MECH 344/M

Machine Element Design

Time: M _ _ _ _14:45 - 17:30

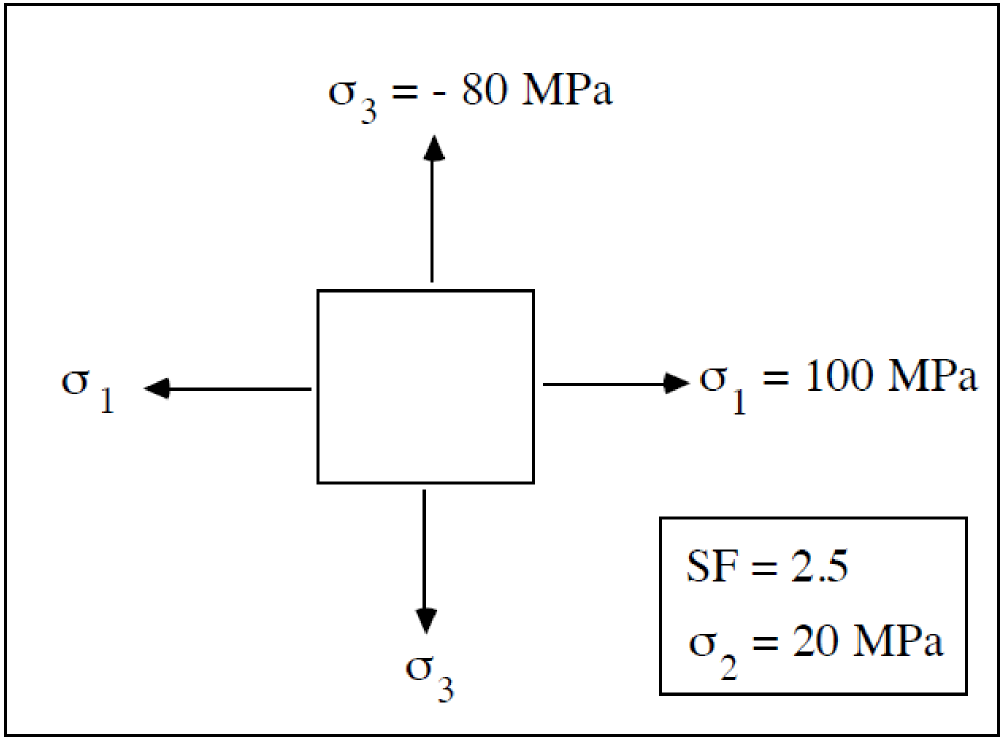
Lecture 3 continued

Static Failure Theories – Safety Factor

The surface of a steel machine member is subjected to stresses of σ_1 = 100 MPa, σ_2 = 20 MPa, and σ_3 = -80 MPa. What tensile yield strength is required to provide a safety factor of 2.5 with respect to initial yielding:

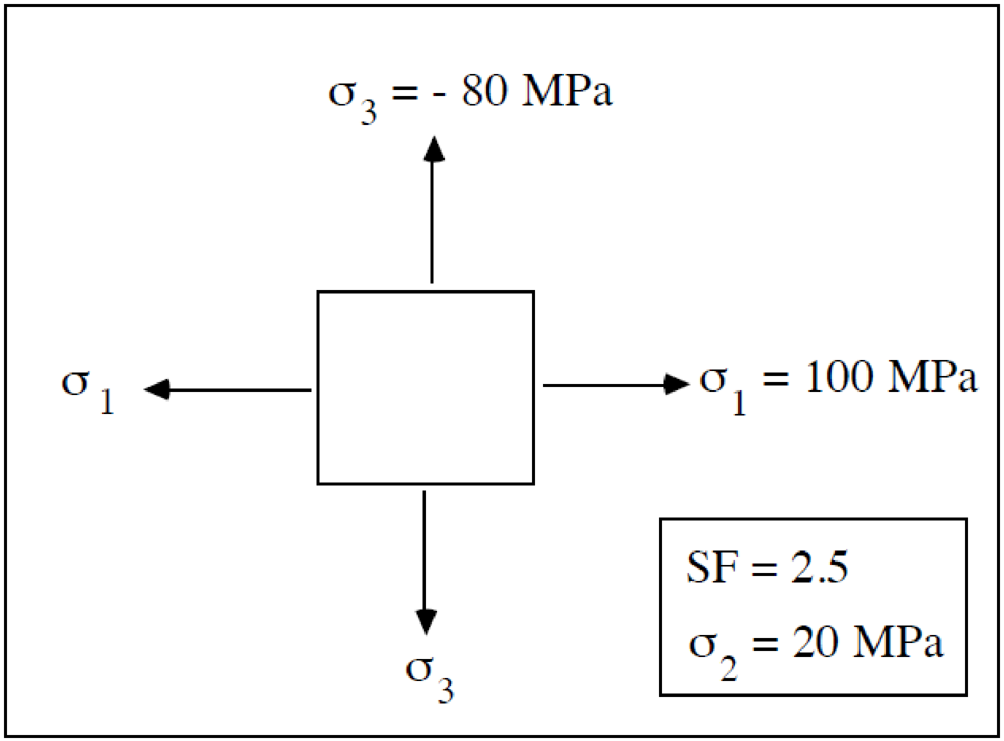
(a) According to the maximum-shear-stress theory?

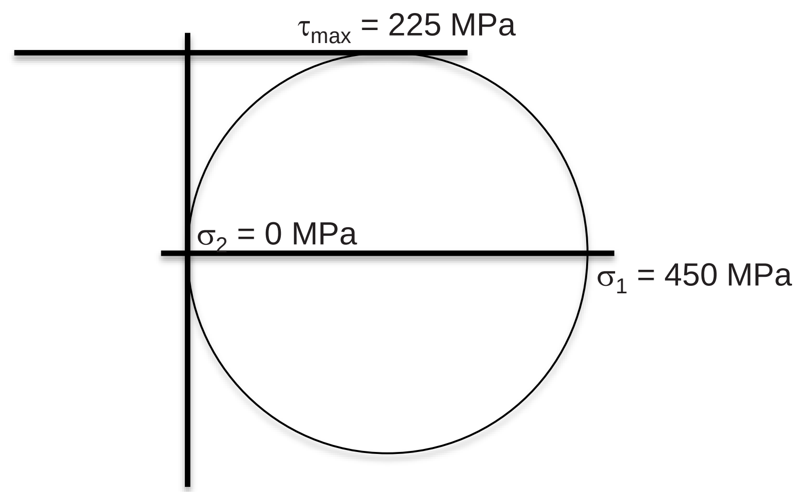
(b) According to the maximum-distortion-energy theory?



Static Failure Theories – Safety Factor

(a) According to the maximum-shear-stress theory?

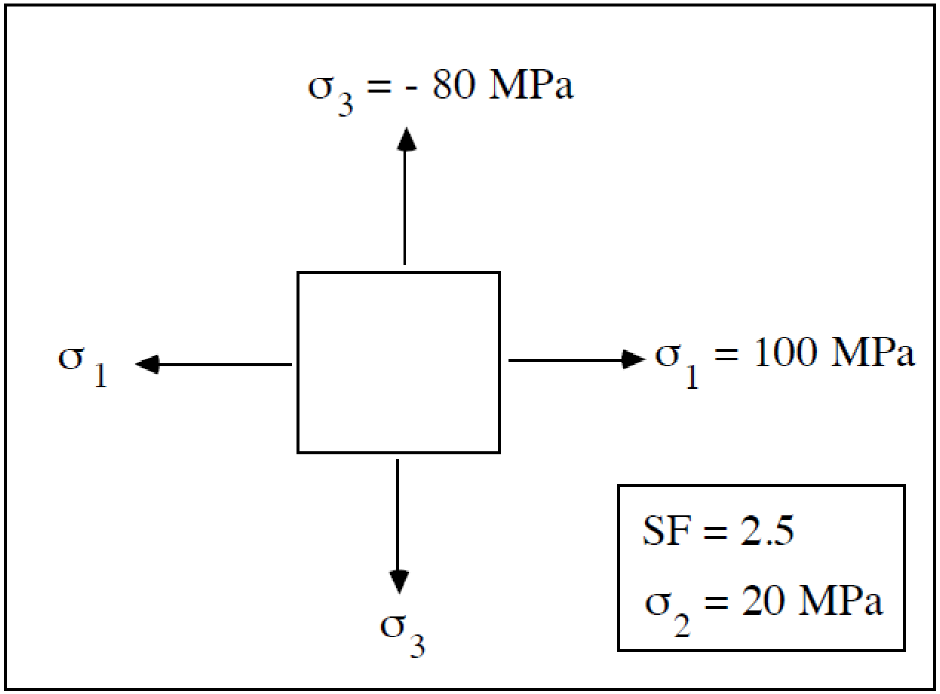


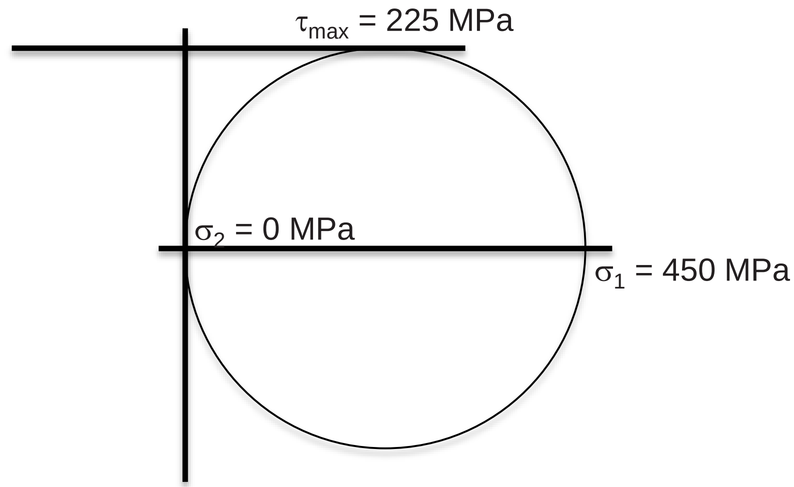


σ_1 = 100 MPa, σ_2 = 20 MPa, and σ_3 = -80 MPa, we have
τ_{max} = (100 + 80)/2 = 90 MPa
Sy = (2. 5)(2)(τ_{max}) = 450 MPa

Static Failure Theories – Safety Factor

(b) According to the maximum-distortion-stress theory?





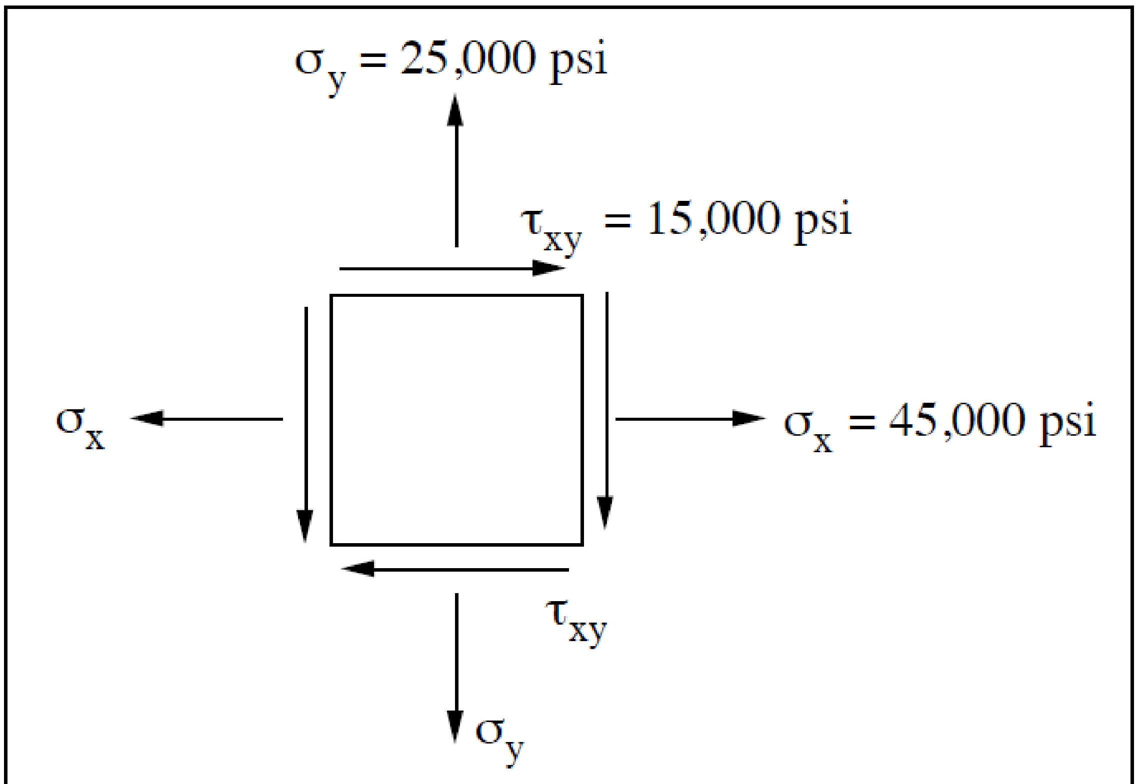
$$\sigma_e = \frac{\sqrt{2}}{2}\left[(\sigma_2-\sigma_1)^2+(\sigma_3-\sigma_1)^2+(\sigma_3-\sigma_2)^2\right]^{1/2} = \frac{\sqrt{2}}{2}\left[(-80)^2+(-180)^2+(-100)^2\right]^{1/2}$$

σ_e = 156 Mpa

The yield strength is $S_y = 2.5(\sigma_e)$ = 390 MPa

Static Failure Theories – Safety Factor

A lawn mower component experiences critical static stresses of σ_x = 45,000 psi, σ_y = 25,000 psi, and τ_{xy} = 15,000 psi. The component is made of 4130 normalized steel with S_{ut} = 97,000 psi and S_y = 63,300 psi. Determine the factor of safety based on predicting failure by the maximum-normal stress theory, the maximum-shear-stress theory, and the distortion energy theory.



Static Failure Theories – Safety Factor

factor of safety based on predicting failure by the maximum-normal stress theory. You can also do this Mohr circle

Maximum-normal-stress theory
From, Eq. (4.16)

$$\sigma_1 = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2\right]^{\frac{1}{2}}$$

$$= \left(\frac{45,000 + 25,000}{2}\right) + \left[\left(\frac{20,000}{2}\right)^2 + (15,000)^2\right]^{\frac{1}{2}}$$

$$= 53,028 \text{ psi}$$

$$SF = S_y / \sigma_1 = 63,300/53,028 = 1.19$$

Static Failure Theories – Safety Factor

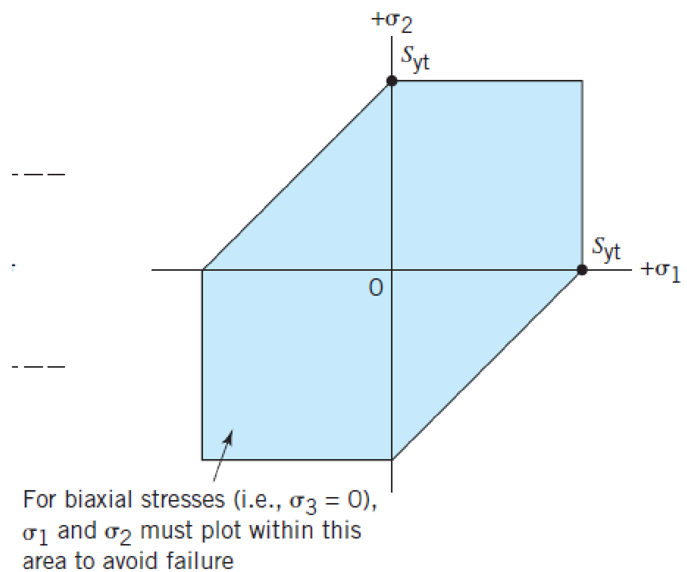
factor of safety based on predicting failure by the maximum-shear-stress theory

Maximum-shear-stress theory
From, Eq. (4.18)

$$\tau_{max} = \left[\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2\right]^{1/2}$$

$$= \left[(15{,}000)^2 + \left(\frac{20{,}000}{2}\right)^2\right]^{1/2} = 18{,}028 \text{ psi}$$

$$SF = \frac{S_y}{2\tau_{max}} = \frac{63{,}300}{2(18{,}028)} = 1.8$$



(b) $\sigma_1 - \sigma_2$ plot

When σ_3 is considered as 0, both theories will have same safety factor.

