MECH 344/M
Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 5

8.8 Summary of Estimated Fatigue Strengths for Completely Reversed Loading

- We have so far emphasized the desirability of obtaining actual fatigue test data that pertain as closely as possible to the application.
- Generalized empirical factors were given for use when such data are not available. These factors can be applied with greatest confidence to steel parts because most of the data on which they are based came from testing steel specimens.
- Five of these factors are involved in the estimate for endurance limit:
- The temperature factor, C_T, accounts for the fact that the strength of a material decreases with increased temperature, and the reliability factor, C_R, acknowledges that a more reliable (> 50%) estimate of endurance limit requires using a lower value of endurance limit.
- Table 8.1 gives a summary of all factors used for estimating the fatigue strength of ductile materials (when subjected to completely reversed loading). It serves as a convenient reference for solving problems.

$$S_n = S'_n C_L C_G C_S C_T C_R \tag{8.1}$$

TABLE 8.1 Generalized Fatigue Strength Factors for Ductile Materials (*S–N* curves)

a. 10^6-cycle strength (endurance limit)[a]

Bending loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Axial loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Torsional loads: $S_n = S'_n C_L C_G C_S C_T C_R$

where S'_n is the R.R. Moore, endurance limit,[b] and

		Bending	Axial	Torsion
C_L	(load factor)	1.0	1.0	0.58
C_G	(gradient factor): diameter < (0.4 in. or 10 mm)	1.0	0.7 to 0.9	1.0
	(0.4 in. or 10 mm) < diameter < (2 in. or 50 mm)[c]	0.9	0.7 to 0.9	0.9
C_S	(surface factor)	see Figure 8.13		
C_T	(temperature factor)	Values are only for steel		
	T ≤ 840 °F	1.0	1.0	1.0
	840 °F < T ≤ 1020 °F	1 - (0.0032T − 2.688)		

C_R (reliability factor):[d]			
50% reliability	1.000	"	"
90% "	0.897	"	"
95% "	0.868	"	"
99% "	0.814	"	"
99.9% "	0.753	"	"

b. 10^3-cycle strength[e, f, g]

Bending loads: $S_f = 0.9S_u C_T$

Axial loads: $S_f = 0.75S_u C_T$

Torsional loads: $S_f = 0.9S_{us} C_T$

where S_u is the ultimate tensile strength and S_{us} is the ultimate shear strength.

[a] For materials not having the endurance limit, apply the factors to the 10^8 or 5×10^8-cycle strength.

[b] $S'_n = 0.5S_u$ for steel, lacking better data.

[c] For (2 in. or 50 mm) < diameter < (4 in. or 100 mm) reduce these factors by about 0.1. For (4 in. or 100 mm) < diameter < (6 in. or 150 mm), reduce these factors by about 0.2.

[d] The factor, C_R, corresponds to an 8 percent standard deviation of the endurance limit. For example, for 99% reliability we shift −2.326 standard deviations, and $C_R = 1\text{-}2.326(0.08) = 0.814$.

[e] No corrections for gradient or surface are normally made, but the experimental value of S_u or S_{us} should pertain to sizes reasonably close to those involved.

[f] No correction is usually made for reliability at 10^3 cycle strength.

[g] $S_{us} \approx 0.8S_u$ for steel; $S_{us} \approx 0.7S_u$ for other ductile metals.

8.9 Effect of Mean Stress on Fatigue Strength

- Machine and structural parts seldom encounter completely reversed stresses; rather, they typically encounter a fluctuating stress static plus reversed stress.
- Fluctuating stress is usually characterized by its mean & alternating components. Also, the terms maximum stress and minimum stress are used. Figure 8.15
- Note that if any two of them are known, the others are readily computed.
- This text uses primarily mean and alternating stress components, as in Figure 8.16. The same information can be portrayed graphically with any combination of two of the stress components shown in Figure 8.15.
- For example, σ_m–σ_{max} coordinates are often found in the literature. For convenience, some graphs use all four quantities, as in Figures 8.17 through 8.19.

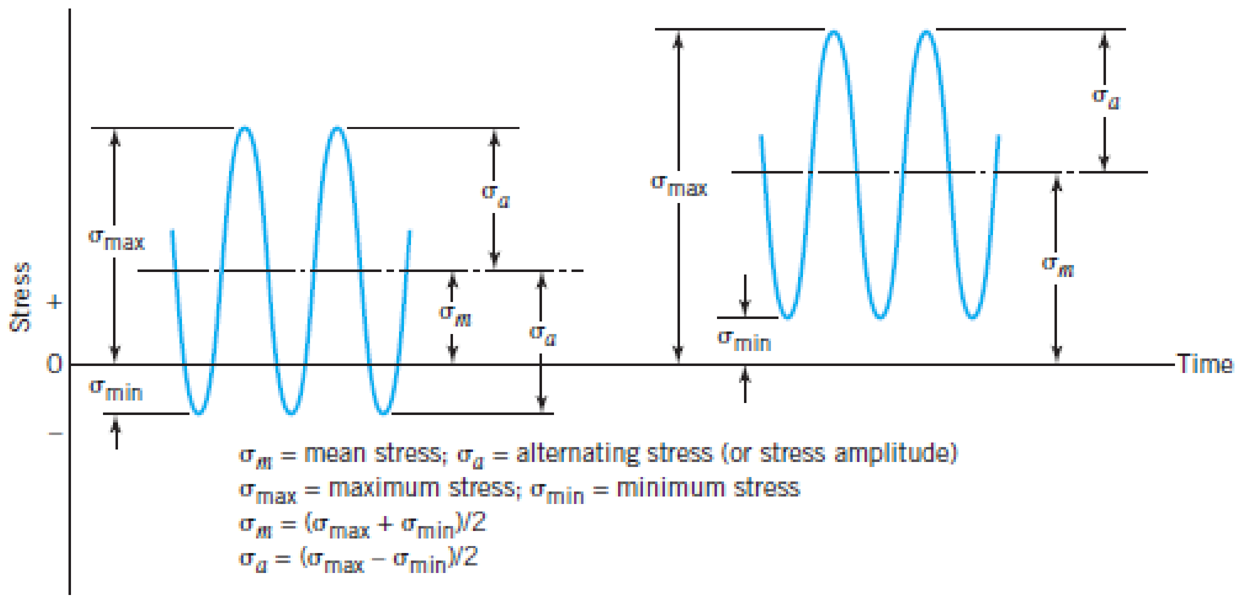


FIGURE 8.15
Fluctuating stress notation illustrated with two examples.

8.9 Effect of Mean Stress on Fatigue Strength

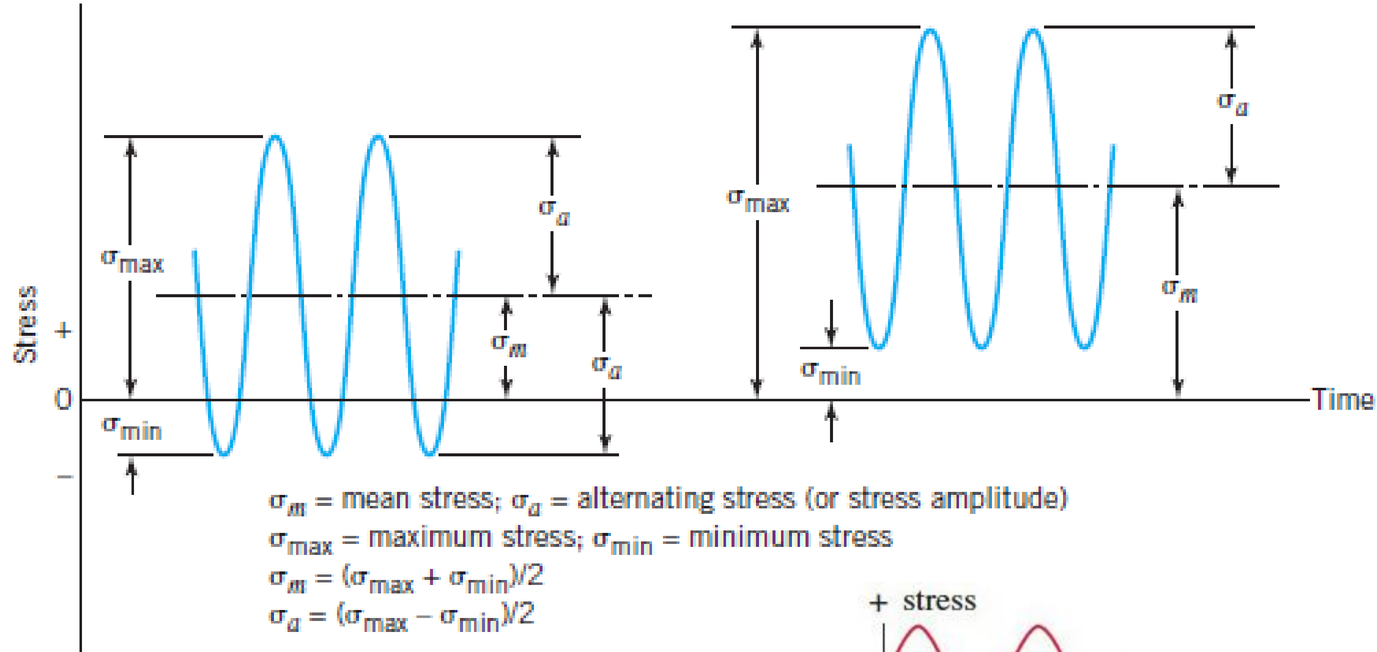
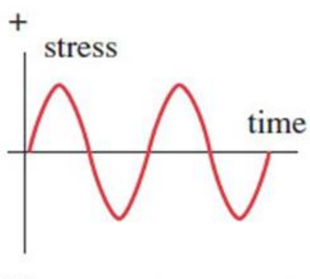


FIGURE 8.15
Fluctuating stress notation illustrated with two examples.



$\sigma_{max} = -\sigma_{min}$

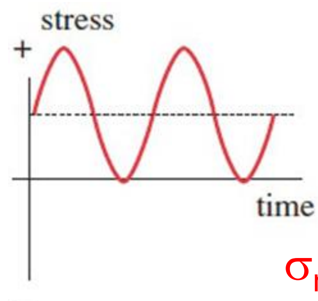
(a) Fully reversed

$\sigma_m = (\sigma_{max} + \sigma_{min})/2$

$\sigma_a = (\sigma_{max} - \sigma_{min})/2$

So $\sigma_m = 0$

$\sigma_{max} = \sigma_a$



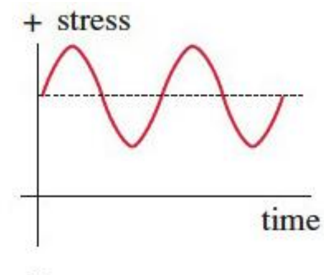
$\sigma_{min} = 0$

(b) Repeated

$\sigma_m = (\sigma_{max} + \sigma_{min})/2$

$\sigma_a = (\sigma_{max} - \sigma_{min})/2$

So $\sigma_m = \sigma_a = \sigma_{max} / 2$



(c) Fluctuating $\sigma_{min} \neq 0$

$\sigma_m = (\sigma_{max} + \sigma_{min})/2$

$\sigma_a = (\sigma_{max} - \sigma_{min})/2$

So $\sigma_m = (\sigma_{max} + \sigma_{min})/ 2$

and $\sigma_a = (\sigma_{max} - \sigma_{min})/ 2$

8.9 Effect of Mean Stress on Fatigue Strength

- Here we use mean σ_m and alternating stress σ_a components, as in Figure 8.16.
- The same information can be portrayed graphically with any combination of two of the stress components shown in Figure 8.15.
- For example, σ_m–σ_{max} coordinates are often found in the literature. For convenience, some graphs use all four quantities, as in Figures 8.17 through 8.19.

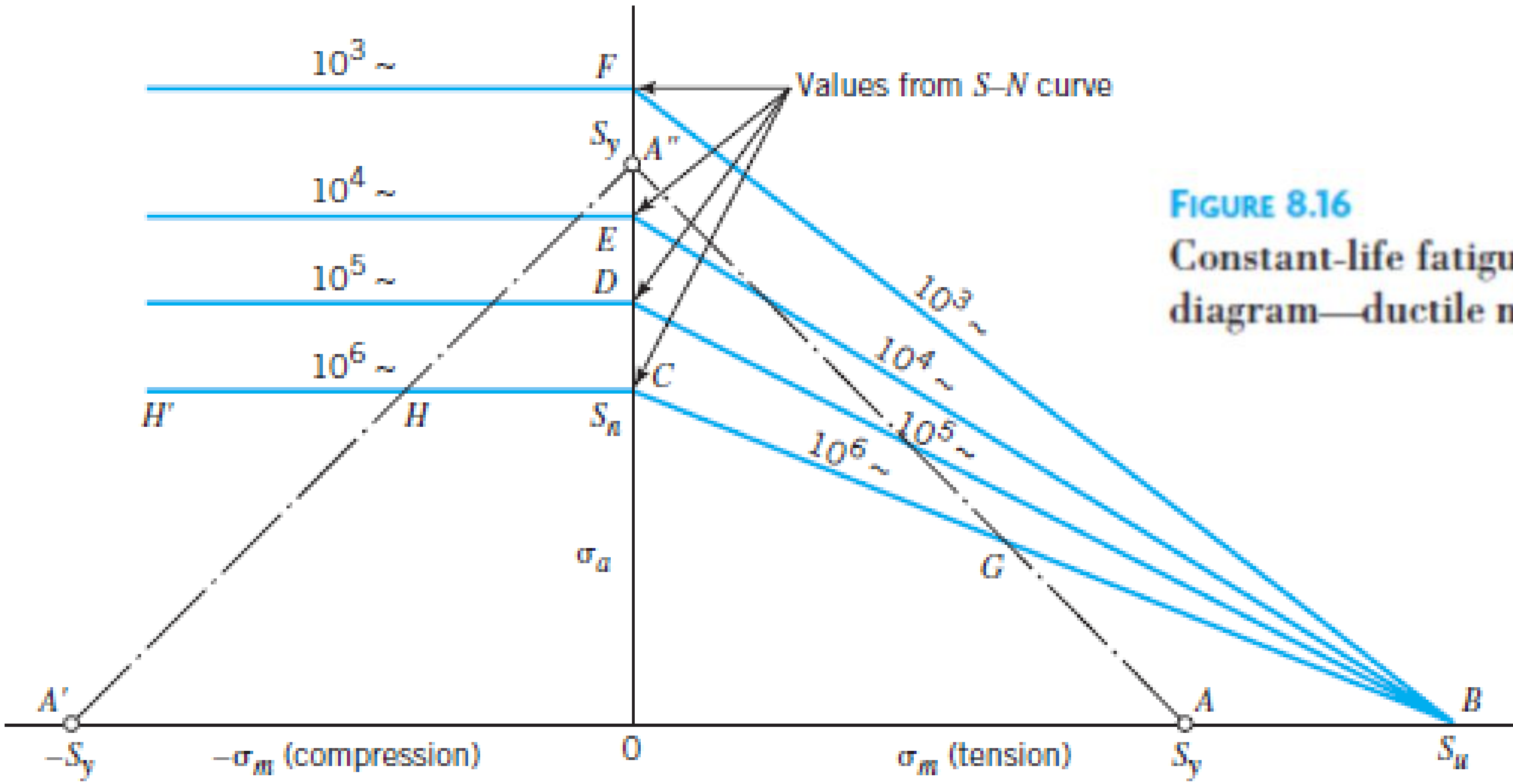


FIGURE 8.16
Constant-life fatigue diagram—ductile materials.

8.9 Effect of Mean Stress on Fatigue Strength

TABLE 8.2 Construction of σ_a versus σ_m Constant Life Fatigue Diagram

Load Type	Directions
Bending	Construct diagram as shown; take points *C*, *D*, and so on from *S*–*N* curve for reversed bending.
Axial	Construct diagram as shown; take points *C*, and so on from *S*–*N* curve for reversed axial loads.
Torsional	Omit left half of diagram (*any* torsional mean stress is considered positive); take points *C* and so on from *S*–*N* curve for reversed torsion; use S_{sy} and S_{us} instead of S_y and S_u. (For steel, $S_{us} \approx 0.8S_u$, $S_{sy} \approx 0.58S_y$.)
General biaxial	Construct the diagram as for *bending* loads, and use it with *equivalent* load stresses, computed as follows. (Note that these equations apply to the generally encountered situation where σ_a and σ_m exist in one direction only. Corresponding equations for the more elaborate general case are also tentatively applicable.) 1. Equivalent alternating bending stress, σ_{ea}, is calculated from the *distortion energy theory* as being *equivalent* to the combination of existing *alternating* stresses: $\sigma_{ea} = \sqrt{\sigma_a^2 + 3\tau_a^2}$ **(a)** 2. Equivalent *mean* bending stress σ_{em}, is taken as the *maximum principal stress* resulting from the superposition of all existing static (mean) stresses. Use Mohr circle, or $\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau_m^2 + \left(\frac{\sigma_m}{2}\right)^2}$ **(b)** [For more complex loading, various other suggested equations for σ_{ea} and σ_{em} are found in the literature.]

8.9 Effect of Mean Stress on Fatigue Strength

- The existence of a static tensile stress reduces the amplitude of reversed stress that can be superimposed. Figure 8.20 illustrates this concept.
- Fluctuation a is a completely reversed stress corresponding to the endurance limit—the mean stress is zero and the alternating stress σ_n. Fluctuation b involves a tensile mean stress. In order to have an equal (in this case, “infinite”) fatigue life, the alternating stress must be < σ_n. In going from b to c, d, e, and f, the mean stress continually increases; hence, the alternating stress must decrease.
- Note that in each case the stress fluctuation is shown as starting from zero, and that the stresses are computed P/A values.
- Microscopic yielding occurs even at a, as has previously been noted.
- Upon reaching d, macroscopic yielding begins. Although load fluctuations e and f give “infinite” life, the part is yielded on the first load application.

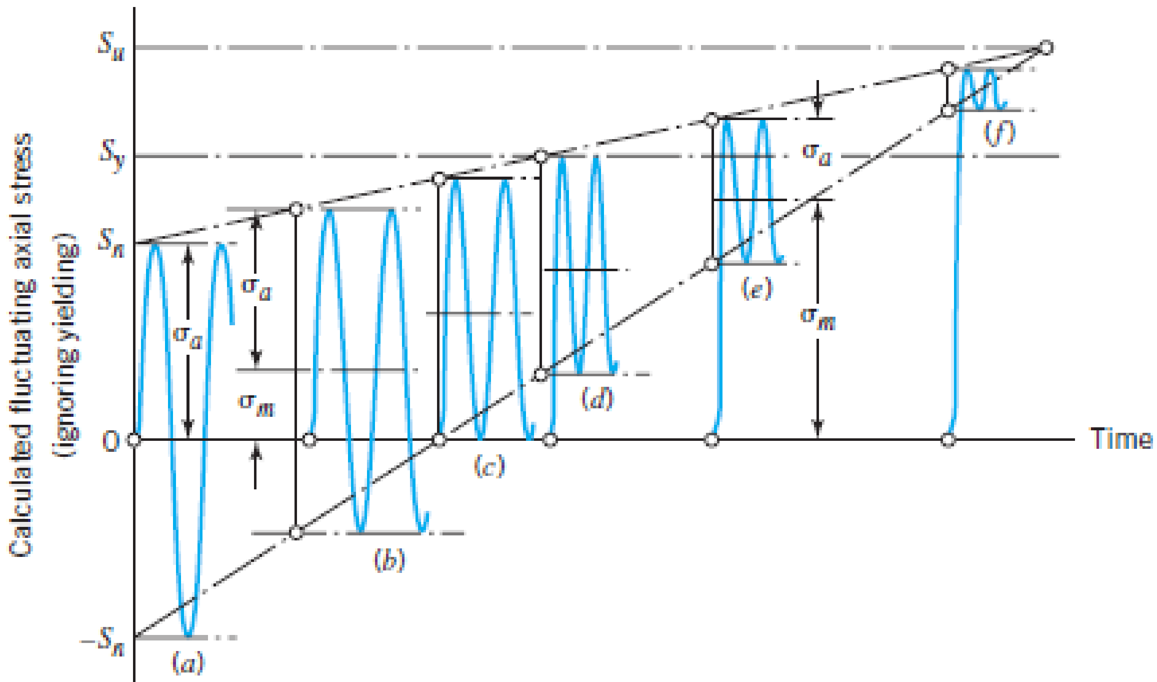


FIGURE 8.20
Various fluctuating uniaxial stresses, all of which correspond to equal fatigue life.

8.9 Effect of Mean Stress on Fatigue Strength

- Figure 8.16 - graphical representation of diff mean and alternating stress in relation to criteria both for yielding and for various fatigue lives. It is often called a constant-life fatigue diagram because it has lines corresponding to a constant 10^6-cycle (or "infinite") life, constant 10^5-cycle life, and so forth.
- To begin the construction of this diagram, put on it first the information that is already known. The horizontal axis ($\sigma_a = 0$) corresponds to static loading.
- Yield and ultimate strengths are plotted at points A and B. For ductile materials, the compressive yield strength is $-S_y$, and this is plotted at point A'.
- If the mean stress is zero and the alternating stress is equal to Sy (point A"), the stress fluctuates between $\pm$Sy
- All points along the line AA" correspond to fluctuations with tensile peak of S_y; and all points on A'A" correspond to compressive peaks = -Sy .
- All combinations of σ_m & σ_a causing no (macroscopic) yielding are contained within triangle AA'A".

