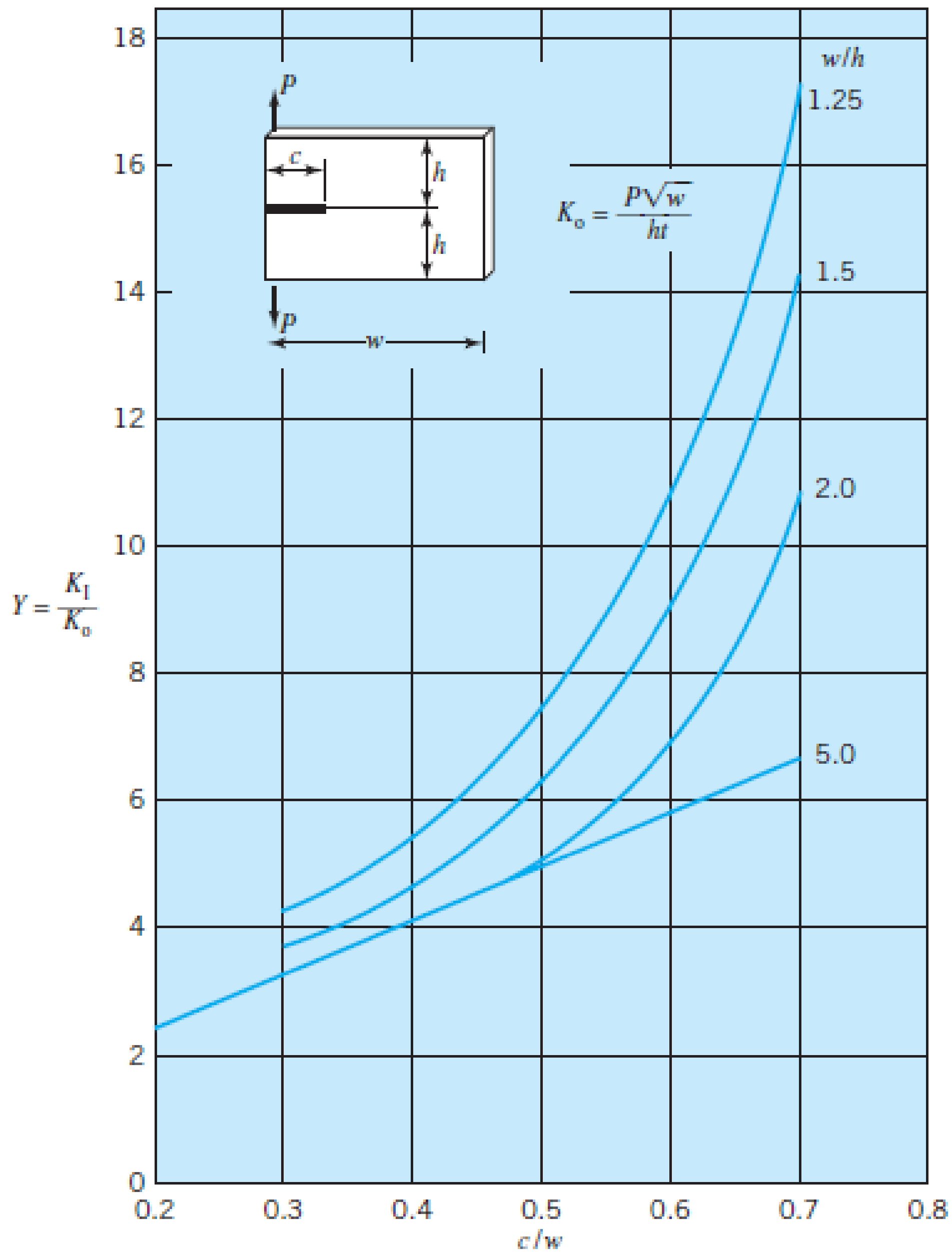


FIGURE 6.5d
Rectangular sheet with edge crack subjected to splitting forces [10].

- Figure 6.5e shows a flat sheet with thickness t, width w, and an edge crack of depth c, for both (i) pure bending and (ii) three-point bending for $c/w \leq 0.6.$
- Curves of, Y, versus c/w are presented for both cases.
- Here, $K_I = YK_o$, and

$$K_0 = 6M\sqrt{\pi c}/(w^2 t).$$

- Since linear elastic fracture mechanics is being used, SIF for other types of Mode I components can be obtained by superposition.

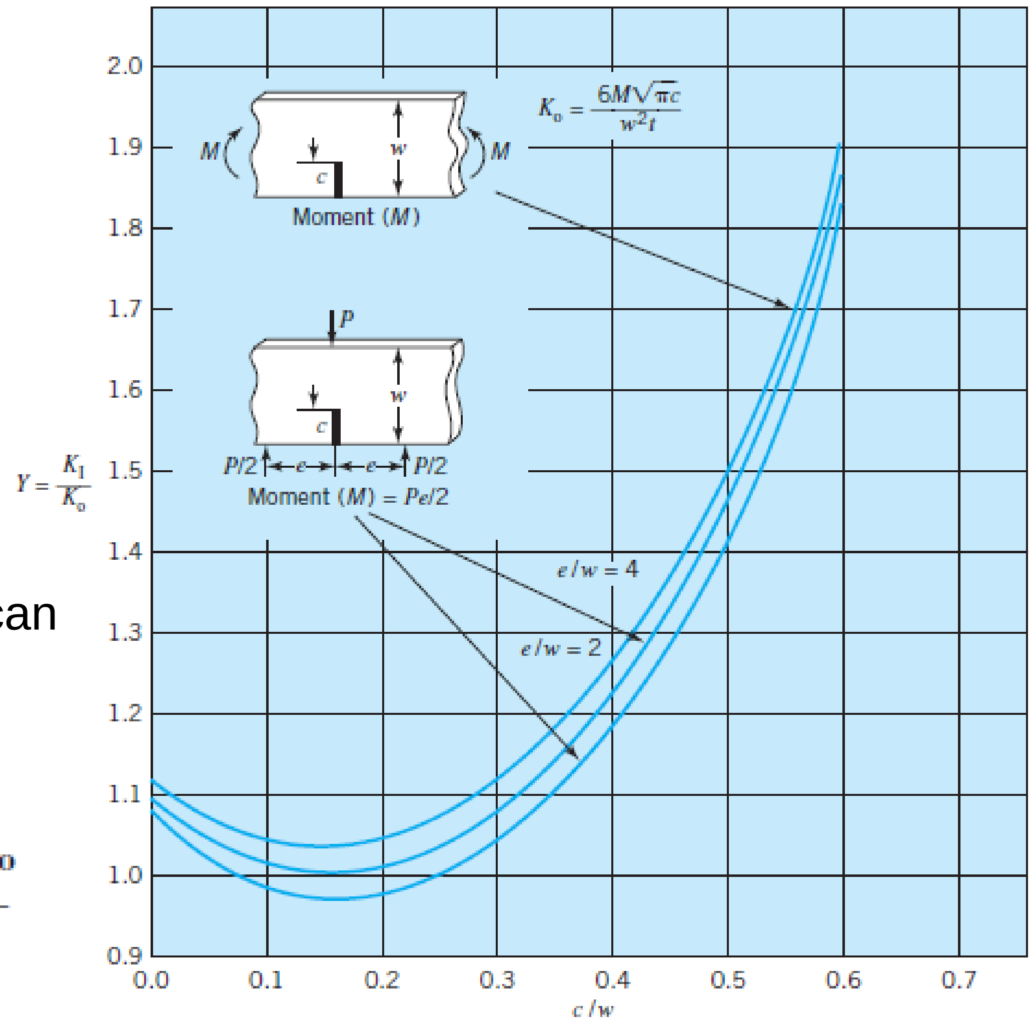


FIGURE 6.5e
Finite width sheet with an edge crack perpendicular to one edge subjected to bending loads which open the crack. K_I is for the edge crack [10].

- For example, for a flat sheet with an edge crack and loaded with uniform σ, and a pure bending moment, the K_I from 6.5c + K_I from 6.5e = K_I for the combined loading.

- Figure 6.5f shows a tube with inner radius r_i and outer radius r_o.
- The tube contains a circumferential crack of depth c extending radially inward from the outside surface.
- Remote from the crack, a uniform tensile stress σ is applied and acts parallel to the tube axis. Presented are curves of Y, versus $c/(r_o - r_i)$ for different r_i/r_o for a long tube.
- Note that in the limiting case of a short circumferential crack in a long tube, the results approach those for a short edge crack in a flat sheet that does not bend (Figure 6.5c).