Static Failure Theories – Safety Factor

factor of safety based on predicting failure by the distortion energy theory

Maximum-distortion-energy theory
From, Eq. (6.7)

$$\sigma_e = \left[\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2\right]^{1/2}$$

$$\sigma_e = \left[(45,000)^2 + (25,000)^2 - (45,000)(25,000) + 3(15,000)^2\right]^{1/2}$$

$$= 46,904 \text{ psi}$$

$$SF = S_y/\sigma_e = 63,300/46,904 = 1.35$$

MECH 344/M
Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 4

8

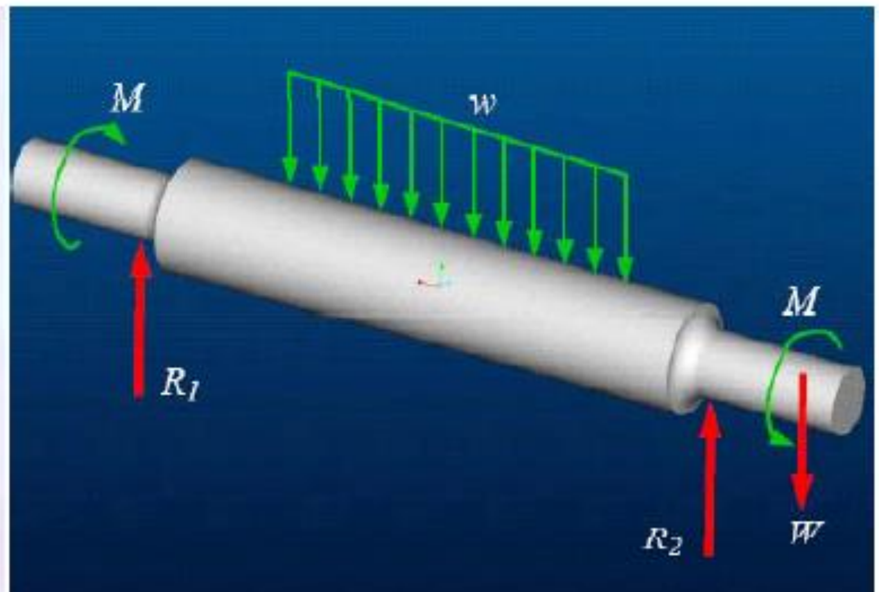
Fatigue

Contents of today's lecture

- Introduction and Basics
- Fatigue Strength
  - Rotating Bending
  - Reversed Bending and Reversed Axial Loading
  - Reversed Torsional Loading
  - Reversed Biaxial Loading
- Surface Science Influence
- Estimated Fatigue Strength in Reversed Loading

What is Fatigue

- Rankine (1843) observed the phenomena (Journal of Railway Axles)
- Wholer (1870) published the results of 12 years of tests on reversed loading on axles – produced as what is known as the S-N diagram
- Rotating parts are subject to fatigue (and all time dependent loaded parts)



What is Fatigue

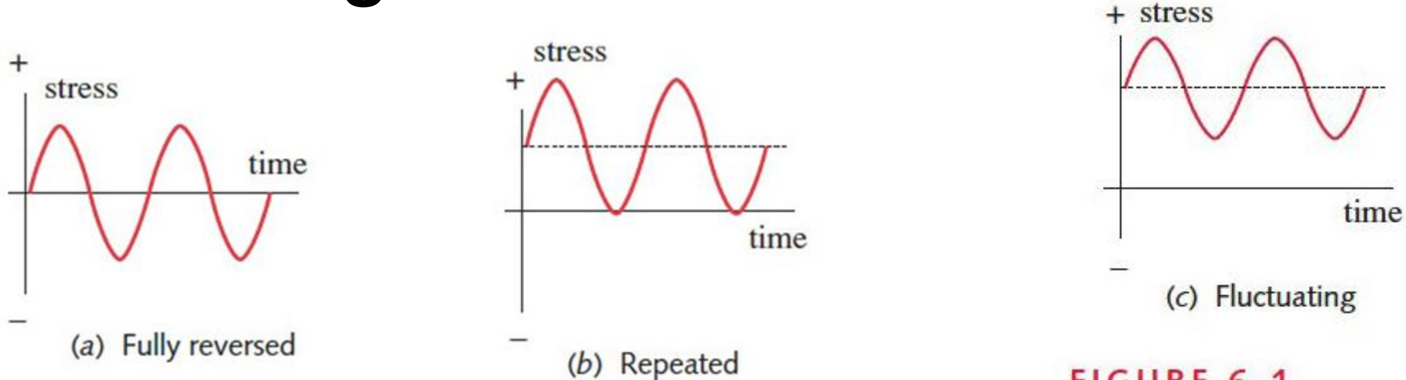


FIGURE 6-1

Time-Varying Stresses

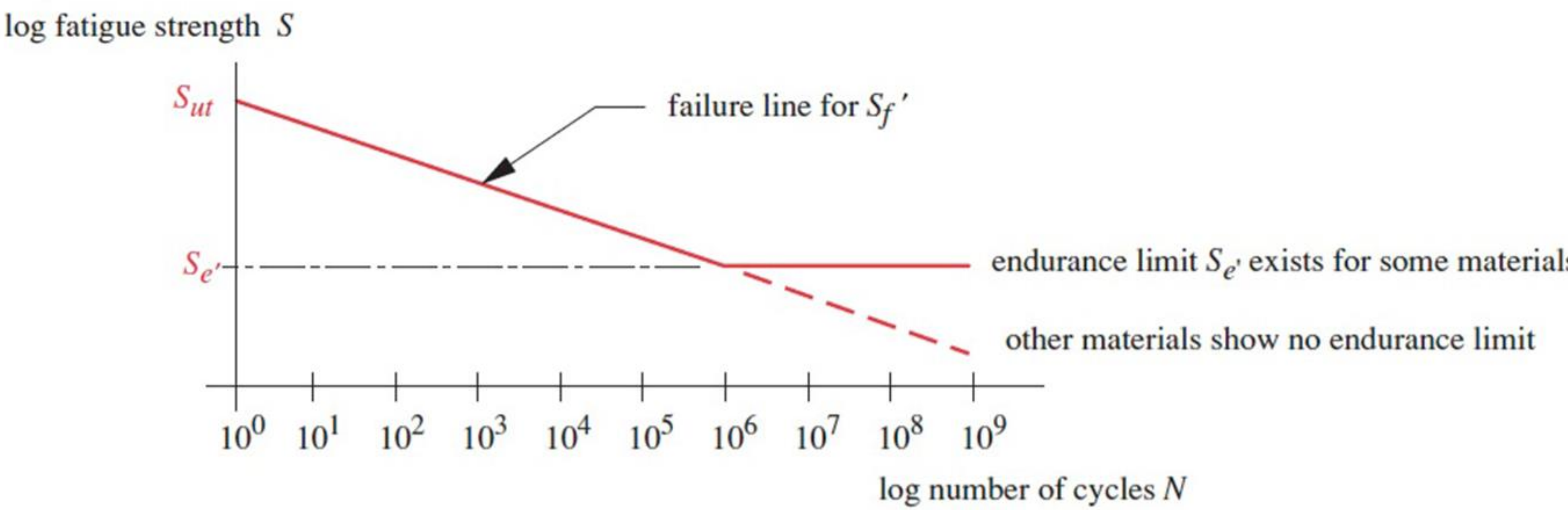


FIGURE 6-2

Wohler Strength-Life or *S-N* Diagram

What is Fatigue

- Fatigue failure can be expensive??? Loss of life
- Fatigue failure of roof resulting in loosing the cabin top. (Hawaiian airlines 1988)
- Humid environment, corrosion, repeated pressure variations due to island hopping, were reason attributed to the failure
- While it failed, it had close to 90,000 trips in 19 years while it was designed for 75000 cycles. (approximately 13 trips a day)


8.1 Introduction

- Until middle of the 19th century engineers treated fluctuating (repeated) loading the same as static loading, except for the use of larger safety factors.
- Use of the term fatigue in these situations appears to have been introduced by Poncelet of France in a book published in 1839.
- “Fatigue” fractures begin with a minute (usually microscopic) crack at a critical area of high local stress. This is almost always at a geometric stress raiser. Additionally, minute material flaws or preexisting cracks are commonly involved
- An inspection of the surfaces after final fracture (as in Figure 8.1) often reveals where the crack has gradually enlarged from one “beach mark” to the next until the section is sufficiently weakened that final fracture occurs on one final load application.
- This can happen when the stress exceeds the ultimate strength, with fracture occurring as in a static tensile test. Usually, however, the final fracture is largely “brittle” and takes place in accordance with the fracture mechanics concepts

8.1 Introduction

- In Figure 8.1, the curvature of the beach marks serves to indicate where the failure originates. The beach-marked area is known as the fatigue zone. It has a smooth, velvety texture developed by the repeated pressing together and separating of the mating crack surfaces.

- This contrasts with the relatively rough final fracture. A distinguishing characteristic of fatigue fracture of a ductile material is that little if any macroscopic distortion occurs during the entire process, whereas failure caused by static overload produces gross distortion.

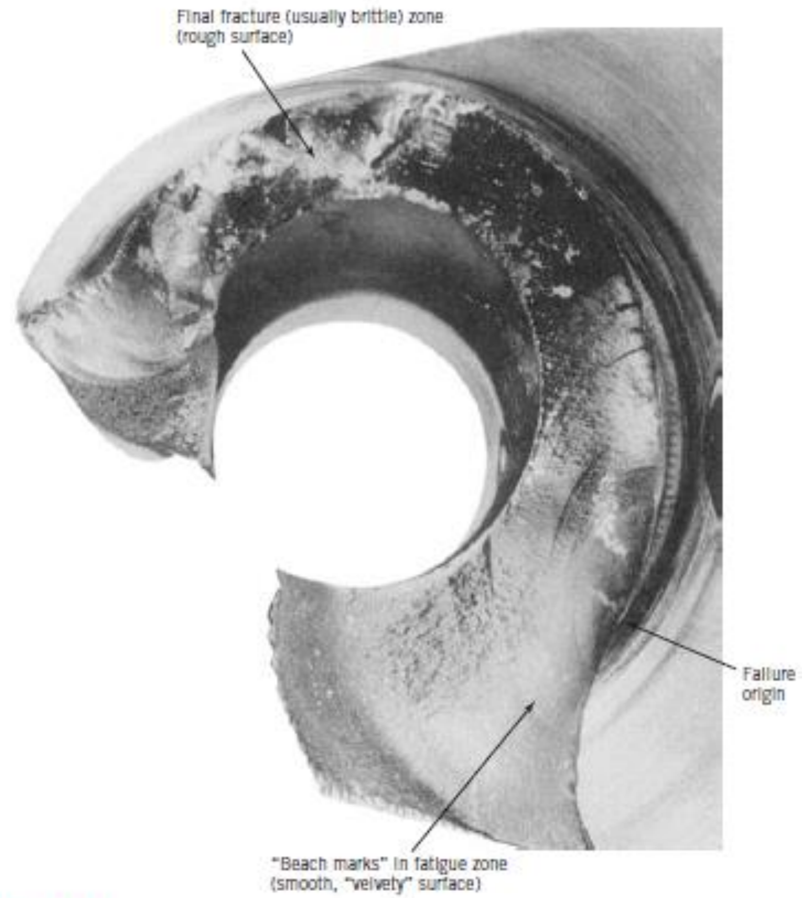
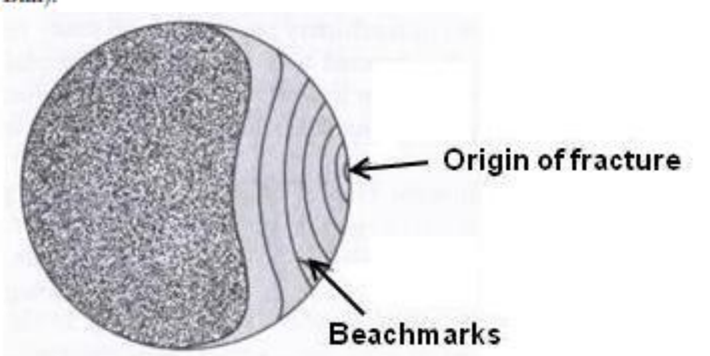


FIGURE 8.1
Fatigue failure originating in the fillet of an aircraft crank-shaft (SAE 4340 steel, 320 Bhn).



Fatigue Fracture with Beachmarks

8.2 Basic Concepts

- Extensive research over the last century has given us a partial understanding of the basic mechanisms associated with fatigue failures. The following are a few fundamental and elementary concepts of observed fatigue behavior.
  1. Fatigue failure results from repeated plastic deformation, such as the breaking of a wire by bending it back and forth repeatedly. Without repeated plastic yielding, fatigue failures cannot occur.
  2. Whereas a wire can be broken after a few cycles of gross plastic yielding, fatigue failures typically occur after thousands or millions of cycles of minute yielding that often exists only on a microscopic level. Fatigue failure can occur at stress levels far below the conventionally determined yield point or elastic limit.
  3. Highly localized plastic yielding can be the beginning of a fatigue failure; attention on all potentially vulnerable locations such as holes, sharp corners, and corrosion is needed. Such a location is shown in Figure. Strengthening these vulnerable locations is as effective as making the entire part from a stronger material.

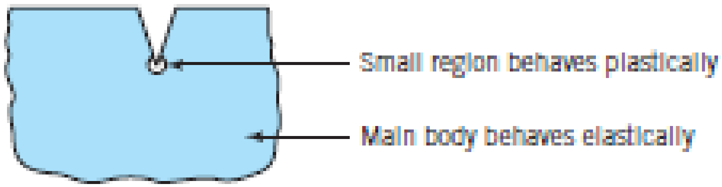


FIGURE 8.2
Enlarged view of a notched region.

8.2 Basic Concepts

4. If the local yielding is sufficiently minute, the material may strain-strengthen, causing the yielding to cease. The part will then have actually benefited from this mild overload. But if the local yielding is any more than this, the repeated load cycling will cause a loss of local ductility until the cyclic strain imposed at the vulnerable spot can no longer be withstood without fracture.

5. The initial fatigue crack usually results in an increase in local stress concentration. As the crack progresses, the material at the crack root at any particular time is subjected to the destructive localized reversed yielding. As the crack deepens, thereby reducing the section and causing increased stresses, the rate of crack propagation increases until the remaining section is no longer able to support a single load application and final fracture occurs, usually in accordance with the principles of fracture mechanics. *(There are situations in which a fatigue crack advances into a region of lower stress, greater material strength, or both, and the crack ceases to propagate, but these situations are unusual.)*

8.2 Basic Concepts

- Present engineering practice relies heavily on the wealth of empirical data that has accumulated from fatigue tests of numerous materials, in various forms, and subjected to various kinds of loading.
- The next section describes the standardized R. R. Moore fatigue test, which is used to determine the fatigue strength characteristics of materials under a standardized and highly restricted set of conditions.
- After the pattern of results obtained from this test has been reviewed, succeeding sections deal with the effects of deviating from the standard test in various ways, thus working toward the completely general case of fatigue.
- The generalizations or patterns of fatigue behavior enables to estimate the fatigue behavior for combinations of materials, geometry, and loading for which test data are not available.
- This estimating of fatigue behavior is an extremely important step
- The preliminary design of critical parts normally encompasses this procedure.
- Then prototypes of the preliminary design are built and fatigue-tested. The results provide a basis for refining the preliminary design to arrive at a final design

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- Figure 8.3 represents a standard R. R. Moore rotating-beam fatigue-testing machine. The reader should verify that the loading imposed by the four symmetrically located bearings causes the center portion of the specimen to be loaded in pure bending (i.e., zero transverse shear), and that the stress at any point goes through a cycle of tension-to-compression-to-tension with each shaft rotation.

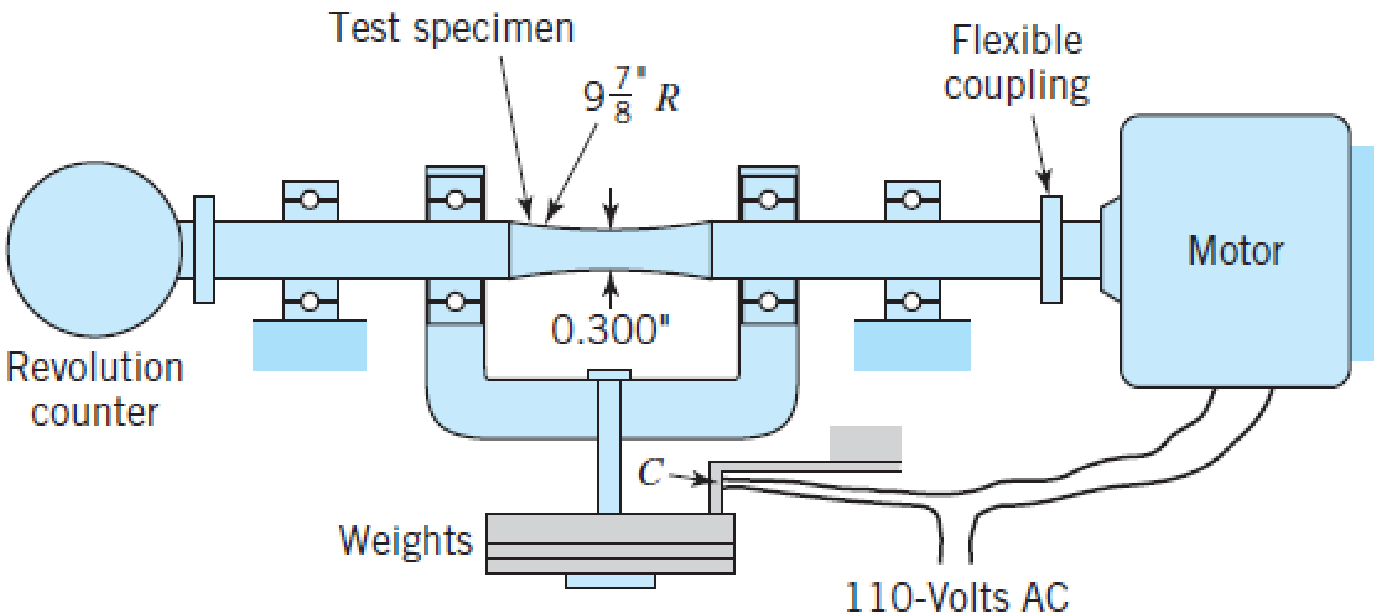


FIGURE 8.3
R. R. Moore rotating-beam fatigue-testing machine.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- The highest level of stress is at the center, where the diameter is a standard 0.300 in. The large radius of curvature prevents a stress concentration. Various weights are chosen to give the desired stress levels. The motor speed is usually 1750 rpm. When the specimen fails, the weight drops, opening contact points C, which stops the motor. The number of cycles to failure is indicated by the revolution counter.