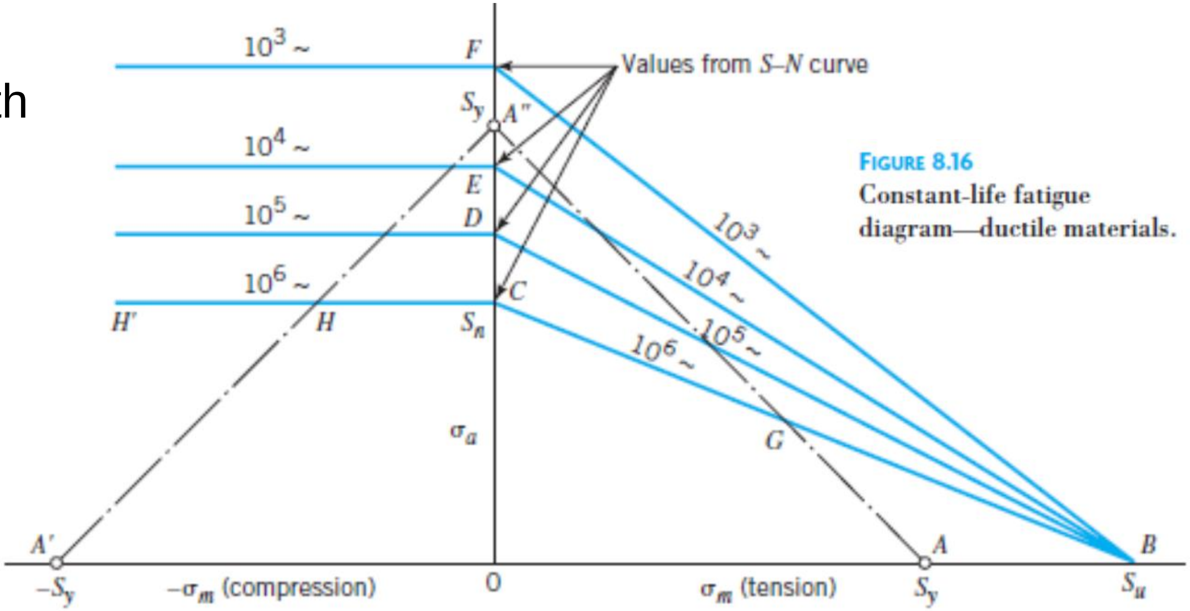


FIGURE 8.16
Constant-life fatigue diagram—ductile materials.

8.9 Effect of Mean Stress on Fatigue Strength

- All S–N curves considered here correspond to $\sigma_m = 0$. Hence, we can read from these curves points like C, D, E, and F for any fatigue life of interest.
- Connecting these points with B gives estimated lines of constant life. This empirical procedure for obtaining constant-life lines is credited to Goodman; hence the lines are commonly called Goodman lines.
- Laboratory tests have consistently indicated that compressive mean stresses do not reduce the amplitude of allowable alternating stress; if anything, they slightly increase it. Figure 8.16 is thus conservative in showing the constant-life lines as horizontal to the left of points C, D, and so on. (The lines apparently extend indefinitely as far as fatigue is concerned, the limitation being only static compression failure.)

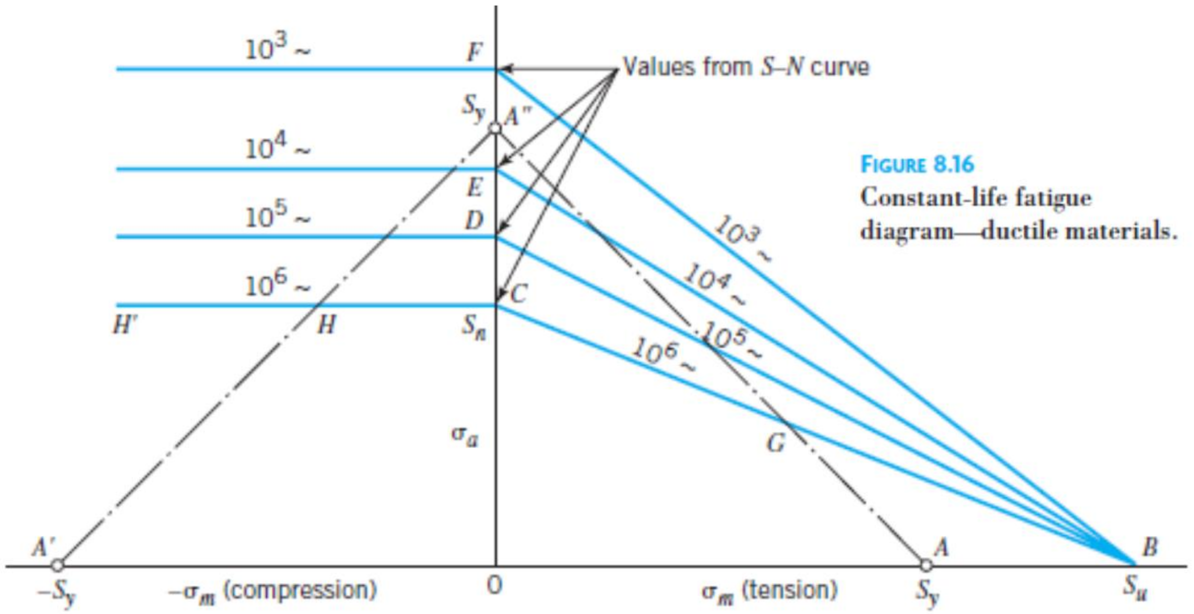


FIGURE 8.16
Constant-life fatigue diagram—ductile materials.

8.9 Effect of Mean Stress on Fatigue Strength

1. If a life >10^6 cycles is required and no yielding is permitted stay inside A'HCGA.
2. If no yielding but <10^6 cycles of life required, work within some or all of HCGA'H.
3. If 10^6 cycles of life are required but yielding acceptable, area AGB (and area to the left of A'H) may be used, in addition to area A'HCGA.
4. Area above A"GB (and above A"HH') corresponds to yielding on the first application of load and fatigue fracture prior to 10^6 cycles of loading.

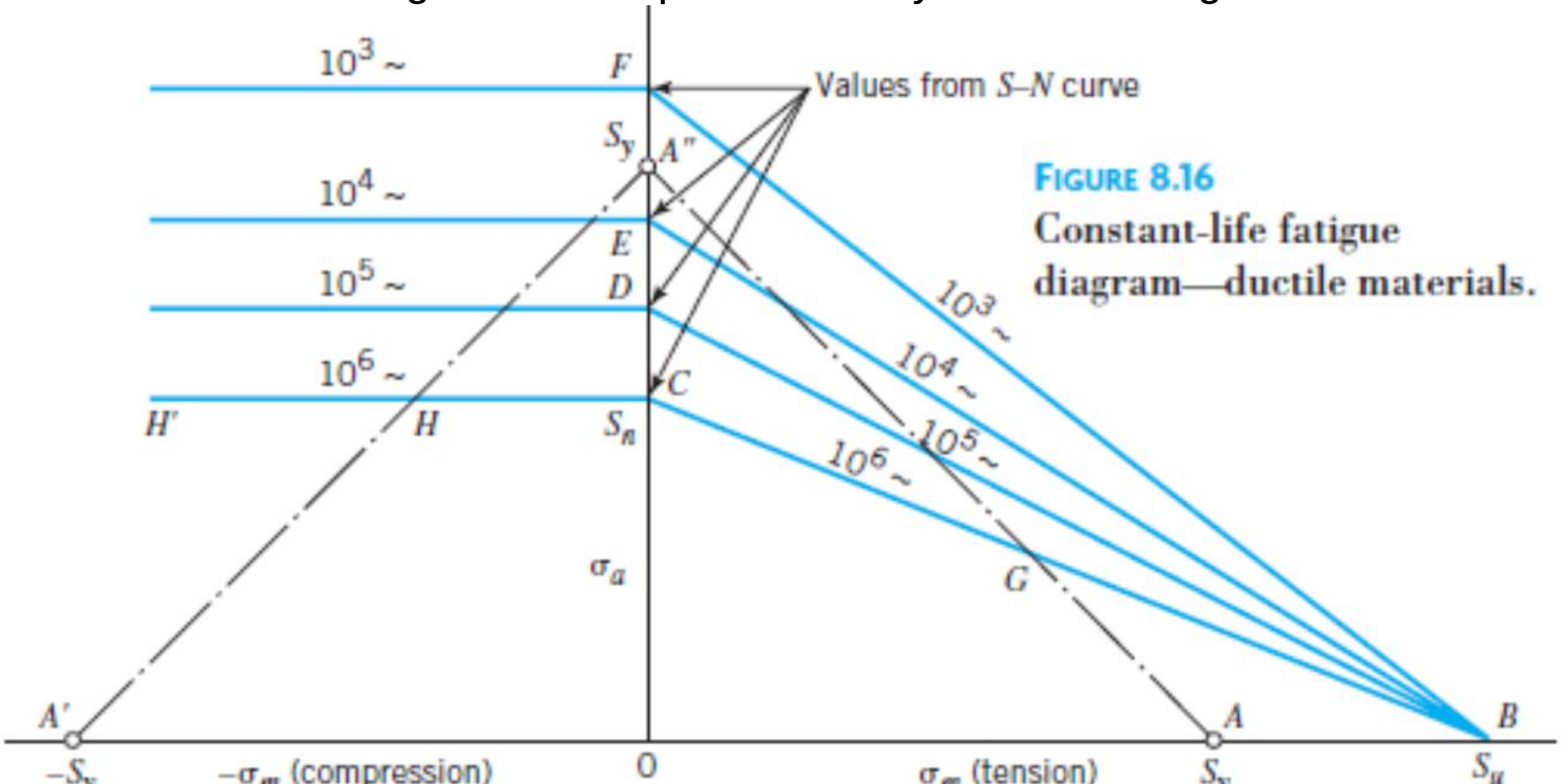


Figure 8.16 Constant-life fatigue diagram—ductile materials.

8.9 Effect of Mean Stress on Fatigue Strength

- The procedure for general biaxial loads given in Figure 8.16 is simplification of a very complex situation. It applies best to situations involving long life, where the loads are all in phase, where the principal axes for σ_m and σ_a are same, and where these axes are fixed with time.
- For an illustration in which these conditions would be fulfilled, consider the example in Figure 4.25 with the shaft stationary, and with the 2000-lb static load changed to a load that fluctuates between 1500 and 2500 lb.
- The static stresses on element A would be unchanged, but alternating stresses would be added. The alternating bending and the alternating torsion would obviously be in phase, the principal planes for σ_m and σ_a would be the same, and these planes would remain the same as the load fluctuated.

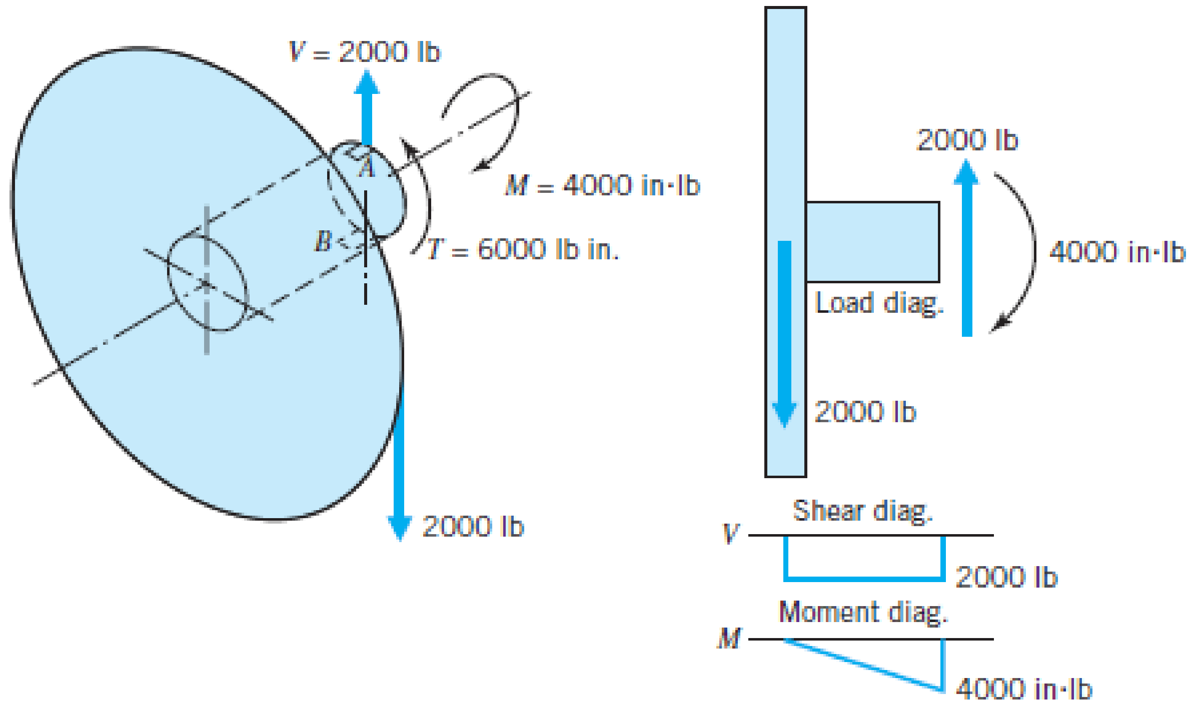


FIGURE 4.25
Free-body and load diagrams.

8.9 Effect of Mean Stress on Fatigue Strength

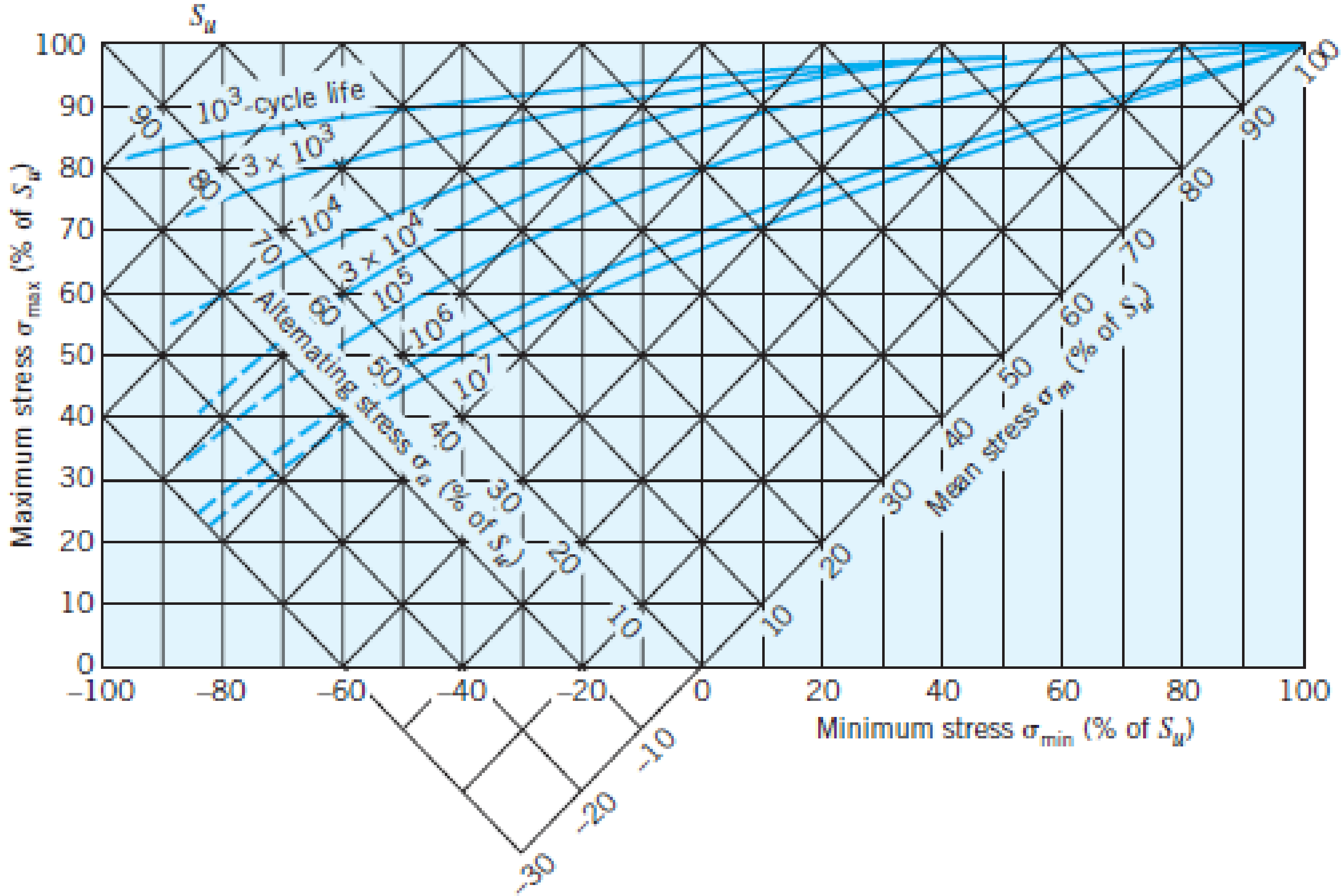


FIGURE 8.17

Fatigue strength diagram for alloy steel, S_u = 125 to 180 ksi, axial loading. Average of test data for polished specimens of AISI 4340 steel (also applicable to other alloy steels, such as AISI 2330, 4130, 8630). (Courtesy Grumman Aerospace Corporation.)

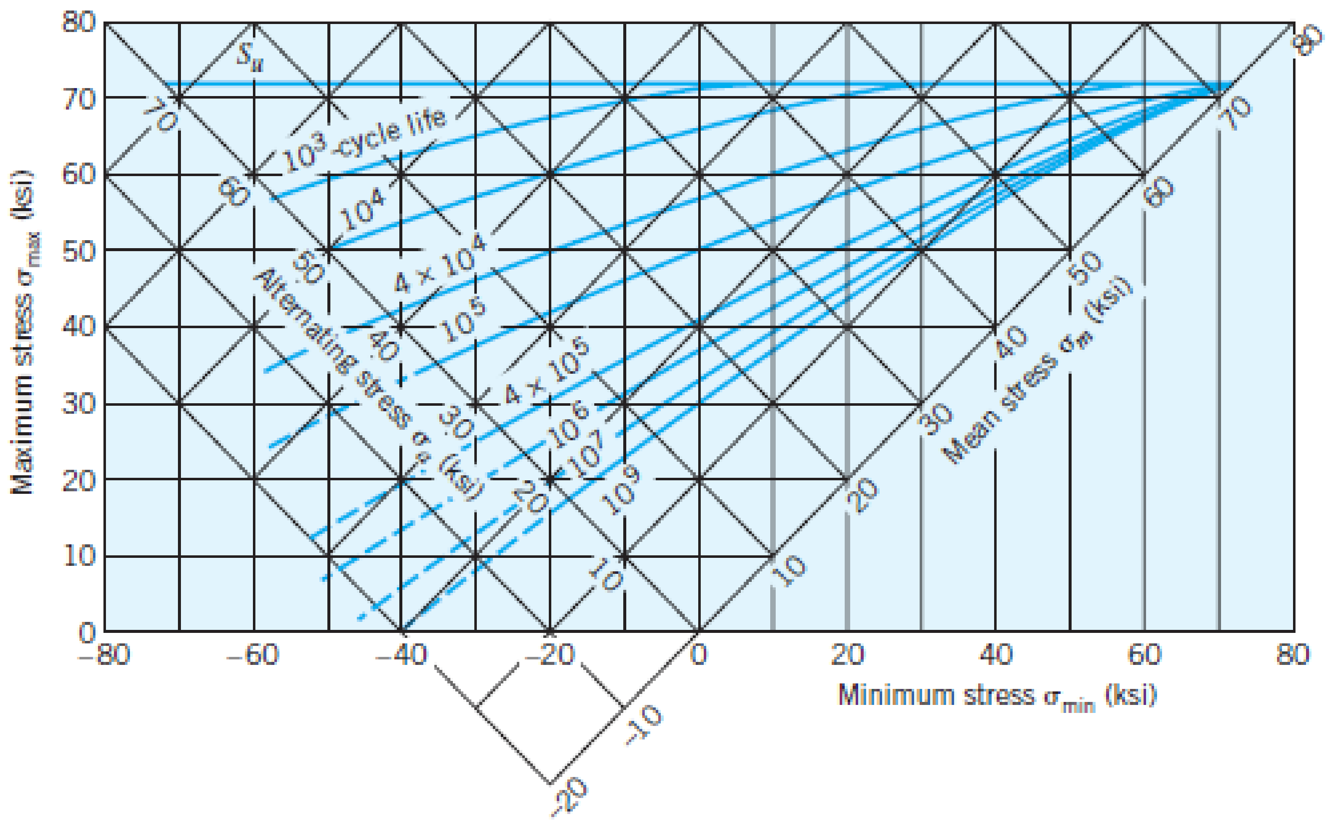


FIGURE 8.18

Fatigue strength diagram for 2024-T3, 2024-T4, and 2014-T6 aluminum alloys axial loading. Average of test data for polished specimens (unclad) from rolled and drawn sheet and bar. Static properties for 2024: $S_u = 72$ ksi, $S_y = 52$ ksi; for 2014: $S_u = 72$ ksi, $S_y = 63$ ksi. (Courtesy Grumman Aerospace Corporation.)

