MECH 344/M
Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 8

Contents of today's lecture

12 Springs

12.1 Introduction

- Springs are elastic members that exert forces, or torques, and absorb energy, which is usually stored and later released.
- Mostly made of metal. Plastics, and rubber are used when loads are light
- For applications requiring compact springs providing very large forces with small deflections, hydraulic springs have proved effective.
- If energy absorption with maximum efficiency (minimum spring mass) is the objective, the ideal solution is an unnotched tensile bar,
- Unfortunately, tensile bars of any reasonable length are too stiff for most spring applications; hence it is necessary to form the spring material so that it can be loaded in torsion or bending.

12.2 Torsion Bar Springs

- Simplest spring is the torsion bar spring
- Used in automotive applications
- Stress, angular deflection and spring rate

$$\tau = \frac{Tr}{J} \qquad \theta = \frac{TL}{JG} \qquad K = \frac{JG}{L}$$

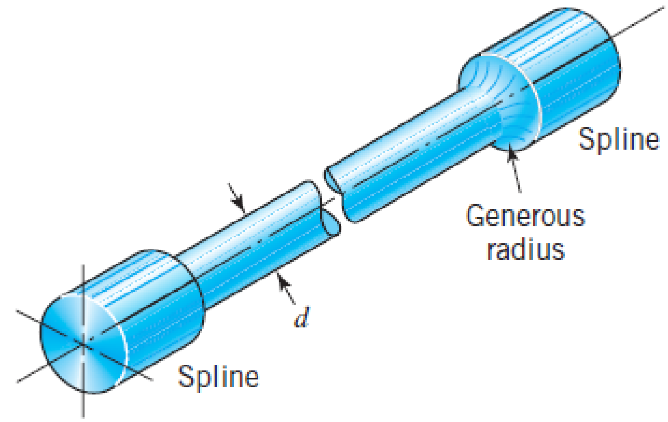
(see Table 5.1)

- For a solid bar of diameter 'd'

$$\tau = \frac{16T}{\pi d^3} \qquad \theta = \frac{32TL}{\pi d^4 G} \qquad K = \frac{\pi d^4 G}{32L}$$

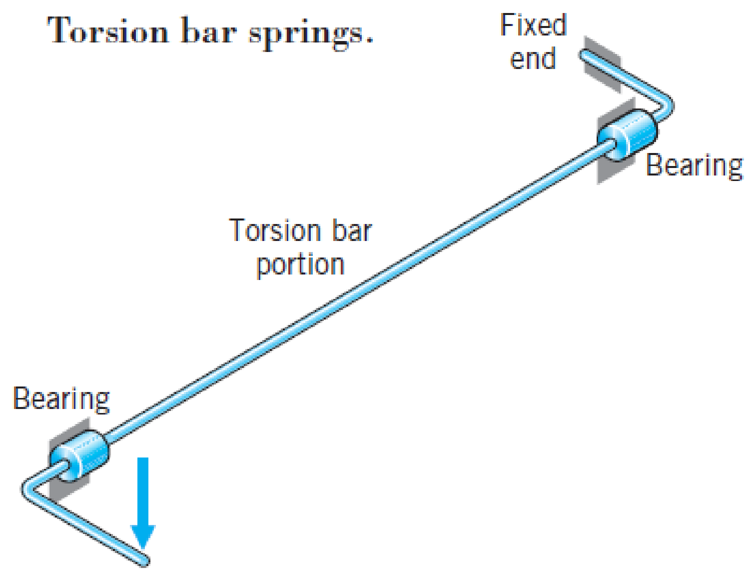
- Shear modulus G is

$$G = \frac{E}{2(1 + \nu)}$$



(a)
Torsion bar with splined ends
(type used in auto suspensions, etc.)

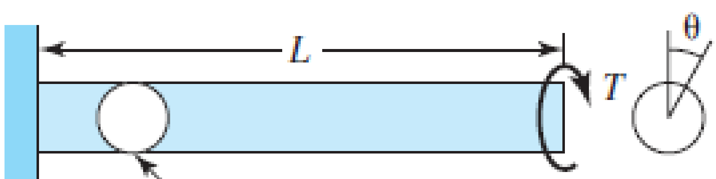
FIGURE 12.1
Torsion bar springs.



(b)
Rod with bent ends serving as torsion bar spring
(type used for auto hood and trunk counterbalancing, etc.)