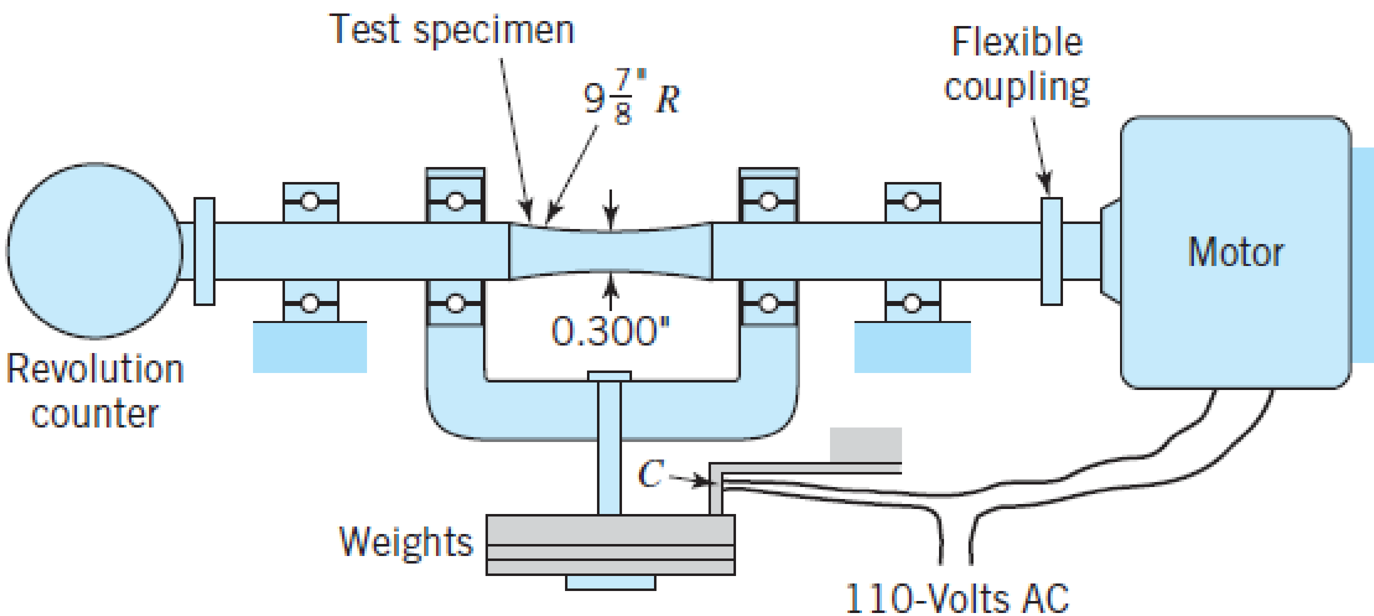
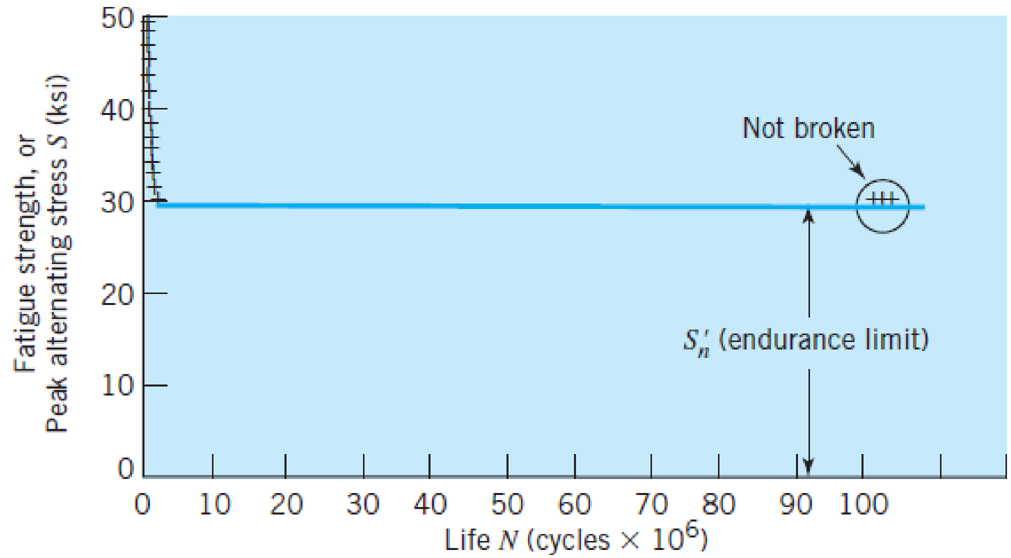


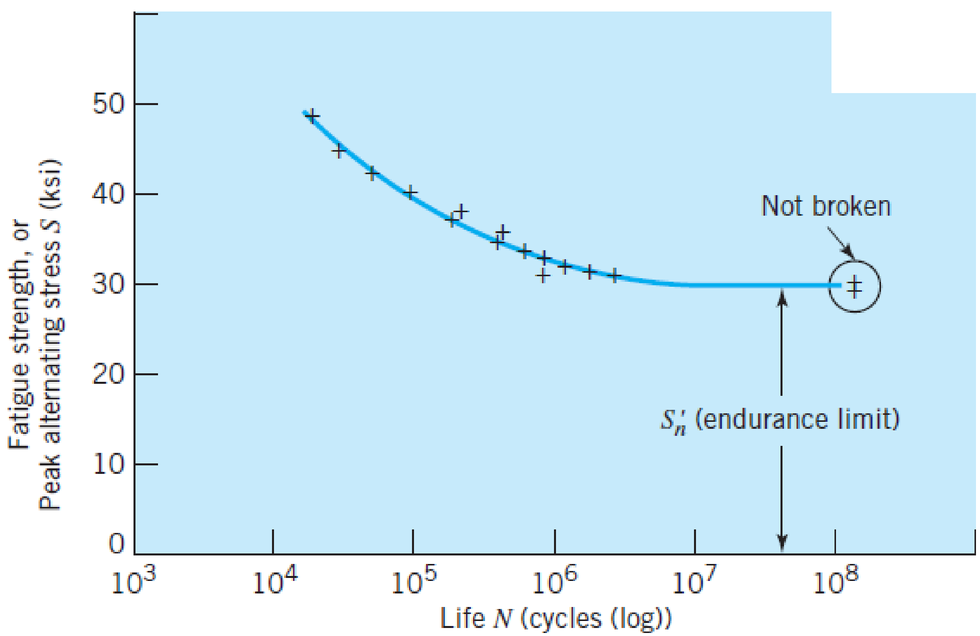
FIGURE 8.3
R. R. Moore rotating-beam fatigue-testing machine.

8.3 Standard Fatigue Strengths (S_n') for Rotating Bending

- From these test results S–N curves are plotted either on semilog or on log-log coordinates.
- Note that the intensity of reversed stress causing failure after a given number of cycles is called the fatigue strength corresponding to that number of loading cycles.



(a) Linear coordinates (not used for obvious reasons)

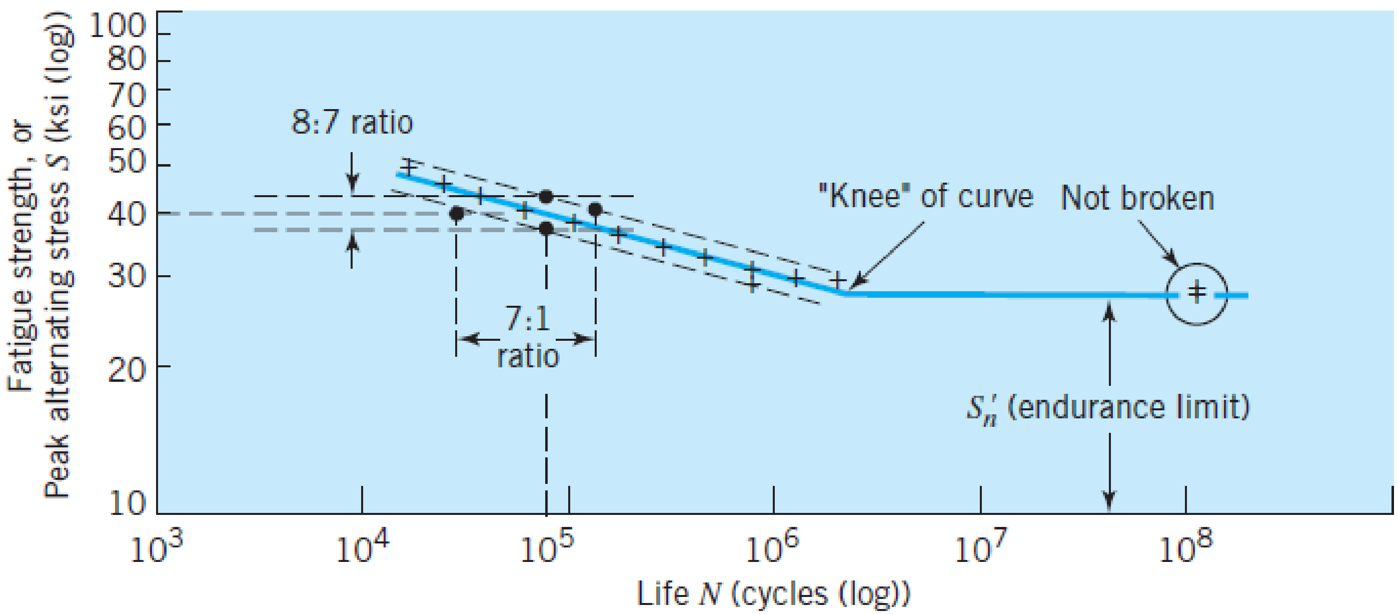


(b) Semilog coordinates

- Numerous tests have established that ferrous materials have an endurance limit, defined as the highest level of alternating stress that can be withstood indefinitely without failure.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- The usual symbol for endurance limit is S_n. It is designated as S'_n in Figure 8.4, where the prime indicates the special case of the standard test illustrated in Figure 8.3. Log-log coordinates are particularly convenient for plotting ferrous S–N curves because of the straight-line relationship shown.



(c) Log-log coordinates

FIGURE 8.4
Three S–N plots of representative fatigue data for 120 Bhn steel.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- Figure 8.4c illustrates the “knee” for materials that have a clear endurance.
- This knee normally occurs around 10^6 and 10^7 cycles. Ferrous materials must not be stressed above the endurance limit if life > 10^6 cycles needed.
- This assumption is illustrated in the generalized S–N curve for steel

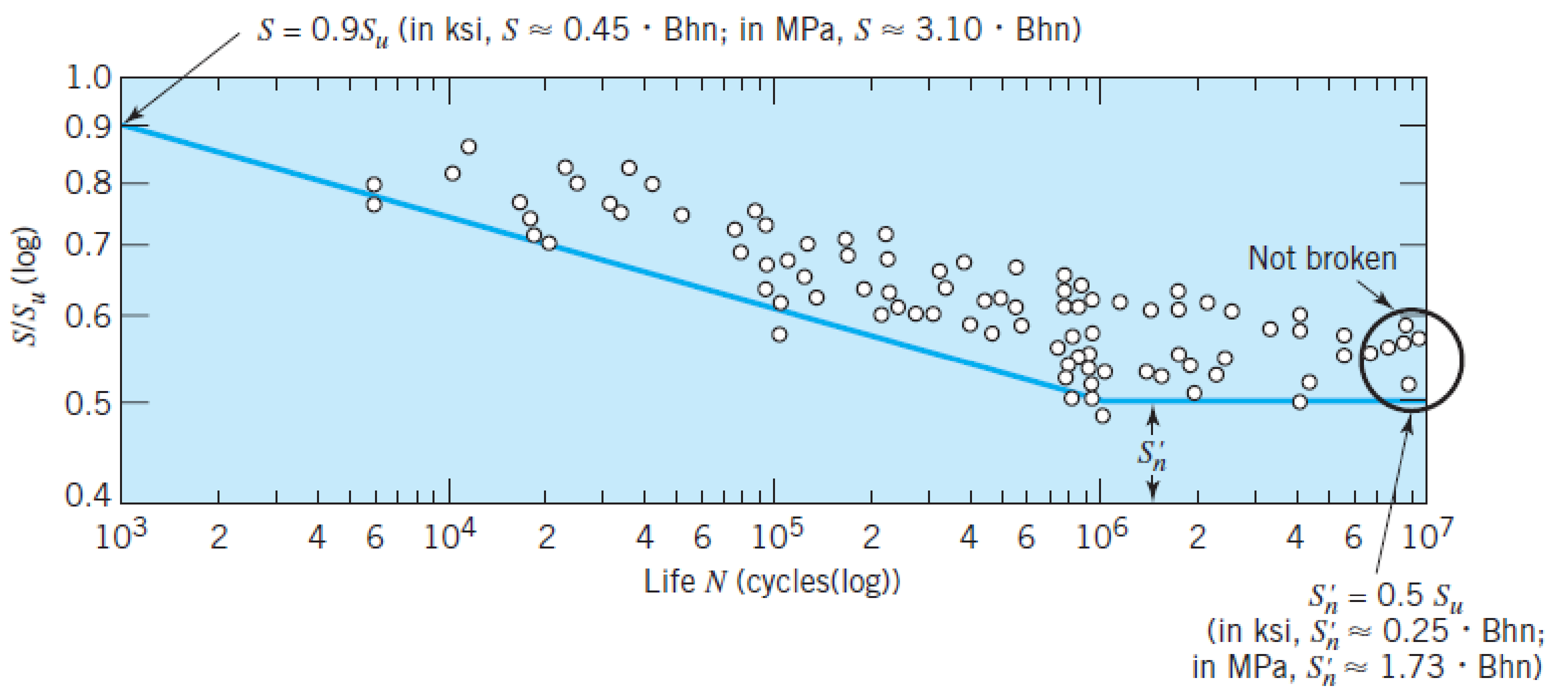
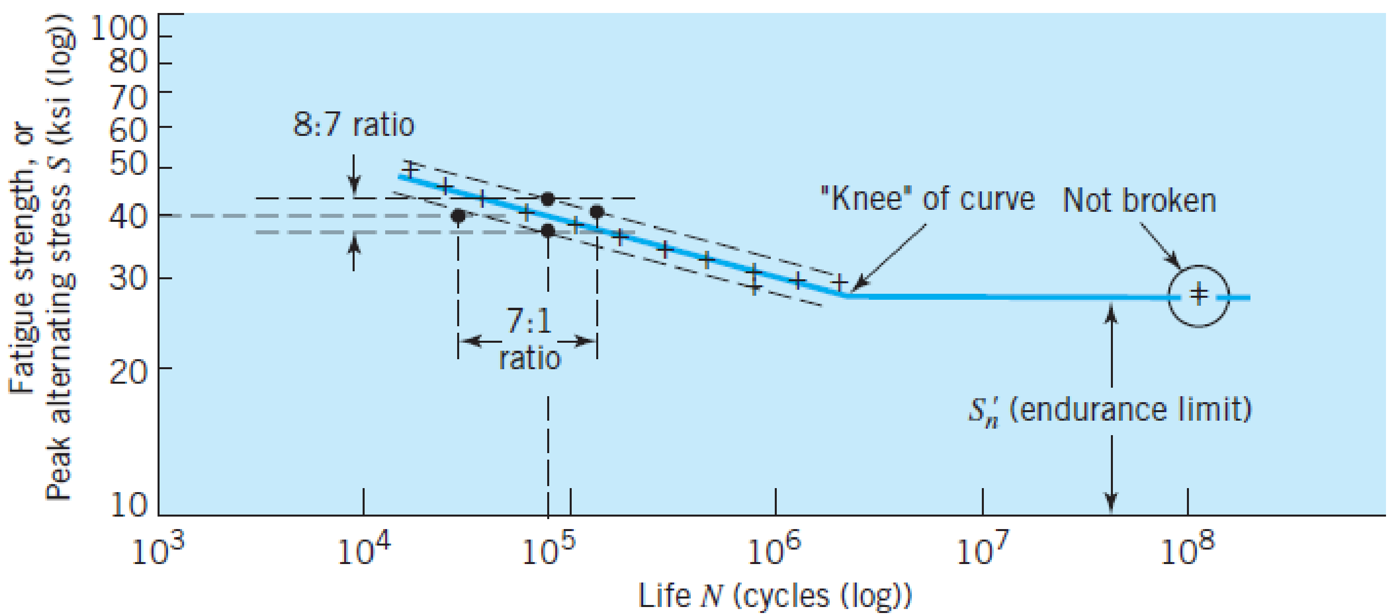


FIGURE 8.5

Generalized S–N curve for wrought steel with superimposed data points [7]. Note that Bhn = H_B = Brinell hardness number.

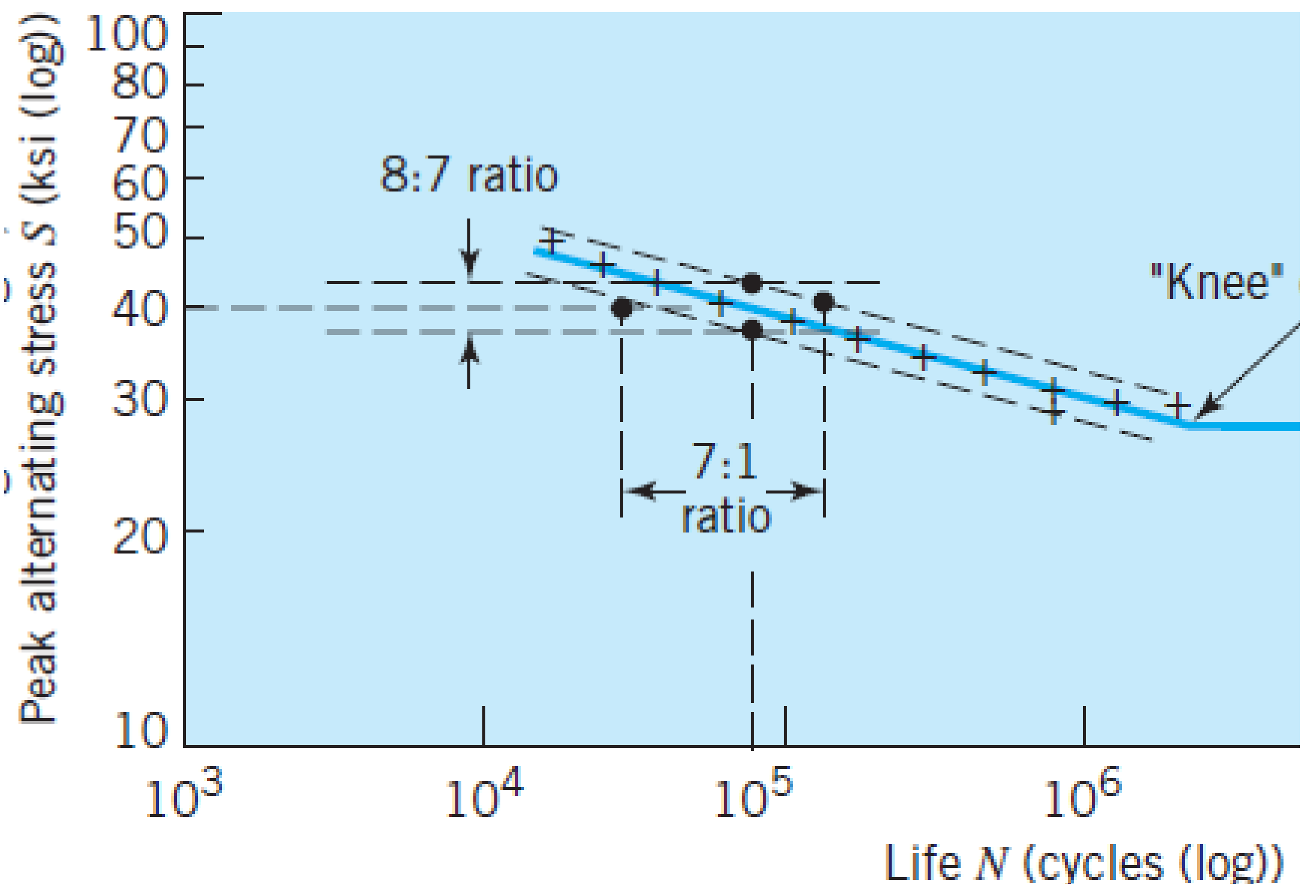
8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- Because fatigue failures originate at local points of relative weakness, the results of fatigue tests have considerably more scatter than do those of static tests.
- For this reason the statistical approach to defining strength takes on greater importance. Standard deviations of endurance limit values are commonly in the range of 4 to 9 % of the nominal value.
- Ideally, the standard deviation is determined experimentally from tests corresponding to the specific application.
- Often, 8 % of the nominal endurance limit is used as a conservative estimate of the standard deviation when more specific information is not available.