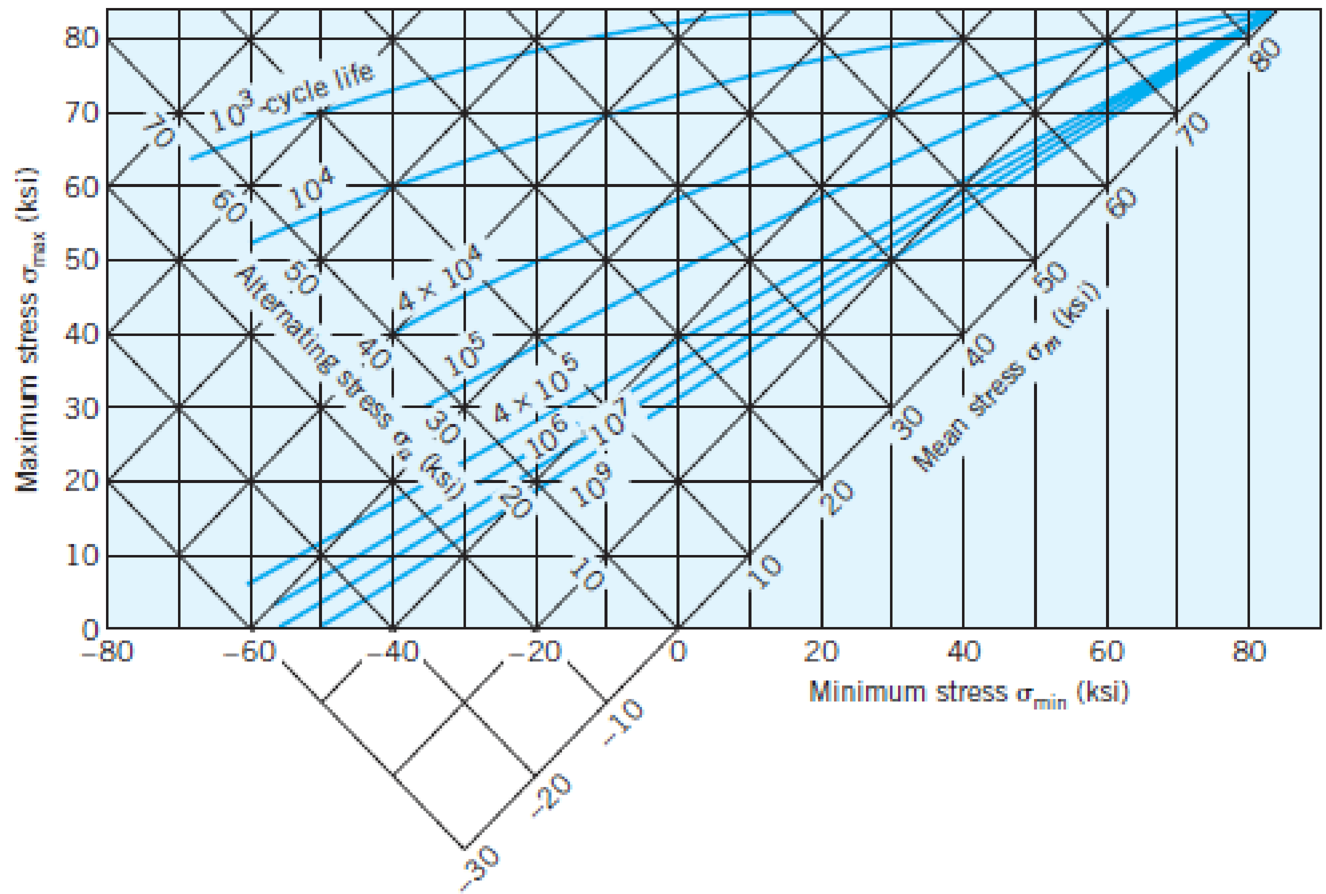


FIGURE 8.19

Fatigue strength diagram for 7075-T6 aluminum alloy, axial loading. Average of test data for polished specimens (unclad) from rolled and drawn sheet and bar. Static properties: $S_u = 82$ ksi, $S_y = 75$ ksi. (Courtesy Grumman Aerospace Corporation.)

8.9 Effect of Mean Stress on Fatigue Strength

- Figures 8.17 through 8.19 give constant-life fatigue strengths for some steel and aluminum materials.
- They differ from Figure 8.16 in the following respects.

1. Figures 8.17 - 8.19 represent actual experimental data for materials involved, while Figure 8.16 shows conservative empirical relationships that are applicable.
2. Figures 8.17 - 8.19 are “turned 45°,” with scales added to show σ_{max} ,$\sigma_{min,}$ σ_m & σ_a.
3. Yield data are not shown on these figures.
4. The experimental constant-life lines shown have some curvature, indicating that Figure 8.16 errs a little on the conservative side in both the straight Goodman lines and in the horizontal lines for compressive mean stress. This conservatism usually exists for ductile but not for brittle materials. Experimental points for brittle materials are usually on or slightly below the Goodman line.

- When experimental data like those given in Figures 8.17 - 8.19 are available, these are to be preferred over the estimated constant-life fatigue curves constructed in Figure 8.16.
- Figure 8.16 and Table 8.1 provide helpful summaries of information pertaining to the solution of a large variety of fatigue problems.

SAMPLE PROBLEM 8.1 Estimation of S–N and Constant-Life Curves from Tensile Test Data

- Using the empirical relationships, estimate the S–N curve and a family of constant-life fatigue curves pertaining to the axial loading of precision steel parts having S_u = 150 ksi, S_y = 120 ksi, and commercially polished surfaces.
- All cross-section dimensions are to be under 2 in.
- **Known**: A commercially polished steel part having a known size and made of a material with specified S_u and S_y is axially loaded
- **Find**: Estimate the S–N curve and construct constant-life fatigue curves.

Schematic and Given Data:

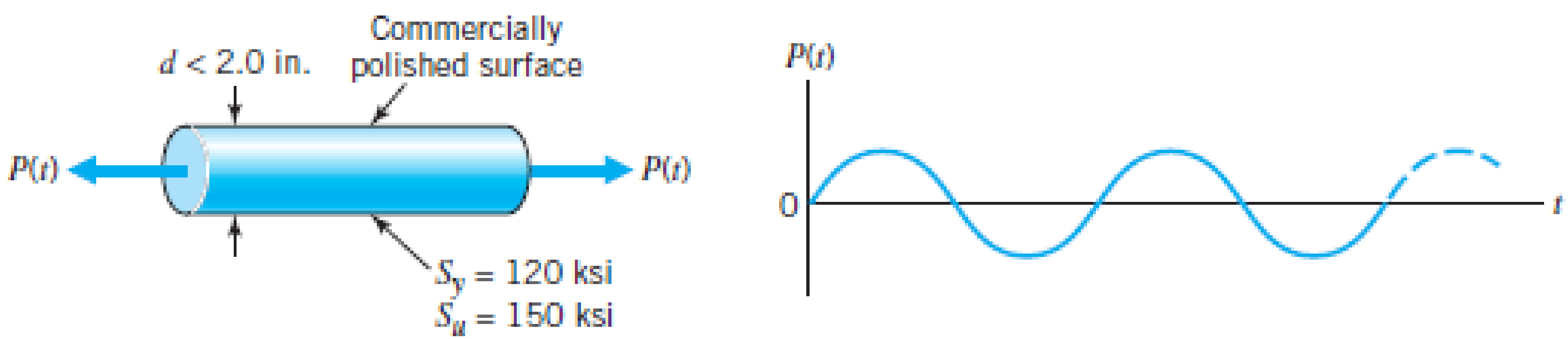


FIGURE 8.21
Axial loading of precision steel part.

SAMPLE PROBLEM 8.1 Estimation of S–N and Constant-Life Curves from Tensile Test Data

Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S–N curve constructed using Table 8.1 and the constant-life fatigue curves constructed according to Figure 8.16 are adequate.
3. The gradient factor, $C_G = 0.9$. The temperature factor, $C_T = 1.0$, and the reliability factor, $C_R = 1.0$.

Analysis:

1. From Table 8.1, the 10^3-cycle peak alternating strength for axially loaded ductile material is $S_{10^3} = 0.75S_u = 0.75(150) = 112$ ksi.
2. Also from Table 8.1, the 10^6-cycle peak alternating strength for axially loaded ductile material is $S_n = S'_n C_L C_G C_S C_T C_R$ where $S'_n = (0.5)(150) = 75$ ksi, $C_L = 1.0$, $C_G = 0.9$, $C_T = 1.0$, $C_R = 1.0$ and from Figure 8.13, $C_S = 0.9$; then $S_n = 61$ ksi.
3. The estimated S–N curve is given in Figure 8.22.
4. From the estimated S–N curve we determine that the peak alternating strengths at 10^4 and 10^5 cycles are, respectively, 92 and 75 ksi.
5. The estimated σ_m–σ_a curves for 10^3, 10^4, 10^5, and 10^6 cycles of life are given in Figure 8.22.

Table 8.1 Generalized Fatigue Strength Factors for Ductile Materials (*S–N* curves)

a. 10^6-cycle strength (endurance limit)[a]

Bending loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Axial loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Torsional loads: $S_n = S'_n C_L C_G C_S C_T C_R$

where S'_n is the R.R. Moore, endurance limit,[b] and

		Bending	Axial	Torsion
C_L	(load factor)	1.0	1.0	0.58
C_G	(gradient factor): diameter $<$ (0.4 in. or 10 mm)	1.0	0.7 to 0.9	1.0
	(0.4 in. or 10 mm) $<$ diameter $<$ (2 in. or 50 mm)[c]	0.9	0.7 to 0.9	0.9
C_S	(surface factor)		see Figure 8.13	
C_T	(temperature factor)		Values are only for steel	
	T $\leq$ 840 °F	1.0	1.0	1.0
	840 °F $<$ T $\leq$ 1020 °F		1 - (0.0032T − 2.688)	
C_R	(reliability factor):[d]			
	50% reliability	1.000	"	"
	90% "	0.897	"	"
	95% "	0.868	"	"
	99% "	0.814	"	"
	99.9% "	0.753	"	"

b. 10^3-cycle strength[e, f, g]

Bending loads: $S_f = 0.9 S_u C_T$

Axial loads: $S_f = 0.75 S_u C_T$

Torsional loads: $S_f = 0.9 S_{us} C_T$

where S_u is the ultimate tensile strength and S_{us} is the ultimate shear strength.

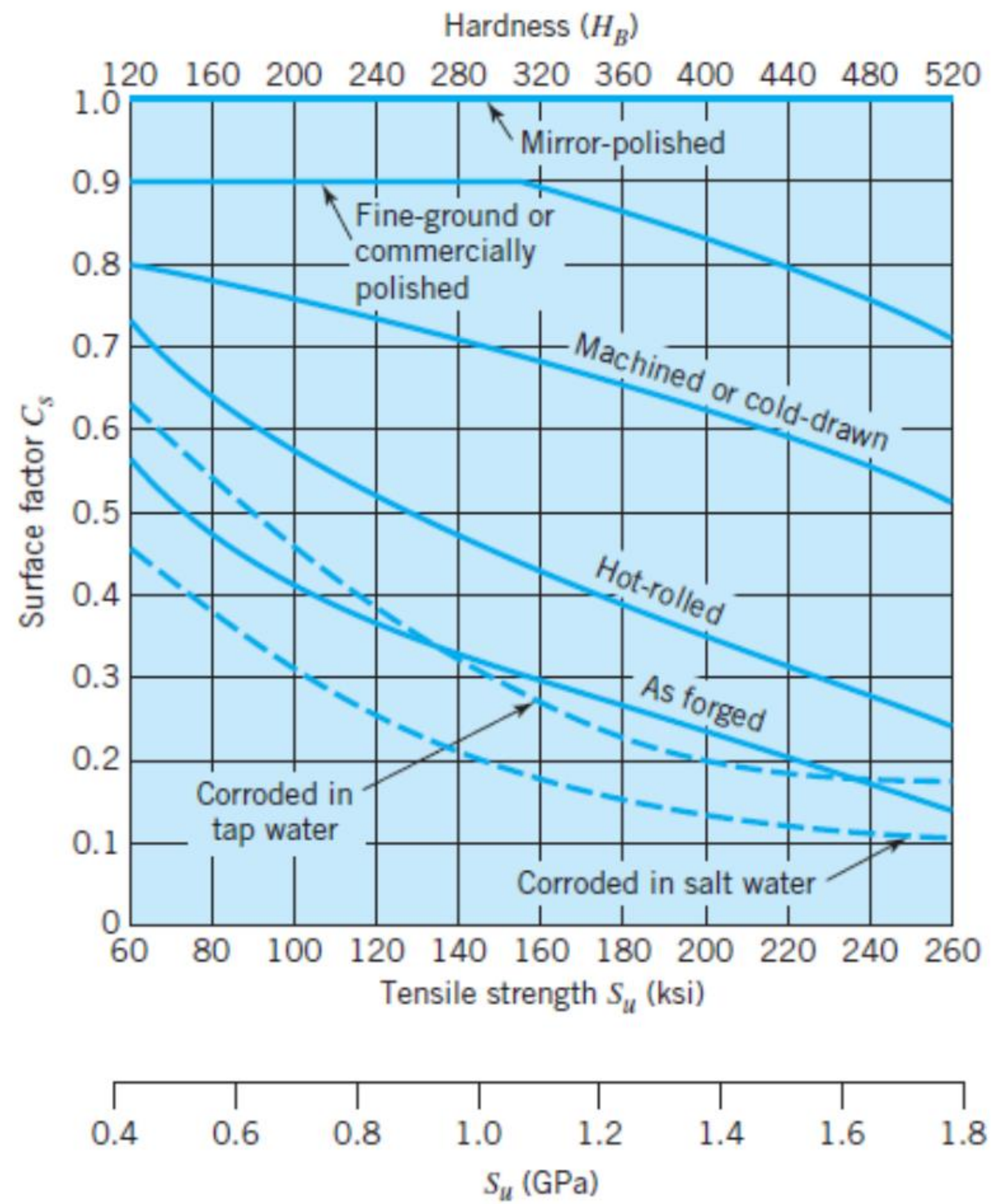


FIGURE 8.13

Reduction in endurance limit owing to surface finish—steel parts; i.e., surface factor versus tensile strength for various surface conditions.

SAMPLE PROBLEM 8.1 Estimation from Tensil

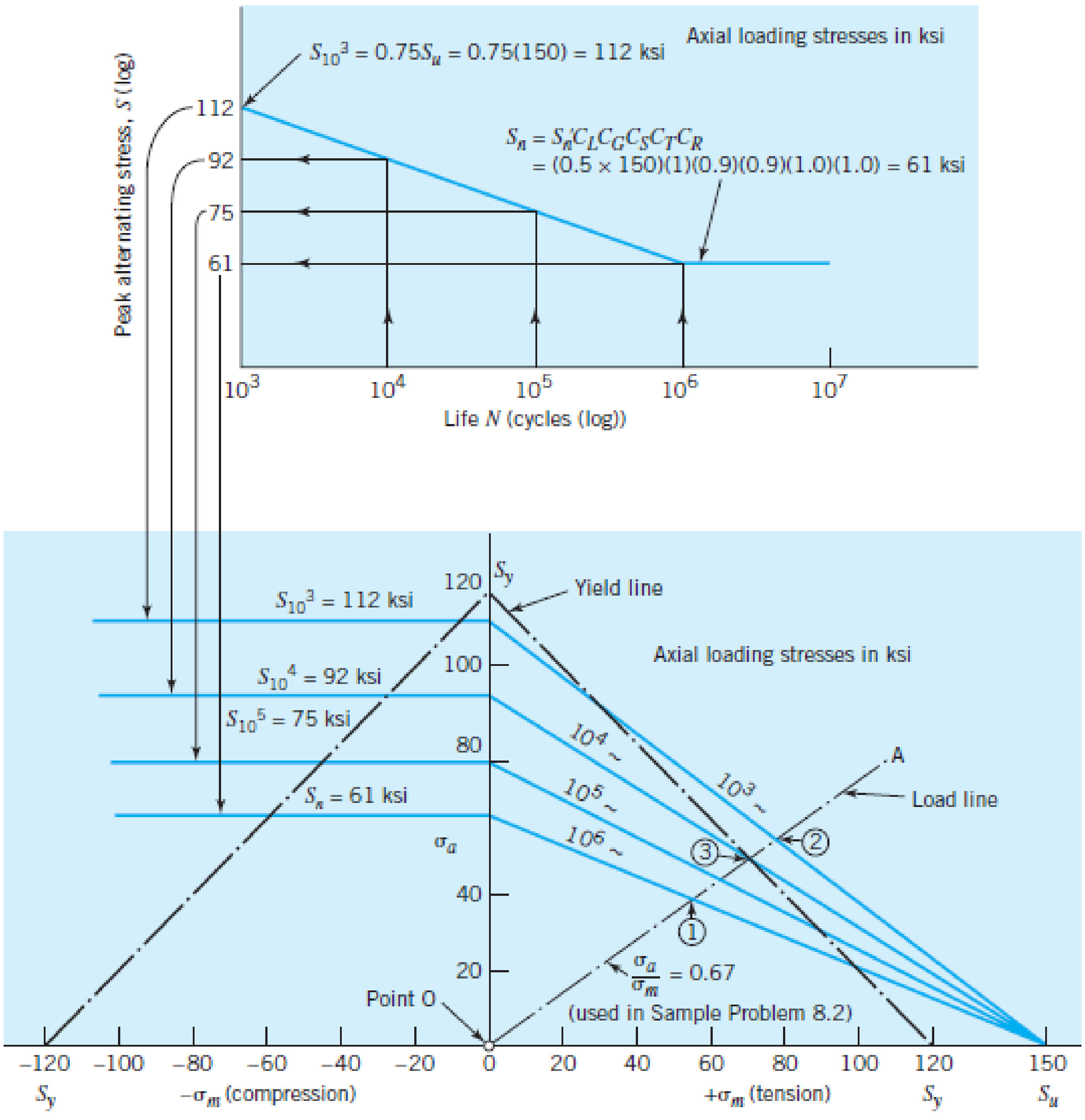


FIGURE 8.22
Sample Problem 8.1—estimate S–N and σ_m–σ_a curves for steel, $S_u = 150$ ksi, axial loading, commercially polished surfaces.

SAMPLE PROBLEM 8.2 Determine the Required Size of a Tensile Link Subjected to Fluctuating Loading

A round tensile link with negligible stress concentration is subjected to a load fluctuating between 1000 and 5000 lb. It is to be a precision member (so that use of $C_G = 0.9$ is justified) with commercially polished surfaces. The material is to be steel, with $S_u = 150$ ksi, $S_y = 120$ ksi. A safety factor of 2 is to be used, applied to all loads.

a. What diameter is required if infinite life is needed?

b. What diameter is required if only 10^3 cycles of life are needed?

SOLUTION

Known: A round steel link with given material properties and a commercially polished surface is to have a safety factor of 2 applied to all loads and is axially loaded with a known fluctuating load.

Find: (a) Determine the required diameter for infinite life. (b) Determine the required diameter for 10^3 cycles of life.

Schematic and Given Data: Figure 8.22 used in Sample Problem 8.1 is applicable.

Assumptions:

1. The diameter is less than 2 in.
2. The gradient factor $C_G = 0.9$.
3. Gross yielding is not permitted.

Analysis:

1. The fatigue strength properties of the material conform to those represented in Figure 8.22, *provided* the diameter comes out to be under 2 in.
2. At the *design overload*: $\sigma_m = SF(F_m/A) = 2(3000)/A = 6000/A$, $\sigma_a = SF(F_a/A) = 2(2000)/A = 4000/A$. Thus, regardless of the tensile link cross-sectional area, A, $\sigma_a/\sigma_m = 4000/6000 = 0.67$. This is represented by line OA on Figure 8.22. Note the interpretation of this line. If area A is infinite, both σ_m and σ_a are zero, and the stresses are represented by the origin, point O. Moving out along load line OA corresponds to progressively decreasing values of A. For part a of the problem we need to determine the cross-sectional area corresponding to the intersection of the load line OA with the infinite-life line (same as 10^6 cycles, in this case), which is labeled ①. At this point, $\sigma_a = 38$ ksi; from $\sigma_a = 4000/A$, A is determined as 0.106 in.^{2} From $A = \pi d^2/4$, $d = 0.367$ in. This is indeed well within the size range for the value of $C_G = 0.9$, which had to be assumed when the diagram was constructed. In many cases, the final answer might be rounded off to $d = \frac{3}{8}$ in.
3. For part b, with only 10^3 cycles of life required, we can move out along line OA of Figure 8.22, seemingly to point ②, where the line intersects the 10^3-cycle life line. However, if point ③ is crossed, the peak design overload of 10,000 lb imposes stresses in excess of the yield strength. In a notch-free tensile bar the stresses are uniform so that gross yielding of the entire link would occur. Assuming that this could not be permitted, we must choose a diameter based on point ③, not point ②. Here, $\sigma_a = 48$ ksi, from which $A = 0.083$ in.^{2}, and $d = 0.326$ in., perhaps rounded off to $d = \frac{11}{32}$ in. This diameter corresponds to an estimated life greater than required, but to make it any smaller than 0.326 in. would cause general yielding on the first overload application.

Sample Problem 8.1 Estimation from Tensil

- Probably the most common use of fatigue strength relationships is in connection with designing parts for infinite (or 5 X 10^8–cycle) life or in analyzing parts intended for infinite fatigue life.