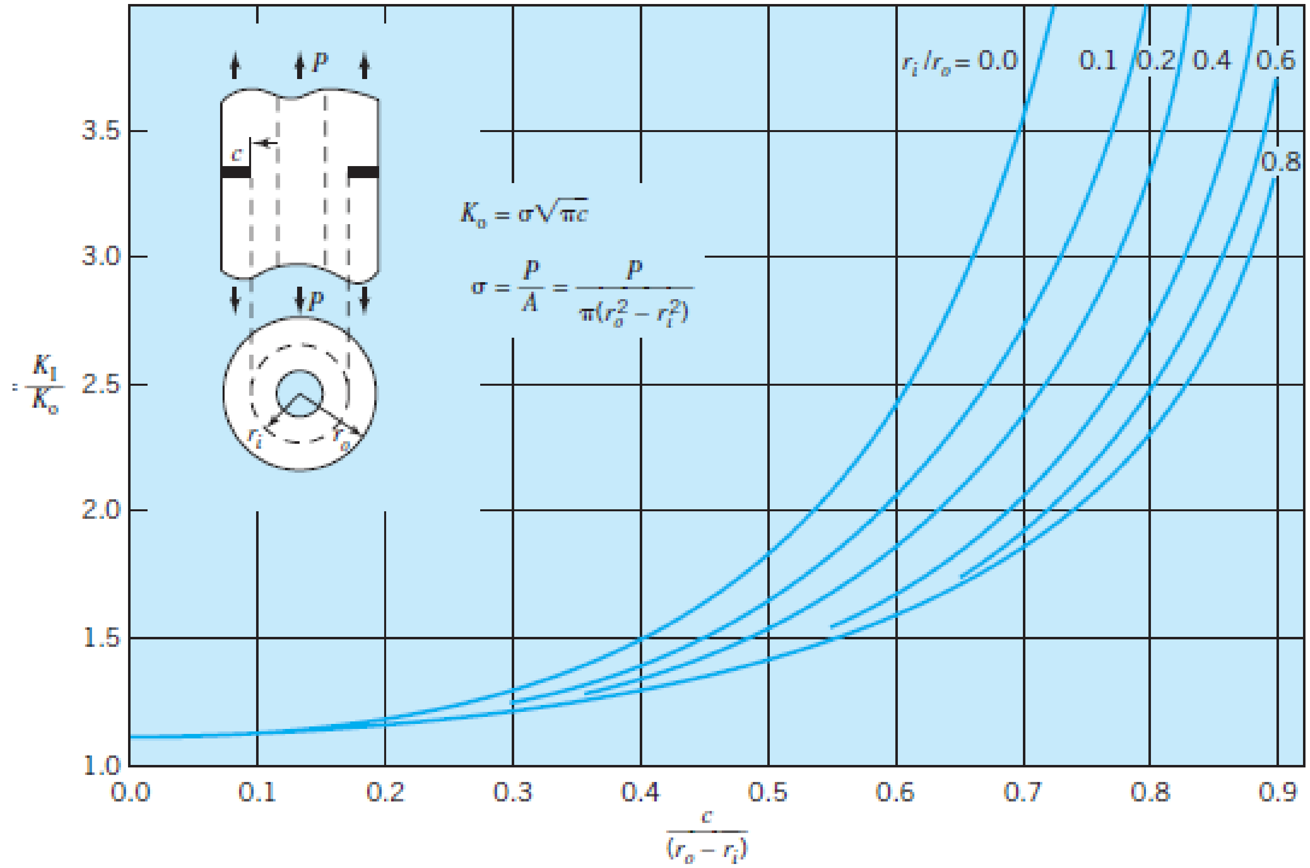


FIGURE 6.5f
Long cylindrical tube with an external circumferential crack subjected to a uniform uniaxial tensile load [10].

- Figure 6.5g shows the CS of a tube with inside radius r_i and outer radius r_o.
- The tube contains a radial crack of length c extending radially inward from the outside cylindrical surface.
- The tube is subjected to an internal pressure p.
- Presented are curves of Y, versus $c/(r_o - r_i)$ for different r_i/r_o for a long tube.
- Here, $K_o = \sigma\sqrt{\pi c}$. Here, $K_I = YK_o$.
- The stress σ_o is equal to the normal tensile stress at the outer surface of the cylinder and is given by

$$\sigma_o = \frac{2pr_i^2}{(r_o^2 - r_i^2)} \qquad \text{(a)}$$

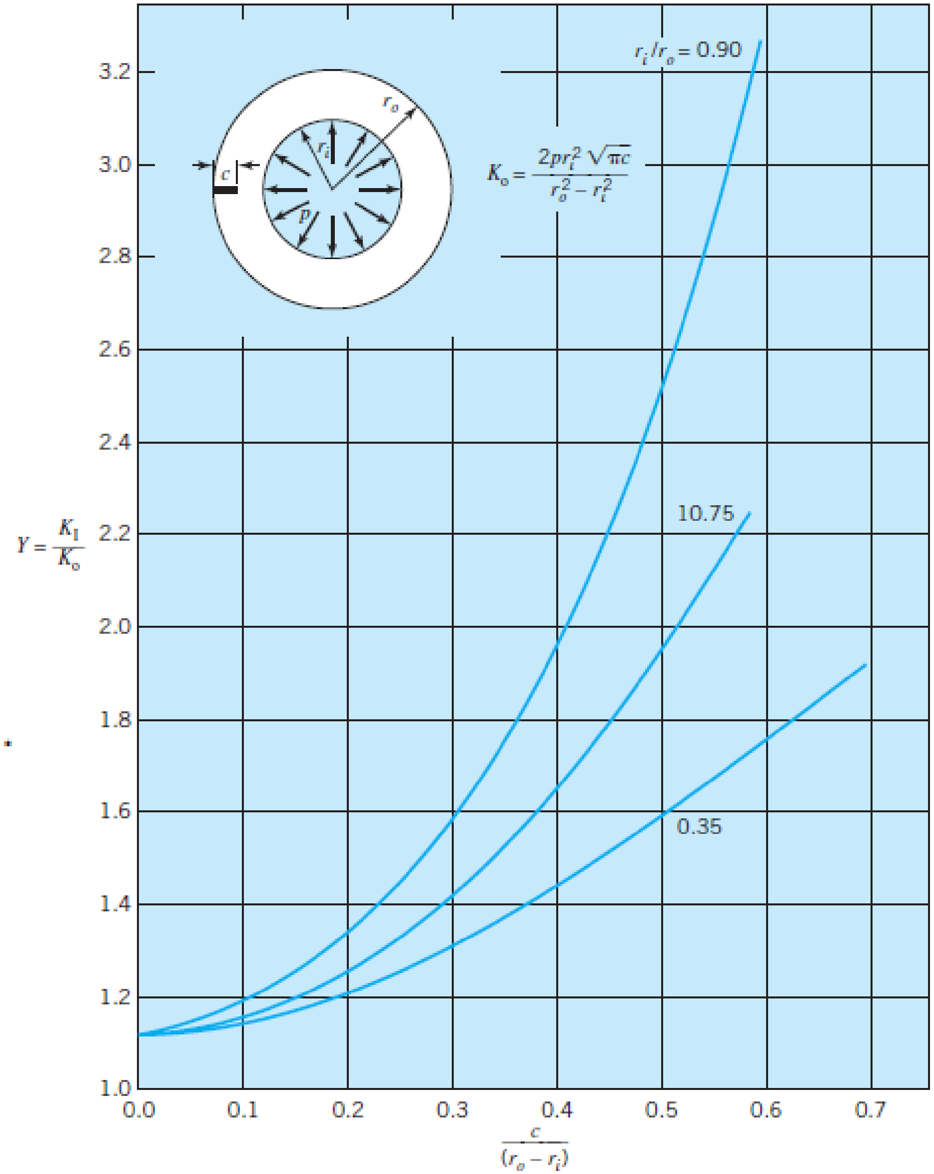


FIGURE 6.5g
Long cylindrical tube with an external radial edge crack extending from the external boundary subjected to a uniform internal pressure. K_I is for the edge crack [10].

- Figure 6.5h pictures a slab of thickness t, with a uniform tensile stress σ acting perpendicular to the plane of a semi-elliptical crack.
- The crack plane is perpendicular to the surface of the slab. The deepest point on the crack front is a distance a, the semi-major axis, from the surface.
- Presented are curves of Y, versus a/t, for the deepest point of the crack (point A), for different a/c.
- Note that in Figure 6.5h, K_o is given by $K_o = \sigma\sqrt{\pi a}$ and $K_I = YK_o$.

- For the design and operation of machine elements, fracture mechanics is important to understand cracks and crack growth.
- Linear elastic fracture mechanics has been used successfully to understand failure, but the procedure requires knowing the SIF for the configuration and loading being considered.