8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- The data scatter illustrated in Figure 8.4 is typical for carefully controlled tests. The scatter band marked on Figure 8.4c illustrates an interesting point:
- The scatter in fatigue strength corresponding to a given life is small; the scatter in fatigue life corresponding to a given stress level is large. Even in carefully controlled tests, these life values can vary over a range of five or ten to one.



8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- A multitude of standard fatigue tests have been conducted, with results tending to conform to certain generalized patterns.
- The most commonly one is in Figure 8.5. With a knowledge of only S_{ut}, a good approximation of S–N curve for steel can quickly be made.
- Also S_{ut} can be estimated from a non-destructive hardness test.

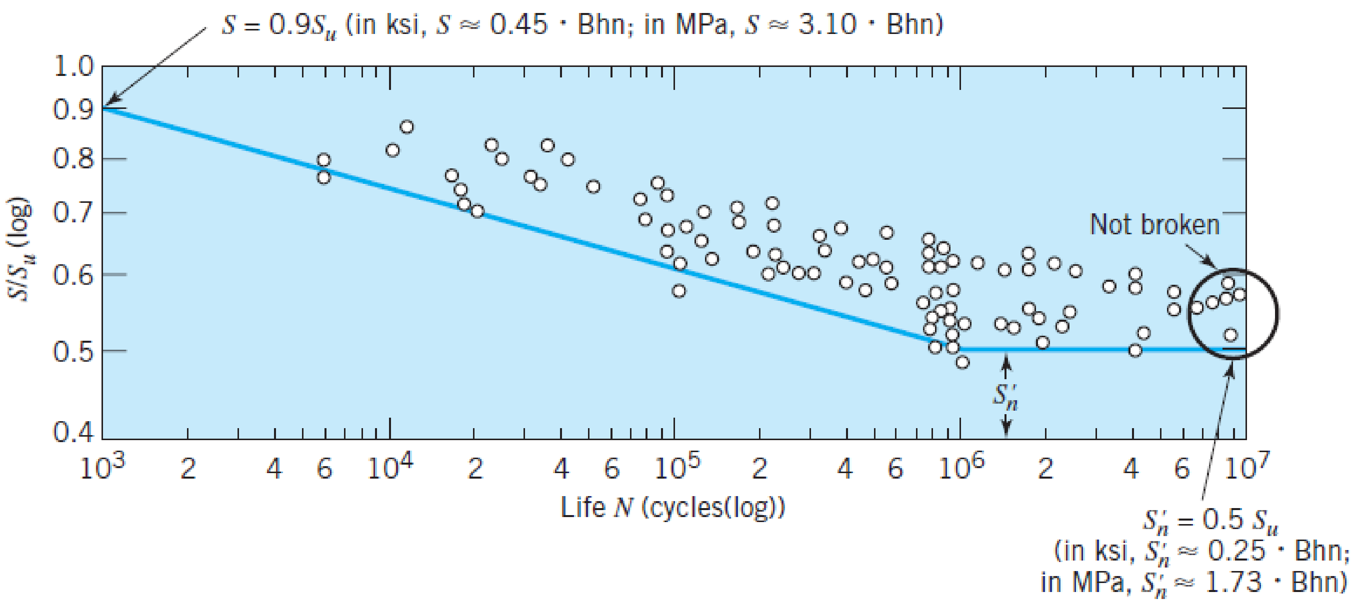


FIGURE 8.5

Generalized S–N curve for wrought steel with superimposed data points [7]. Note that Bhn = H_B = Brinell hardness number.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- For steels, S_{ut} in psi is about 500 times the Brinell hardness ir in ksi about .5 times the BHn; hence, a conservative estimate of endurance limit is about 250 * H_B in psi or 0.25 * H_B in ksi . (Only upto 400 H_B)

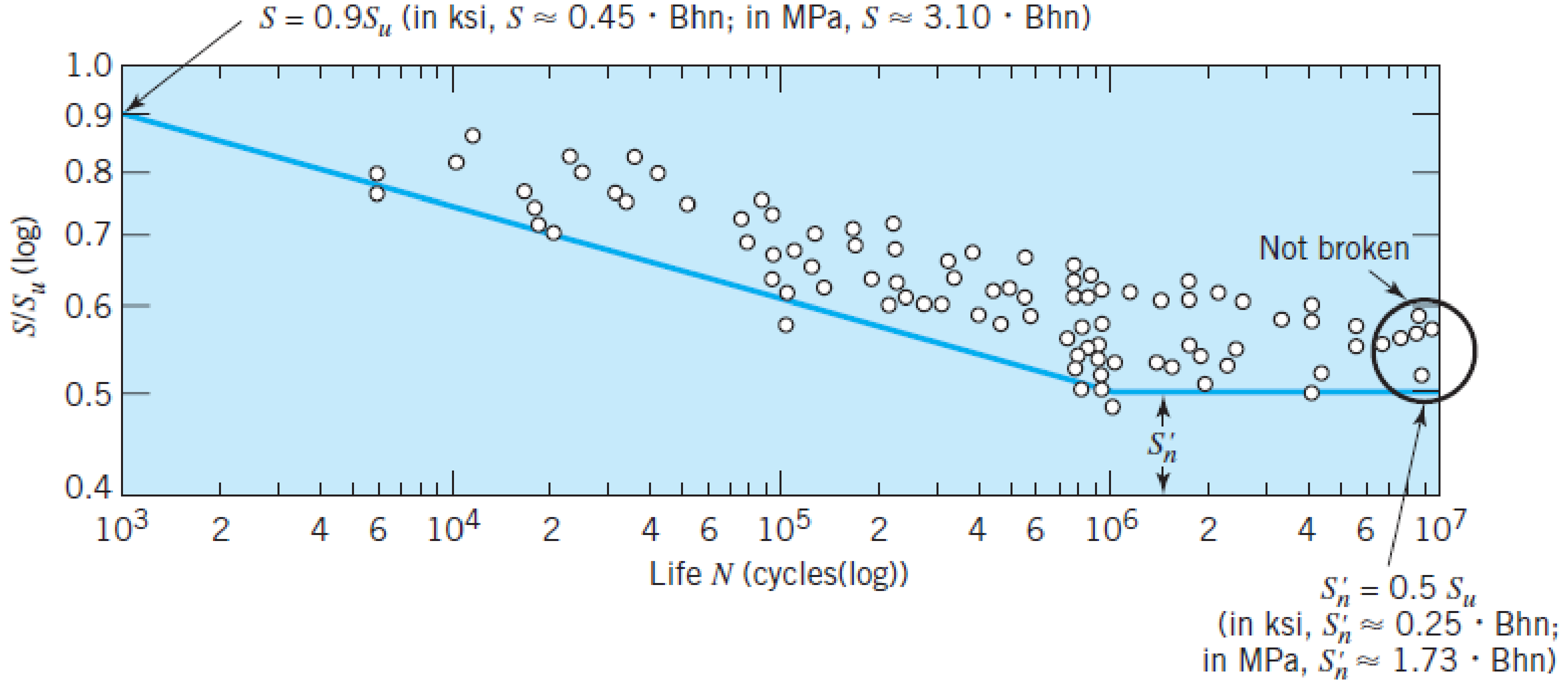


FIGURE 8.5

Generalized S–N curve for wrought steel with superimposed data points [7]. Note that Bhn = H_B = Brinell hardness number.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- The endurance limit may or may not continue to increase for greater hardnesses, depending on the composition of the steel.
- This is illustrated in Figure 8.6.

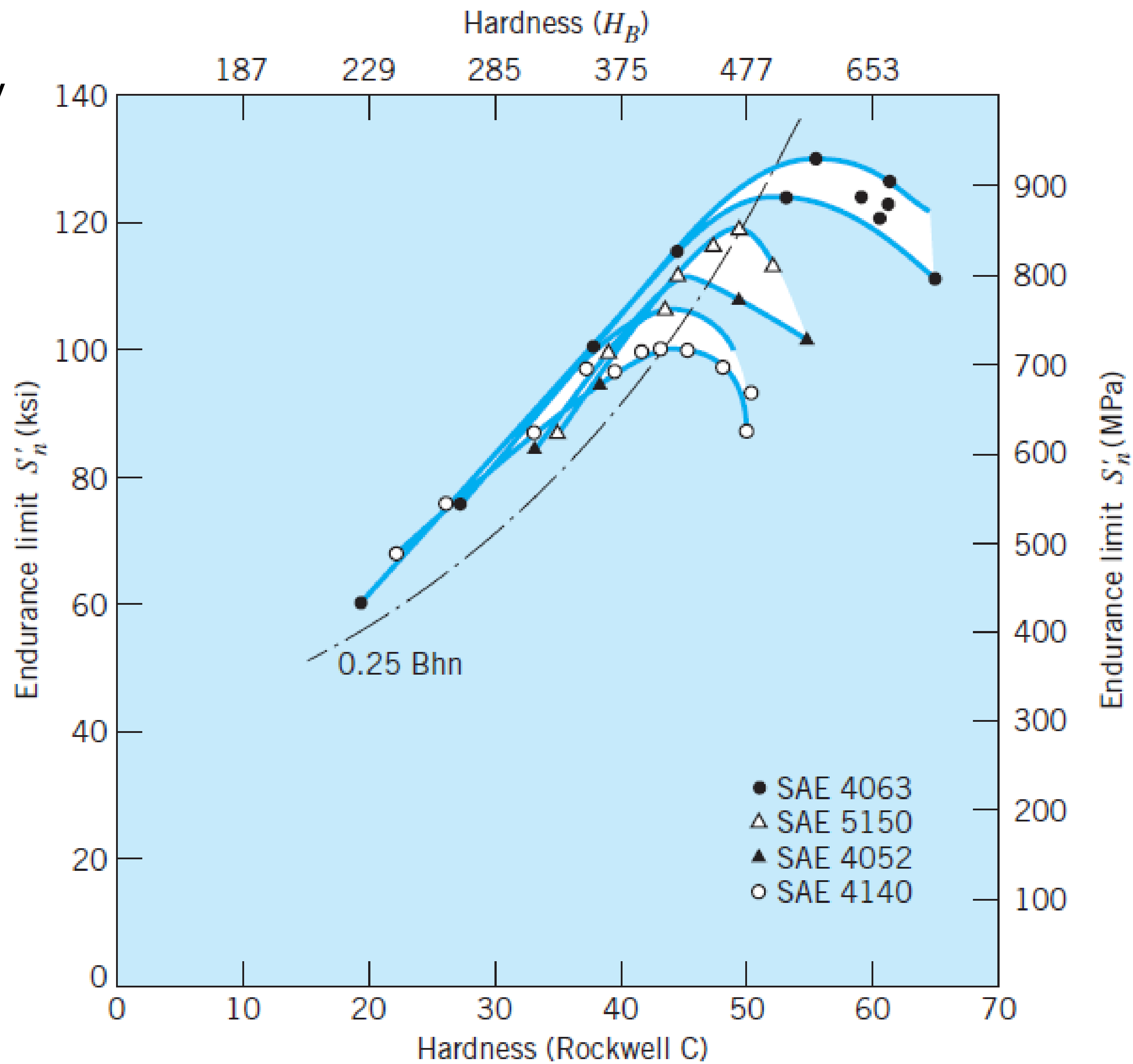
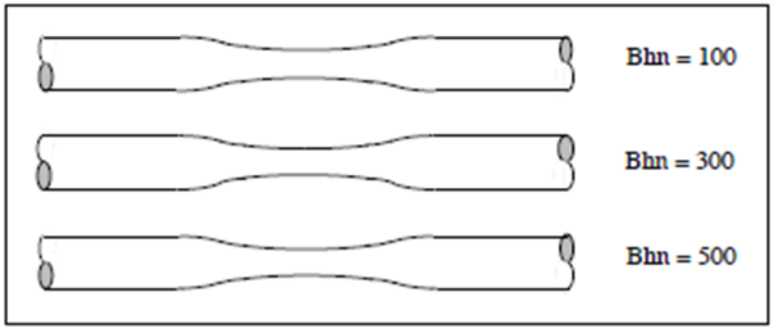


FIGURE 8.6
Endurance limit versus hardness for four alloy steels. (From M. F. Garwood, H. H. Zurburg, and M. A. Erickson, *Interpretation of Tests and Correlation with Service*, American Society for Metals, 1951, p. 13.)

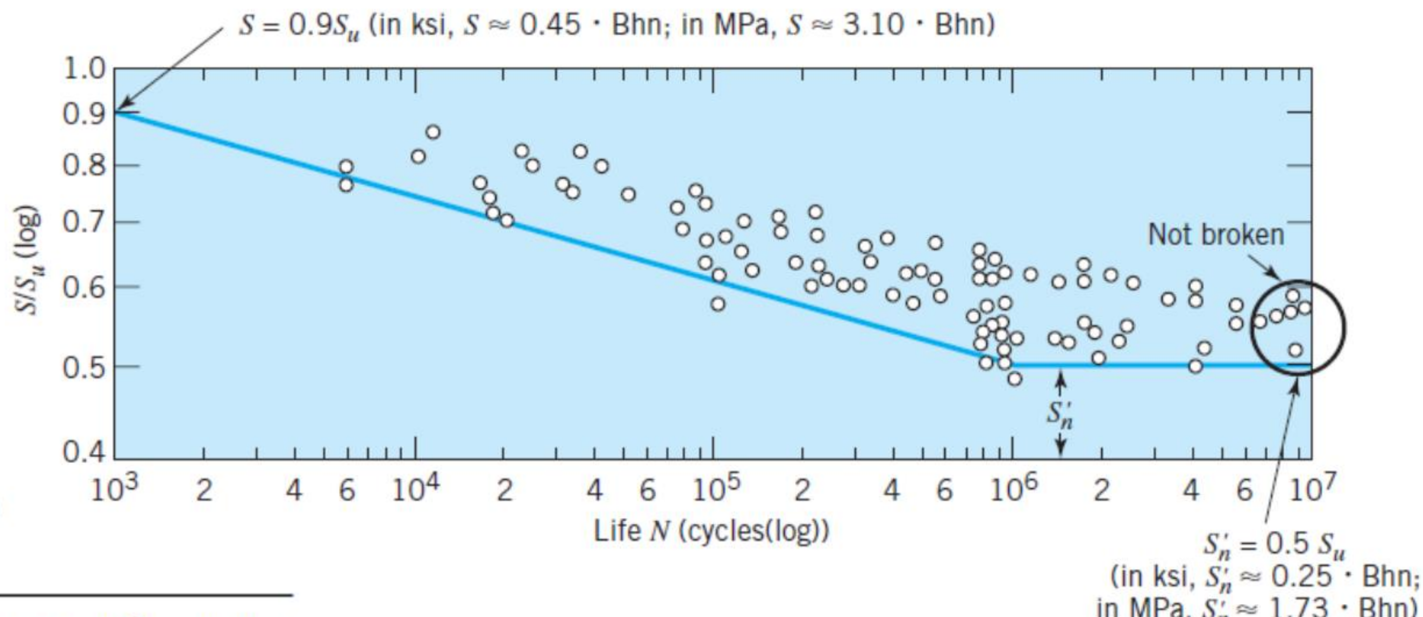
Known: Standard R.R. Moore test specimens are made of steels having known Brinell hardness.

Find: Estimate the rotating bending endurance limit and also the 10^3 cycle fatigue strength.

Schematic and Given Data:



Assumptions:
1. For steel, the tensile strength in psi is 500 times the Brinell hardness.
2. The curve in Fig. 8.5 is an accurate representation of the S-N data for steel.
3. For steel, the endurance limit in psi is 250 times the Brinell hardness.
4. For steel, the endurance limit for 10^3 cycle is 90% of the ultimate strength.



Analysis:
1. $S_u = 500$ Bhn in psi.
2. $S_n' = 0.25$ Bhn in ksi.
3. S for 10^3 cycle $= 0.9S_u$
4.

Bhn	S_u (ksi)	S_n' (ksi)	S for 10^3 cycle (ksi)
100	50	25	45
300	150	75	135
500	250	100~125	225

Comments:
1. The relationship $S_n' = 0.25$ Bhn is accurate only to Brinell hardness values of about 400.
2. For 10^3-cycle fatigue strength, actual stress is not as high as calculated values because of significant yielding.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- Although the 10^3-cycle fatigue strength in Figure 8.5 is computed as being about 90 % of the ultimate strength, the actual stress is not that high.
- The reason is that fatigue strength values corresponding to the test points in Figure 8.4 are computed according to the elastic formula, $\sigma = Mc/I$.
- Loads large enough to cause failure in 1000 cycles usually cause significant yielding, resulting in actual stresses that are lower than calculated values. This point is illustrated in Figure 8.7.