- In these situations no S–N curve is required. Only the estimated endurance limit need be calculated and the infinite-life Goodman line plotted

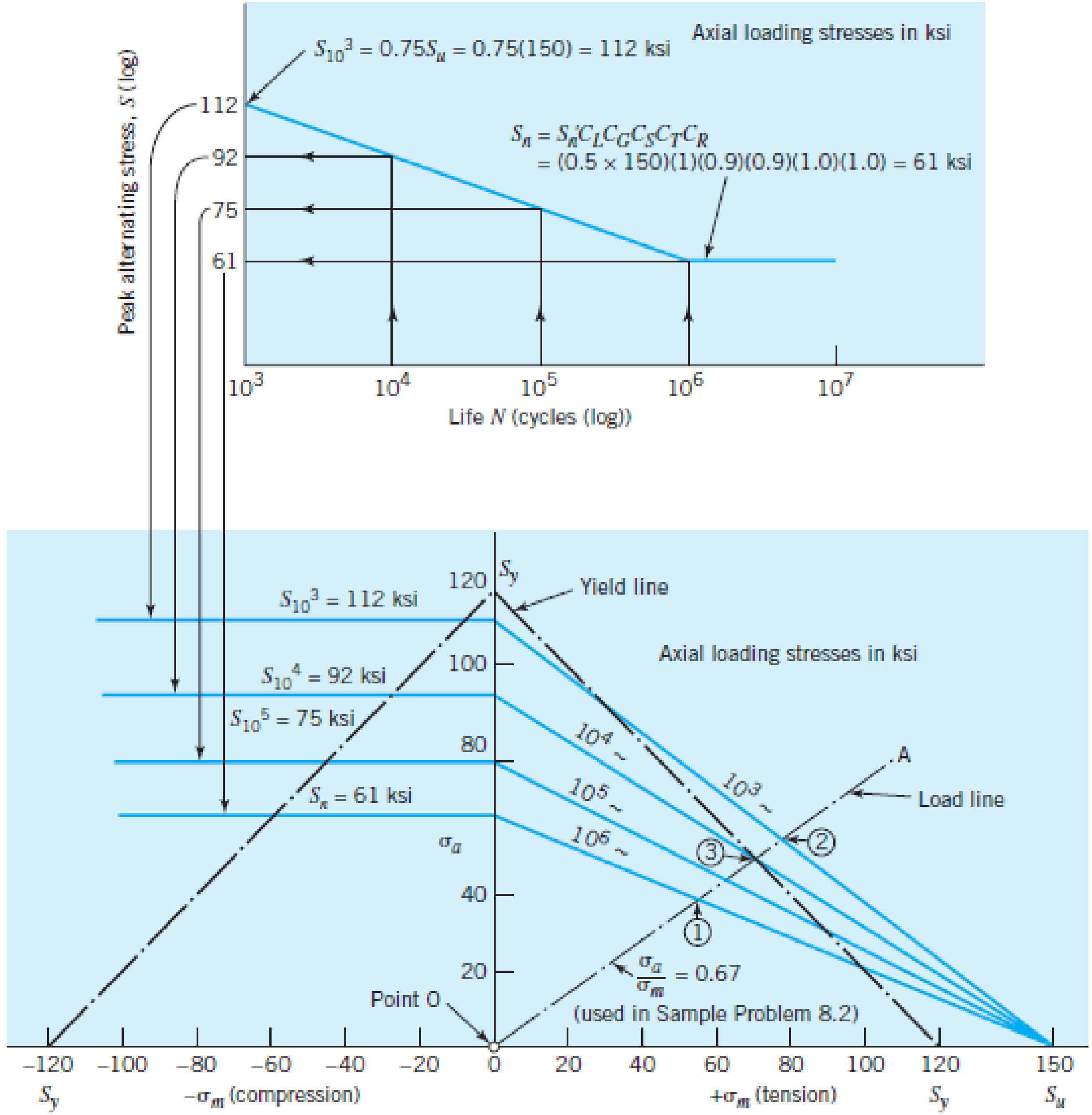
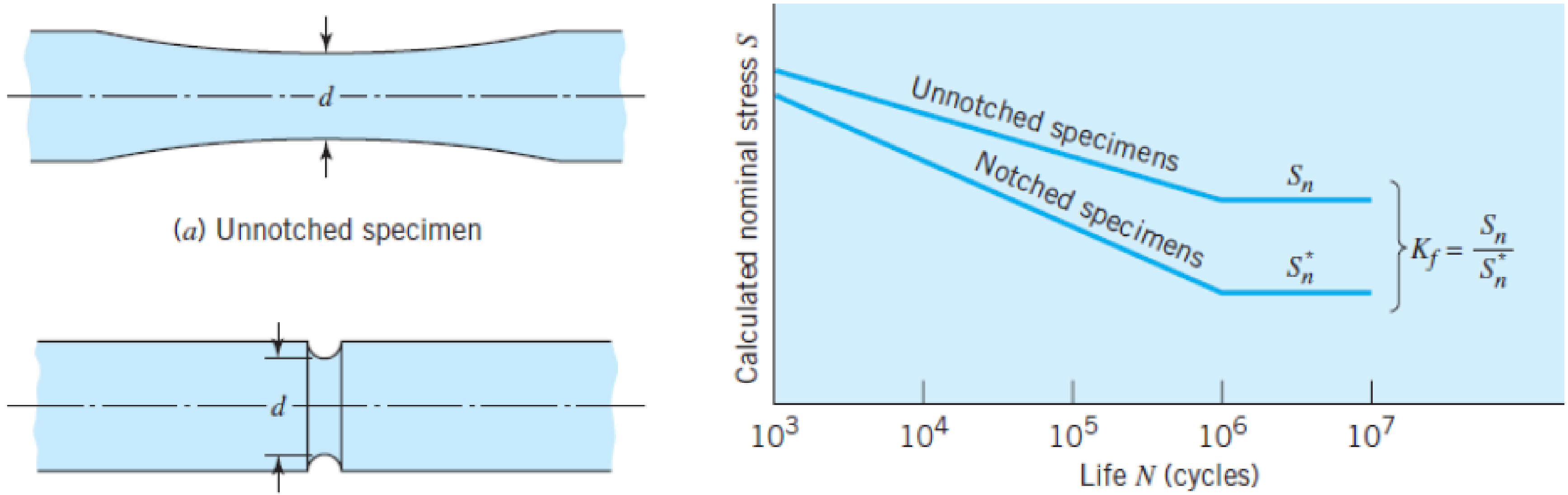


Figure 8.22
Sample Problem 8.1—estimate S–N and σ_m–σ_a curves for steel, $S_u = 150$ ksi, axial loading, commercially polished surfaces.

8.10 Effect of Stress Concentration with Completely Reversed Fatigue Loading

- Figure 8.23 shows typical S–N curves for unnotched and notched specimens
- Unlike other S–N curves, the stresses plotted are nominal stresses
- The specimen dimensions where fatigue fractures occur are the same
- Any given load causes the same calculated stress in both cases.
- Ratio of the unnotched to notched endurance limit is fatigue stress concentration factor, designated as Kf . $K_f = S_n/S'_n$



(a) Unnotched specimen

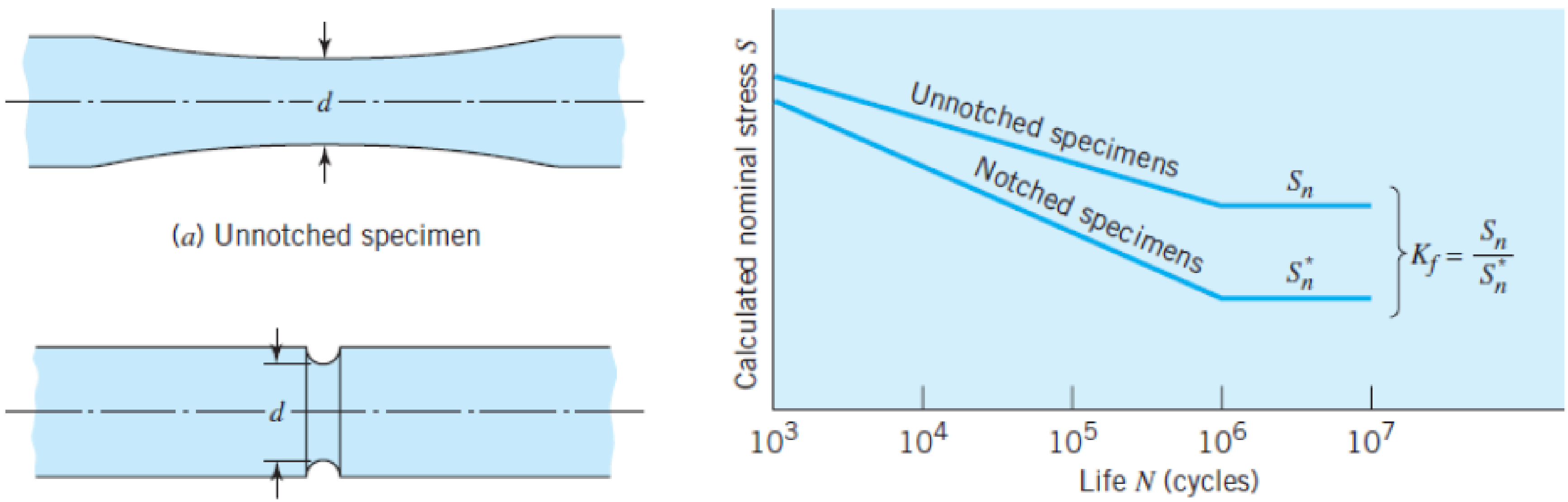
(b) Notched specimen (*)

(c) Illustration of fatigue stress concentration factor, K_f

FIGURE 8.23
Reversed-load fatigue tests, notched (*) versus unnotched specimens.

8.10 Effect of Stress Concentration with Completely Reversed Fatigue Loading

- Theoretically, K_f can be equal to the theoretical or geometric factor K_t. Experimentally $K_f < K_t$.
- This is apparently due to internal irregularities in the structure of the material.
- Cast Iron is insensitive to notches so $K_f \cong 1$. If material is highly sensitive to notches $K_f = K_t$
- This can be used to define notch sensitivity factor (q) as $K_f = 1 + (K_t - 1)q$
- If q is 0, $K_f = 1$ and if q is 1 $K_f = K_t$



(a) Unnotched specimen

(b) Notched specimen (*)

(c) Illustration of fatigue stress concentration factor, K_f

FIGURE 8.23

Reversed-load fatigue tests, notched (*) versus unnotched specimens.

8.10 Effect of Stress Concentration with Completely Reversed Fatigue Loading

- q depends on the material + relative notch radius. Notch radii so small approaching the imperfection size give zero notch sensitivity.
- Figure shows that a given steel is a little more notch-sensitive for torsional loading than for bending and axial loading.
- 0.04-in.-radius notch in a 160-Bhn steel part has a notch sensitivity of about 0.71 if the loading is bending or axial and about 0.76 if the load is torsional.

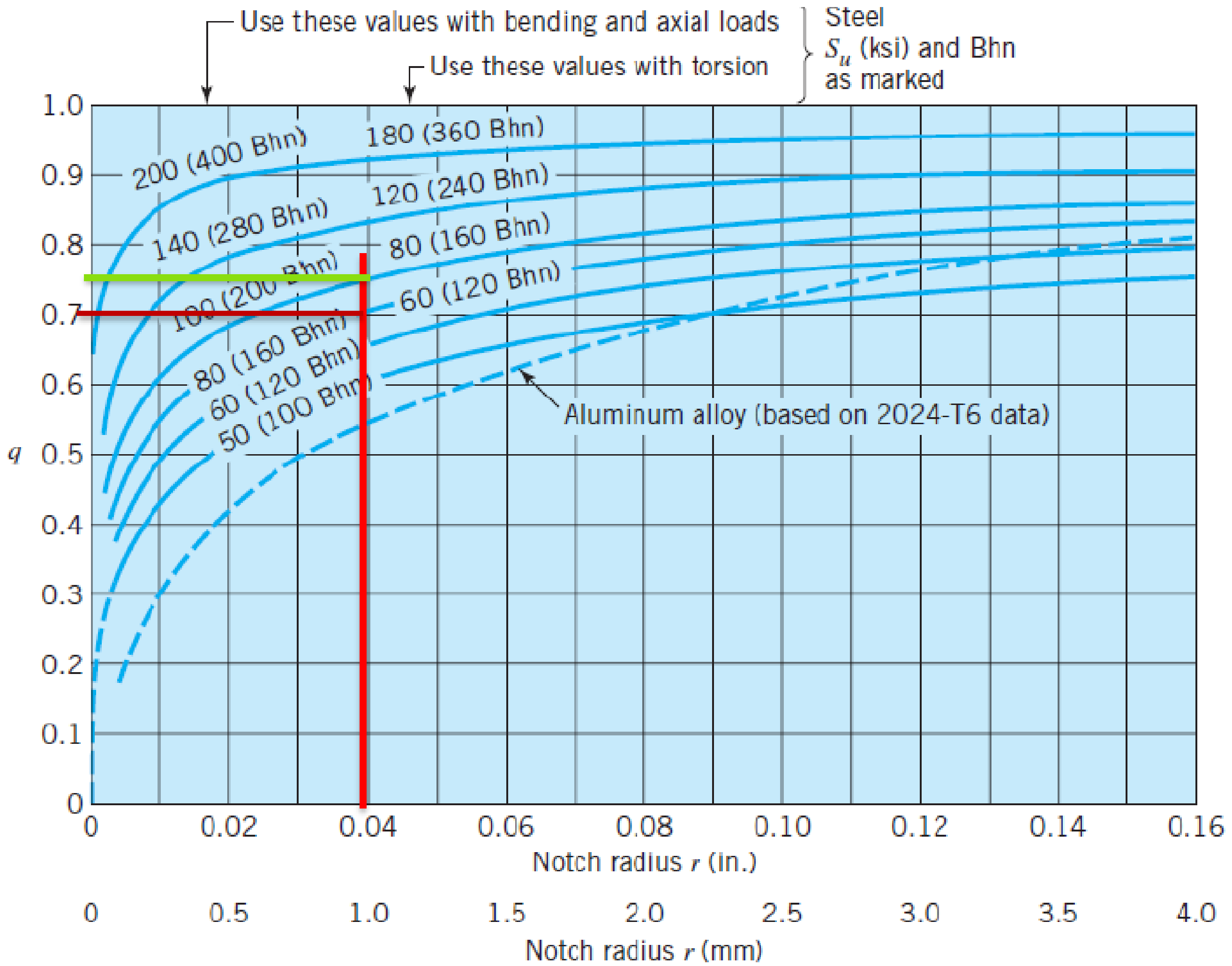


FIGURE 8.24
Notch sensitivity curves (after [9]). Note: (1) Here r is the radius at the point where the potential fatigue crack originates. (2) For $r > 0.16$ in., extrapolate or use $q \approx 1$.

8.10 Effect of Stress Concentration with Completely Reversed Fatigue Loading

- recommended that K_f used in all cases, though conservative in low cycle fatigue
- K_f will be regarded as a stress concentration factor.
- The notched endurance limit can be calculated as

$$S'_n C_L C_G C_S C_T C_R / K_f$$

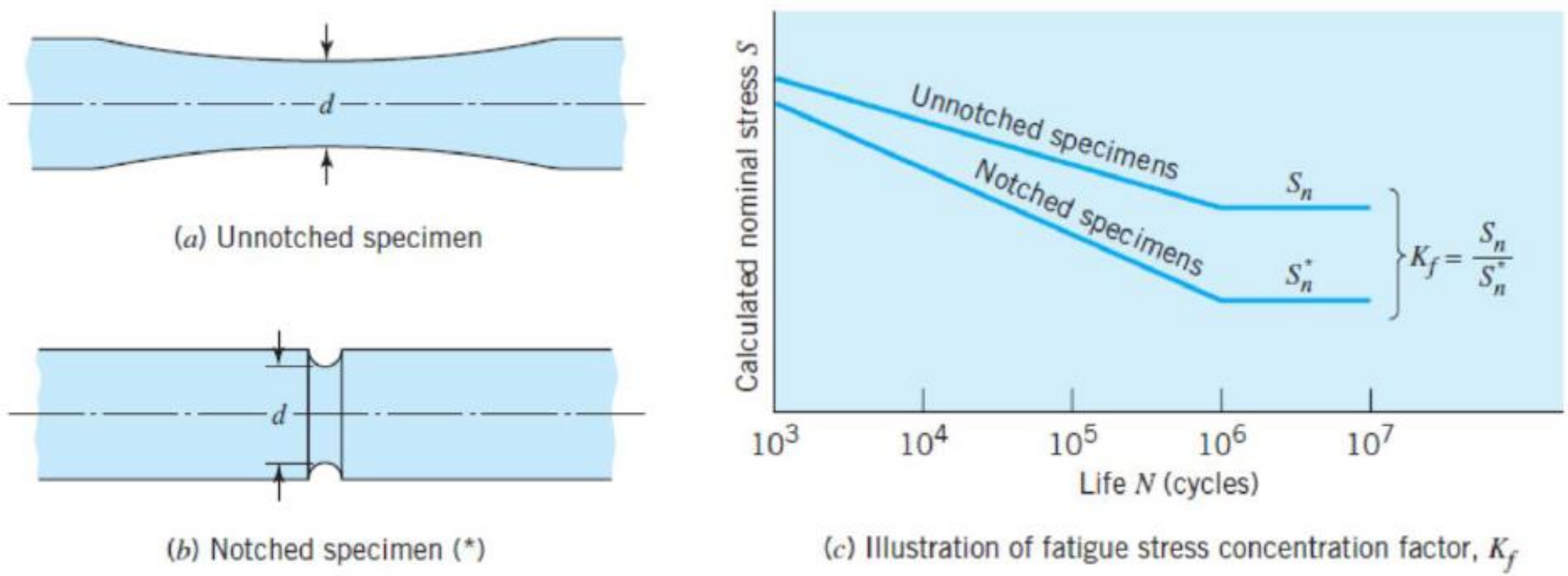


FIGURE 8.23
Reversed-load fatigue tests, notched (*) versus unnotched specimens.

8.11 Effect of Stress Concentration with Mean Plus Alternating Loads

- the procedure recommended here for fatigue life prediction of notched parts subjected to combinations of mean and alternating stress is
    - All stresses (both mean and alternating) are multiplied by the fatigue stress concentration factor K_f
    - correction is made for yielding and resultant residual stresses if the calculated peak stress exceeds the material yield strength.
- This procedure is sometimes called the residual stress method because of the recognition it gives to the development of residual stresses.

Sample Problem 8.3 Determine the Required Diameter of a Shaft Subjected to Mean and Alternating Torsion

A shaft must transmit a torque of 1000 N · m, with superimposed torsional vibration causing an alternating torque of 250 N · m. A safety factor of 2 is to be applied to both loads. A heat-treated alloy steel is to be used, having $S_u = 1.2$ GPa, and $S_y = 1.0$ GPa (unfortunately, test data are not available for S_{us} or S_{ys}). It is required that the shaft have a shoulder, with $D/d = 1.2$ and $r/d = 0.05$ (as shown in Figure 4.35). A good-quality commercial ground finish is to be specified. What diameter is required for infinite fatigue life?

Solution

Known: A commercial ground shaft made from steel with known yield and ultimate strengths and having a shoulder with known D/d and r/d ratios transmits a given steady and superimposed alternating torque with a safety factor of 2 applied to both torques (see Figure 8.26).

Find: Estimate the shaft diameter d required for infinite life.

Schematic and Given Data:

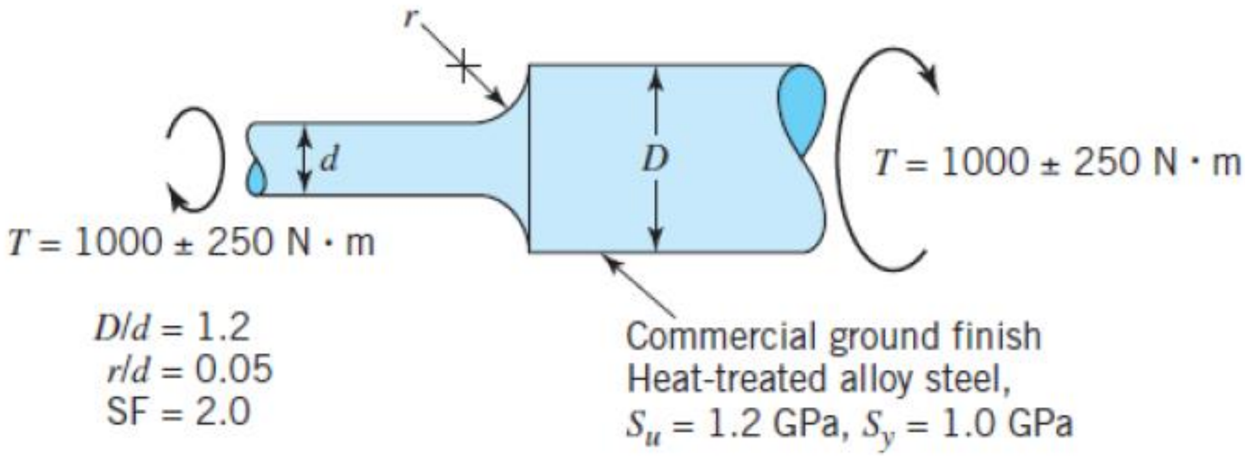


Figure 8.26
Shaft subjected to mean and alternating torsion.

Assumptions/Decisions:

1. The shaft is manufactured as specified with regard to the critical fillet and the shaft surface finish.
2. The shaft diameter will be between 10 and 50 mm.