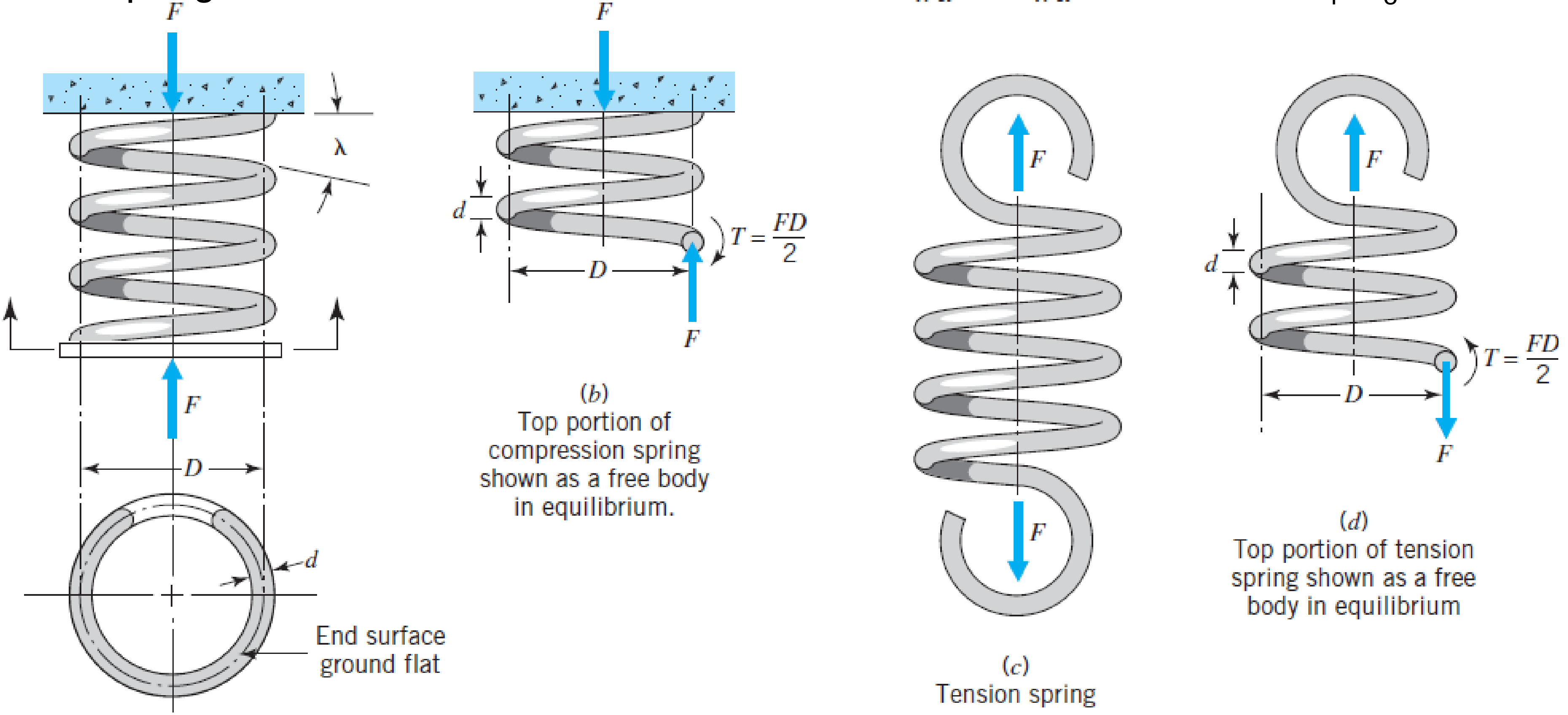
12.2 Torsion Bar Springs

TABLE 5.1 Deflection and Stiffness Formulas for Straight Bars (Rods, Beams) of Uniform Section

Number	Case	Deflection	Spring Rate
2.	Torsion  K'^{a} = section property. For solid round section, $K' = J = \pi d^4/32$.	$\theta = \dfrac{TL}{K'G}$ For solid round bar and deflection in degrees, $\theta^\circ = \dfrac{584TL}{d^4G}$	$K = \dfrac{T}{\theta} = \dfrac{K'G}{L}$

12.3 Coil Spring Stress and Deflection Equations

- Figure shows compression and extension springs of small helix angle λ
- Force F applied along helix axis, and on the whole length the wire experiences F (transverse force) and FD/2 (torsion force)
- For spring of solid wire with dia 'd' $\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{8FD}{\pi d^3}$ where D = d_i+d_o/2



(a) Compression spring (ends squared and ground)

(b) Top portion of compression spring shown as a free body in equilibrium.