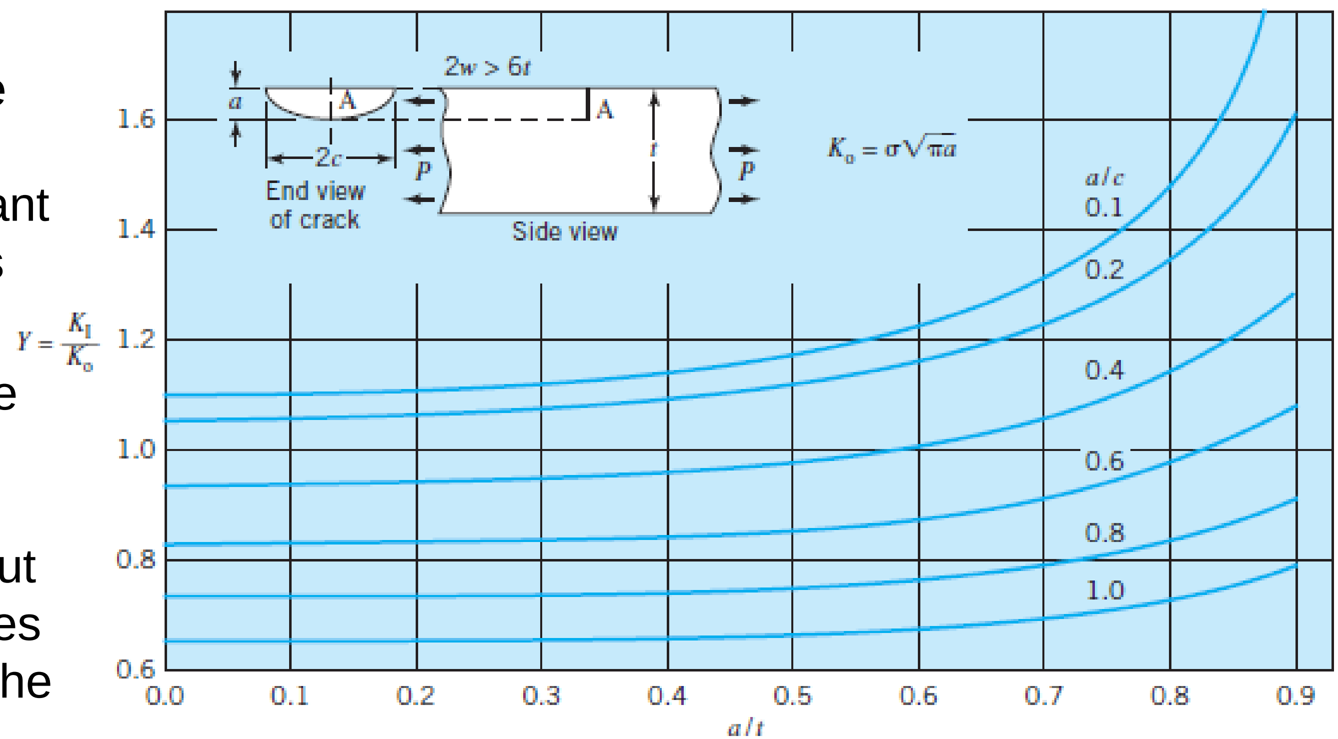


FIGURE 6.5h
Slab with a plane semi-elliptical surface crack subjected to a uniform uniaxial tensile load. K_I is for point A on the semi-elliptical edge crack [10].

6.5 The "Theory" of Static Failure Theories

- in Figure 6.6: A proposed application has a combination of static loads that produce, at a critical location, stresses of σ_1 = 80 ksi, σ_2 = -40 ksi, and σ_3 = 0. The material being considered was found to fail on a tensile test at a stress of 100 ksi.
- Will this material fail in the proposed application?
- It is impractical to test all material & all stress combination σ_1, σ_2, and σ_3, a failure theory is needed based on material's performance on the tensile test to know how strong it will be under any other conditions of static loading.
- The "theory" behind the various classical failure theories is that whatever is responsible for failure in the standard tensile test will also be responsible for failure under all other conditions of static loading.

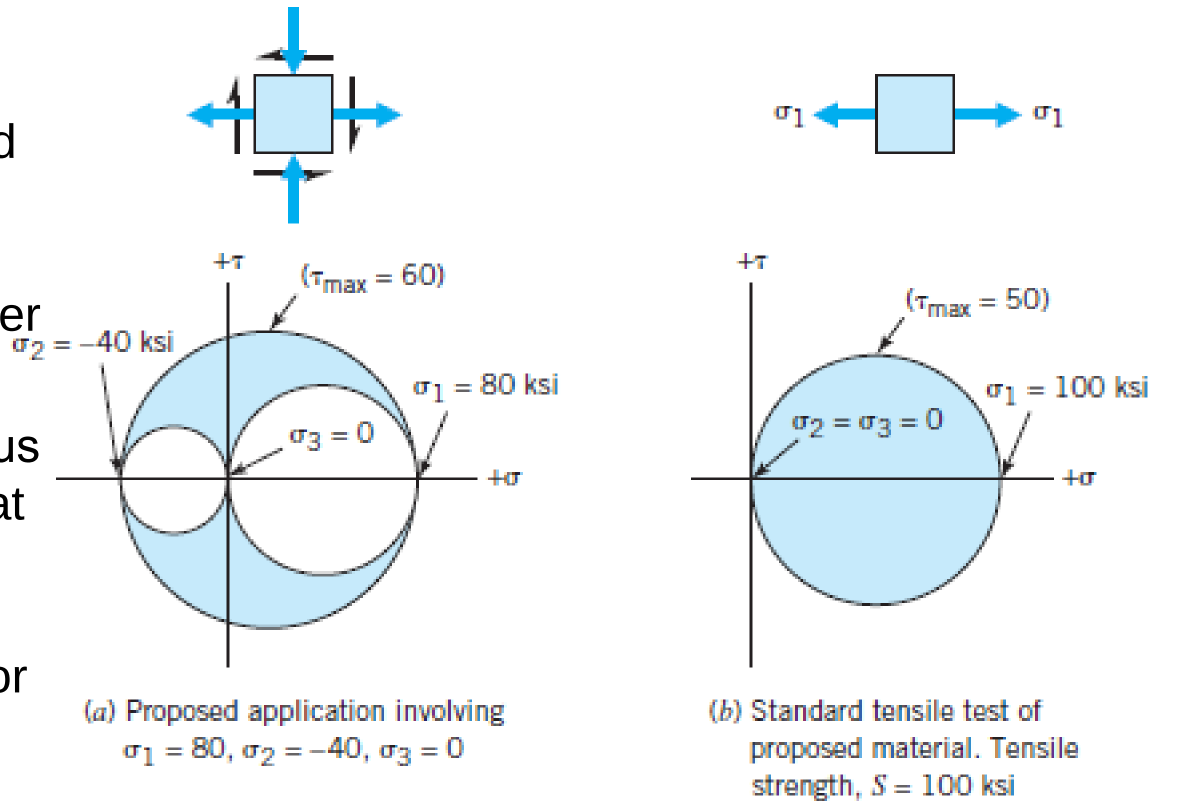


FIGURE 6.6
Typical situation requiring a failure theory.

6.5 The "Theory" of Static Failure Theories

- Suppose the theory is that failure occurred during the tensile test represented by Figure 6.6b simply because the material is unable to withstand a tensile stress above 100 ksi. The theory then predicts that under any conditions of loading, the material will fail if, and only if, σ_1 exceeds 100 ksi. Since the proposed application in Figure 6.6a has a maximum tensile stress of only 80 ksi, no failure is predicted.
- On the other hand, suppose that it is postulated that failure during the tensile test occurred because the material is limited by its inherent capacity to resist shear stress, and that, based on the tensile test, the shear stress capacity is 50 ksi. On this basis, failure would be predicted in Figure 6.6a.
- The preceding examples illustrate the maximum-normal-stress and maximum-shear-stress theories
- Other theories have been advanced that would interpret the information in Figure 6.6b as establishing limiting values of other allegedly critical quantities, such as normal strain, shear strain, total energy absorbed, and distortion energy absorbed.
- Sometimes one of these theories is modified empirically in order to obtain better agreement with experimental data. It should be emphasized that the failure theories presented in this chapter apply only to situations in which the same type of failure (i.e., ductile or brittle) occurs in the application as in the standard test.

6.6 Maximum-Normal-Stress Theory

- maximum normal stress theory credited to W. J. M. Rankine, is the simplest
- Failure will occur whenever the greatest tensile stress is > the uniaxial tensile strength, or largest compressive stress is > the uniaxial compressive strength.
- WRT Mohr circle plot in Figure, failure is predicted for any stress for which the principal Mohr circle extends beyond either of the dotted vertical boundaries.
- On the σ_1–σ_2 plot for biaxial stresses (i.e., $\sigma_3 = 0$) shown in Figure 6.7b, failure is predicted for all combinations of σ_1 and σ_2 falling outside the shaded area.
- This theory has been found to correlate reasonably well with test data for brittle fractures.
- It is not suited for predicting ductile failures. For this reason, the test points in Figure 6.7 have been marked S_{ut} and S_{uc}, ultimate strengths in tension and compression, respectively, of an assumed brittle material.

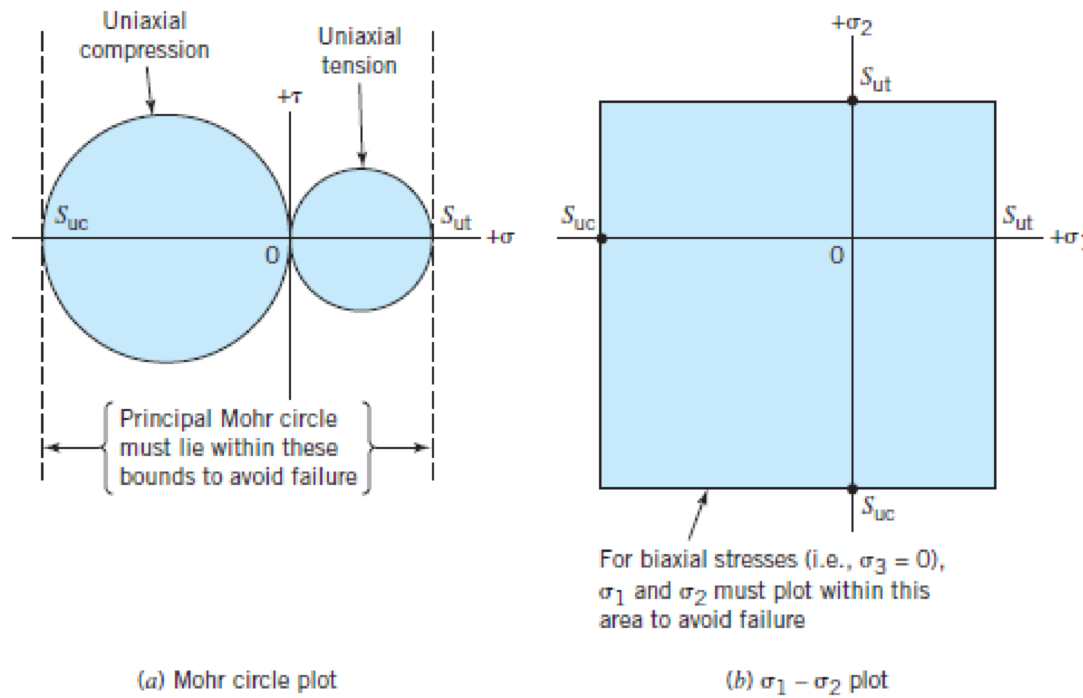


FIGURE 6.7
Two graphical representations of the maximum-normal-stress theory.