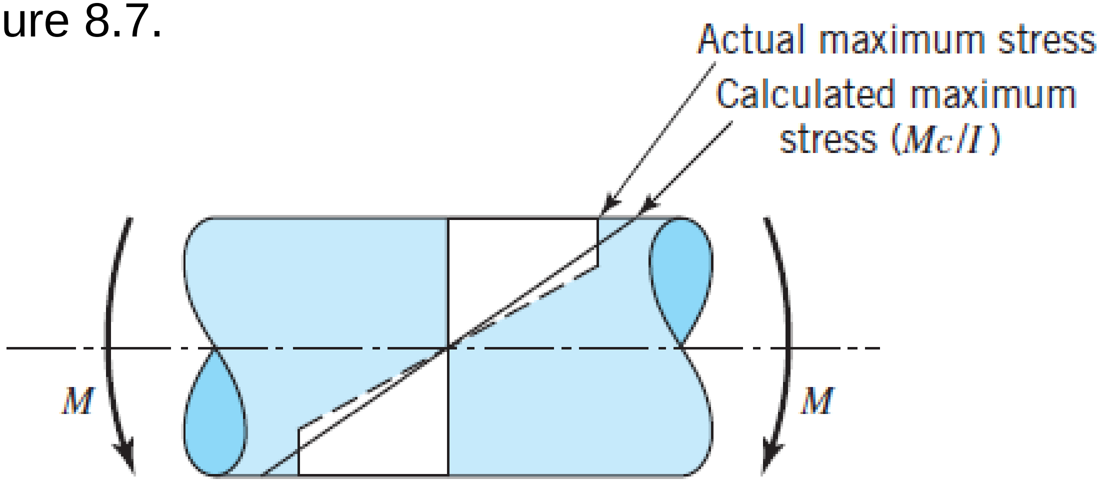


FIGURE 8.7

Representation of maximum bending stress at low fatigue life (at 1000 cycles). (Note: *Calculated* maximum stress is used in S–N plots.)

8.3 Standard Fatigue Strengths (S_n') for Rotating Bending

- The fatigue strength characteristics of cast iron are similar to those of steel, with the exception that the endurance limit corresponds to about 0.4 (rather than 0.5) times the ultimate strength.

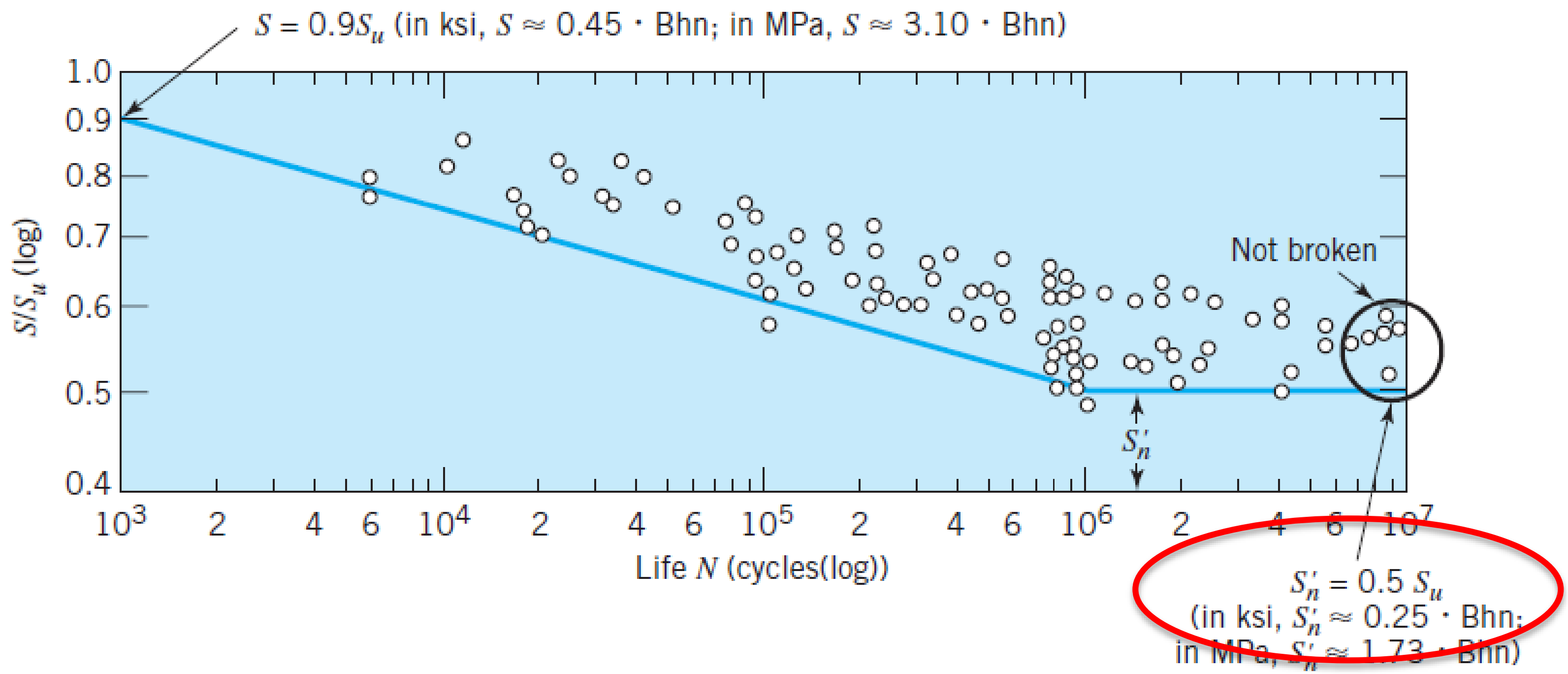


FIGURE 8.5

Generalized S–N curve for wrought steel with superimposed data points [7]. Note that Bhn = H_B = Brinell hardness number.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

- Representative S–N curves for various aluminum alloys are shown in Figure Note the absence of a sharply defined “knee” and true endurance limit.
- This is typical of nonferrous metals.

for the average grade cast aluminum, the fatigue strength at 5×10^8 cycles is 60 MPa for sand cast and 85 MPa for permanent mold cast.

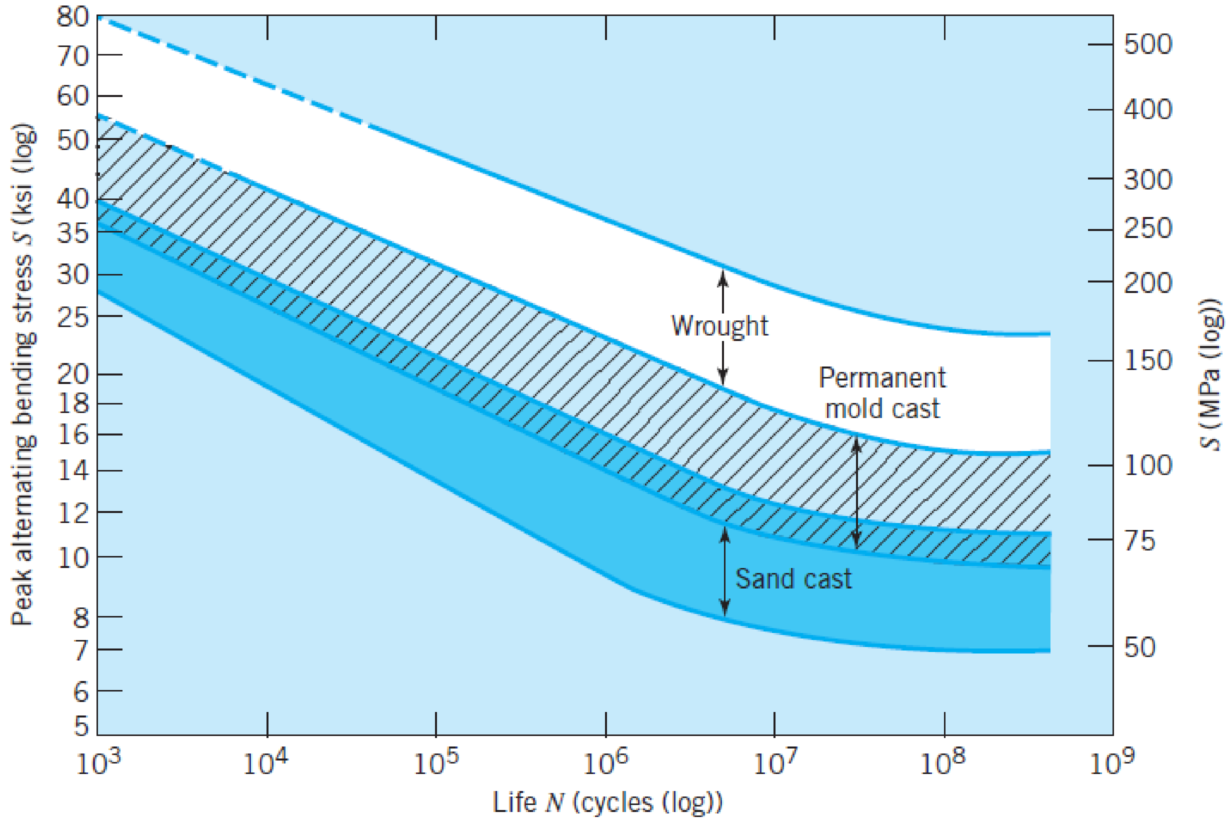


FIGURE 8.8

S–N bands for representative aluminum alloys, excluding wrought alloys with $S_u < 38$ ksi.

8.3 Standard Fatigue Strengths (S_n') for Rotating Bending

- In the absence of an endurance limit, the fatigue strength at 10^8 or 5 * 10^8 cycles is often used. (an automobile would typically travel nearly 400,000 miles before any one of its cylinders fired 5 * 10^8 times.)

- For typical wrought-aluminum alloys, the 5 * 10^8–cycle fatigue strength is related to the static tensile strength

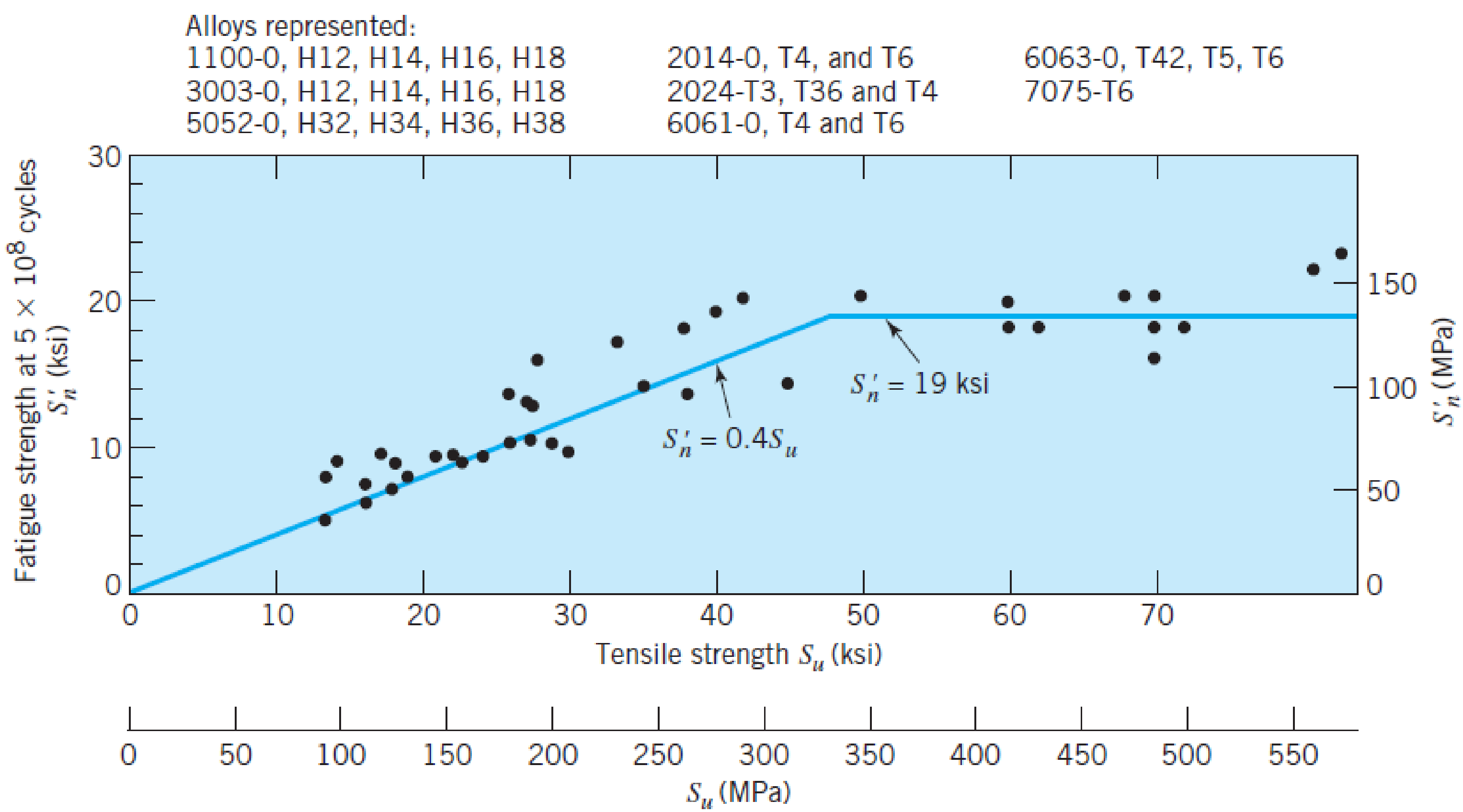


FIGURE 8.9
Fatigue strength at 5 × 10^8 cycles versus tensile strength for common wrought-aluminum alloys.

8.3 Standard Fatigue Strengths (S'_n) for Rotating Bending

From Fig, for the wrought aluminum having S_u = 250 MPa, the fatigue strength at 5 × 10^8 cycles is 100MPa
wrought aluminum having S_u = 450 MPa, the fatigue strength at 5 × 10^8 cycles is 130 MPa.

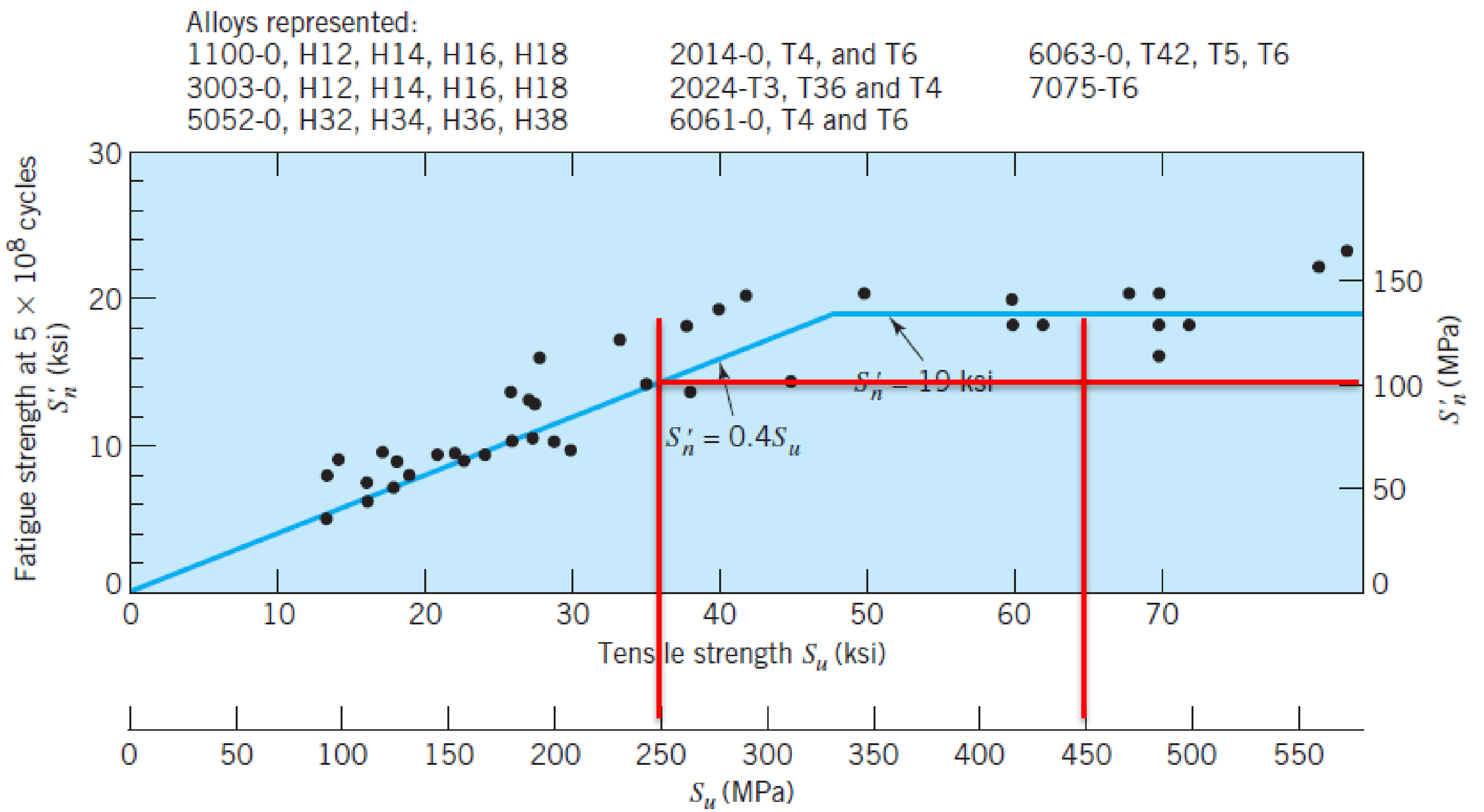
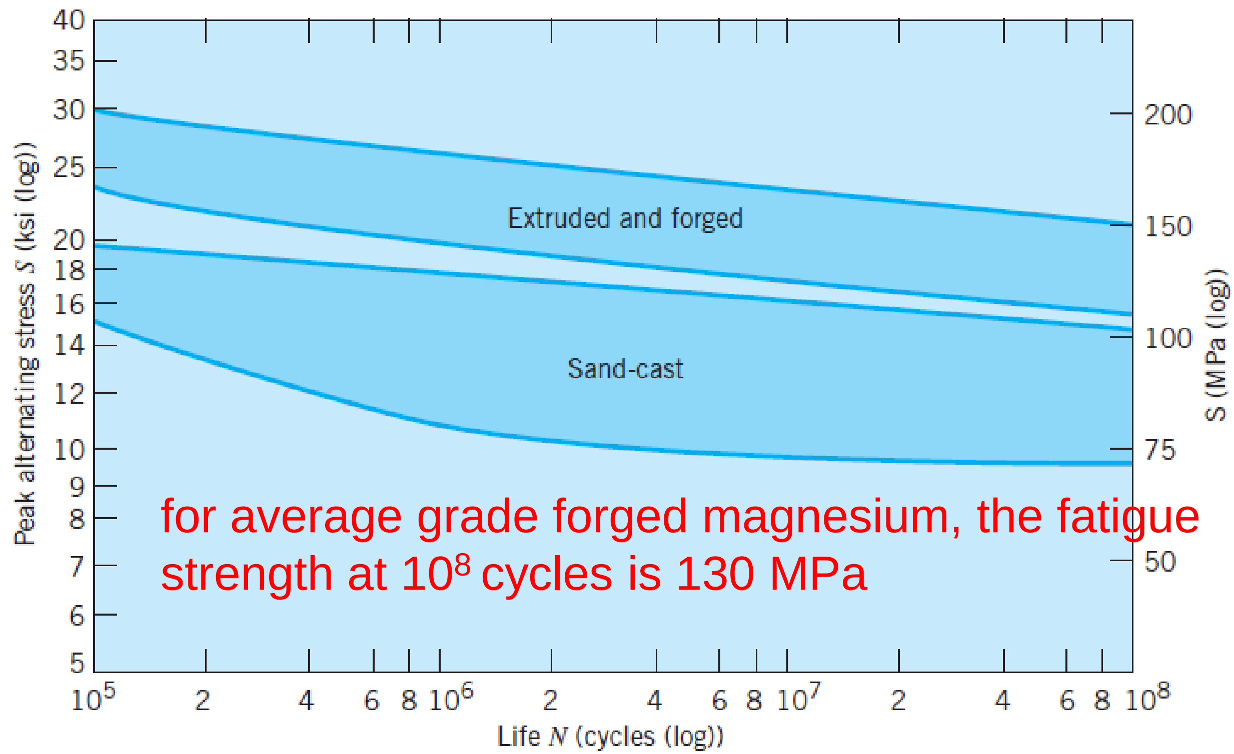


FIGURE 8.9
Fatigue strength at 5 × 10^8 cycles versus tensile strength for common wrought-aluminum alloys.