(c) Tension spring

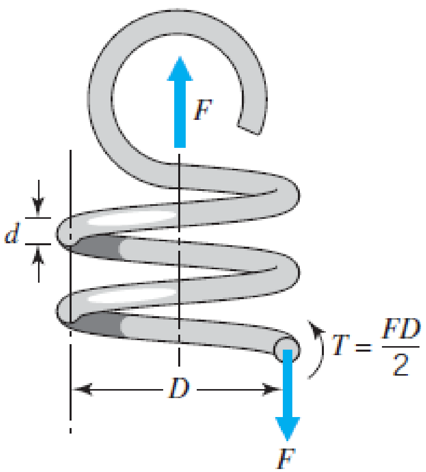
(d) Top portion of tension spring shown as a free body in equilibrium

FIGURE 12.2

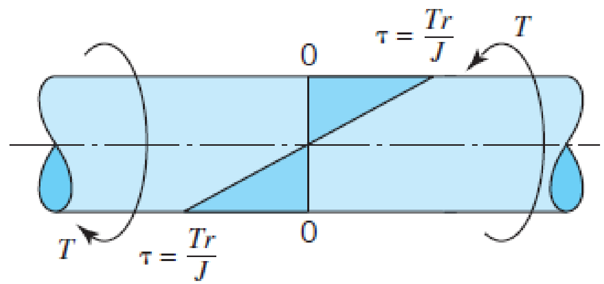
Helical (coil) compression and tension springs.

12.3 Coil Spring Stress and Deflection Equations

- The curvature of the spring, implies that there is an additional stress on the inside of the coil (fig)
- This effect is severe for small values of spring index C (ie C=D/d)
- The analysis was first published by Wahl and hence called Wahl factor K_w which is multiplied with τ to get the stress on the inside of the spring

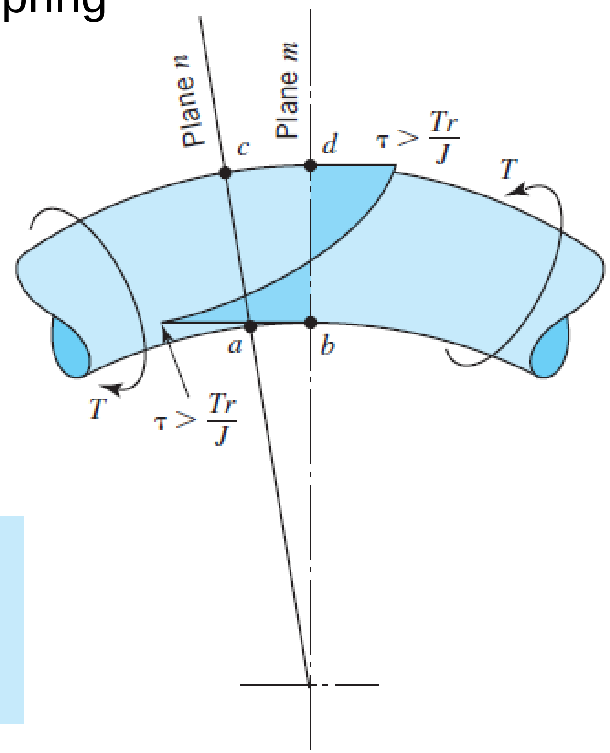


(d)
Top portion of tension spring shown as a free body in equilibrium



(a)
Straight torsion bar

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$



(b)
Curved torsion bar

FIGURE 12.2
Helical (coil) compression and tension springs.

12.3 Coil Spring Stress and Deflection Equations

- When static loading the first term can be 1 (considered as stress concentration)
- $$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$ so $$K_s = 1 + \frac{0.615}{C}$$
- Which reduces to $$\tau = \frac{8FD}{\pi d^3} K_s$$ if initial yielding occurs in static loading and after which the loads remain uniform the K_s can be approximated to $$K_s = 1 + \frac{0.5}{C}$$
- Use K_s for static and K_w for fatigue for normal springs C>3, $\lambda<12^\circ$
- In case of fatigue loading $$\tau = \frac{8FD}{\pi d^3} K_w = \frac{8F}{\pi d^2} C K_w$$
- In case of static loading $$\tau = \frac{8FD}{\pi d^3} K_s = \frac{8F}{\pi d^2} C K_s$$
- The values of K_s, K_w, CK_s, CK_w are shown in fig

12.3 Coil Spring Stress and Deflection Equations

- These equations are derived neglecting the following
- **The bending stresses**, if λ>15, the bending of the coil is > D/4, then bending needs to be considered
- **Load eccentricity**. Load acting away from spring axis causes the stresses on one side of the spring to be higher than indicated by equations
- **Axial loading**. In addition to creating a transverse shear stress, a small component of force F produces axial compression of the spring wire. In critical spring designs involving relatively large values of λ, this factor may warrant consideration.