6.7 Maximum-Shear-Stress Theory

- This theory in generalized form states that a material subjected to any combination of loads will fail (by yielding or fracturing) whenever the maximum shear stress exceeds the shear strength (yield or ultimate) of the material.
- The shear strength, is usually assumed to be determined from the standard uniaxial tension test.
- Note carefully in Figure that in the I and III quadrants the 0 principal stress is involved in the principal Mohr circle, whereas it is not in the II and IV quadrants.
- The single test point is marked S_{yt} , yield strength in tension, of a ductile material
- If the material behaves in accordance with the maximum-shear-stress theory, all test data would agree on the level of shear stress associated with failure.
- This theory correlates reasonably well with the yielding of ductile materials.

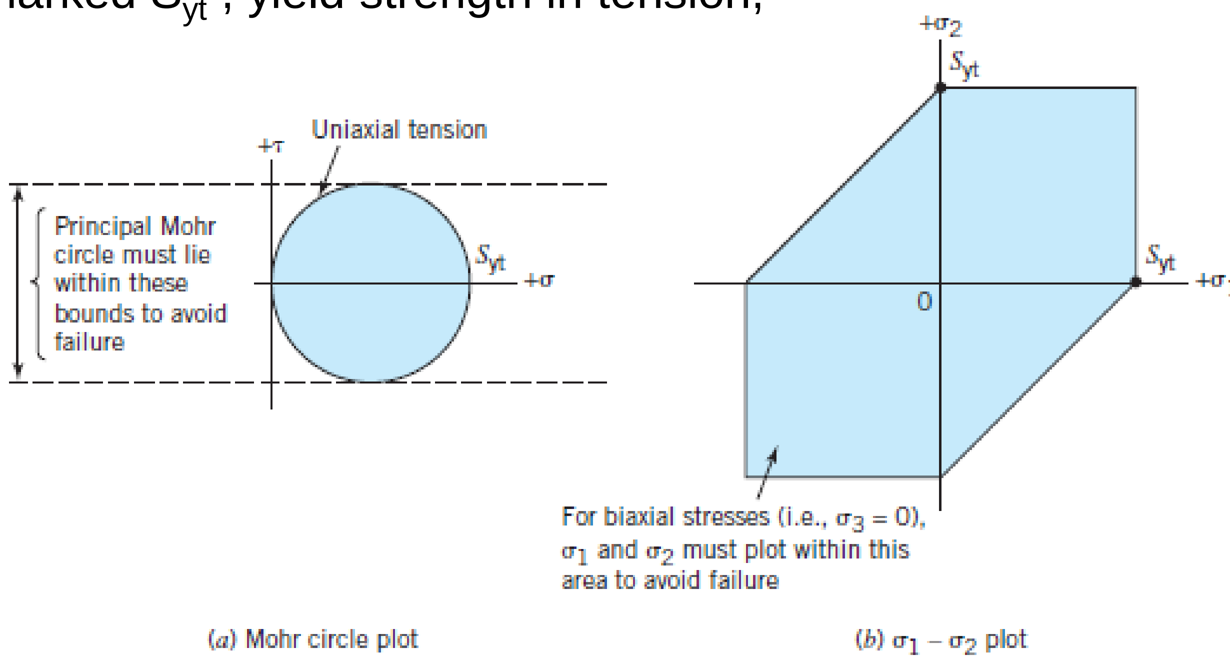


FIGURE 6.8
Two graphical representations of the maximum-shear-stress theory.

6.8 Maximum-Distortion-Energy Theory (Maximum-Octahedral-Shear-Stress Theory)

- maximum-distortion-energy theory is that any elastically stressed material undergoes a (slight) change in shape, volume, or both. The energy required to produce this change is stored in the material as elastic energy.
- It was recognized that engineering materials could withstand enormous hydrostatic pressures (i.e., $\sigma_1 = \sigma_2 = \sigma_3$ = large compression) without damage.
- a given material has a definite limited capacity to absorb energy of distortion (i.e., energy tending to change shape but not size), and that attempts to subject the material to greater amounts of distortion energy result in yielding.
- when using this theory to work with an equivalent stress, σ_e , defined as the value of uniaxial tensile stress that would produce the same level of distortion energy (the same likelihood of failure) as the actual stresses involved. In terms of the existing principal stresses, the equation for equivalent stress is

$$\sigma_e = \frac{\sqrt{2}}{2}[(\sigma_2 - \sigma_1)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_3 - \sigma_2)^2]^{1/2} \qquad (6.5)$$

- For the case of biaxial stress, where σ_1 & σ_2 are the nonzero principal stresses, this reduces to

$$\sigma_e = (\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{1/2} \qquad (6.6)$$

6.8 Maximum-Distortion-Energy Theory (Maximum-Octahedral-Shear-Stress Theory)

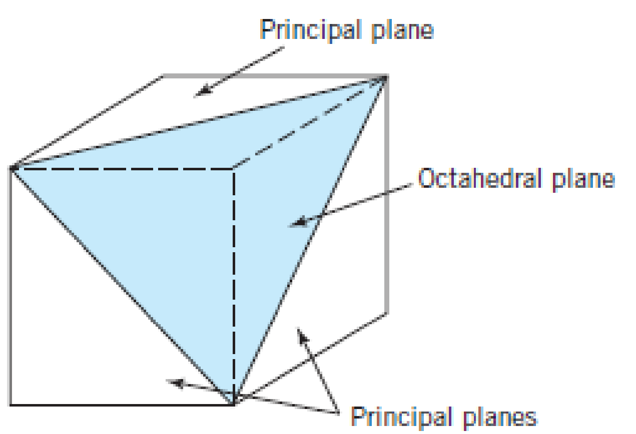
- If the direct stresses σ_x & σ_y , and τ_{xy} are readily obtainable, a convenient form of the equivalent stress equation is

$$\sigma_e = (\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2)^{1/2} \qquad (6.7)$$

- If only σ_x and τ_{xy} are present, the equation reduces to

$$\sigma_e = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2} \qquad (6.8)$$

- Once the equivalent stress is obtained, this is compared with the yield strength from the standard tensile test. If σ_e is > S_{yt} , yielding is predicted.
- These same equations can readily be derived on the basis of shear stress on an octahedral plane. Figure shows the relationship of an octahedral plane to the faces of a principal element.
- There are 8 octahedral planes, having same intensity of normal and shear stress.
- Then, σ_e can be defined as that value of uniaxi tensile stress that produces the same level of shear stress on the octahedral planes (hence, the same likelihood of failure) as do the actual stresses involved.



6.8 Maximum-Distortion-Energy Theory (Maximum-Octahedral-Shear-Stress Theory)

- Figure 6.10 shows that a σ_1 - σ_2 plot for maximum-distortion-energy theory is an ellipse. This is shown in comparison with plots for the maximum-shear-stress and maximum-normal-stress theories for ductile mat'l having S_{yt} = S_{yc} = 100 ksi.

- The distortion energy and the shear stress theories agree fairly well, with the distortion energy theory giving the material credit for 0 to 15 % more strength, depending on the ratio of σ_1 to σ_2.

- Also shown in figure is the shear diagonal, corresponding to pure shear ($\sigma_1 = - \sigma_2$; $\sigma_3 = 0$). It is interesting to note the wide variation in shear strengths predicted by the various theories.

- Actual tests of ductile materials usually agree quite well with the distortion energy theory, which predicts S_{sy} , is $0.58S_y$.