- Typical S–N curves for mg alloys; the 10^8–cycle fatigue strength is about 0.35 times tensile strength for most wrought and cast alloys.
- For copper alloys, the ratio of 10^8–cycle fatigue strength to static tensile strength ranges between 0.25 and 0.5.
- For nickel alloys it is between 0.35 and 0.5.
- Titanium and its alloys behave like steel in that they tend to exhibit a true endurance limit in the range of 10^6 to 10^7 cycles, with the endurance limit being between 0.45 and 0.65 times the tensile strength.



for average grade forged magnesium, the fatigue strength at 10^8 cycles is 130 MPa

FIGURE 8.10
General range of S–N curves for magnesium alloys.

S-N curve approximation

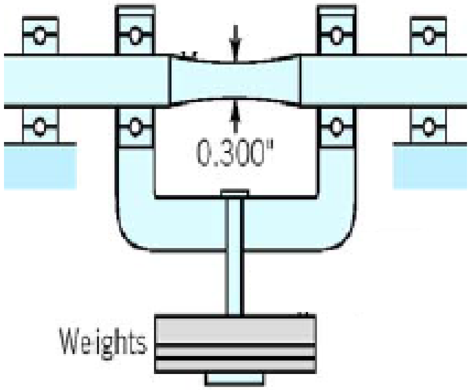
Endurance limit

Steel	$S_n' = 0.5 \times S_u$	@ $N=10^6$
Titanium	$S_n' = 0.45 \ldots 0.6 \times S_u$	
Cast Iron	$S_n' = 0.4 \times S_u$	
Aluminum		@ $N=10^8$
Magnesium	$S_n' = 0.35 \times S_u$	
Nickel alloys	$S_n' = 0.35 \ldots 0.5 \times S_u$	
Cooper alloys	$S_n' = 0.25 \ldots 0.5 \times S_u$	

8.4 Fatigue Strengths for Reversed Bending and Reversed Axial Loading

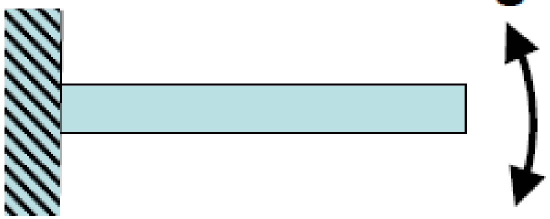
- If a test specimen, similar to the one used in the R. R. Moore testing machine, is not rotated but mounted horizontally with one end fixed and the other pushed alternately up and down, reversed bending stresses are produced.
- Here the maximum stresses are limited to the top and bottom, whereas in rotating bending maximum stresses are all around the circumference.
- In rotating bending, a fatigue failure will originate from the weakest point on the surface; in reversed bending there is an excellent statistical probability that the weakest point will not be at exactly the top or bottom.
- This means fatigue strength in reversed bending is > in rotating bending.
- The difference is small and usually neglected. Thus, for problems involving reversed bending, a small error on the conservative side is deliberately introduced.

Rotating Bending (Moore testing)



maximum stresses **on surface**
weakest point → fatigue start

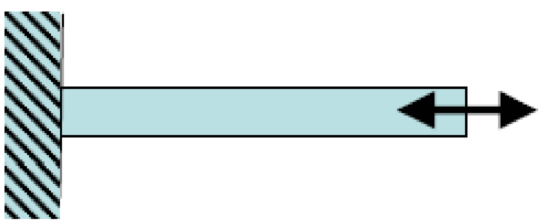
Reversed Bending (not rotating bending like in Moore testing)



maximum stresses **only @ top and bottom**
high probability not weakest point

Fatigue strength usually slightly greater
deliberately neglected → safe side

Reversed Axial Loading



maximum stresses **entire cross section**
no reserve !

Fatigue strength about 10% less
eccentric loads about 20…30% less

C_G=0.7…0.9
gradient factor

8.4 Fatigue Strengths for Reversed Bending and Reversed Axial Loading

- Similar reasoning indicates that a reversed axial loading—which subjects the entire CS to maximum stress—should give lower fatigue strengths than rotating bending.
- This is indeed the case, and this difference should be taken into account.
- Axial or “push–pull” tests give endurance limits about 10 % < rotating bending.
- Furthermore, if the supposedly axial load is just a little off center (non-precision parts with as-cast or as-forged surfaces), slight bending is introduced which causes stresses on one side to be a little higher than P/A.
- Determine the load eccentricity and calculate the peak alternating stress as $P/A + Mc/I$, but the magnitude of unwanted eccentricity is often not known.
- In such cases it is customary to take this into account by using only the P/A stress, and reducing the rotating bending endurance limit by a little more than 10 % (perhaps by 20 to 30 %).

8.4 Fatigue Strengths for Reversed Bending and Reversed Axial Loading

- Since this reduction of 10 % or more in endurance limit for rotating bending is associated with differences in stress gradient, take this into account by multiplying the basic endurance limit S_n', by a gradient constant, C_G,
  - where, $C_G = 0.9$ for pure axial loading of precision parts and
  - C_G ranges from 0.7 to 0.9 for axial loading of nonprecision parts.
- Stress gradient is also responsible for the 10^3-cycle fatigue strength being lower for axial loading than for bending loads.
- From Figure 8.5, that the $0.9S_u$ strength for rotating bending was in most cases an artificial calculated value that neglected the effect of yielding at the surface.

- Yielding cannot reduce the surface stress in the case of axial loading. Accordingly, tests indicate that the 10^3-cycle strength for this loading is only about $0.75S_u$.

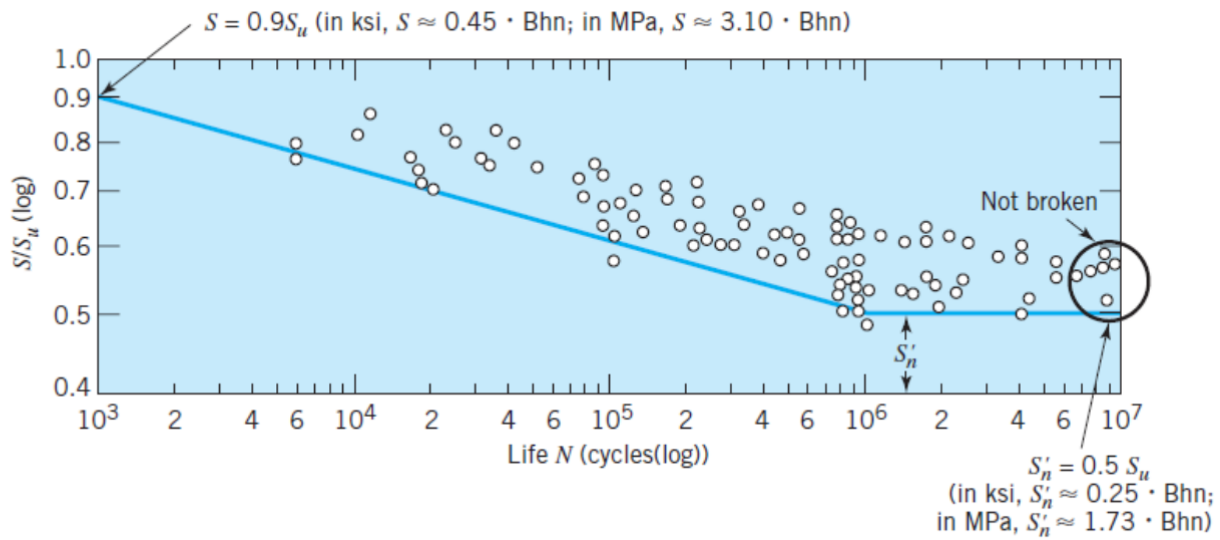


FIGURE 8.5
Generalized S–N curve for wrought steel with superimposed data points [7]. Note that Bhn = H_B = Brinell hardness number.

8.4 Fatigue Strengths for Reversed Bending and Reversed Axial Loading

- The preceding points are illustrated in Figure 8.11. The top two show comparative estimated S–N curves for bending and axial loading.
- The bottom curve shows a comparative estimated S–N curve for torsion loading.
- For axial fatigue, another possible approach is to rely on the data in MIL-HDBK-5J (see Appendix F). Among the data in MIL-HDBK-5J are S–N data points and the associated curve fits for many different metals, including steel, aluminum, and titanium.

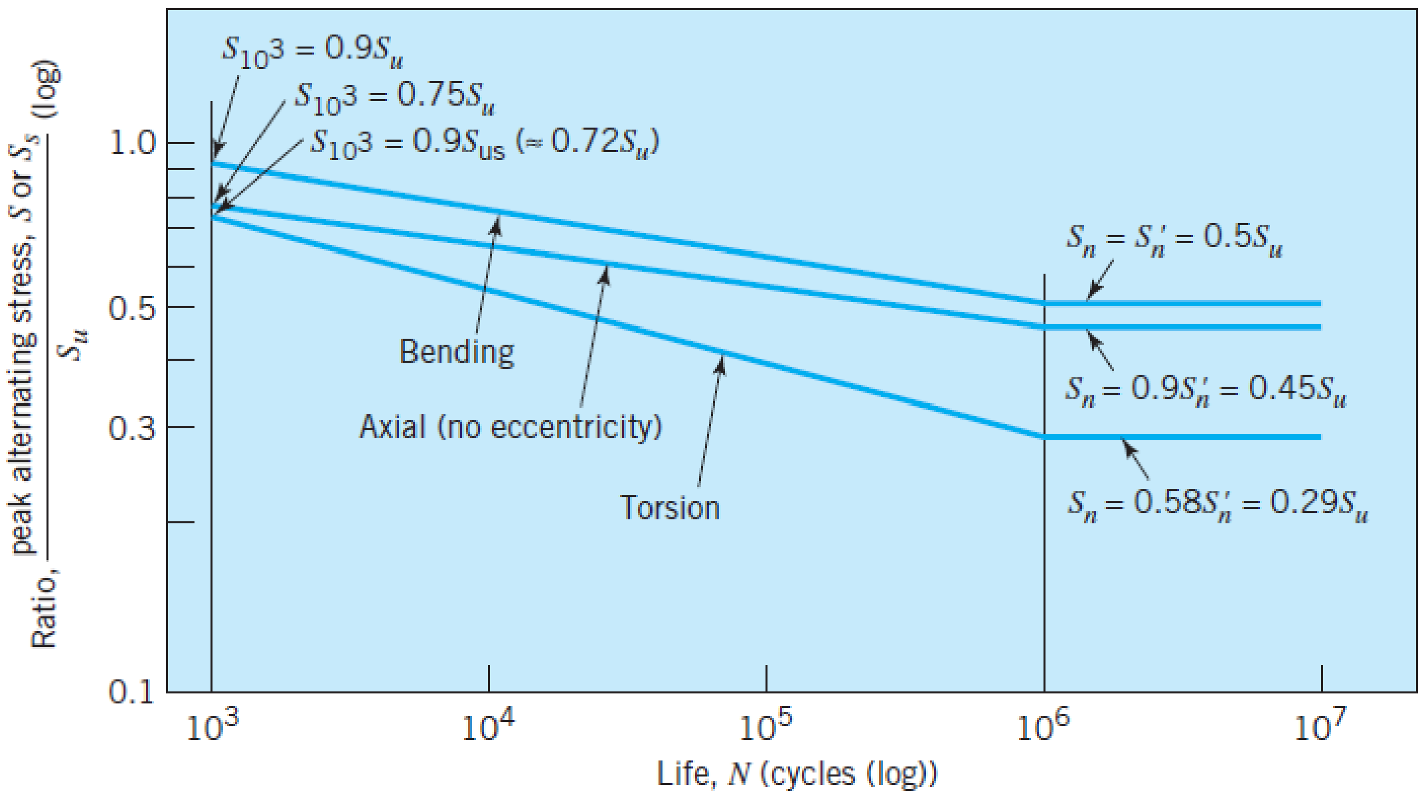


FIGURE 8.11
Generalized S–N curves for polished 0.3-in.-diameter steel specimens (based on calculated elastic stresses, ignoring possible yielding).

8.4 Fatigue Strengths for Reversed Bending and Reversed Axial Loading

- The loading consists of both fully reversed axial fatigue and axial fatigue with non–zero mean stress.
- In some cases the stress concentrations are also included.
- It should be noted that MIL-HDBK-5J only presents axial fatigue data, as other types of fatigue testing such as bending and torsion are seldom used for aerospace applications

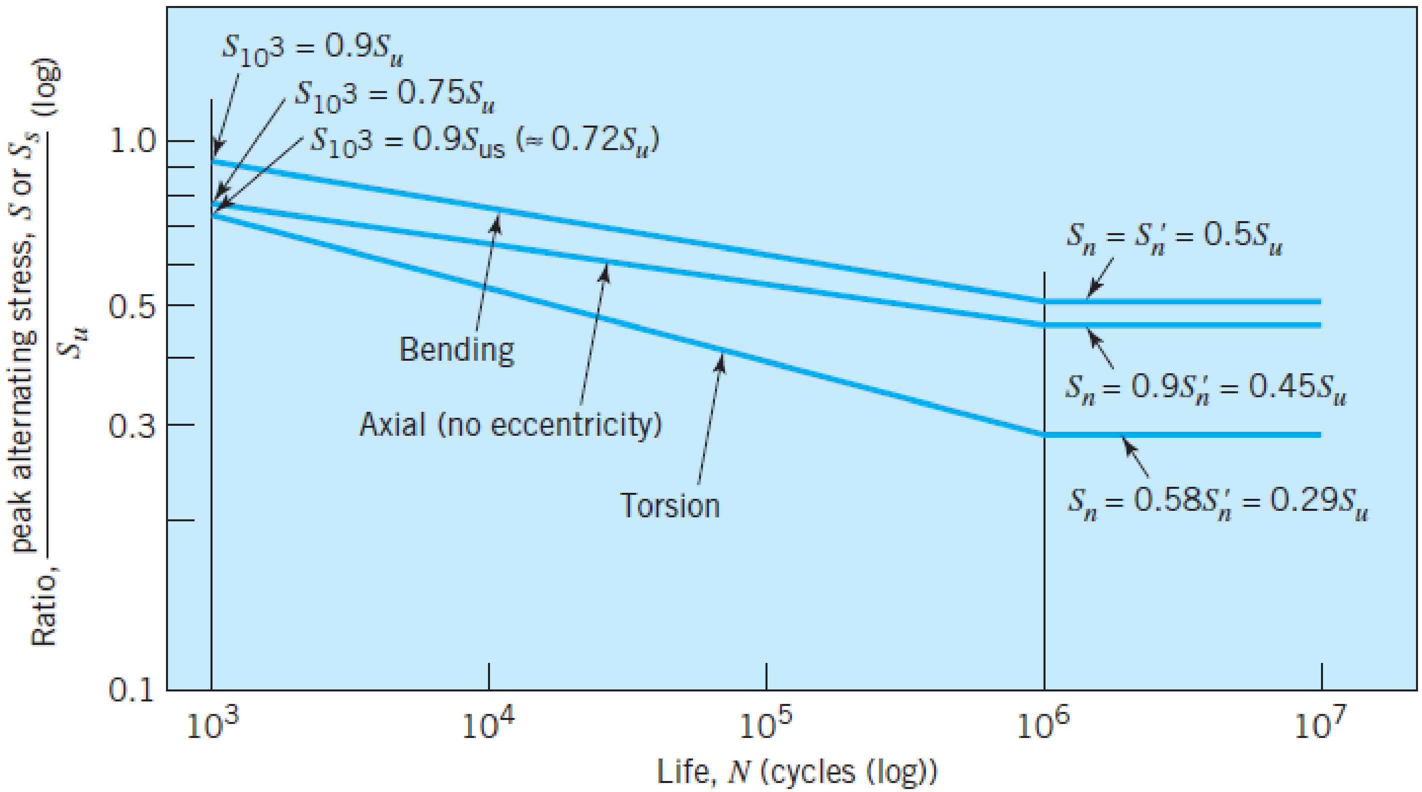


FIGURE 8.11
Generalized S–N curves for polished 0.3-in.-diameter steel specimens (based on calculated elastic stresses, ignoring possible yielding).

8.5 Fatigue Strength for Reversed Torsional Loading

- Fatigue failures are associated with highly localized yielding, and yielding of ductile materials correlates well with maximum distortion energy theory
- This theory is useful in predicting the endurance limit of ductile materials under various combinations of reversed biaxial loading, including torsion. Figure 8.12.