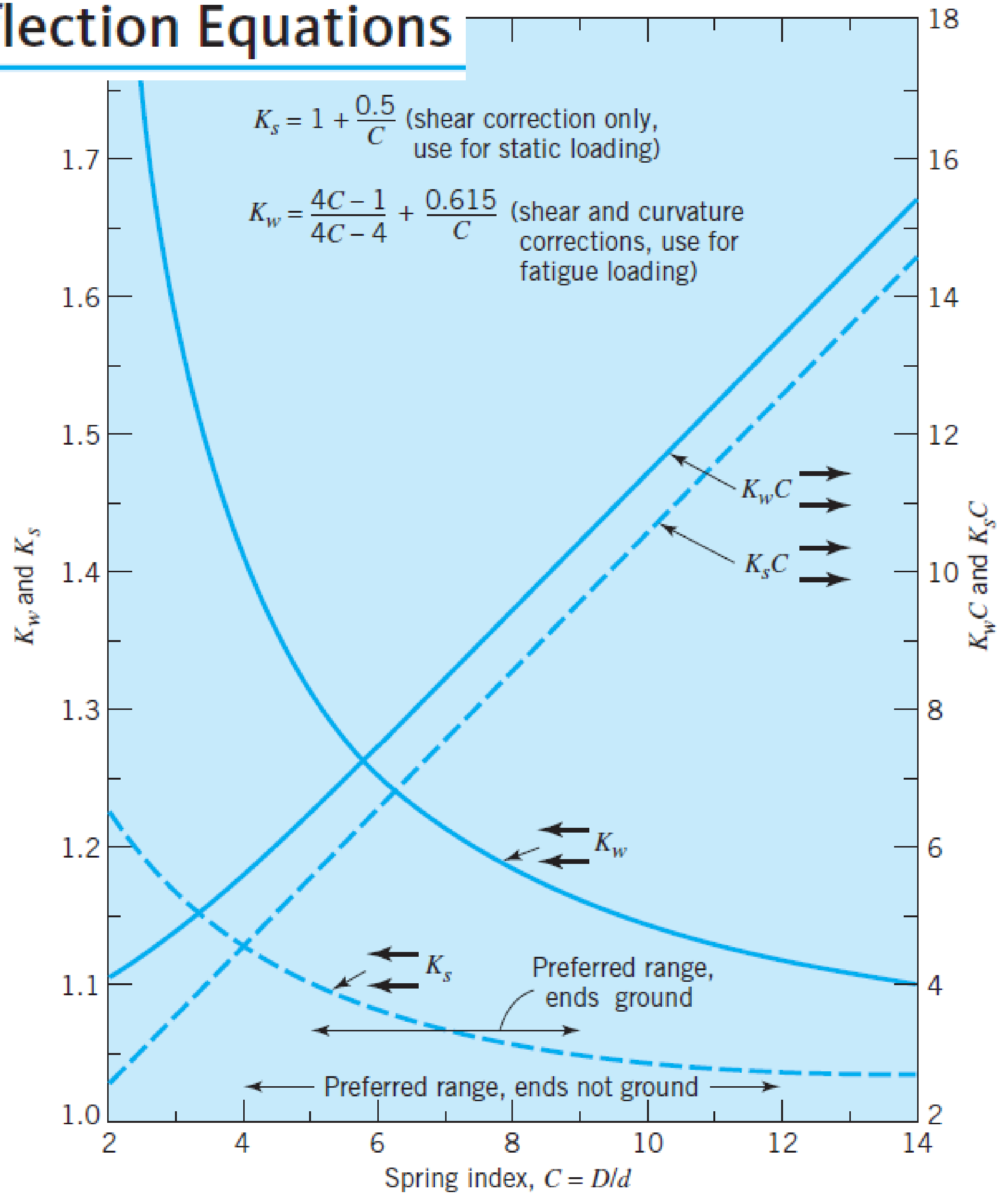


Figure 12.4
Stress correction factors for helical springs.