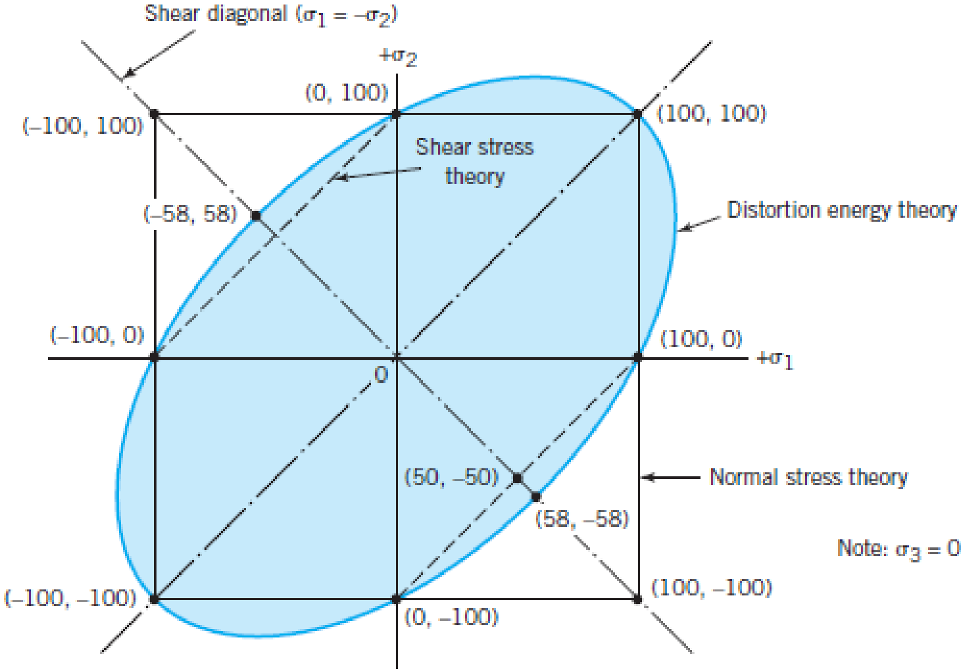
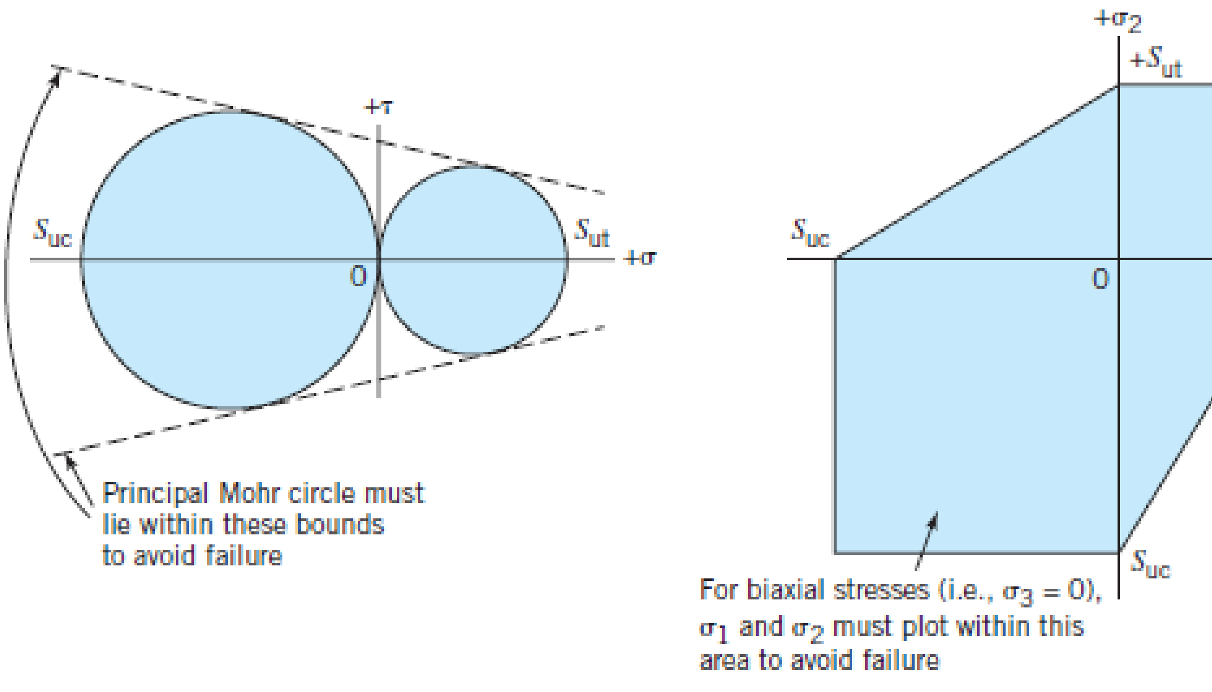


FIGURE 6.10

A σ_1–σ_2 plot of distortion energy theory and other theories for a ductile material of $S_{yt} = S_{yc} = 100$ ksi. (The distortion energy theory predicts failure for all points outside the ellipse.) Note that the point (58, –58) is actually 100 times ($\sqrt{3}/3, -\sqrt{3}/3$). Distortion energy theory predicts that shear yield strength is $\sqrt{3}/3$, or 0.577 times tensile yield strength, whereas shear stress theory predicts 0.500 times tensile yield, and normal-stress theory predicts 1.0 times tensile yield strength.

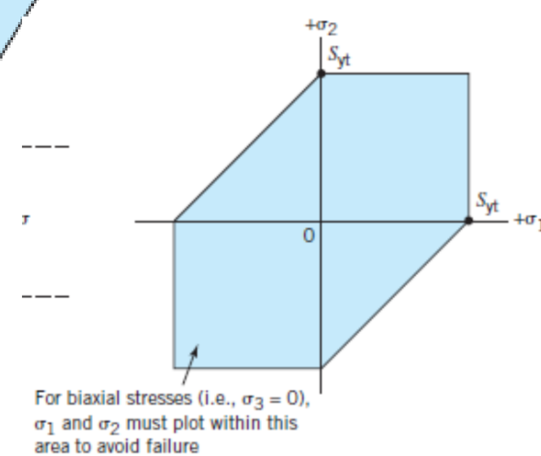
6.9 Mohr Theory and Modified Mohr Theory

- Various empirical modifications to the basic failure theories have been proposed, one of which is the Coulomb–Mohr theory, represented in Figure
- This theory was suggested for brittle materials, for which the compressive strength far exceeds the tensile strength. (the theory is generally thought of as an empirical modification of the maximum-shear stress theory, using experimental values for both tensile and compressive strengths



(a) Mohr circle plot

(b) $\sigma_1 - \sigma_2$ plot



(b) $\sigma_1 - \sigma_2$ plot

FIGURE 6.11
Two graphical representations of the Mohr (or Coulomb–Mohr) theory.

aximum-shear-stress theory.

6.9 Mohr Theory and Modified Mohr Theory

- A modification of the Mohr theory, illustrated in Figure 6.12, is recommended for predicting the fracture of brittle materials. It correlates better with most experimental data than do the Mohr or maximum-normal-stress theories, which are also used.
- failure theory is a substitute for good test data pertaining to the actual material and the combination of stresses involved.
- Any additional good test data can be used to improve a theoretical failure theory curve for a given material.

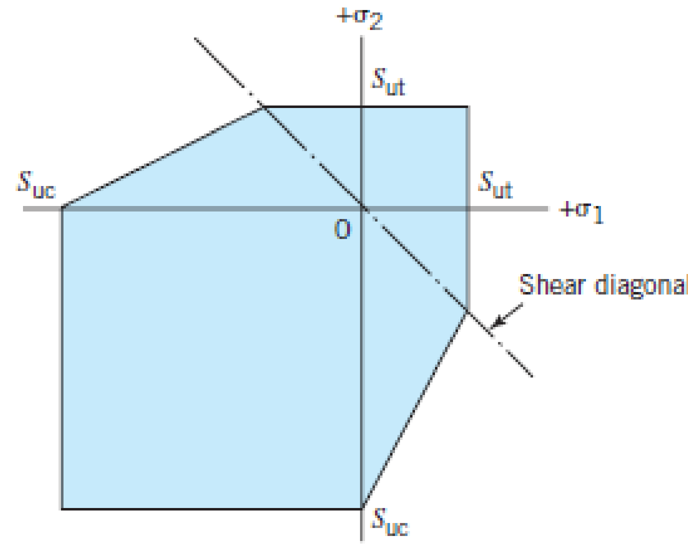
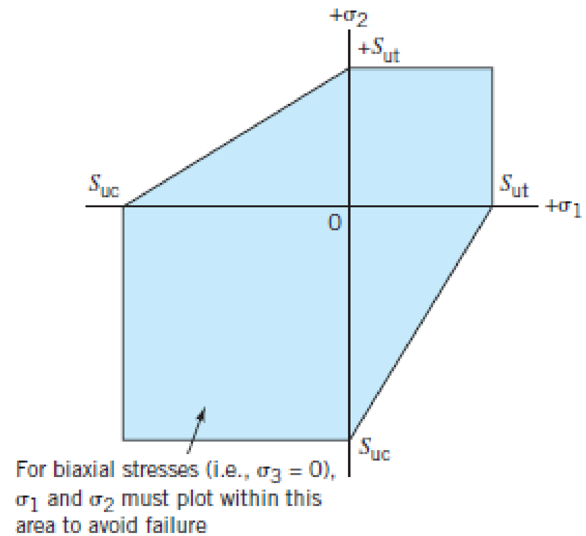


FIGURE 6.12
Graphical representation of the modified Mohr theory for biaxial stresses ($\sigma_3 = 0$).



(b) $\sigma_1 - \sigma_2$ plot

- Experimental data shows the unsuitability of Max NST
- And the CM theory being too conservative.
- Leading to modified CM theory – *which is the preferred theory for failure of uneven brittle materials in static loading*