Analysis:

1. Construct the fatigue strength diagram shown in Figure 8.27. (Since infinite life is required, there is no need for an *S–N* curve.) In computing an estimated value for S_n, we assumed that the diameter will be between 10 and 50 mm. If it is not, the solution will have to be repeated with a more appropriate value of C_G.
2. The *calculated* notch root stresses (i.e., not yet taking any possible yielding into account) are

$$\tau_m = (16T_m/\pi d^3)K_f$$

τ=Tr/J

$$\tau_a = (16T_a/\pi d^3)K_f$$

In order to find K_f from Eq. 8.2, we must first determine K_t and q. We find K_t from Figure 4.35*c* as 1.57, but the determination of q from Figure 8.24 again requires an assumption of the final diameter. This presents little difficulty, however, as the curve for torsional loading of steel of this strength (S_u = 1.2 GPa = 174 ksi, or very close to the top curve of the figure) gives $q \approx 0.95$ for $r \geq 1.5$ mm, which in this case corresponds to $d \geq 30$ mm. With the given loading, intuition (or subsequent calculation) tells us that the shaft will have to be at least this large. Substitution of these values, together with the given values for design overload (nominal load times safety factor), gives

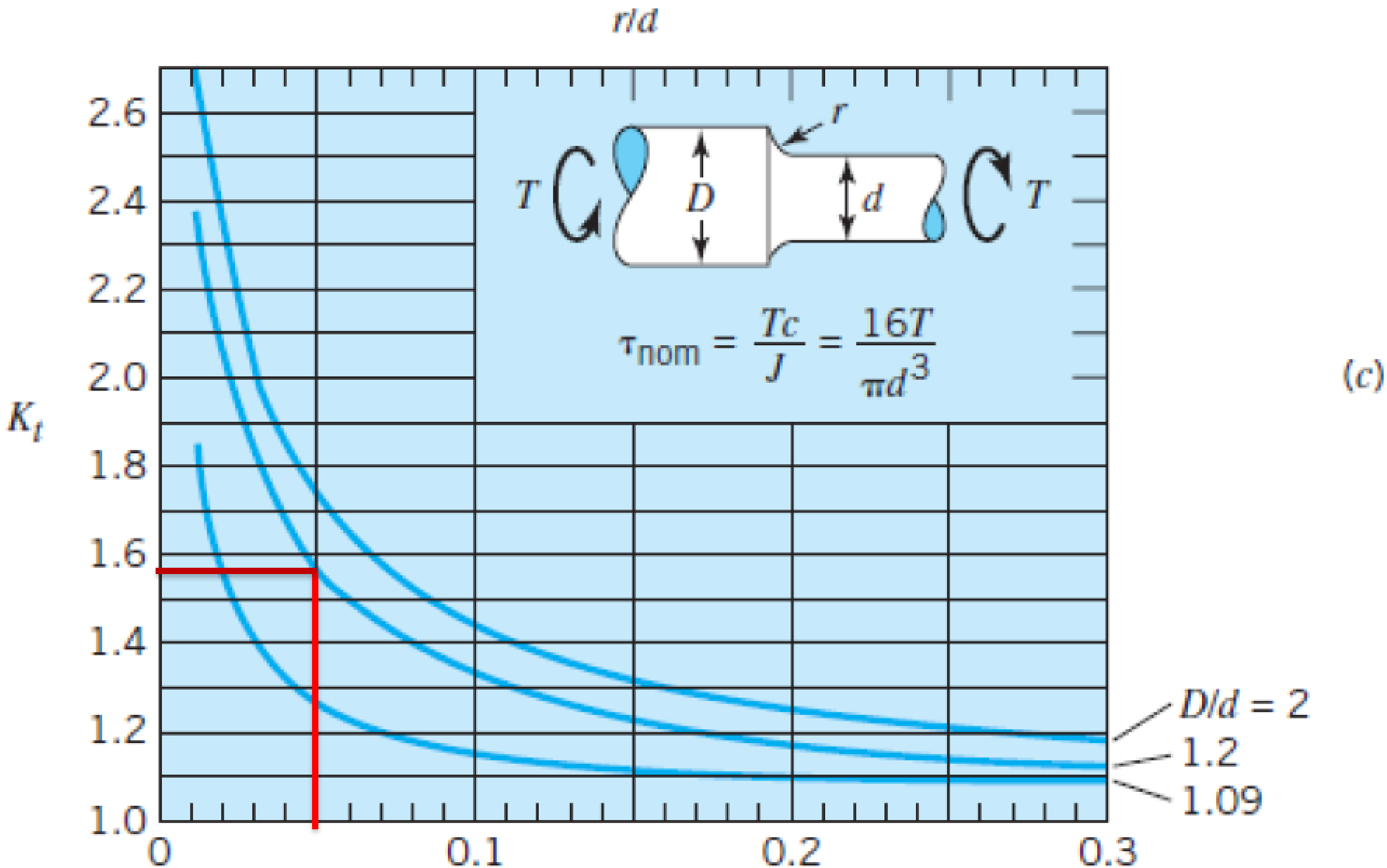


FIGURE 4.35
Shaft with fillet (*a*) bending; (*b*) axial load; (*c*) torsion [7].

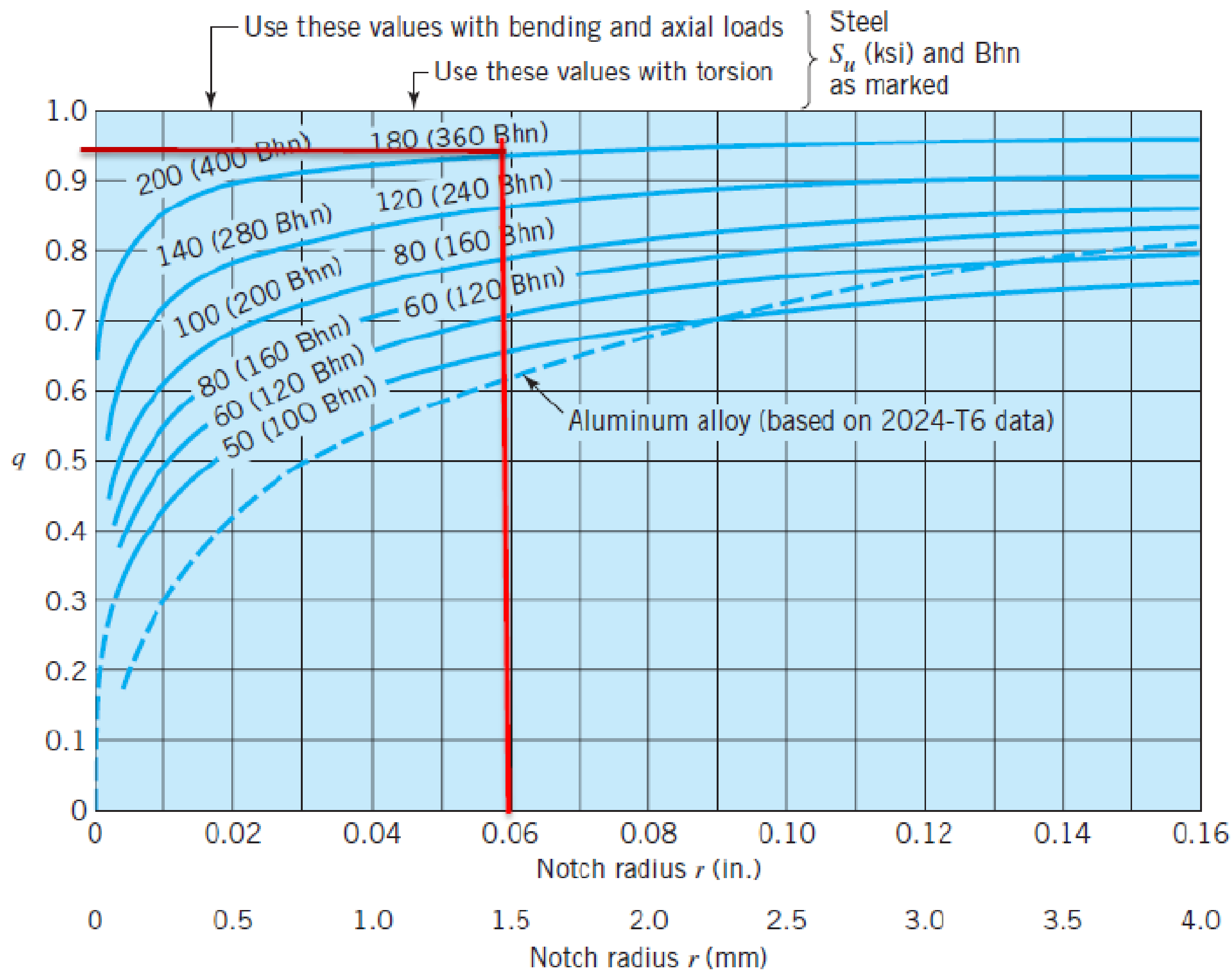


FIGURE 8.24

Notch sensitivity curves (after [9]). Note: (1) Here r is the radius at the point where the potential fatigue crack originates. (2) For $r > 0.16$ in., extrapolate or use $q \approx 1$.

$$K_f = 1 + (K_t - 1)q = 1 + (1.57 - 1)0.95 = 1.54$$

$$\tau_m = [(16 \times 2 \times 1000 \text{ N} \cdot \text{m})/\pi d^3]1.54 = 15{,}685/d^3$$

$$\tau_a = [(16 \times 2 \times 250 \text{ N} \cdot \text{m})/\pi d^3]1.54 = 3922/d^3$$

and $\tau_a/\tau_m = 0.25$.

3. Starting at the origin of Figure 8.27 (which corresponds to making the diameter infinite) and moving to the right along the line of slope = 0.25, we tentatively stop at point A. If no yielding is to be permitted, the stresses can go no higher than this. At A, $\tau_a = 116$ MPa or 0.116 GPa. Thus, $3922/d^3 = 0.116$ or $d = 32.2$ mm.
4. In most situations, perhaps a little yielding in the localized zone of the fillet under "design overload" conditions could be permitted. If so, the diameter can be further reduced until the *calculated* stresses reach point B on Figure 8.27, because yielding and residual stresses bring the *actual* stresses back to point B', which is right on the infinite life line. Yielding did not affect the alternating stress magnitude, so the equation for alternating stress can be equated to 150 MPa, giving $d = 29.7$ mm.
5. Before accepting either the $d = 32.3$ mm or the $d = 29.7$ mm answer, it is important to go back and see whether the values for C_G and q are consistent with the diameter finally chosen. In this case they are.

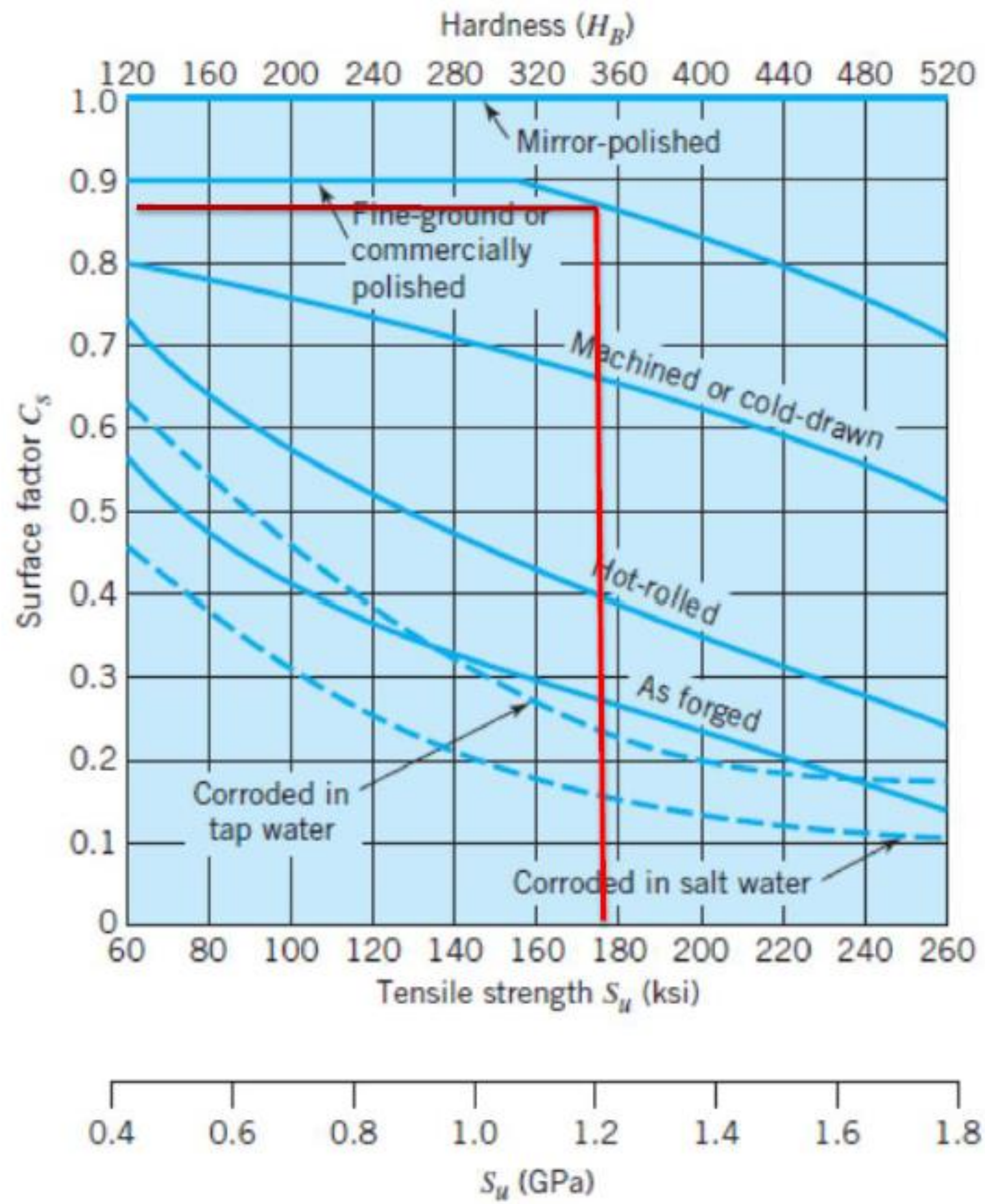


FIGURE 8.13

Reduction in endurance limit owing to surface finish—steel parts; i.e., surface factor versus tensile strength for various surface conditions.

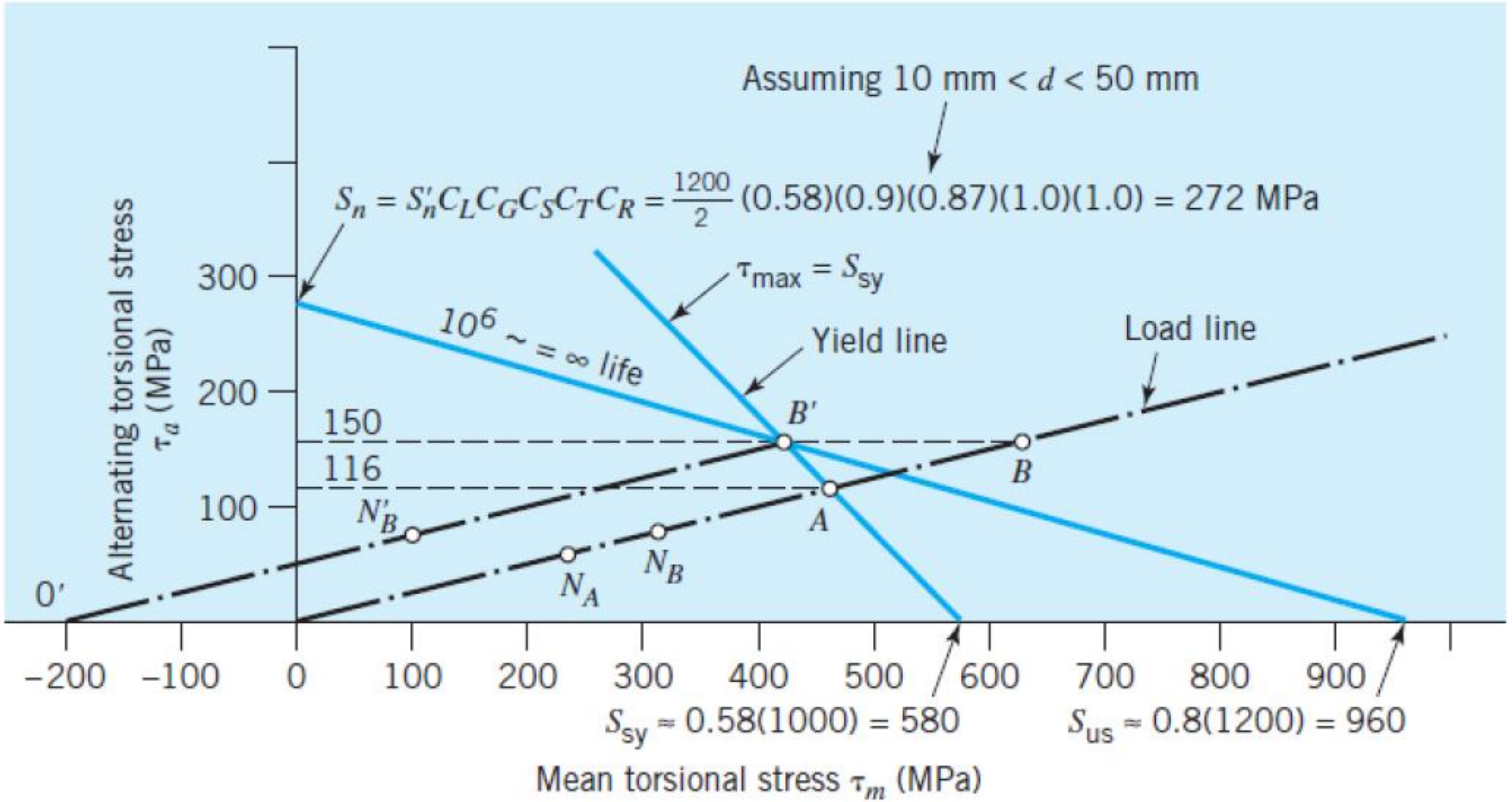


FIGURE 8.27

Fatigue strength diagram for Sample Problem 8.3.

SAMPLE PROBLEM 8.4 Estimate the Safety Factor of a Disk Sander Shaft

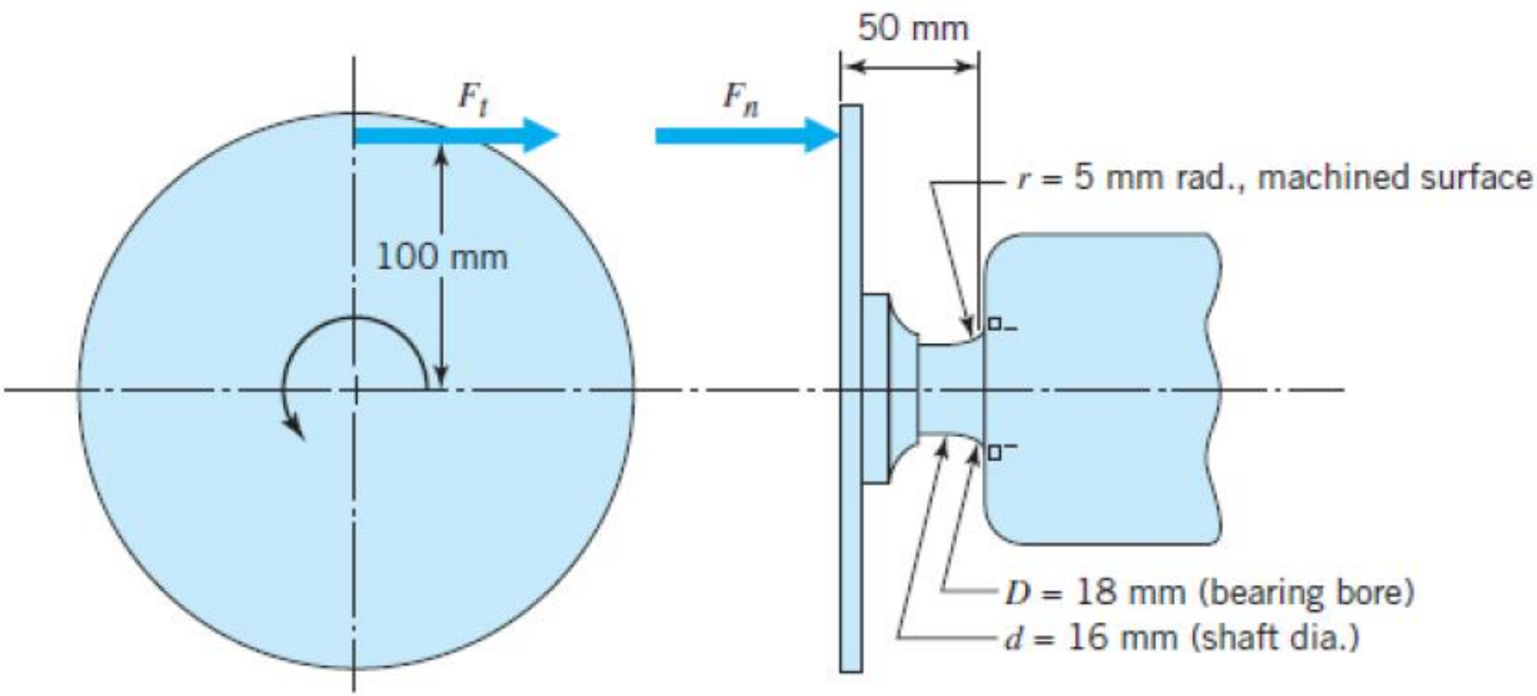
Figure 8.28 pertains to the shaft of a disk sander that is made of steel having $S_u = 900$ MPa, and $S_y = 750$ MPa. The most severe loading occurs when an object is held near the periphery of the disk (100-mm radius) with sufficient force to develop a friction torque of 12 N·m (which approaches the stall torque of the motor). Assume a coefficient of friction of 0.6 between the object and the disk. What is the safety factor with respect to eventual fatigue failure of the shaft?

SOLUTION

Known: A shaft with given geometry and loading is made of steel having known ultimate and yield strengths.

Find: Determine the safety factor for eventual failure by fatigue.

Schematic and Given Data:



$f = 0.6$ (between the object and the disk)
$T = 12\ \text{N} \cdot \text{m}$ (friction torque)

$S_u = 900$ MPa

$S_y = 750$ MPa

Assumption: The 50-mm disk shaft overhang is necessary.