12.3 Coil Spring Stress and Deflection Equations

- Catigliano's methods, and considering the torsional load as the major contribution towards spring deflection δ,

$$\delta = \frac{8FD^3N}{d^4G}$$

- Where N is the "active" number of coils (end coils that do not contribute to deflection, are not counted)
- And the spring rate k which is F/δ is

$$k = \frac{d^4G}{8D^3N}, \quad k = \frac{dG}{8NC^3}$$

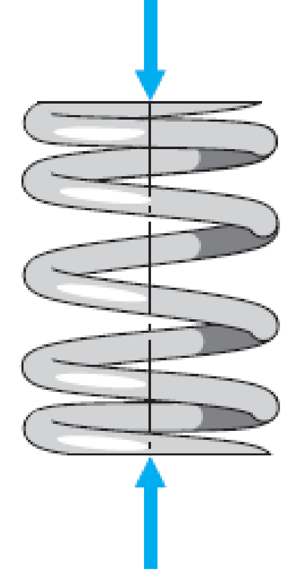


FIGURE 12.5
Helical compression spring of unequal pitch.

- As the spring is loaded, the coils bottom out slowly becoming inactive thereby reducing N and increasing K
- In case of conical springs, the solid height will be same of spring diameter.
- In this case the torque (which is function of D) will not be uniform
- The deflection and spring constant (stiffness) of conical spring can be approximated using the same euqations considering the average value of D

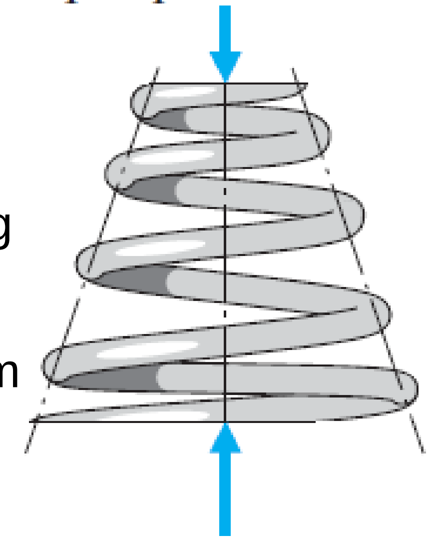


FIGURE 12.6
Conical compression coil spring.

12.4 Stress and Strength Analysis for Helical Compression Springs—Static Loading

- Helical springs are wound from wire of solid round cross section and manufactured in standard “gage” diameters. The relative costs and minimum tensile strengths of commonly used spring wire materials are given in Table 12.1 and Figure 12.7,
- For spring design allowable values of shear stress are needed for use with Eq. 12.5.

$$\tau = \frac{8FD}{\pi d^3} K_w = \frac{8F}{\pi d^2} C K_w \qquad (12.5)$$

Table 12.1 Relative Cost[a] of Common Spring Wire of 2-mm (0.079-in.) Diameter

Wire Material	ASTM Specification	Relative Cost
Patented and cold-drawn steel	A227	1.0
Oil-tempered steel	A229	1.3
Music (steel)	A228	2.0
Carbon steel valve spring	A230	2.5
Chrome silicon steel valve	A401	4.0
Stainless steel (Type 302)	A313 (302)	6.2
Phosphor bronze	B159	7.4
Stainless steel (Type 631)	A313 (631)	9.9
Beryllium copper	B197	22.
Inconel alloy X-750		38.

[a] Average of mill and warehouse quantities [2].