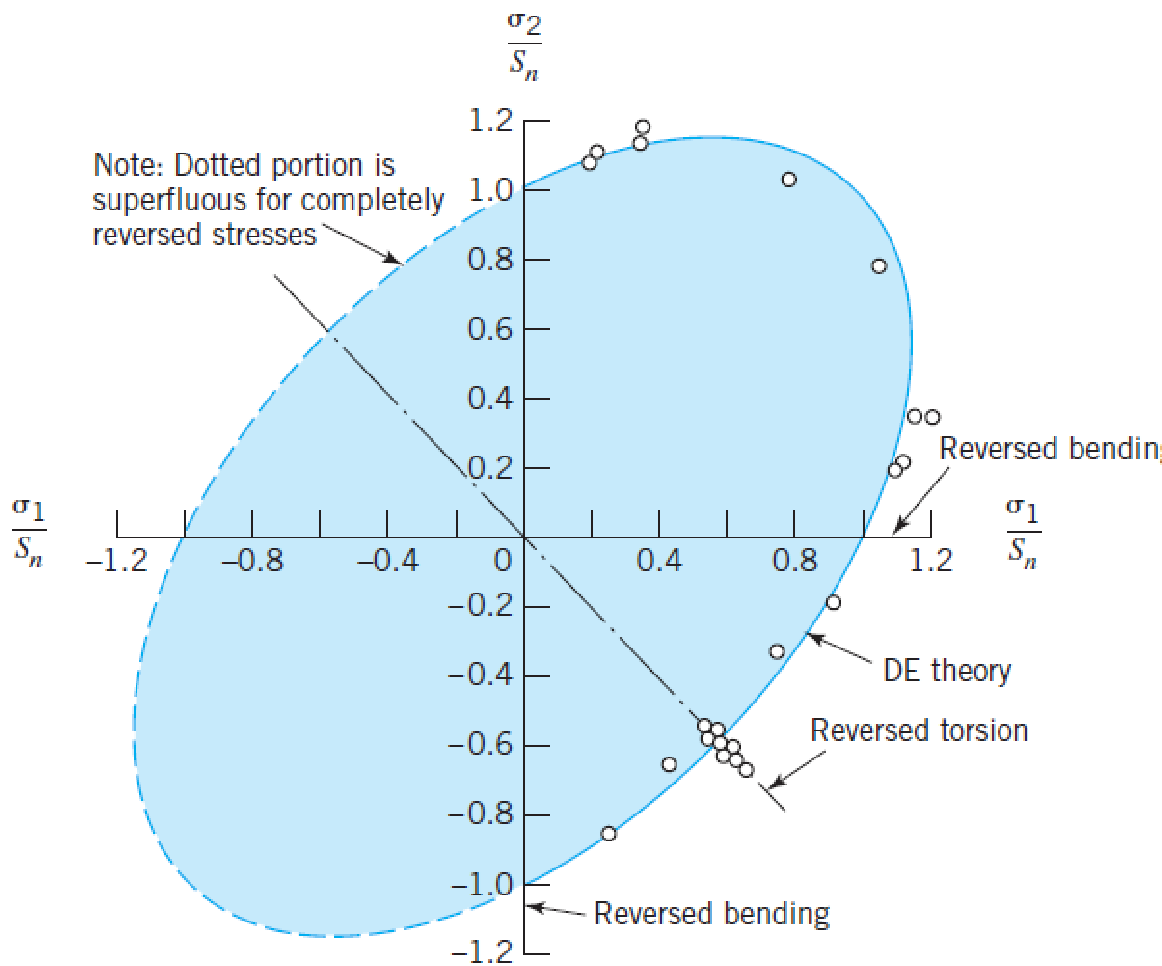


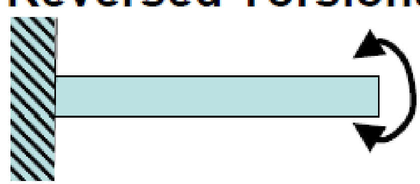
FIGURE 8.12
A σ_1–σ_2 plot for completely reversed loading, ductile materials. [Data from Walter Sawert, Germany, 1943, for annealed mild steel; and H. J. Gough, "Engineering Steels under Combined Cyclic and Static Stresses," *J. Appl. Mech.*, **72:** 113–125 (March 1950).]

8.5 Fatigue Strength for Reversed Torsional Loading

- Thus, for ductile metals, the endurance limit in reversed torsion is about 58% of endurance limit in reversed bending.
- This is taken into account by multiplying the basic endurance limit by a load factor $C_L = 0.58$.
- Since torsional stresses involve stress gradients similar to bending, the 10^3-cycle fatigue strength is generally about 0.9 times the appropriate ultimate strength.
- Thus, for reversed torsion the 10^3-cycle strength is approximately 0.9 times the ultimate shear strength.
- Experimental values for ultimate torsional shear strength should be used if they are available. If not, they may be roughly approximated as

$$
\begin{aligned}
S_{us} &= 0.8 S_u \quad \text{(for steel)} \\
&= 0.7 S_u \quad \text{(for other ductile metals)}
\end{aligned}
$$

Reversed Torsional Loading



maximum stresses **on surface**

shear stresses → fatigue start

reversed biaxial stress

distortion energy theory → **58%** } **C_L=0.58** *load factor*

8.6 Fatigue Strength for Reversed Biaxial Loading

- Figure 8.12 illustrates the good general agreement of the distortion energy theory with the endurance limit of ductile materials subjected to all combinations of reversed loading.

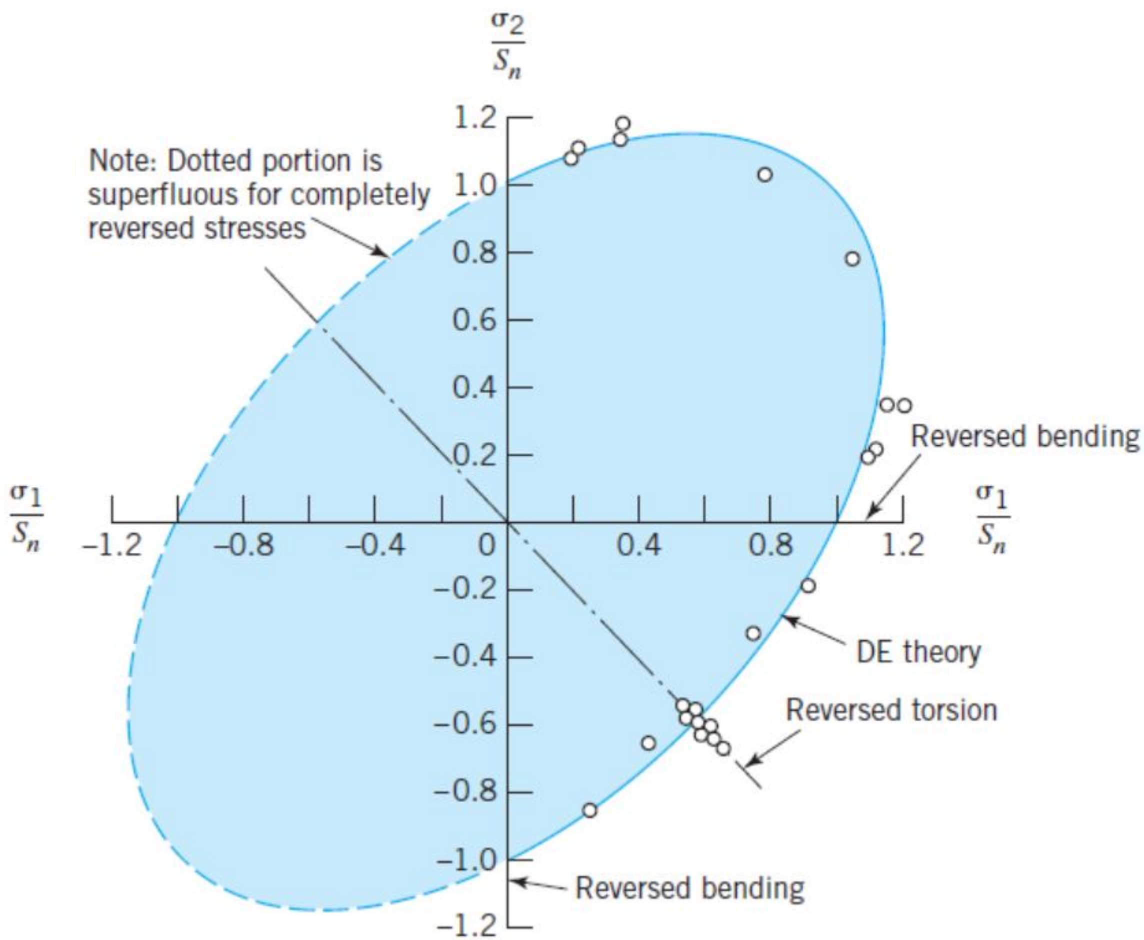


FIGURE 8.12
A σ_1–σ_2 plot for completely reversed loading, ductile materials. [Data from Walter Sawert, Germany, 1943, for annealed mild steel; and H. J. Gough, "Engineering Steels under Combined Cyclic and Static Stresses," *J. Appl. Mech.*, **72**: 113–125 (March 1950).]

8.5 Fatigue Strength for Reversed Torsional Loading

- The bottom curve of Figure 8.11 shows an estimated torsional S–N curve for steel based on the preceding relationships.
- There are fewer data available to support a generalized procedure for estimating torsional S–N curves for brittle materials, and this makes it all the more desirable to obtain actual experimental fatigue data for the specific material and loading condition in the problem at hand.
- In the absence of such data, torsional S–N curves for brittle materials are sometimes estimated on the basis of
    - (1) assuming an endurance limit at 10^6 cycles of 0.8 times the standard reversed bending endurance limit (this correlates somewhat with using the Mohr theory of failure to relate bending and torsion in the same way that the distortion energy theory is used for ductile materials) and
    - (2) assuming a 10^3-cycle strength of $0.9S_{us}$, the same as for ductile materials.

8.6 Fatigue Strength for Reversed Biaxial Loading

- Figure 8.12 illustrates the good general agreement of the distortion energy theory with the endurance limit of ductile materials subjected to all combinations of reversed biaxial loading.
- For shorter life fatigue strengths of ductile materials, and for brittle materials, we are not in a very good position to make fatigue strength predictions without the benefit of directly applicable experimental data.

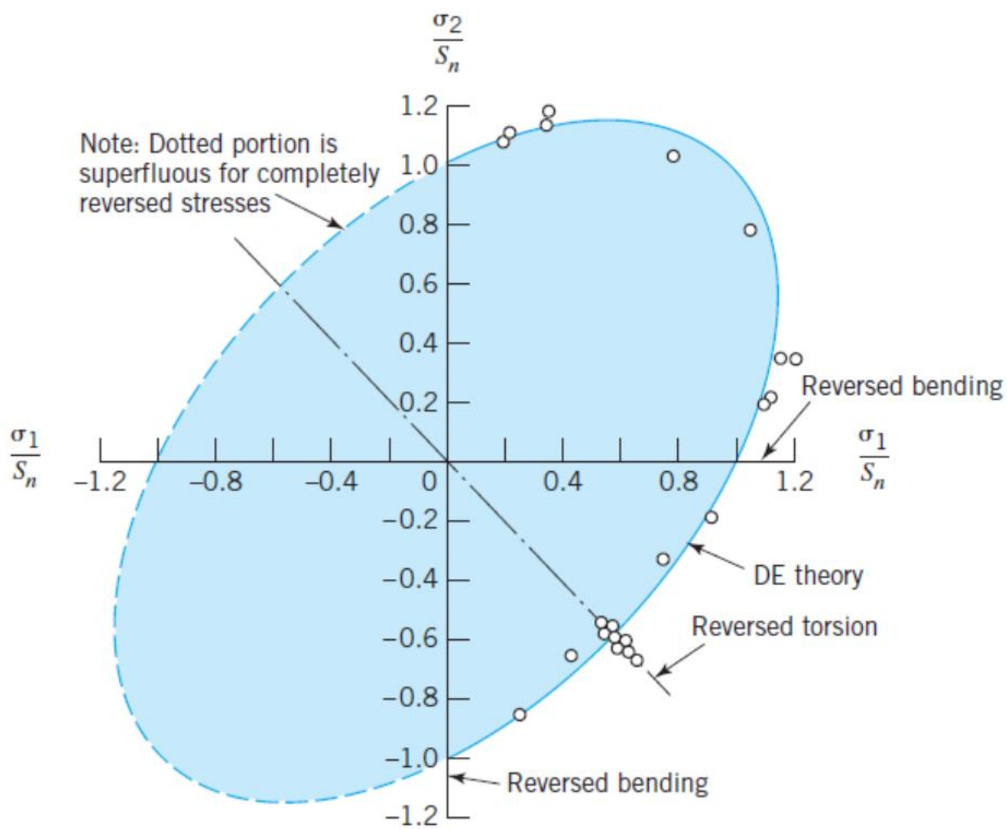
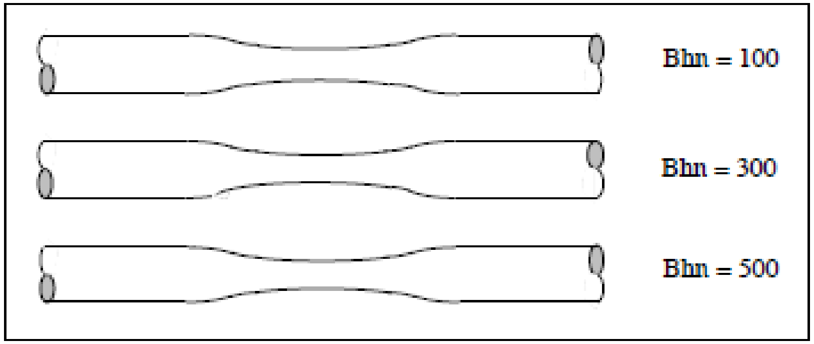


FIGURE 8.12
A σ_1–σ_2 plot for completely reversed loading, ductile materials. [Data from Walter Sawert, Germany, 1943, for annealed mild steel; and H. J. Gough, "Engineering Steels under Combined Cyclic and Static Stresses," *J. Appl. Mech.*, **72**: 113–125 (March 1950).]

Known: Standard R.R. Moore test specimens are made of steels having known Brinell hardness.

Find: Estimate the endurance limit and also the 10^3 cycle fatigue strength for reversed torsional loading.

Schematic and Given Data:



Assumption: Figs. 8.5 and 8.11 can be used to estimate the endurance limit and 10^3 cycle fatigue strength for reversed torsional loading.

Analysis:

1. $S_u = 500$ Bhn in psi.
2. $S_n' = 0.25$ Bhn in ksi.
3. $S_n = 0.58\ S_n'$
4. S for 10^3 cycle $= 0.9\ S_{us}$ where $S_{us} = 0.8\ S_u$ for steel
5.

Bhn	S_u (ksi)	S_n (ksi)	S for 10^3 cycle (ksi)
100	50	25(0.58) = 14.5	36
300	150	75(0.58) = 43.5	108
500	250	(100~125)0.58 = 58 to 72	180

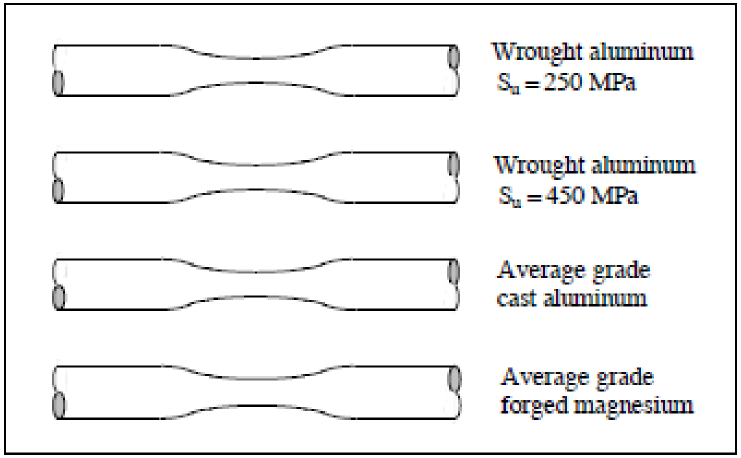
Comment: The relationship $S_n' = 0.25$ Bhn is accurate only to Brinell hardness values of about 400.

SOLUTION (8.16)

Known: Four known standard R.R. Moore specimens are given.

Find: Estimate the long-life fatigue strength for reversed torsional loading. (State whether it is for 10^8 or 5×10^8 cycles.)

Schematic and Given Data:



Assumption: Figs. 8.8, 8.9, and 8.10 can be used to estimate long life fatigue strength for reversed torsional loading.