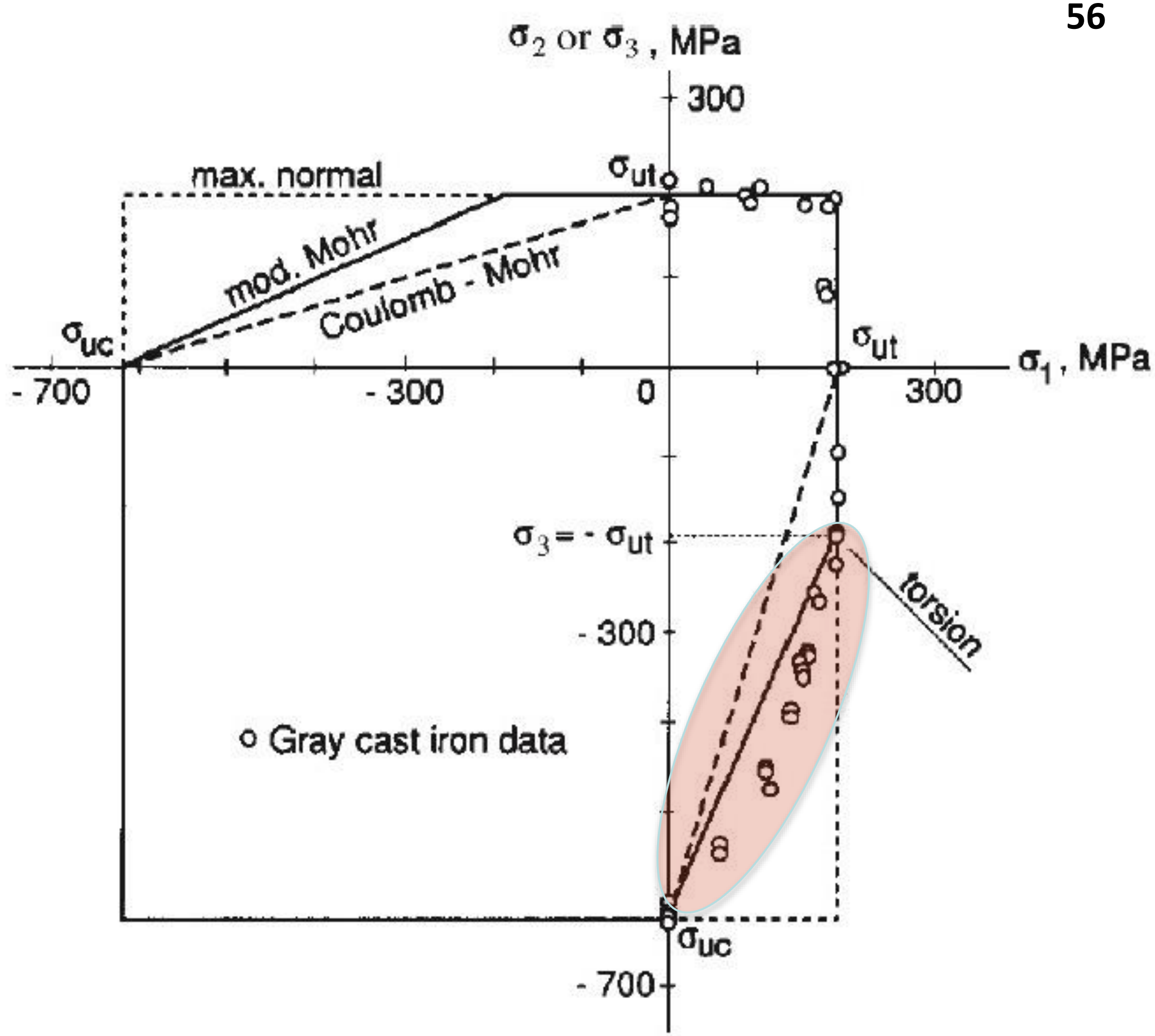


FIGURE 5-12

Biaxial Fracture Data of Gray Cast Iron Compared to Various Failure Criteria *(From Fig 7.13, p. 255, in Mechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993. Data from R. C. Grassi and I. Cornet, "Fracture of Gray Cast Iron Tubes under Biaxial Stresses," J. App. Mech, v. 16, p.178, 1949)*

- suppose that the material involved in Figure 6.10 was known to have an experimentally determined torsional yield strength of 60 ksi
- (but before accepting this value, one should be well aware of the inherent difficulty of making an accurate experimental determination of S_{sy}).
- We might then conclude that the material did indeed appear to behave in general accordance with the distortion energy theory, but not exactly.
- By empirically modifying the ellipse just enough that it passes through the experimental point, we would have a presumably better failure theory curve for making predictions in the second and fourth quadrants.

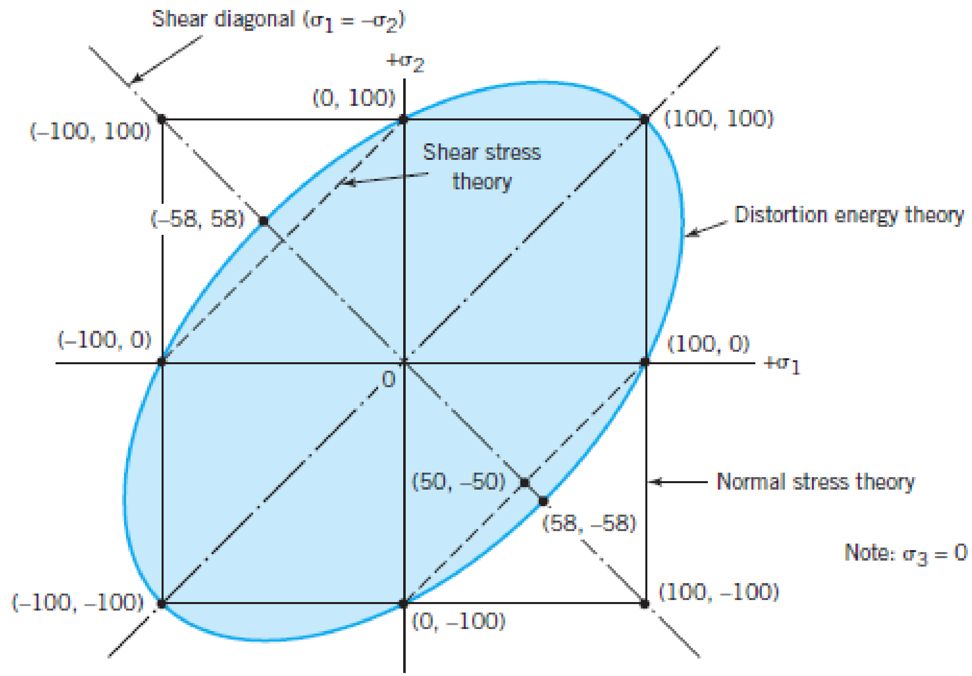


FIGURE 6.10

A σ_1–σ_2 plot of distortion energy theory and other theories for a ductile material of $S_{yt} = S_{yc} = 100$ ksi. (The distortion energy theory predicts failure for all points outside the ellipse.) Note that the point (58, −58) is actually 100 times $(\sqrt{3}/3, -\sqrt{3}/3)$. Distortion energy theory predicts that shear yield strength is $\sqrt{3}/3$, or 0.577 times tensile yield strength, whereas shear stress theory predicts 0.500 times tensile yield, and normal-stress theory predicts 1.0 times tensile yield strength.

6.9 Mohr Theory and Modified Mohr Theory

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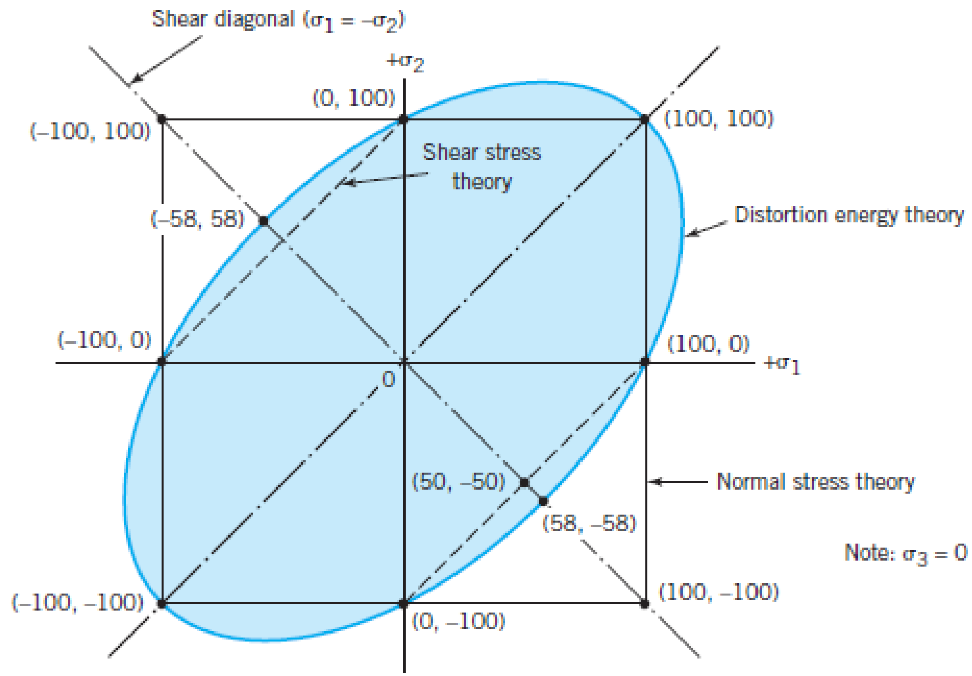