Analysis:

1. The 12 N · m torque specification requires that the tangential force F_t be 120 N. With a coefficient of friction of 0.6, this requires a normal force F_n of 200 N.
2. These two force components produce the following loading at the shaft fillet:

   Torque: $T = 12 \text{ N} \cdot \text{m} = 12{,}000 \text{ N} \cdot \text{mm}$

   Axial load: $P = 200 \text{ N}$

   Bending: In the horizontal plane, $M_h = 120 \text{ N} \times 50 \text{ mm}$
   In the vertical plane, $M_v = 200 \text{ N} \times 100 \text{ mm}$
   The resultant is $M = \sqrt{M_h^2 + M_v^2} = 20{,}900 \text{ N} \cdot \text{mm}$

3. From Figure 4.35, geometric stress concentration factors for torsion, axial, and bending loads are about

$$K_{t(t)} = 1.10, \qquad K_{t(a)} = 1.28, \qquad K_{t(b)} = 1.28$$

   From Figure 8.24, estimated notch sensitivities q are 0.93 for torsion and 0.91 for bending and axial loads. From Eq. 8.2, values of K_f are estimated as 1.09, 1.25, and 1.25 for torsional, axial, and bending loads, respectively.

$$\boxed{K_f = 1 + (K_t - 1)q}$$

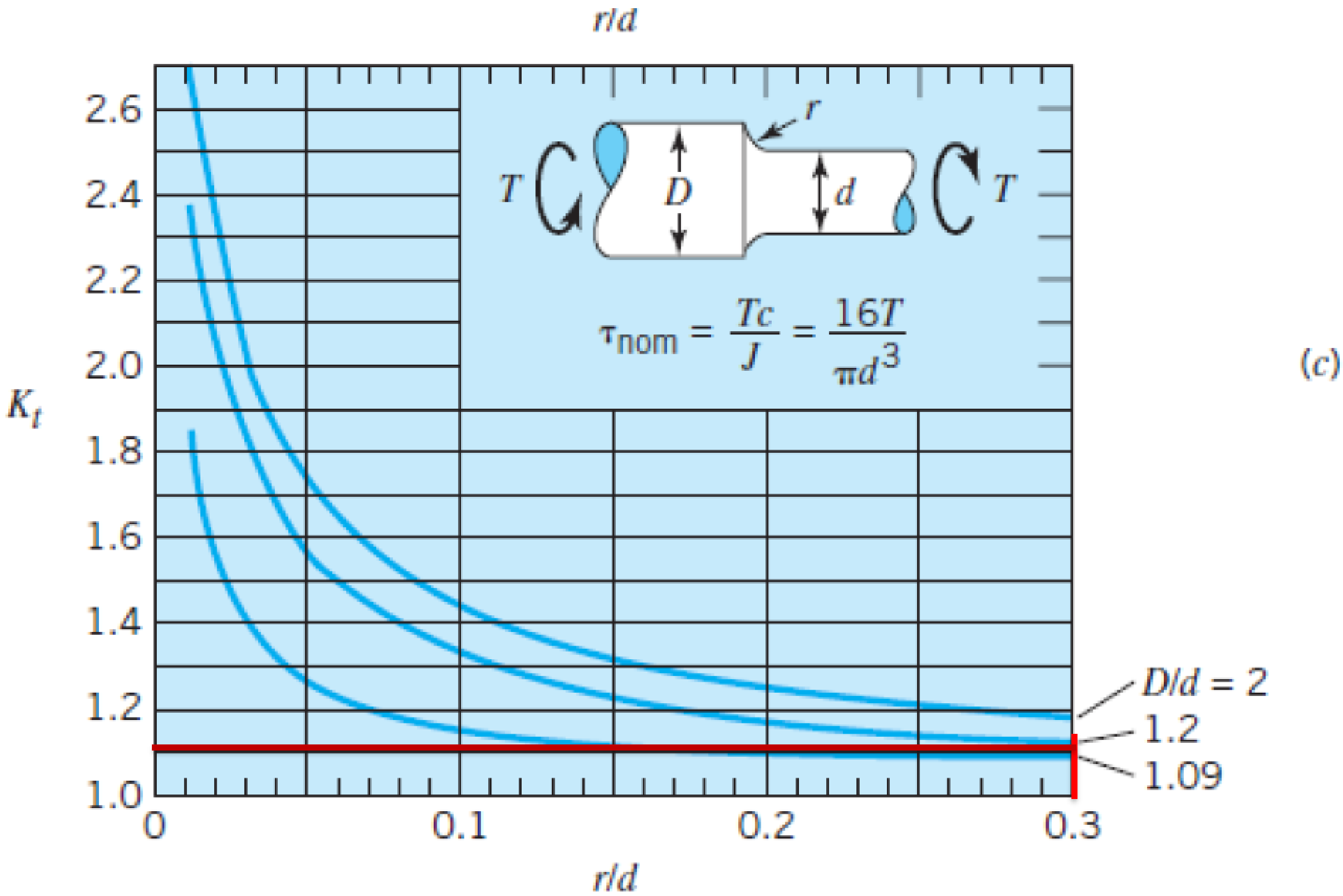


FIGURE 4.35

Shaft with fillet (*a*) bending; (*b*) axial load; (*c*) torsion [7].

130 KSI for .9GPA S_u extrapolated for r>5mm

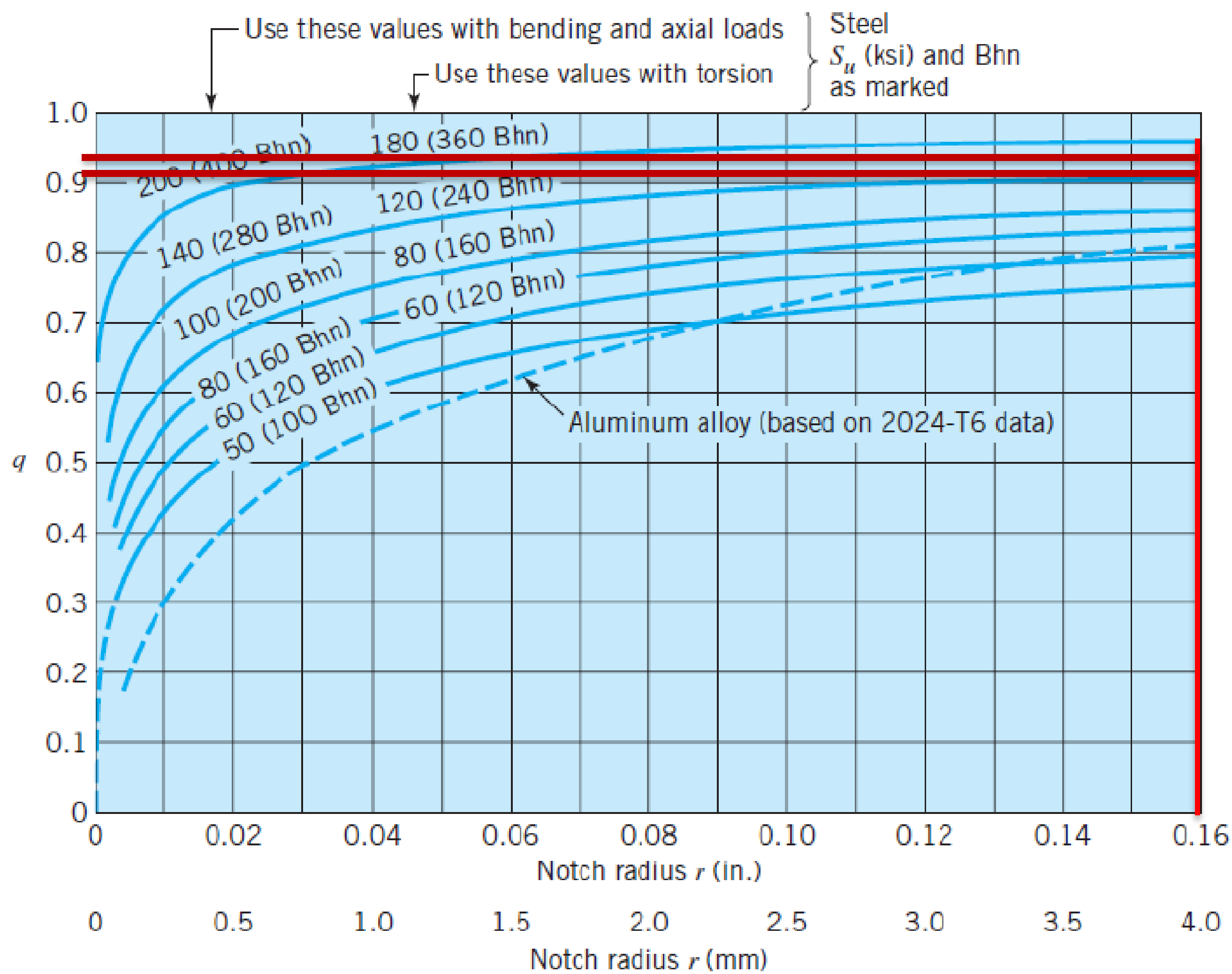


FIGURE 8.24
Notch sensitivity curves (after [9]). Note: (1) Here r is the radius at the point where the potential fatigue crack originates. (2) For $r > 0.16$ in., extrapolate or use $q \approx 1$.

4. The three stress components at the fillet are

$$\tau = \frac{16T}{\pi d^3}K_{f(t)} = \frac{16(12{,}000)}{\pi(16)^3}(1.09) = 16.3 \text{ MPa}$$

$$\sigma_{(a)} = \frac{P}{A}K_{f(a)} = \frac{-200(4)}{\pi(16)^2}(1.25) = -1.24 \text{ MPa}$$

$$\sigma_{(b)} = \frac{32M}{\pi d^3}K_{f(b)} = \frac{32(20{,}900)}{\pi(16)^3}(1.25) = 65.0 \text{ MPa}$$

5. Applying the procedure specified for "general biaxial loads" in Table 8.2, we construct in Figure 8.28 an estimated infinite-life Goodman line for *bending* loads. Next, an "operating point" that corresponds to the *equivalent* mean and *equivalent* alternating bending stresses is placed on the diagram. Of the three stress components determined, torsional and axial stresses are constant for steady-state operating conditions; the bending stress is completely reversed (the bending stress at any point on the fillet goes from tension-to-compression-to-tension during each shaft revolution). Using the recommended procedure to determine the equivalent mean and alternating stresses, we have

$$\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau^2 + \left(\frac{\sigma_m}{2}\right)^2}$$

$$= \frac{-1.24}{2} + \sqrt{(16.3)^2 + \left(\frac{-1.24}{2}\right)^2} = 15.7 \text{ MPa}$$

$$\sigma_{ea} = \sqrt{\sigma_a^2 + 3\tau_a^2} + \sqrt{(65.0)^2 + 0} = 65.0 \text{ MPa}$$

6. By drawing a line through the origin and the "operating point," we see that all stresses would have to be increased by a factor of about 4 to reach the estimated "failure point" where conditions would be on the verge of causing eventual fatigue failure. Hence, the estimated safety factor is 4.

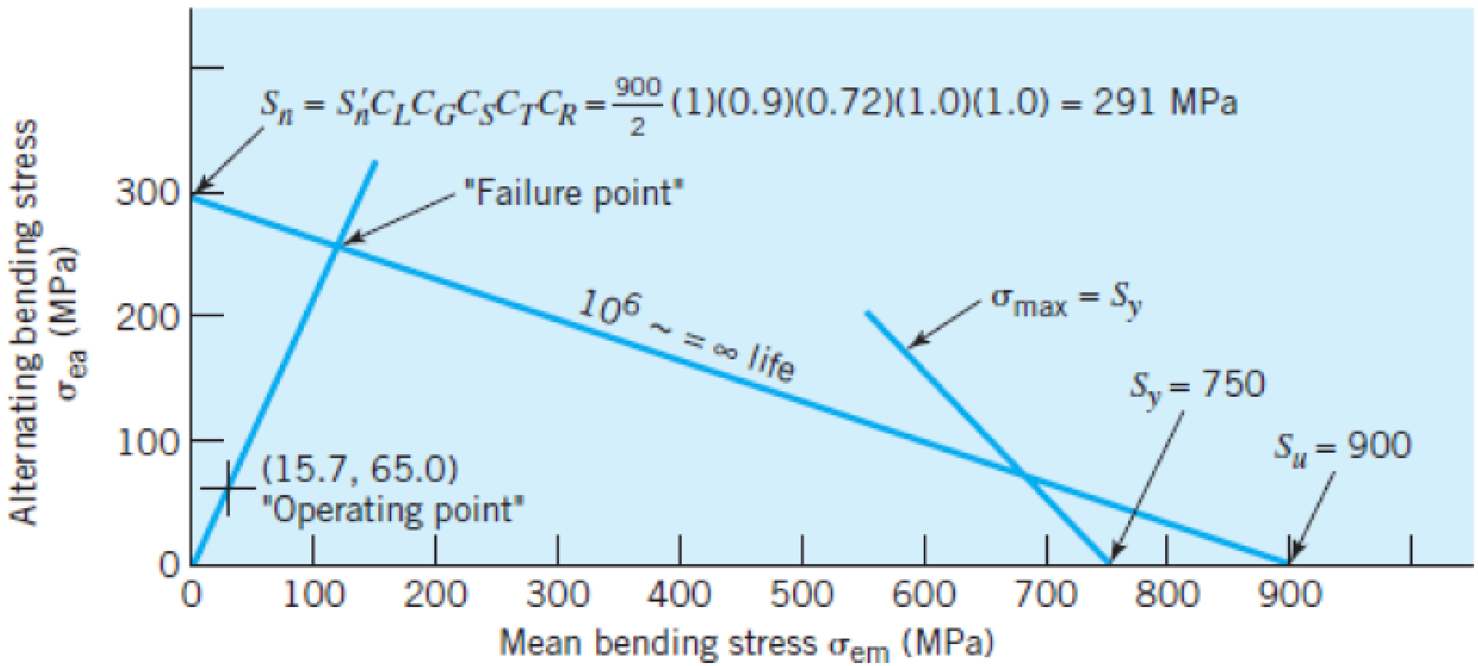


FIGURE 8.28

Sample Problem 8.4—disk sander.

8.12 Fatigue Life Prediction with Randomly Varying Loads

- Predicting the life of parts stressed above the endurance limit is a rough method.
- This point is illustrated by the typical scatter band of 7 : 1 life ratio in Fig 8.4c.
- For the large percentage of mechanical and structural parts subjected to randomly varying stress cycle intensity (e.g., automotive suspension and aircraft structural components), the prediction of fatigue life is further complicated.
- The procedure is often called the linear cumulative-damage rule
- the Palmgren or Miner concept is that if a part is cyclically loaded at a stress level causing failure in 10^5 cycles, each cycle of this loading consumes one part in 10^5 of the life of the part. 10^4 cycles consumes one part in 10^4 life and if 100% life is consumed fatigue is predicted
- The rule is expressed by the following equation in which $n_1, n_2, \ldots, n_k$ represent the number of cycles at specific overstress levels, and $N_1, N_2, \ldots, N_k$ represent the life (in cycles) at these overstress levels, as taken from S–N curve.
- Fatigue failure is predicted when

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots + \frac{n_k}{N_k} = 1 \quad \text{or} \quad \sum_{j=1}^{j=k} \frac{n_j}{N_j} = 1 \tag{8.3}$$

SAMPLE PROBLEM 8.5 Fatigue Life Prediction with Randomly Varying, Completely Reversed Stresses

Stresses (including stress concentration factor K_f) at the critical notch of a part fluctuate randomly as indicated in Figure 8.29*a*. The stresses could be bending, torsional, or axial—or even equivalent bending stresses resulting from general biaxial loading. The plot shown represents what is believed to be a typical 20 seconds of operation. The material is steel, and the appropriate *S–N* curve is given in Figure 8.29*b*. This curve is corrected for load, gradient, and surface. Estimate the fatigue life of the part.

SOLUTION

Known: A stress-versus-time history corrected for stress concentration, load, gradient, and surface is given for a 20-second test of a steel part.

Find: Determine the fatigue life of the part.

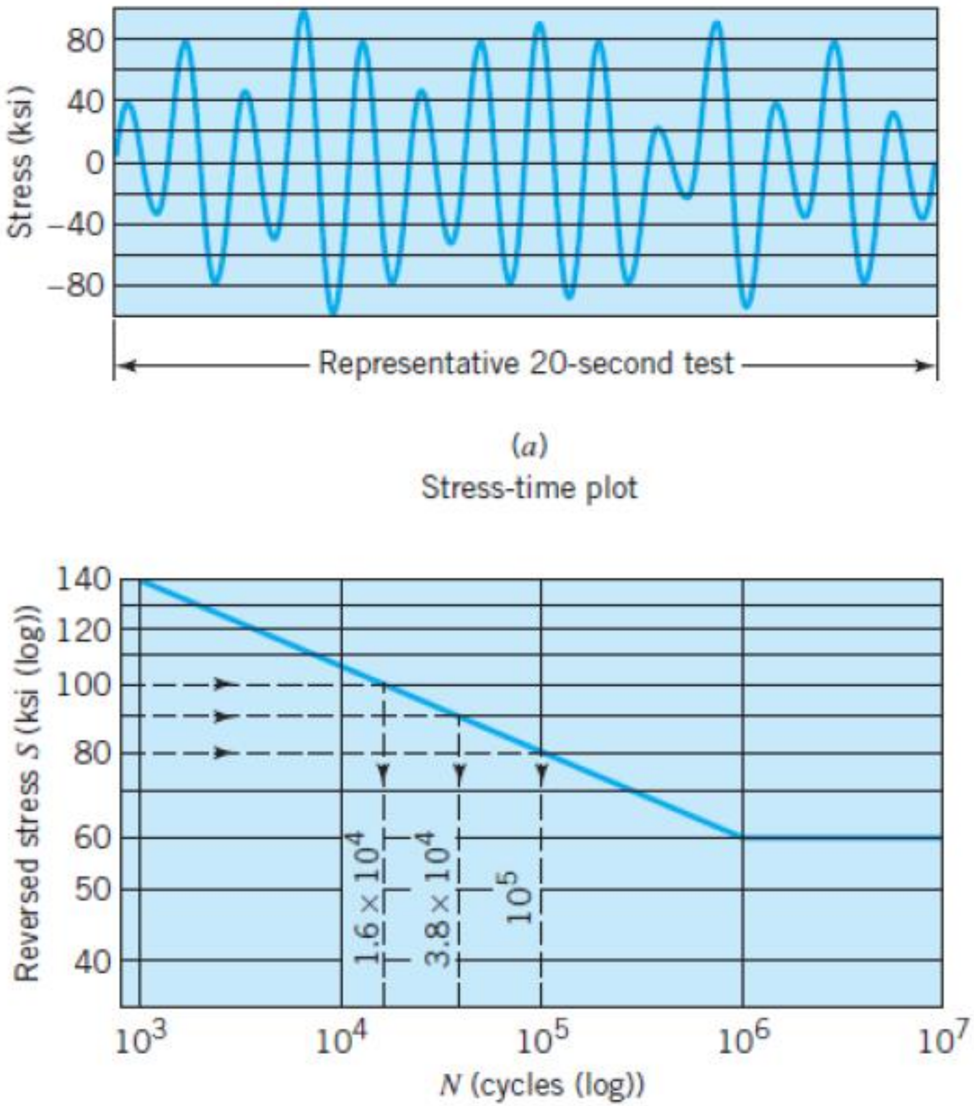
Assumptions:

1. The representative 20-second test result for stress will repeat until the part fails by eventual fatigue.
2. The linear cumulative-damage rule applies.

Analysis: In Figure 8.29*a* there are eight stress cycles above the endurance limit of 60 ksi: five at 80 ksi, two at 90 ksi, and one at 100 ksi. The *S–N* curve shows that each 80-ksi cycle uses one part in 10^5 of the life, each at 90 ksi uses one part in 3.8×10^4, and the one at 100 ksi uses one part in 1.6×10^4. Adding these fractions of life used gives

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = \frac{5}{10^5} + \frac{2}{3.8 \times 10^4} + \frac{1}{1.6 \times 10^4} = 0.0001651$$

For the fraction of life consumed to be unity, the 20-second test time must be multiplied by $1/0.0001651 = 6059$. This corresponds to 2019 minutes, or about *30 to 35 hours*.



(*a*)
Stress-time plot

(*b*)
S-N curve

FIGURE 8.29
Sample Problem 8.5—fatigue life prediction, reversed stresses.

10 Threaded Fasteners and Power Screws

Introduction

- Multitude of fasteners are available raging from nuts and bots to different varieties. Only a small sample is shown here
- Limit our discussion to design and selection of conventional fasteners (screws, nuts & bolts).
- Primarily used in machine design applications and lot of stresses are encountered.
- Used primarily for holding, or moving (lead screw)
- Loads are tensile, or shear or both
- The economic implications are tremendous.
- the airframe of a large jet aircraft has approximately 2.4 Million fasteners costing about $750,000 in 1978 dollars.

10.2 Thread Forms, Terminology, and Standards

- Figure 10.1 illustrates the basic arrangement of a helical thread wound around a cylinder, as used on screw-type fasteners, power screws, and worms.
- Pitch, lead, lead angle, and hand-of-thread are defined by the illustrations.
- Virtually all bolts and screws have a single thread, but worms and power screws sometimes have double, triple, and even quadruple threads.
- Unless otherwise noted, all threads are assumed to be right-hand.

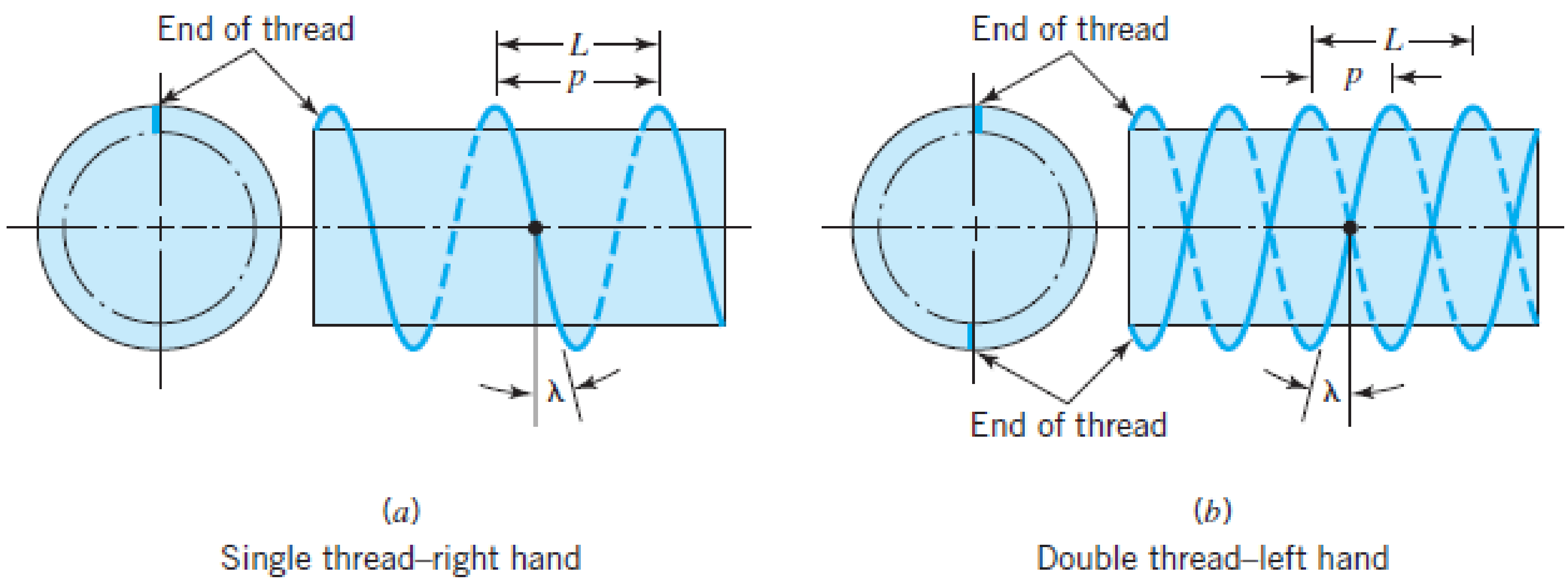


FIGURE 10.1
Helical threads of pitch p, lead L, and lead angle λ.