Analysis:

1. From Fig. 8.9, for the wrought aluminum having $S_u = 250$ MPa, the rotating bending fatigue strength at 5×10^8 cycles is $S_n' = 95$ MPa. Since, for reversed torsional loading $S_n = 0.58\ S_n'$, $S_n = 0.58(95) = 55$ MPa ■
2. From Fig. 8.9, for the wrought aluminum having $S_u = 450$ MPa, the rotating bending fatigue strength at 5×10^8 cycles is $S_n' = 130$ MPa. Thus, for reversed torsional loading, $S_n = 0.58(130) = 75$ MPa ■
3. From Fig. 8.8, for average grade cast aluminum, the rotating bending fatigue strength at 5×10^8 cycles is 60 MPa for sand cast and 85 MPa for permanent mold cast. Thus, for reversed torsional loading,
   $S_n = 0.58(60) = 35$ MPa for sand cast
   $S_n = 0.58(85) = 49$ MPa for permanent mold cast
4. From Fig. 8.10, for average grade forged magnesium, the rotating bending fatigue strength at 10^8 cycles is 130 MPa. Thus, for reversed torsional loading,
   $S_n = 0.58(130) = 75$ MPa ■

8.6 Fatigue Strength for Reversed Biaxial Loading

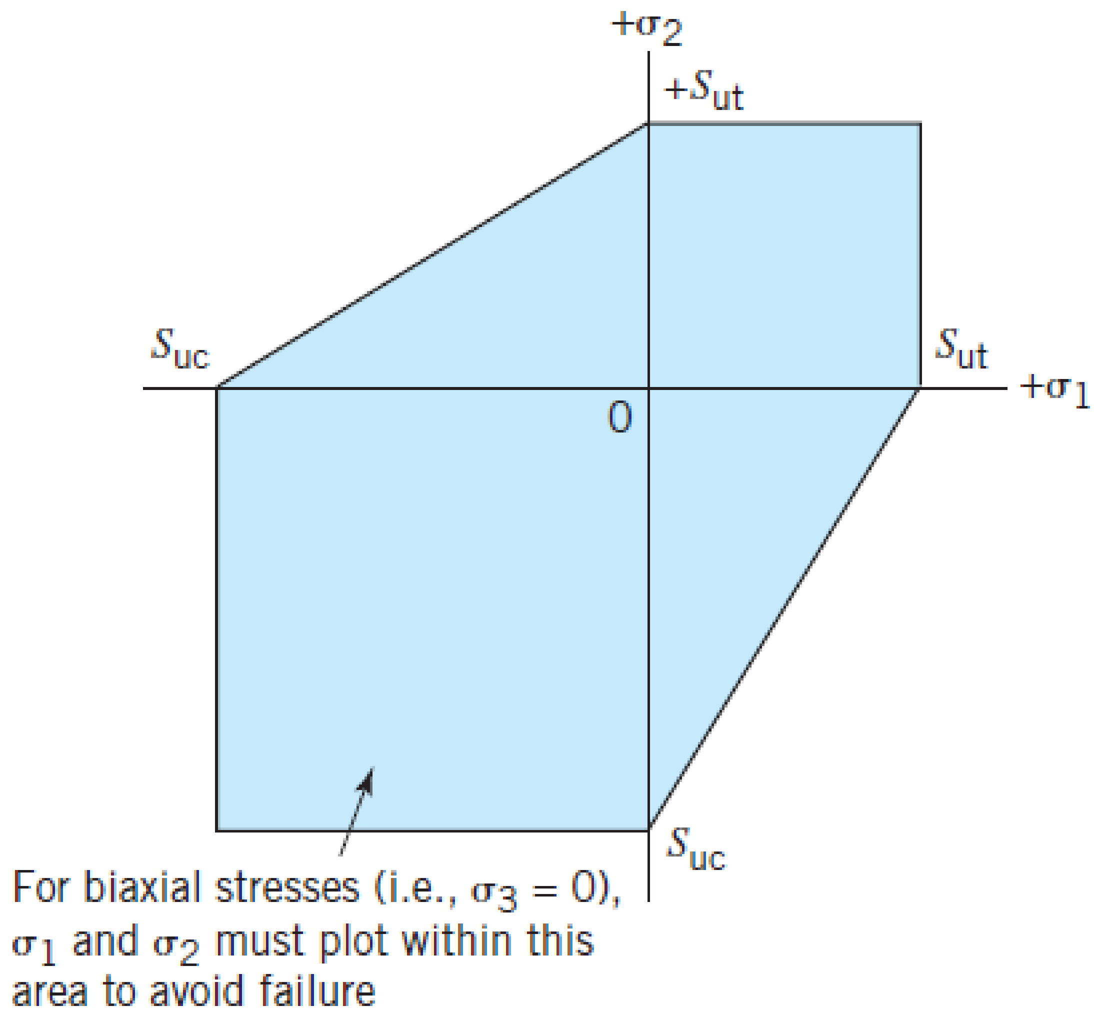
- The following procedure is tentatively recommended.

1. For ductile materials, use the distortion energy theory (usually Eq. 6.8) to convert from the actual load stresses to an equivalent stress that is regarded as a reversed bending stress. Then proceed to relate this stress to the fatigue properties of the material (i.e., the S–N curve) in reversed bending.
2. For brittle materials, use the Mohr theory to obtain an equivalent reversed stress that is regarded as a reversed bending stress, and relate this to the bending fatigue properties (i.e., S–N curve) of the material.

- (A convenient graphical procedure for determining the equivalent bending stress is to draw a $\sigma_1 - \sigma_2$ plot like that in Figure 6.11b for the material, and then plot the point corresponding to the actual reversed stresses.
- Next, draw a line through this point and parallel to the failure line. The intersection of this line with the σ_1 axis gives the desired equivalent bending stress.)

$$\sigma_e = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2} \qquad \textbf{(6.8)}$$

8.6 Fatigue Strength for Reversed Biaxial Loading



(b) $\sigma_1 - \sigma_2$ plot

8.7 Influence of Surface and Size on Fatigue Strength

8.7.1 Surface Factor

- So far fatigue strength have assumed the surface to have a special “mirror polish” finish. This requires a costly laboratory procedure but serves to minimize
  1. surface scratches and other irregularities acting as stress concentration
  2. differences in the metallurgical character of the surface layer and the interior,
  3. any residual stresses produced by the surface finishing procedure.
- Normal commercial surface finishes usually have localized points of greater fatigue vulnerability: hence lower fatigue strengths.
- The amount of “surface damage” caused by the commercial processes depends not only on the process but also on the susceptibility of the material to damage.
- Figure 8.13 gives estimated values of surface factor, C_S, for various finishes applied to steels of various hardnesses.
- In all cases the endurance limit for the laboratory polished surface X C_S to obtain the corresponding endurance limit for commercial finish.
- It is standard practice not to make any surface correction for the 10^3-cycle strength—the reason being that this is close to the strength for static loads and that the static strength of ductile parts is not influenced by surface finish

8.7 Influence of Surface and Size on Fatigue Strength

8.7.1 Surface Factor

- The surface factor for ordinary gray cast iron is approx 1. The reason is that even mirror-polished samples have surface discontinuities because of the graphite flakes in the CI matrix, so adding surface scratches does not make the situation much worse.
- There is little published information on surface factors for other mat'ls.
- For critical parts, actual fatigue tests of the material and surface in question must be conducted.

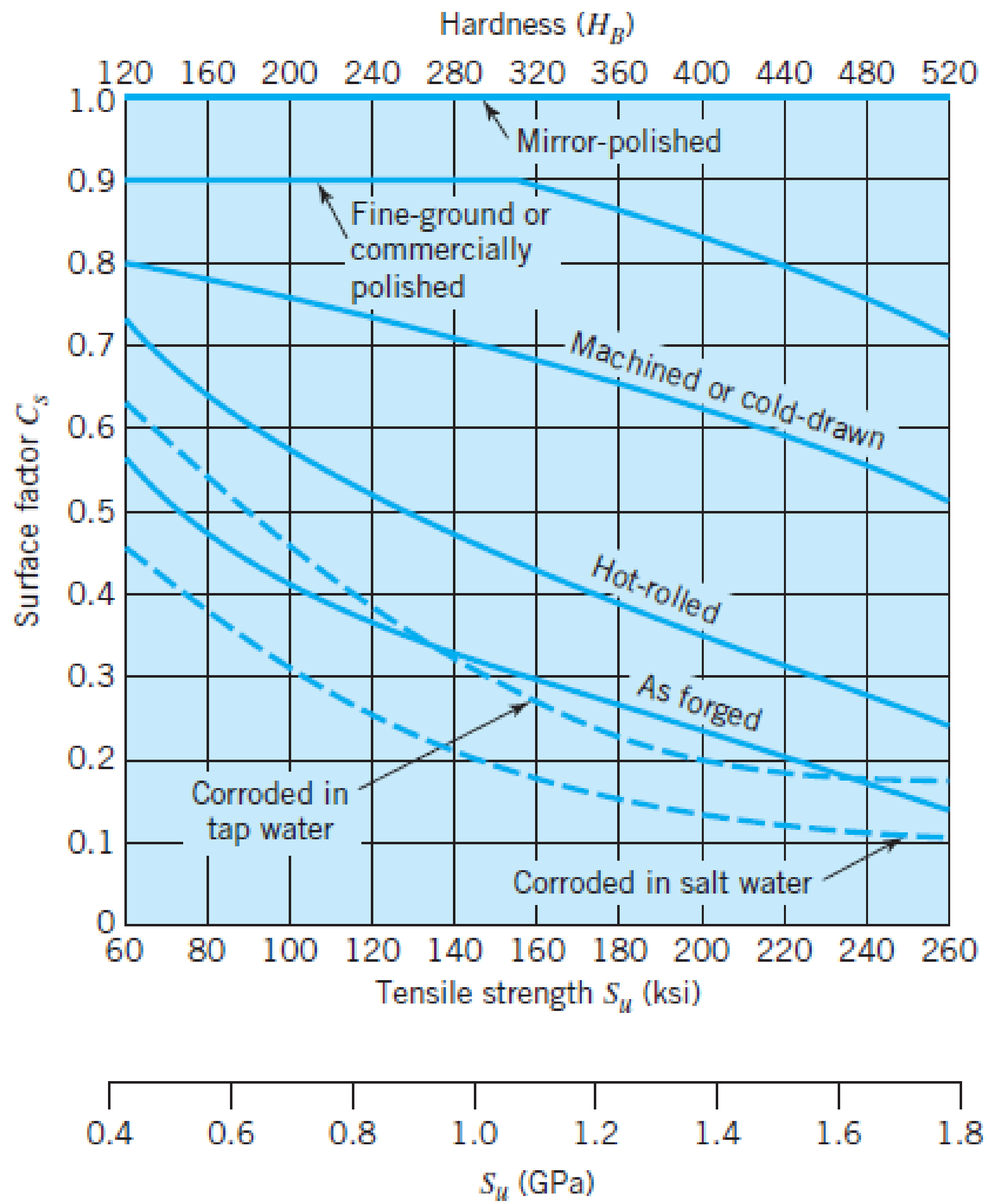


FIGURE 8.13
Reduction in endurance limit owing to surface finish—steel parts; i.e., surface factor versus tensile strength for various surface conditions.

8.7 Influence of Surface and Size on Fatigue Strength

8.7.2 Size Factor (Gradient Factor)

- In Section 8.4 it was pointed out that the endurance limit for reversed axial load is about 10 % < for reversed bending because of stress gradient.
- For the 0.3-in.-diameter bending specimen, the rapid drop in stress level below the surface is somehow beneficial. The 0.3-in.-dia axial specimen does not enjoy this benefit.
- A comparison of stress gradients in Figures 8.14a and 8.14b shows that large specimens in bending or torsion do not have the same favorable gradients as the standard 0.3-in. specimen.
- Experiments show that if the dia is > 0.4 in. (or 10 mm) , most of the beneficial gradient effect is lost. Hence a gradient factor C_G of 0.9 (even in bending/torsion)
- Figure 8.14c shows that very small parts have an even more favorable gradient than the standard R. R. Moore specimen. Thus, we might expect the endurance limit for such parts to be >for 0.3-in.-diameter parts.
- Sometimes this has been found to be the case—but unless specific data are available to substantiate this increase, it is best to use a C_G of 1 for small parts.

8.7.2 Size Factor (Gradient Factor)

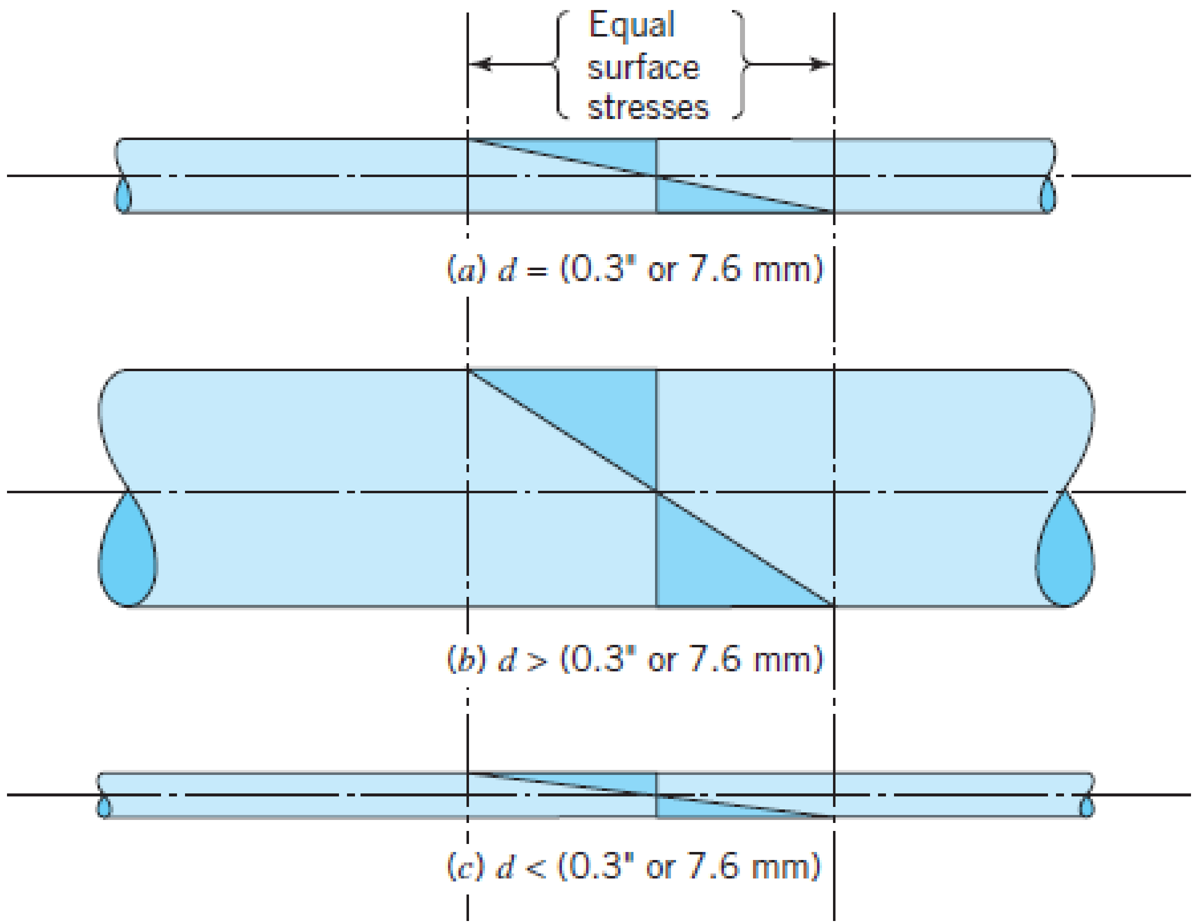


FIGURE 8.14
Stress gradients versus diameter for bending and torsion.

8.7 Influence of Surface and Size on Fatigue Strength

8.7.2 Size Factor (Gradient Factor)

- Consider the question of what C_G to use with the bending of a rectangular section, say 6 mm * 12 mm. If the bending is about the neutral axis that places the tension and compression surfaces 6 mm apart, use $C_G = 1$; if the tension and compression surfaces are 12 mm apart, use $C_G = 0.9$.
- Thus, the gradient factor is determined on the basis of an equivalent round section having the same stress gradient as the actual part.
- Recall that a C_G of 0.9 (or lower) was specified (Section 8.4) for all axially loaded parts because the stress gradient is unfavorable, regardless of size.
- Parts with sections larger than about 50-mm equivalent diameter are usually found to have lower endurance limits than those computed using the gradient factors recommended previously.
- This is due in part to metallurgical factors such as hardenability, for the interior of large-section parts is usually metallurgically different from the surface metal.
- The extent to which the endurance limit of very large parts is reduced varies substantially, and generalizations are hardly warranted.
- If the part in question is critical, there is no substitute for pertinent test data. A very rough guide for the values sometimes used is given in Table 8.1.

8.7 Influence of Surface anc

8.7.2 Size Factor (Gradient Factor)

Table 8.1 Generalized Fatigue Strength Factors for Ductile Materials (S–N curves)

a. 10^6-cycle strength (endurance limit)[a]

Bending loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Axial loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Torsional loads: $S_n = S'_n C_L C_G C_S C_T C_R$

where S'_n is the R.R. Moore, endurance limit,[b] and

		Bending	Axial	Torsion
C_L	(load factor)	1.0	1.0	0.58
C_G	(gradient factor): diameter < (0.4 in. or 10 mm)	1.0	0.7 to 0.9	1.0
	(0.4 in. or 10 mm) < diameter < (2 in. or 50 mm)[c]	0.9	0.7 to 0.9	0.9
C_S	(surface factor)	see Figure 8.13		
C_T	(temperature factor)	Values are only for steel		
	T ≤ 840 °F	1.0	1.0	1.0
	840 °F < T ≤ 1020 °F	1 - (0.0032T − 2.688)		
C_R	(reliability factor):[d]			
	50% reliability	1.000	"	"
	90% "	0.897	"	"
	95% "	0.868	"	"
	99% "	0.814	"	"
	99.9% "	0.753	"	"

b. 10^3-cycle strength[e, f, g]

Bending loads: $S_f = 0.9 S_u C_T$

Axial loads: $S_f = 0.75 S_u C_T$

Torsional loads: $S_f = 0.9 S_{us} C_T$

where S_u is the ultimate tensile strength and S_{us} is the ultimate shear strength.

[a] For materials not having the endurance limit, apply the factors to the 10^8 or 5×10^8-cycle strength.

[b] $S'_n = 0.5 S_u$ for steel, lacking better data.

[c] For (2 in. or 50 mm) < diameter < (4 in. or 100 mm) reduce these factors by about 0.1. For (4 in. or 100 mm) < diameter < (6 in. or 150 mm), reduce these factors by about 0.2.

[d] The factor, C_R, corresponds to an 8 percent standard deviation of the endurance limit. For example, for 99% reliability we shift −2.326 standard deviations, and $C_R = 1\text{-}2.326(0.08) = 0.814$.

[e] No corrections for gradient or surface are normally made, but the experimental value of S_u or S_{us} should pertain to sizes reasonably close to those involved.

[f] No correction is usually made for reliability at 10^3 cycle strength.

[g] $S_{us} \approx 0.8 S_u$ for steel; $S_{us} \approx 0.7 S_u$ for other ductile metals.

8.8 Summary of Estimated Fatigue Strengths for Completely Reversed Loading

- The foregoing sections have emphasized the desirability of obtaining actual fatigue test data that pertain as closely as possible to the application.
- Generalized empirical factors were given for use when such data are not available. These factors can be applied with greatest confidence to steel parts because most of the data on which they are based came from testing steel specimens.
- Five of these factors are involved in the estimate for endurance limit:
- The temperature factor, C_T, accounts for the fact that the strength of a material decreases with increased temperature, and the reliability factor, C_R, acknowledges that a more reliable (> 50%) estimate of endurance limit requires using a lower value of endurance limit.
- Table 8.1 gives a summary of all factors used for estimating the fatigue strength of ductile materials (when subjected to completely reversed loading). It serves as a convenient reference for solving problems.

$$S_n = S'_n C_L C_G C_S C_T C_R \tag{8.1}$$