FIGURE 6.10

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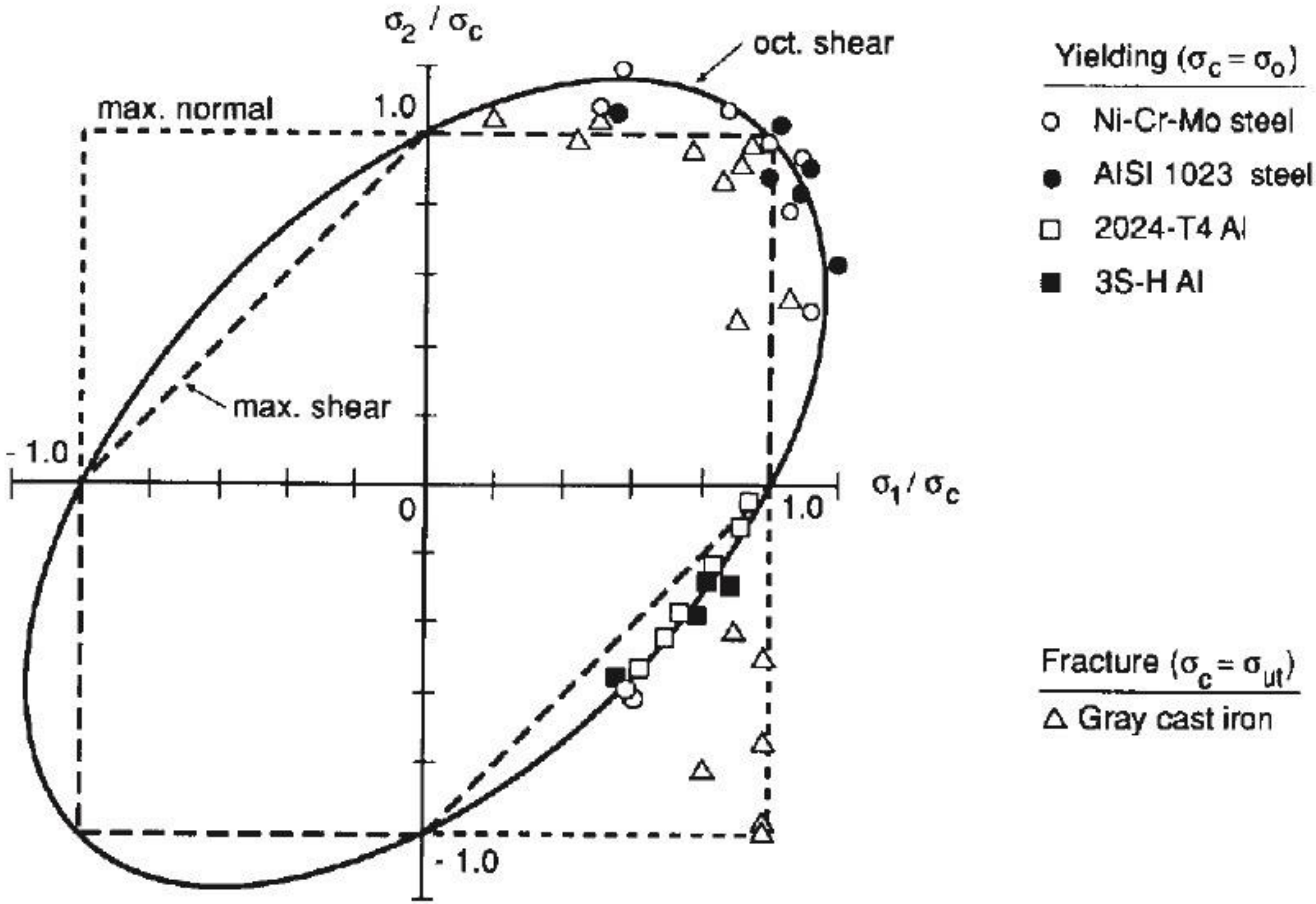


FIGURE 5-8

Experimental Data from Tensile Tests Superposed on Three Failure Theories *(Reproduced from Fig. 7.11, p. 252, in Mechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993)*

6.10 Selection and Use of Failure Theories

- In situations where an overloaded part in service fails in the same manner as the standard tensile test bar made of the same material, it is recommended that (1) the maximum-distortion-energy theory be used to predict ductile yielding and (2) the modified Mohr theory be used to predict brittle fracture.

SAMPLE PROBLEM 6.3 Estimate the Safety Factor of a Steel Part

Strain gage tests have established that the critical location on the surface of a steel part is subjected to principal stresses of $\sigma_1 = 35$ ksi and $\sigma_2 = -25$ ksi. (Because the surface is exposed and unloaded, $\sigma_3 = 0$.) The steel has a yield strength of 100 ksi. Estimate the safety factor with respect to initial yielding, using the preferred theory. As a matter of interest, compare this with results given by other failure theories.

SOLUTION

Known: The constant stress at a point is $\sigma_1 = 35$ ksi, $\sigma_2 = -25$ ksi, $\sigma_3 \cong 0$; and the yield strength is given (see Figure 6.13).

Find: Determine the safety factor based on (a) distortion energy theory, (b) shear stress theory, and (c) normal-stress theory.

Schematic and Given Data:

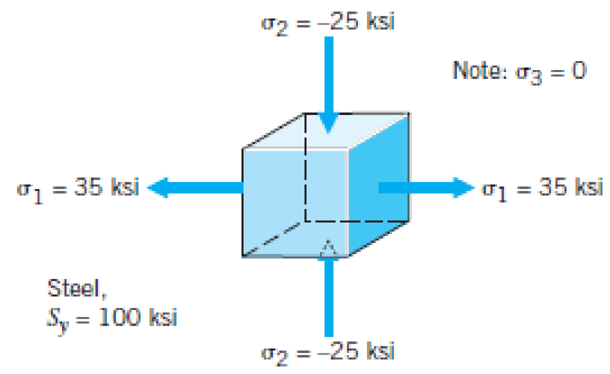


FIGURE 6.13
Stresses on the surface of a part for Sample Problem 6.3.

6.10 Selection and Use of Failure Theories

Analysis: Figure 6.14 depicts a graphical solution. Starting from the "nominal load point," the stresses can be proportionally increased until σ_1 reaches values of 58 ksi, 66 ksi, and 100 ksi, according to the shear stress, distortion energy, and normal-stress theories, respectively. Corresponding safety factor estimates are $SF = 58/35 = 1.7$, $66/35 = 1.9$, and $100/35 = 2.9$. (Final answers were given with only two significant figures to emphasize that neither the inherent validity of the theories nor the accuracy of graphical construction justifies an implication of highly precise answers.) It is concluded that

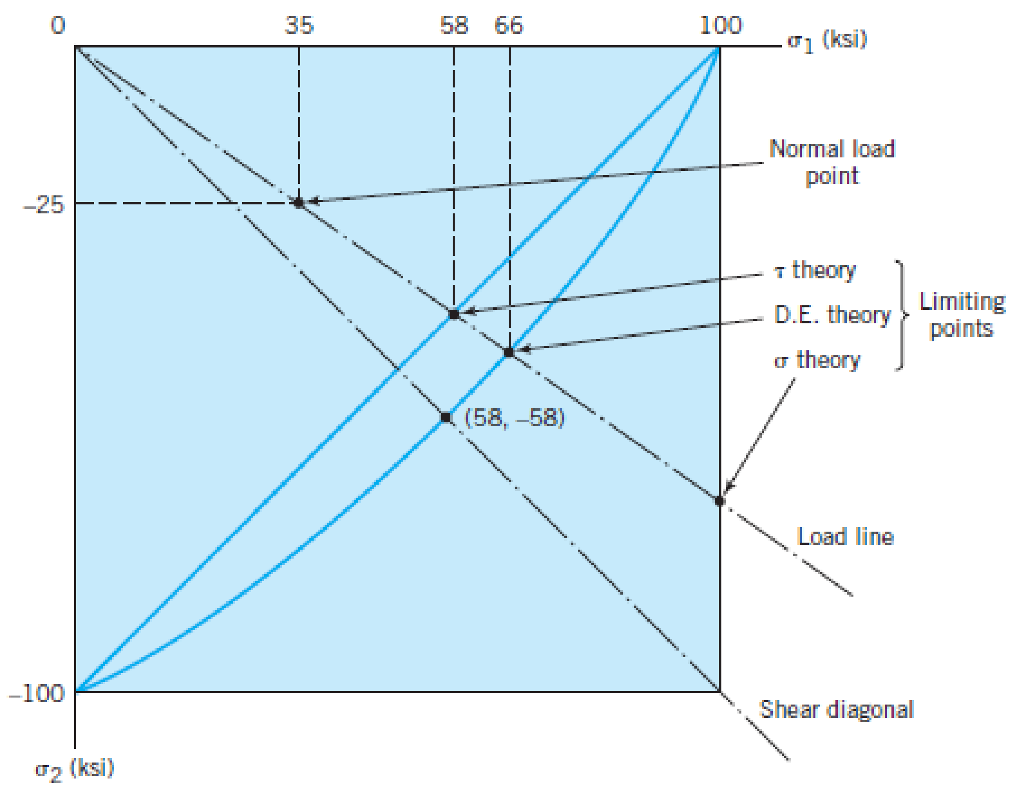


Figure 6.14
Graphical solution to Sample Problem 6.3.

Schematic and Given Data:

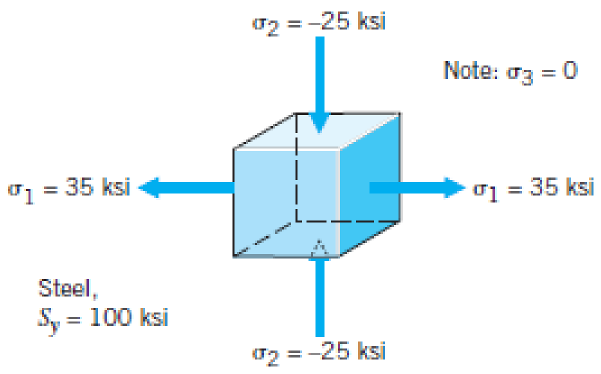


Figure 6.13
Stresses on the surface of a part for Sample Problem 6.3.

a. The "best" prediction of safety factor is 1.9, based on the distortion energy theory.
b. The shear stress theory is in reasonably good agreement (it is often used by engineers to obtain quick estimates).
c. The normal-stress theory has no validity in this case. (To use it mistakenly would give an answer indicating a degree of safety that does not exist.)