10.2 Thread Forms, Terminology, and Standards

- Figure 10.2 shows the standard geometry of screw threads used on fasteners.
- This is basically the same for both Unified (inch) and ISO (metric) threads.
- Standard sizes for the two systems are given in Tables 10.1 and 10.2.
- The pitch diameter, d_p, is the diameter of a cylinder on a perfect thread where the width of the thread and groove are equal.
- The stress area tabulated is based on the average of the pitch and root diameters.
- This is the area used for "P/A" stress calculations.
- It approximates the smallest possible fracture area, considering the presence of the helical thread.

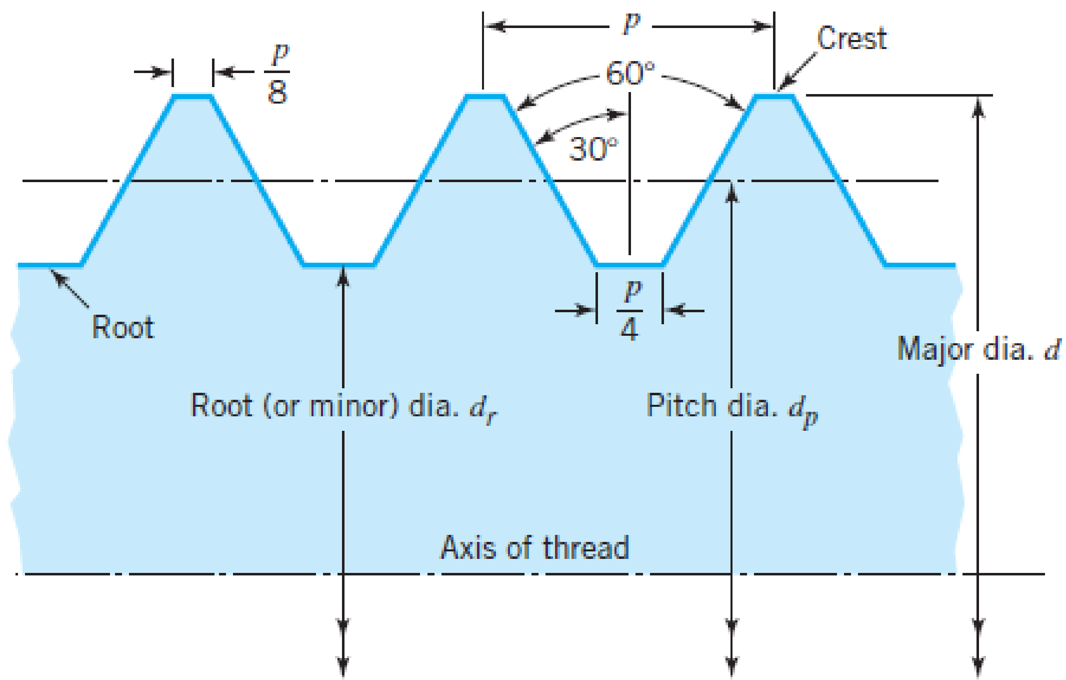


FIGURE 10.2
Unified and ISO thread geometry. The basic profile of the external thread is shown.

Table 10.1 Basic Dimensions of Unified Screw Threads

Size	Major Diameter d (in.)	Coarse Threads—UNC Threads per Inch	Coarse Threads—UNC Minor Diameter of External Thread d_r (in.)	Coarse Threads—UNC Tensile Stress Area A_t (in.^{2})	Fine Threads—UNF Threads per Inch	Fine Threads—UNF Minor Diameter of External Thread d_r (in.)	Fine Threads—UNF Tensile Stress Area A_t (in.^{2})
0(.060)	0.0600	—	—	—	80	0.0447	0.00180
1(.073)	0.0730	64	0.0538	0.00263	72	0.0560	0.00278
2(.086)	0.0860	56	0.0641	0.00370	64	0.0668	0.00394
3(.099)	0.0990	48	0.0734	0.00487	56	0.0771	0.00523
4(.112)	0.1120	40	0.0813	0.00604	48	0.0864	0.00661
5(.125)	0.1250	40	0.0943	0.00796	44	0.0971	0.00830
6(.138)	0.1380	32	0.0997	0.00909	40	0.1073	0.01015
8(.164)	0.1640	32	0.1257	0.0140	36	0.1299	0.01474
10(.190)	0.1900	24	0.1389	0.0175	32	0.1517	0.0200
12(.216)	0.2160	24	0.1649	0.0242	28	0.1722	0.0258
$\frac{1}{4}$	0.2500	20	0.1887	0.0318	28	0.2062	0.0364
$\frac{5}{16}$	0.3125	18	0.2443	0.0524	24	0.2614	0.0580
$\frac{3}{8}$	0.3750	16	0.2983	0.0775	24	0.3239	0.0878
$\frac{7}{16}$	0.4375	14	0.3499	0.1063	20	0.3762	0.1187
$\frac{1}{2}$	0.5000	13	0.4056	0.1419	20	0.4387	0.1599
$\frac{9}{16}$	0.5625	12	0.4603	0.182	18	0.4943	0.203
$\frac{5}{8}$	0.6250	11	0.5135	0.226	18	0.5568	0.256
$\frac{3}{4}$	0.7500	10	0.6273	0.334	16	0.6733	0.373
$\frac{7}{8}$	0.8750	9	0.7387	0.462	14	0.7874	0.509
1	1.0000	8	0.8466	0.606	12	0.8978	0.663
$1\frac{1}{8}$	1.1250	7	0.9497	0.763	12	1.0228	0.856
$1\frac{1}{4}$	1.2500	7	1.0747	0.969	12	1.1478	1.073
$1\frac{3}{8}$	1.3750	6	1.1705	1.155	12	1.2728	1.315
$1\frac{1}{2}$	1.5000	6	1.2955	1.405	12	1.3978	1.581
$1\frac{3}{4}$	1.7500	5	1.5046	1.90			
2	2.0000	$4\frac{1}{2}$	1.7274	2.50			
$2\frac{1}{4}$	2.2500	$4\frac{1}{2}$	1.9774	3.25			
$2\frac{1}{2}$	2.5000	4	2.1933	4.00			
$2\frac{3}{4}$	2.7500	4	2.4433	4.93			
3	3.0000	4	2.6933	5.97			
$3\frac{1}{4}$	3.2500	4	2.9433	7.10			
$3\frac{1}{2}$	3.5000	4	3.1933	8.33			
$3\frac{3}{4}$	3.7500	4	3.4433	9.66			
4	4.0000	4	3.6933	11.08			

Note: See ANSI standard B1.1-1974 for full details. Unified threads are specified as "$\frac{1}{2}$ in.–13UNC," "1 in.–12UNF."

Table 10.1 Basic Dimensions of Unified Screw Threads

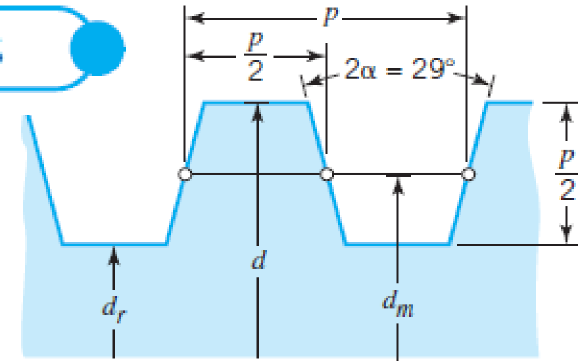
		Coarse Threads—UNC			Fine Threads—UNF		
Size	Major Diameter d (in.)	Threads per Inch	Minor Diameter of External Thread d_r (in.)	Tensile Stress Area A_t (in.^{2})	Threads per Inch	Minor Diameter of External Thread d_r (in.)	Tensile Stress Area A_t (in.^{2})
0(.060)	0.0600	—	—	—	80	0.0447	0.00180
1(.073)	0.0730	64	0.0538	0.00263	72	0.0560	0.00278
2(.086)	0.0860	56	0.0641	0.00370	64	0.0668	0.00394
3(.099)	0.0990	48	0.0734	0.00487	56	0.0771	0.00523
4(.112)	0.1120	40	0.0813	0.00604	48	0.0864	0.00661
5(.125)	0.1250	40	0.0943	0.00796	44	0.0971	0.00830
6(.138)	0.1380	32	0.0997	0.00909	40	0.1073	0.01015
8(.164)	0.1640	32	0.1257	0.0140	36	0.1299	0.01474
10(.190)	0.1900	24	0.1389	0.0175	32	0.1517	0.0200
12(.216)	0.2160	24	0.1649	0.0242	28	0.1722	0.0258
$\frac{1}{4}$	0.2500	20	0.1887	0.0318	28	0.2062	0.0364
$\frac{5}{16}$	0.3125	18	0.2443	0.0524	24	0.2614	0.0580
$\frac{3}{8}$	0.3750	16	0.2983	0.0775	24	0.3239	0.0878
$\frac{7}{16}$	0.4375	14	0.3499	0.1063	20	0.3762	0.1187
$\frac{1}{2}$	0.5000	13	0.4056	0.1419	20	0.4387	0.1599

Table 10.2 Basic Dimensions of ISO Metric Screw Threads

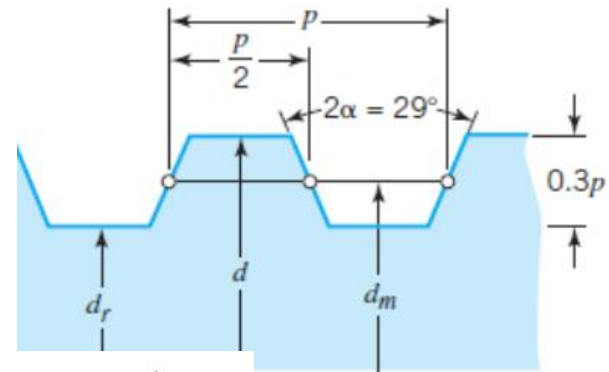
	Coarse Threads			Fine Threads		
Nominal Diameter d (mm)	Pitch p (mm)	Minor Diameter d_r (mm)	Stress Area A_t (mm^2)	Pitch p (mm)	Minor Diameter d_r (mm)	Stress Area A_t (mm^2)
3	0.5	2.39	5.03			
3.5	0.6	2.76	6.78			
4	0.7	3.14	8.78			
5	0.8	4.02	14.2			
6	1	4.77	20.1			
7	1	5.77	28.9			
8	1.25	6.47	36.6	1	6.77	39.2
10	1.5	8.16	58.0	1.25	8.47	61.2
12	1.75	9.85	84.3	1.25	10.5	92.1
14	2	11.6	115	1.5	12.2	125
16	2	13.6	157	1.5	14.2	167
18	2.5	14.9	192	1.5	16.2	216
20	2.5	16.9	245	1.5	18.2	272
22	2.5	18.9	303	1.5	20.2	333
24	3	20.3	353	2	21.6	384
27	3	23.3	459	2	24.6	496
30	3.5	25.7	561	2	27.6	621
33	3.5	28.7	694	2	30.6	761
36	4	31.1	817	3	32.3	865
39	4	34.1	976	3	35.3	1030

Note: Metric threads are identified by diameter and pitch as "M8 × 1.25."

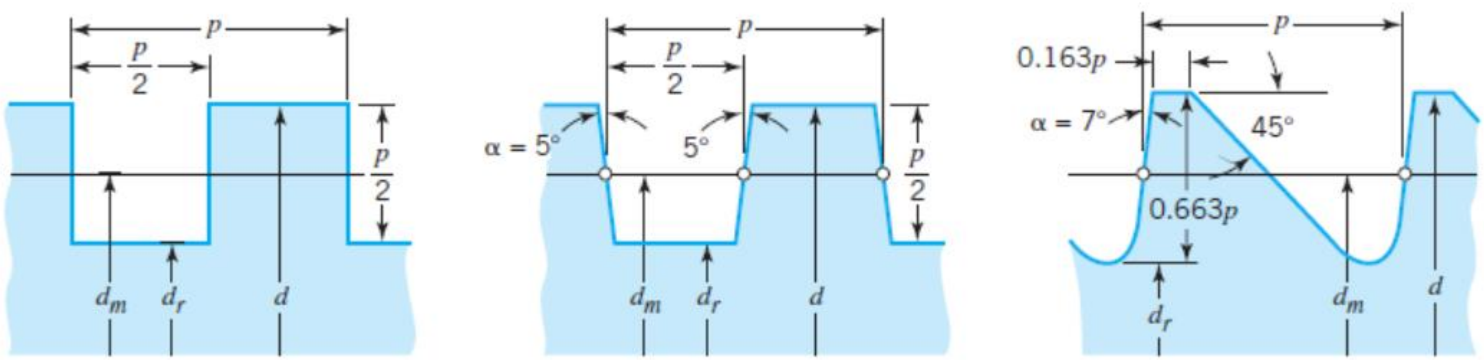
10.2 Thread Forms, Terminology, and Standards



- Standard form of power screws
- Acme is the oldest. Acme stub is easier to heat treat
- Square gives more efficiency but 0° angle difficult
- Modified square with 5° is commonly used
- Buttress is used to resist large axial force in one direction
- For power screws with multiple threads, the number of threads per inch is defined as the reciprocal of the pitch, not the reciprocal of the lead.



) Acme stub



(c) Square

(d) Modified square

(e) Buttress

FIGURE 10.4

Power screw thread forms. [Note: All threads shown are external (i.e., on the screw, not on the nut); d_m is the mean diameter of the thread contact and is approximately equal to $(d + d_r)/2$.]

TABLE 10.3 Standard Sizes of Power Screw Threads

Major Diameter d (in.)	Threads per Inch: Acme and Acme Stub[a]	Threads per Inch: Square and Modified Square	Threads per Inch: Buttress[b]
$\frac{1}{4}$	16	10	
$\frac{5}{16}$	14		
$\frac{3}{8}$	12		
$\frac{3}{8}$	10	8	
$\frac{7}{16}$	12		
$\frac{7}{16}$	10		
$\frac{1}{2}$	10	$6\frac{1}{2}$	16
$\frac{5}{8}$	8	$5\frac{1}{2}$	16
$\frac{3}{4}$	6	5	16
$\frac{7}{8}$	6	$4\frac{1}{2}$	12
1	5	4	12
$1\frac{1}{8}$	5		
$1\frac{1}{4}$	5	$3\frac{1}{2}$	10
$1\frac{3}{8}$	4		10
$1\frac{1}{2}$	4	3	10
$1\frac{3}{4}$	4	$2\frac{1}{2}$	8
2	4	$2\frac{1}{4}$	8
$2\frac{1}{4}$	3	$2\frac{1}{4}$	8
$2\frac{1}{2}$	3	2	8
$2\frac{3}{4}$	3	2	6
3	2	$1\frac{3}{4}$	6
$3\frac{1}{2}$	2	$1\frac{5}{8}$	6
4	2	$1\frac{1}{2}$	6
$4\frac{1}{2}$	2		5
5	2		5

10.3 Power Screws

- Nut turned with applied torque of T lifts load P
- To compensate for the friction between nut and the base, thrust bearings are used
- Another application is shown below
  - For accurate positioning of the nut, based on rotation of the lead screw by servomotor

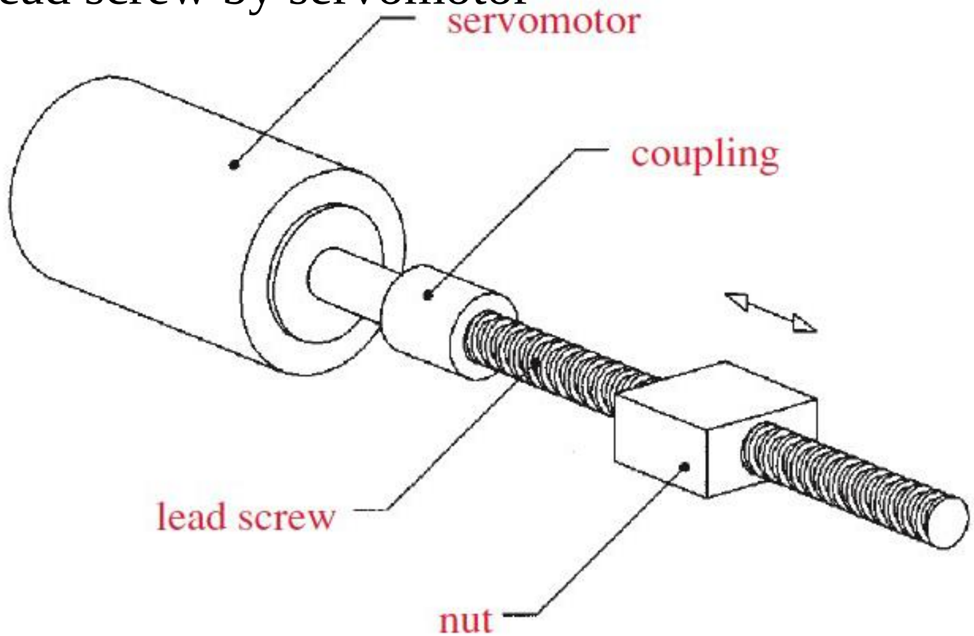


FIGURE 15-5

Servomotor-Driven Lead Screw for Use as a Positioning Device *Courtesy of J. Karsberg,*

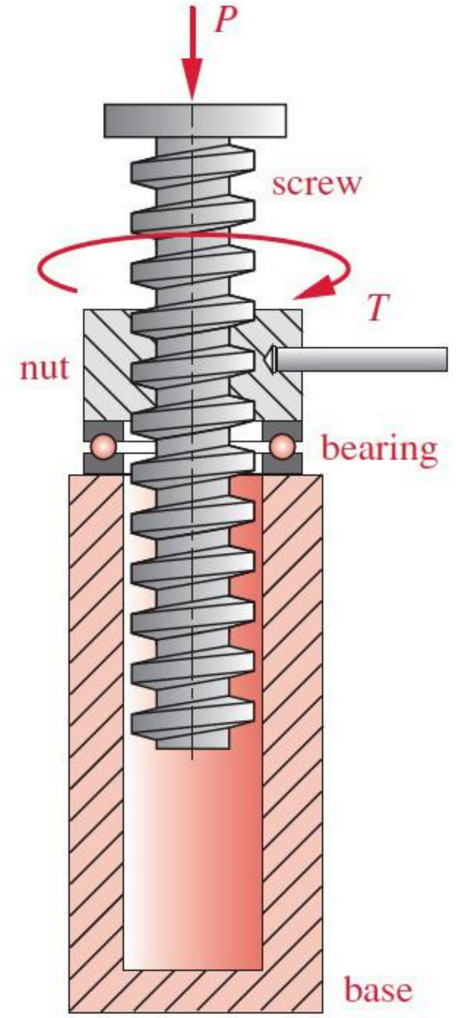


FIGURE 15-4

An Acme-Thread Power-Screw Jack

10.3 Power Screws

- shaded member connected to the handle rotates, and that a ball thrust bearing transfers the axial force from the rotating to a nonrotating member.
- All 3 jacks being same, Figure 10.5c for determining the torque, Fa, that must be applied to the nut in order to lift a given weight.
- Turning the nut in Figure 10.5c forces each portion of the nut thread to climb an inclined plane.

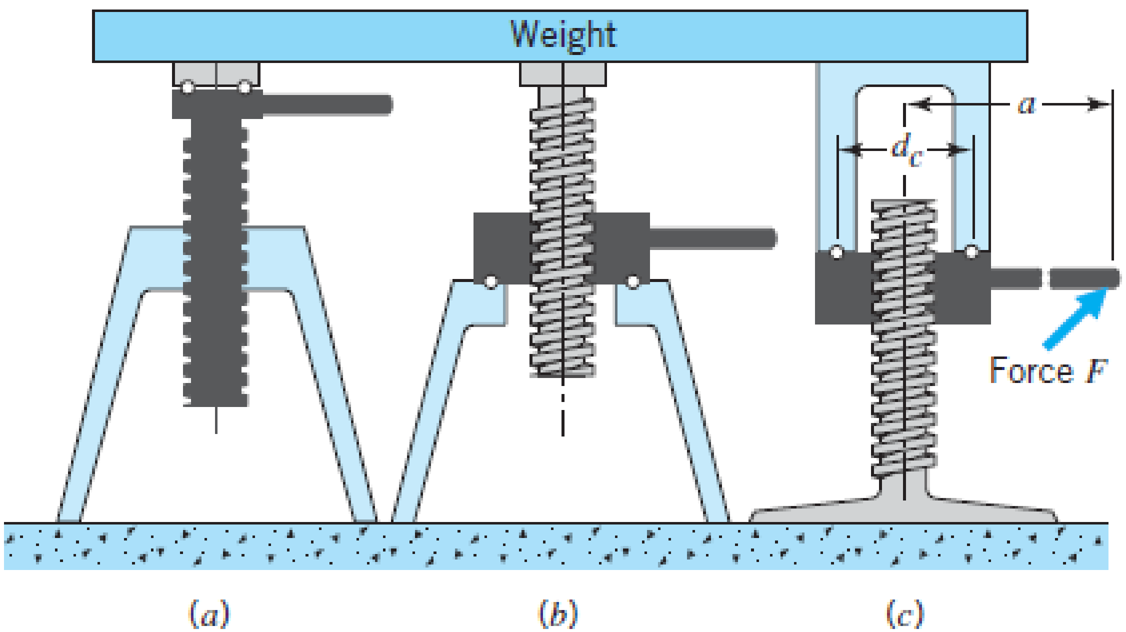


FIGURE 10.5
Weight supported by three screw jacks. In each screw jack, only the shaded member rotates.

10.3 Power Screws

- Turning the nut forces each portion of the nut thread to climb an inclined plane.
- If a full turn were developed, a triangle would be formed, illustrating tan λ

$$\tan \lambda = \frac{L}{\pi d_m} \tag{10.1}$$

- A segment of the nut is represented by the small block acted upon by load w, normal force n, friction force fn, and tangential force q.
- force q times $d_m/2$ represents the torque applied to the nut segment.

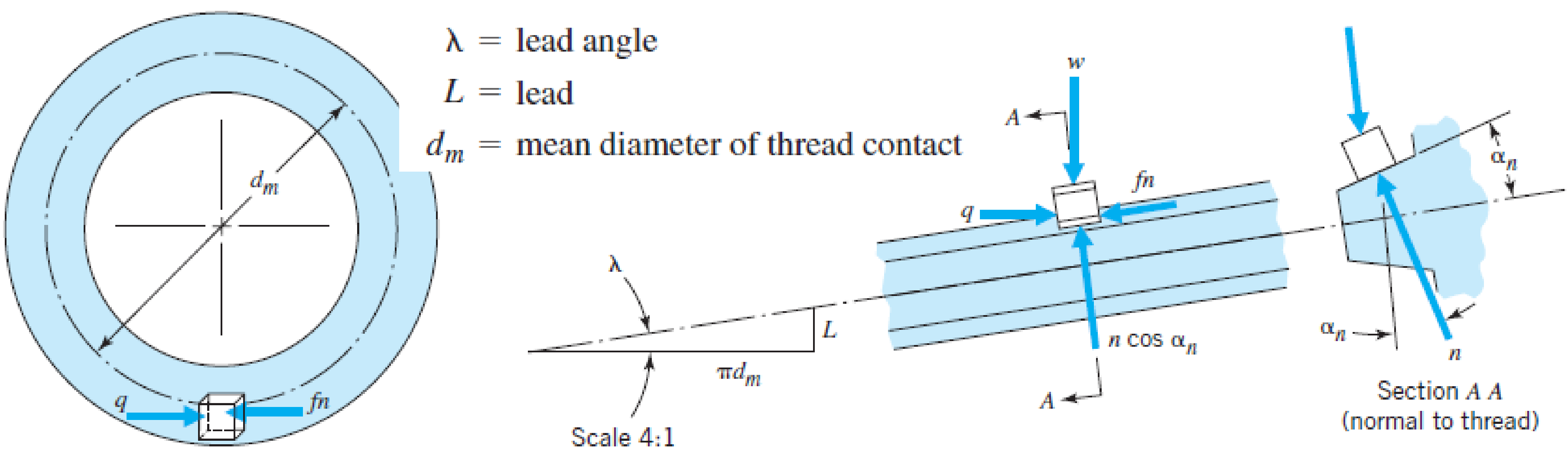


FIGURE 10.6
Screw thread forces.

10.3 Power Screws

- Summing the tangential forces

$$\Sigma F_t = 0: \quad q - n(f \cos \lambda + \cos \alpha_n \sin \lambda) = 0 \qquad \textbf{(a)}$$

.

- Summing the axial forces

$$\Sigma F_a = 0: \quad w + n(f \sin \lambda - \cos \alpha_n \cos \lambda) = 0$$

$$n = \frac{w}{\cos \alpha_n \cos \lambda - f \sin \lambda} \qquad \textbf{(b)}$$

Combining Eqs. a and b, we have

$$q = w \frac{f \cos \lambda + \cos \alpha_n \sin \lambda}{\cos \alpha_n \cos \lambda - f \sin \lambda} \qquad \textbf{(c)}$$

- With torque for q being $q(d_m/2)$ and q, n, w are acting on a small segment of the nut, integrating this to full nut and changing the notations to Q, N, W, the torque T required to lift a load W is

$$T = Q\frac{d_m}{2} = \frac{Wd_m}{2} \frac{f \cos \lambda + \cos \alpha_n \sin \lambda}{\cos \alpha_n \cos \lambda - f \sin \lambda} \qquad \textbf{(10.2)}$$

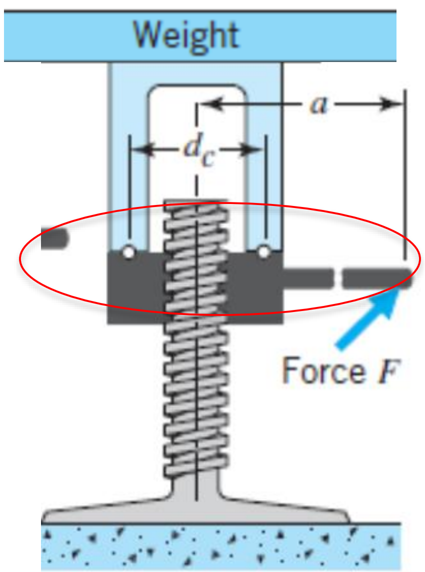
10.3 Power Screws

- Since L is more commonly referred to in threads than λ, dividing the numerator and denominator by cos λ and then substituting L/πdm for tan λ.

$$T = \frac{Wd_m}{2}\frac{f\pi d_m + L\cos\alpha_n}{\pi d_m\cos\alpha_n - fL} \qquad (10.3)$$

- Since a bearing or thrust washer (dia d_c) is used friction adds to the torque required
- If the coefficient of friction of the collar washer or bearing is f_c then

$$T = \frac{Wd_m}{2}\frac{f\pi d_m + L\cos\alpha_n}{\pi d_m\cos\alpha_n - fL} + \frac{Wf_c d_c}{2} \qquad (10.4)$$



- For a square thread, $\cos\alpha_n = 1$ this simplifies to

$$T = \frac{Wd_m}{2}\frac{f\pi d_m + L}{\pi d_m - fL} + \frac{Wf_c d_c}{2} \qquad (10.4a)$$

10.3 Power Screws

- For lowering the load, the directions of q and fn are reversed giving

$$T = \frac{Wd_m}{2}\frac{f\pi d_m - L\cos\alpha_n}{\pi d_m \cos\alpha_n + fL} + \frac{Wf_c d_c}{2} \tag{10.5}$$

- For a square thread, this simplifies to

$$T = \frac{Wd_m}{2}\frac{f\pi d_m - L}{\pi d_m + fL} + \frac{Wf_c d_c}{2} \tag{10.5a}$$

- f_c can be (because very low) neglected if ball or roller thrust bearing is used and the second portion of the term does not come into play

- f & f_c can vary between .08 to .2 if plain thrust collar is used

10.3 Power Screws

- Self locking implies positive torque to lower the load

$$T = \frac{Wd_m}{2}\frac{f\pi d_m - L\cos\alpha_n}{\pi d_m\cos\alpha_n + fL} + \frac{Wf_c d_c}{2} \qquad \textbf{(10.5)}$$

- Neglecting collar friction, screw can be self locking if T>=0

$$f \geqq \frac{L\cos\alpha_n}{\pi d_m} \qquad \textbf{(10.7)}$$

- For square threads

$$T = \frac{Wd_m}{2}\frac{f\pi d_m - L}{\pi d_m + fL} + \frac{Wf_c d_c}{2} \qquad \textbf{(10.5a)}$$

$$f \geqq \frac{L}{\pi d_m}, \qquad \text{or} \quad f \geqq \tan\lambda$$

10.3 Power Screws

- Work output divided by work input is the efficiency
- Work output in 1 revolution is load times distance which is WL
- Work input is the torque in one revolution which is $2\pi T$
- So efficiency e = WL/ $2\pi T$

$$\text{Efficiency, } e = \frac{L}{\pi d_m} \frac{\pi d_m \cos \alpha_n - fL}{\pi f d_m + L \cos \alpha_n} \qquad (10.8)$$

- For a square thread

$$e = \frac{L}{\pi d_m} \frac{\pi d_m - fL}{\pi f d_m + L} \qquad (10.8a)$$

- Simplified to

$$e = \frac{\cos \alpha_n - f \tan \lambda}{\cos \alpha_n + f \cot \lambda} \qquad (10.9)$$

for square threads

$$e = \frac{1 - f \tan \lambda}{1 + f \cot \lambda}$$

10.3 Power Screws

- As f increases; e lowers
- Efficiency tends to 0 as lead angle approaches 0, as load does not move much in the vertical plane
- Efficiency tends to 0 as lead angle approaches 90, as the plane more perpendicular and requires a lot of torque to move the object even slightly
- Ball bearing screws reduce f

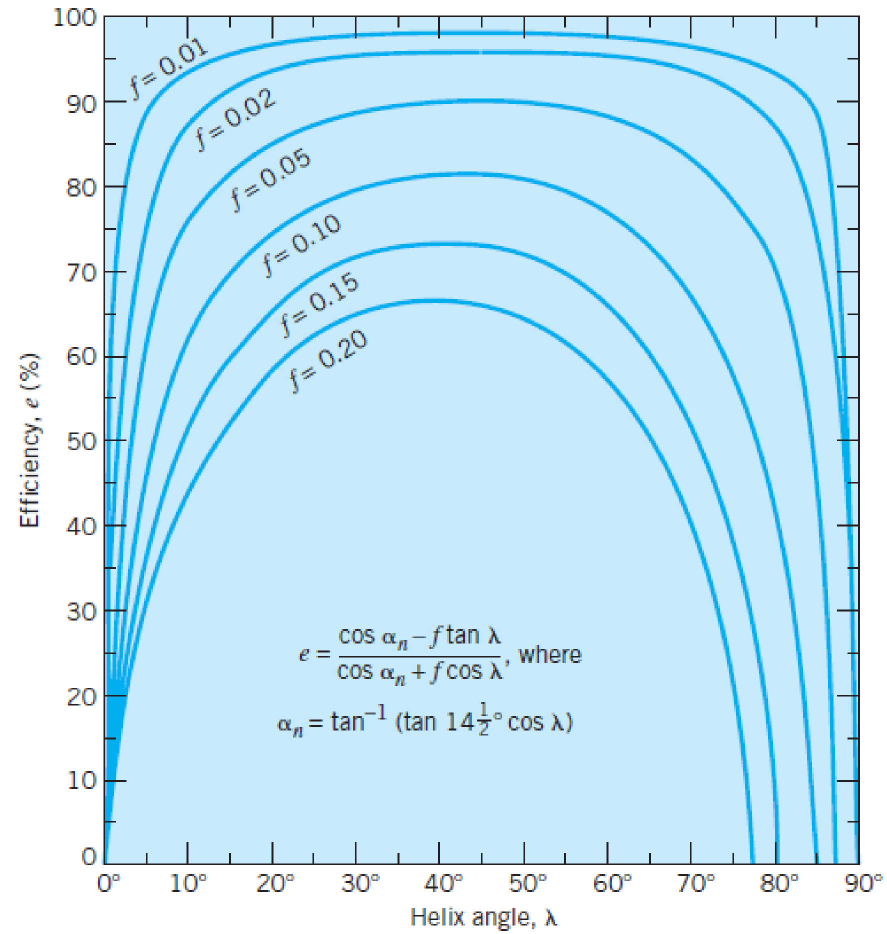


FIGURE 10.8
Efficiency of Acme screw threads when collar friction is negligible. (Note: Values for square threads are higher by less than 1 percent.)

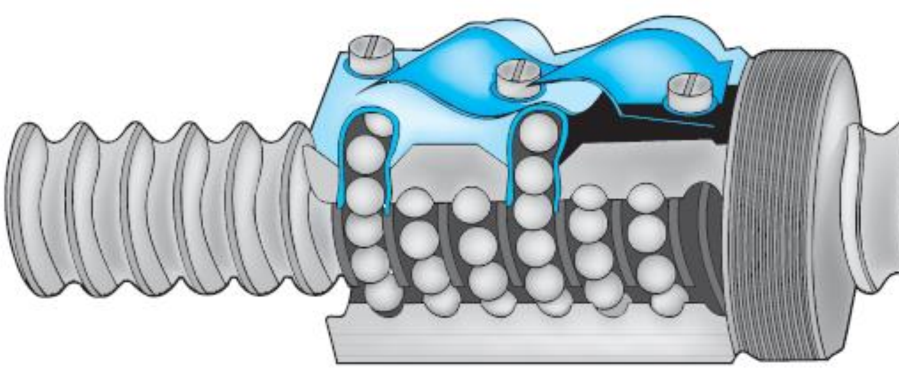


FIGURE 10.9
Ball-bearing screw assembly with a portion of the nut cut away to show construction. (Courtesy Saginaw Steering Gear Division, General Motors Corporation.)

Sample Problem 10.1 Acme Power Screw

A screw jack (Figure 10.10) with a 1-in., double-thread Acme screw is used to raise a load of 1000 lb. A plain thrust collar of $1\frac{1}{2}$-in. mean diameter is used. Coefficients of running friction are estimated as 0.12 and 0.09 for f and f_c, respectively.

a. Determine the screw pitch, lead, thread depth, mean pitch diameter, and helix angle.

b. Estimate the starting torque for raising and for lowering the load.

c. Estimate the efficiency of the jack when raising the load.

Solution

Known: A double-thread Acme screw and a thrust collar, each with known diameter and running friction coefficient, are used to raise a specified load.

Find:

a. Determine the screw pitch, lead, thread depth, mean pitch diameter, and helix angle.

b. Estimate the starting torque for raising and lowering the load.

c. Calculate the efficiency of the jack when raising the load.

TABLE 10.3 Standard Sizes of Power Screw Threads

Major Diameter d (in.)	Threads per Inch: Acme and Acme Stub[a]
$\frac{1}{4}$	16
$\frac{5}{16}$	14
$\frac{3}{8}$	12
$\frac{3}{8}$	10
$\frac{7}{16}$	12
$\frac{7}{16}$	10
$\frac{1}{2}$	10
$\frac{5}{8}$	8
$\frac{3}{4}$	6
$\frac{7}{8}$	6
1	5

Schematic and Given Data:

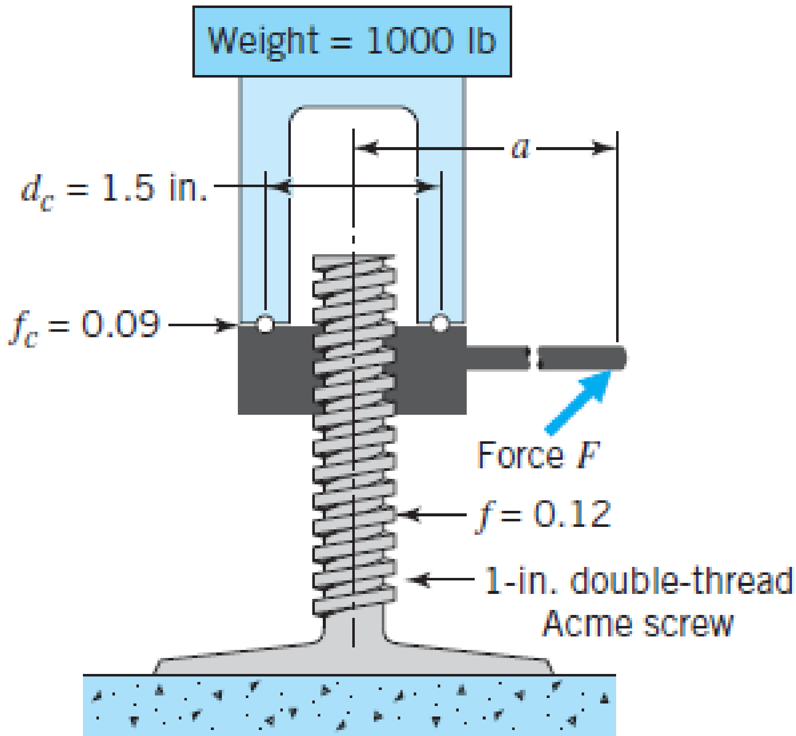


FIGURE 10.10
Screw jack lifting a nonrotating load.

Assumptions:

1. The starting and running friction remain steady.
2. Starting friction is about one-third higher than running friction.

Analysis:

a. From Table 10.3, there are five threads per inch, hence $p = 0.2$ in.
Because of the double thread, $L = 2p$, or $L = 0.4$ in.
From Figure 10.4*a*, thread depth $= p/2 = 0.1$ in.
From Figure 10.4*a*, $d_m = d - p/2 = 1 \text{ in.} - 0.1 \text{ in.} = 0.9$ in.
From Eq. 10.1, $\lambda = \tan^{-1} L/\pi d_m = \tan^{-1} 0.4/\pi(0.9) = 8.05°$.

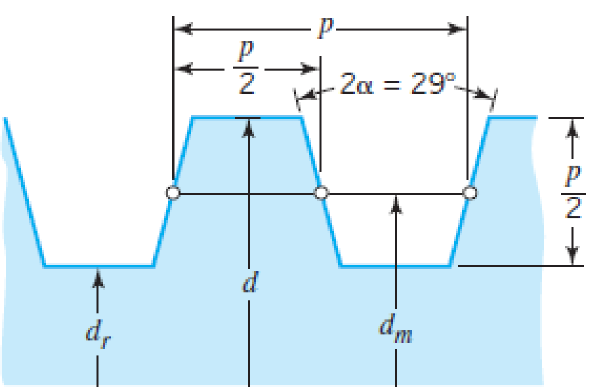
b. For starting, increase the given coefficients of friction by about one-third, giving $f = 0.16$ and $f_c = 0.12$. Equation 10.4a for square threads could be used with sufficient accuracy, but we will illustrate the complete solution, using Eq. 10.4 for the general case.

First, find α_n from Eq. 10.6:

$$\begin{aligned}\alpha_n &= \tan^{-1}(\tan \alpha \cos \lambda)\\ &= \tan^{-1}(\tan 14.5° \cos 8.05°) = 14.36°\end{aligned}$$

Then, substituting in Eq. 10.4 gives

$$\begin{aligned}T &= \frac{Wd_m}{2}\frac{f\pi d_m + L\cos\alpha_n}{\pi d_m \cos\alpha_n - fL} + \frac{Wf_c d_c}{2}\\ &= \frac{1000(0.9)}{2}\frac{0.16\pi(0.9) + 0.4\cos 14.36°}{\pi(0.9)\cos 14.36° - 0.16(0.4)} + \frac{1000(0.12)(1.5)}{2}\\ &= 141.3 + 90; \qquad T = 231.3 \text{ lb} \cdot \text{in.}\end{aligned}$$



(*a*) Acme

(*Comment:* With reference to Figure 10.10, this would correspond to a force of 19.3 lb on the end of a 12-in. handle. If Eq. 10.4a is used, the answer is only slightly less: 228.8 lb · in.). For lowering the load, use Eq. 10.5:

$$T = \frac{Wd_m}{2}\frac{f\pi d_m - L\cos\alpha_n}{\pi d_m\cos\alpha_n + fL} + \frac{Wf_c d_c}{2}$$

$$= \frac{1000(0.9)}{2}\frac{0.16\pi(0.9) - 0.4\cos 14.36°}{\pi(0.9)\cos 14.36° + 0.16(0.4)} + 90$$

$$= 10.4 + 90; \qquad T = 100.4\ \text{lb}\cdot\text{in.}$$

(*Comment:* Equation 10.5a gives a torque of 98.2 lb · in.)

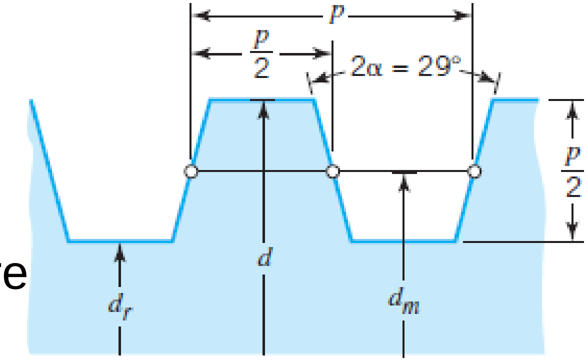
c. Repeating the substitution in Eq. 10.4, but changing the coefficient of friction to the running values of 0.12 and 0.09, indicates that to raise the load, once motion is started, the torque must be 121.5 + 67.5 = 189 lb · in. Substituting once more in Eq. 10.4, but changing both friction coefficients to zero, indicates that the torque must be 63.7 + 0 = 63.7 lb · in. to raise the load. Efficiency is the ratio of friction-free torque to actual torque, or

$$e = \frac{63.7}{189} = 33.7\ \text{percent}$$

Comment: If a ball thrust bearing were used so that collar friction could be neglected, the efficiency would increase to 63.7/121.5 = 52 percent. This would correspond to the efficiency of the screw itself and agrees with the plotted value in Figure 10.8.

10.4 Static Screw Stresses

10.4.1 Torsion



(a) Acme

- For power screws and threaded fasteners the stress are

- **<u>Torsion</u>** while tightening

$$\tau = \frac{Tc}{J} = \frac{16T}{\pi d^3} \qquad (4.3, 4.4)$$

- where d is root diameter, d_r , obtained from Figure 10.4 (for power screws) or Tables 10.1 and 10.2 (for threaded fasteners).

- If the screw or bolt is hollow, where d_i represents the inside diameter.

- Where collar friction is negligible, the torque transmitted through a power screw is the full applied torque.
- With threaded fasteners, the equivalent of substantial collar friction is normally present, in which case it is customary to assume that the torque transmitted through the threaded section is approximately half the wrench torque.

$$J = \pi\,(d_r^4 - d_i^4)/32$$

10.4 Static Screw Stresses

10.4.2 Axial Load

- Power screws are subjected to direct P/A tensile and compressive stresses; threaded fasteners are normally subjected only to tension.
- The effective area for fasteners is the tensile stress area A_t (Table 10.1 & 10.2).
- For power screws axial stresses are not critical; so A_t, approximated based on d_r
- Threaded fasteners should always have enough ductility to permit local yielding at thread roots without damage. So non uniform load distribution is ok for static stresses. But not fatigue.

10.4.3 Combined Torsion and Axial Load

- The combination of the stresses can be the distortion energy theory used as a criterion for yielding.
- With threaded fasteners, it is normal for some yielding to occur at the thread roots during initial tightening.