The graphical solution was illustrated because it is quick, it is at least as accurate as the theories themselves, and it gives us a good intuitive feel for what is going on. Analytical solutions are, of course, equally valid, and are as follows.

a. For the distortion energy theory, Eq. 6.6 gives

$$\begin{aligned}\sigma_e &= (\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{1/2} \\ &= [(35)^2 + (-25)^2 - (35)(-25)]^{1/2} = 52.2\end{aligned}$$

Thus the given stresses are *equivalent* to a simple tensile stress of 52.2 ksi. The tensile test established that the material can withstand a tensile stress of 100 ksi. The safety factor is therefore 100/52.2, or 1.9.

b. When we use the shear stress theory, the principal stresses define a principal Mohr circle having a diameter of 60 and a radius of 30. Thus the maximum shear stress in the part is 30 ksi. The standard tensile test gave a principal Mohr circle having a *radius* of 50. Thus the material is capable of withstanding a shear stress of 50 ksi. The safety factor is therefore 50/30, or 1.7.

b. When we use the shear stress theory, the principal stresses define a principal Mohr circle having a diameter of 60 and a radius of 30. Thus the maximum shear stress in the part is 30 ksi. The standard tensile test gave a principal Mohr circle having a *radius* of 50. Thus the material is capable of withstanding a shear stress of 50 ksi. The safety factor is therefore 50/30, or 1.7.

c. When we use the normal-stress theory, the application involves a maximum normal stress of 35 ksi, whereas the standard tensile test established that the material is capable of withstanding a normal stress of 100 ksi. The safety factor (by wishful thinking!) is therefore 100/35, or 2.9.

Comment: In many situations the exposed surface of parts are subjected to atmospheric pressure ($p = 14.7$ psi). Relative to other principal stresses (e.g., in this problem $\sigma_1 = 35$ ksi and $\sigma_2 = -25$ ksi), $\sigma_3 = p = 14.7$ psi is zero stress.

The preceding discussion of failure theories applies to *isotropic* materials. For anisotropic materials subjected to various combinations of stress, the reader is referred to special references, as [6].

6.11 Safety Factors—Concept and Definition

- A safety factor was a number by which the S_{UT} of a material was divided in order to obtain a value of "working design stress." for use in simplified calculations that made no allowance for stress concentration, impact, fatigue and so on. So safety factor recommendations as high as 20 to 30 were common.
- Modern engineering gives a rational accounting for all factors possible, leaving few items of uncertainty to be covered by a safety factor, in the range of 1.25 to 4.
- Modern engineering practice also bases the safety factor on the significant strength of the material—not the static tensile strength.
- If failure involves static yielding, the safety factor relates to the static stress caused by the load, called the significant stress, to the static yield strength of the material, called the significant strength, as shown in Sample Problem 6.3.
- If the significant stresses involve fatigue, then the safety factor is based on the fatigue strength;
- if brittle fracture is expected, then the factor is based on tensile strength, and so forth.
- Thus the SF can be defined as

$$SF = \frac{\text{significant strength of the material}}{\text{corresponding significant stress, from normally expected loads}} \tag{6.9}$$

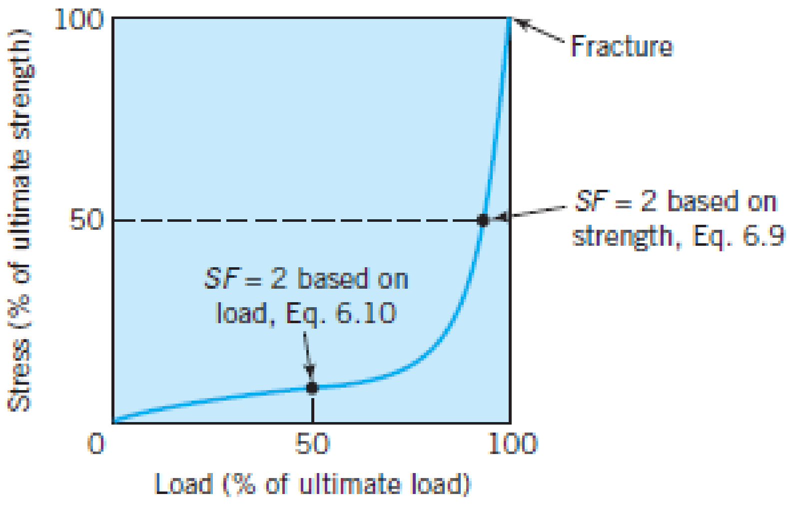
Safety factor can also be defined in terms of loads:

$$SF = \frac{\text{design overload}}{\text{normally expected load}} \tag{6.10}$$

where the *design overload* is defined as being just sufficient to cause failure.

6.11 Safety Factors—Concept and Definition

- In most situations, the two definitions of safety factor are equivalent. For example, if a material has a significant strength of 200 MPa and the significant stress is 100 MPa, the safety factor is 2.
- Looking at it the other way, the design overload needed to bring the stress up to the limiting value of 200 MPa is twice the expected normal load, thus giving a safety factor of 2.
- Although the distinction between Eqs. 6.9 and 6.10 may seem trivial, it is recommended that the design overload concept and Eq. 6.10 be used. These are always valid, whereas there are instances in which Eq. 6.9 cannot be properly applied.
- For example, consider the design of a slender column for a safety factor of 2.
- Figure 6.15 shows how load and stress increase to buckling failure, and where operation with a safety factor of 2 would be calculated both ways.
- The discrepancy is due to nonlinearity of the load–stress curve. It is clear which interpretation is most conducive to the engineer's peace of mind.



6.12 Safety Factors—Selection of a Numerical Value

- After going as far as is practical in determining the significant strength of the actual fabricated part and the details of the loading to which it will be subjected, there always remains some margin of uncertainty that must be covered by a safety factor.
- The part must be designed to withstand a “design overload” somewhat larger than the normally expected load. In the last analysis, selection of the safety factor comes down to engineering judgment based on experience. Sometimes these selections are formalized into design codes covering specific situations—for example, the ASME Pressure Vessel Codes, the various building codes etc.

6.12.1 Factors in the Selection of a Safety Factor

- Degree of uncertainty about loading. In some situations loads can be determined with virtual certainty. The centrifugal forces in the rotor of an AC motor cannot exceed those calculated for synchronous speed. But what loads should be used for the design of automotive suspension components, whose loads can vary tremendously depending on the severity of use and abuse? The greater the uncertainty, the more conservative the safety factor.
- Degree of uncertainty about material strength. Ideally, the engineer would have available data pertaining to the strength of the material as fabricated into the actual parts, and tested at temperatures and environments similar to those actually encountered. But this is seldom the case. More often, the available material strength data pertain to samples smaller than the actual part and which have been tested at room temperature in ordinary air. Furthermore, the material properties may sometimes change significantly over the service life of the part. The greater the uncertainty about all these factors, the larger the safety factor that must be used.
- Uncertainties in relating applied loads to material strength via stress analysis. There are number of possible uncertainties, such as (a) validity of the assumptions involved in the standard equations for calculating nominal stresses, (b) accuracy in determining the effective stress concentration factors, (c) accuracy in estimating residual stresses, and (d) suitability of any failure theories and other relationships used to estimate “significant strength” from available laboratory strength test data.

6.12.1 Factors in the Selection of a Safety Factor

- Consequences of failure—human safety and economics. If the consequences of failure are catastrophic, relatively large safety factors must, of course, be used. If the failure of some inexpensive part could cause extensive shutdown of a major assembly line, simple economics dictates increasing the cost of this part several fold in order to virtually eliminate the possibility of its failure.
- An important item is the nature of a failure. If failure is caused by ductile yielding, the consequences are likely to be less severe than if caused by brittle fracture. Accordingly, safety factors recommended in handbooks are invariably larger for brittle materials.
- Cost of providing a large safety factor. This cost involves a monetary consideration and may also involve important consumption of resources. In some cases, a safety factor larger than needed may have serious consequences. A dramatic example is a hypothetical aircraft with excessive safety factors making it too heavy to fly!
- WRT the design of an automobile, it would be possible to increase safety factors on structural components to the point that a "maniac" driver could hardly cause a failure even when trying. But to do so would penalize "sane" drivers by requiring them to pay for stronger components than they can use.
- More likely, of course, it would motivate them to buy competitor's cars!

6.12.2 Recommended Values for a Safety Factor

- as a guide, some suggestions for "ball park" values of safety factor are suggested. These safety factors are based on yield strength.

1. $SF = 1.25$ to 1.5 for exceptionally reliable materials used under controllable conditions and subjected to loads and stresses that can be determined with certainty—used almost invariably where low weight is a particularly important consideration.
2. $SF = 1.5$ to 2 for well-known materials, under reasonably constant environmental conditions, subjected to loads and stresses that can be determined readily.
3. $SF = 2$ to 2.5 for average materials operated in ordinary environments and subjected to loads and stresses that can be determined.
4. $SF = 2.5$ to 3 for less tried materials or for brittle materials under average conditions of environment, load, and stress.
5. $SF = 3$ to 4 for untried materials used under average conditions of environment, load, and stress.
6. $SF = 3$ to 4 should also be used with better known materials that are to be used in uncertain environments or subjected to uncertain stresses.
7. Repeated loads: The factors established in items 1 to 6 are acceptable but must be applied to the *endurance limit* rather than to the yield strength of the material.
8. Impact forces: The factors given in items 3 to 6 are acceptable, but an *impact factor* should be included.
9. Brittle materials: Where the ultimate strength is used as the theoretical maximum, the factors presented in items 1 to 6 should be approximately doubled.
10. Where higher factors might appear desirable, a more thorough analysis of the problem should be undertaken before deciding on their use.