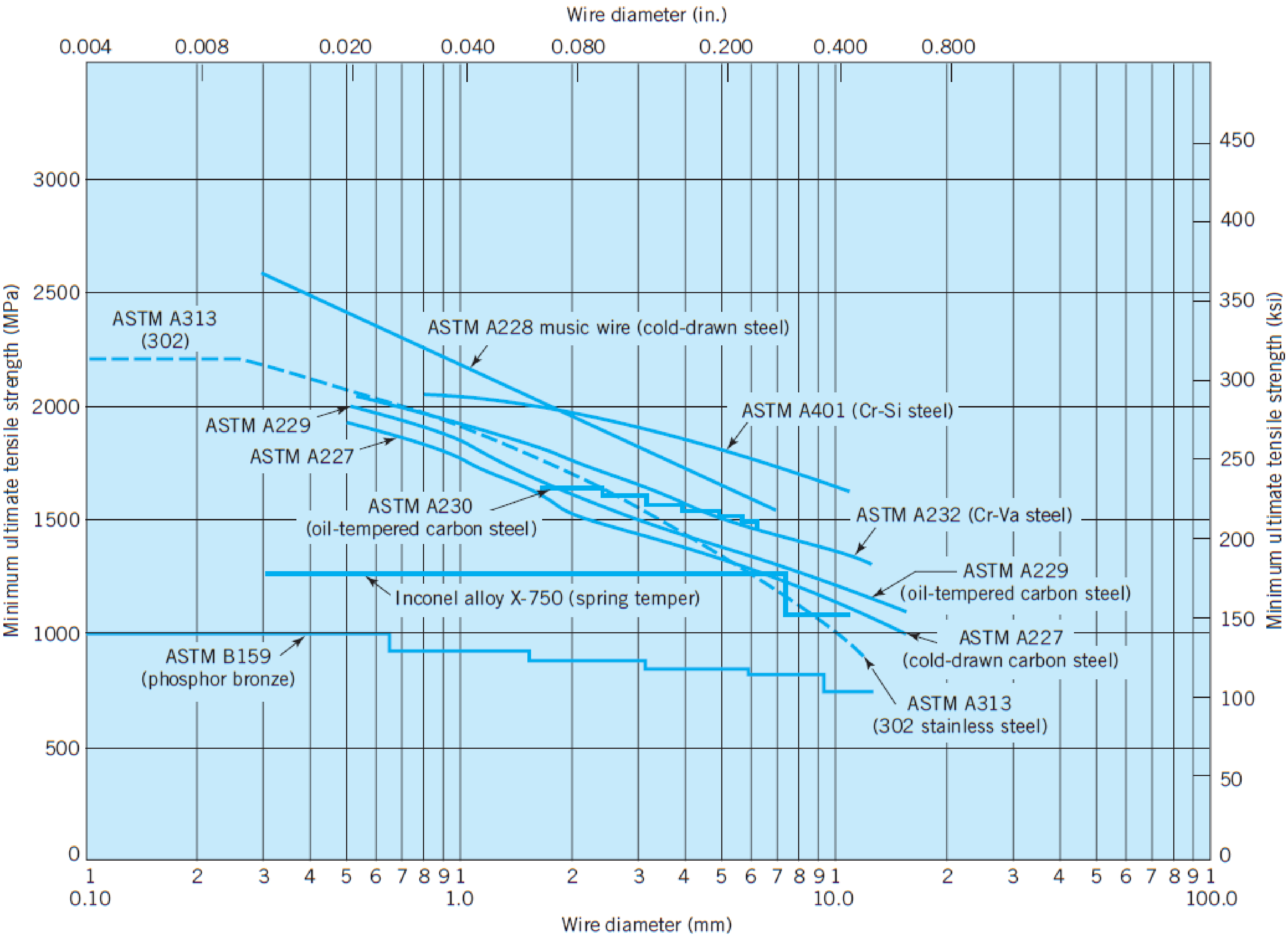


FIGURE 12.7

Tensile strengths of various spring wire materials and diameters, minimum values [2].

12.4 Stress and Strength Analysis for Helical Compression Springs—Static Loading

- First step in designing springs for static loading is avoiding set, or long-term shortening $S_{sy} = 0.53S_u$.
- Max stress on a compression helical spring is loading it to its solid height (all coils touching). never should be experienced in service, but can happen during installation or removal. Typically then, τ (calculated with F equal to the load required to close the spring solid) < S_{sy}, or, less than $0.53S_u$
- Less than 2% long-term "set" will occur in springs designed for τ_s (where subscript s denotes spring "solid") equal to $0.45S_u$ for ferrous spring, or $0.35S_u$ for nonferrous and austenitic stainless steel springs.
- If we use the pervious step the safety factor is .53/.45 about 1.18 (good enough for known load and high quality spring manufacturing) also little over 2% set is not a major cause of concern as well
- Springs can be designed for working loads that brings the spring close to solid. So a clash allowance of 10% is provided so even when small fluctuations in load will not close the spring solid
- Since compression springs are loaded in compression, the residual stress is favorable. Initially coiling the spring more than required and allow to yield slightly and this is called presetting.

12.4 Stress and Strength Analysis for Helical Compression Springs—Static Loading

- taking maximum advantage of presetting permits the design stress to be increased from the $0.45S_u$ and $0.35S_u$ values to $0.65S_u$ and $0.55S_u$.
- To limit long-term set in compression coil springs to less than 2 %, shear stresses calculated from Eq. 12.6 (normally with force F corresponding to spring "solid") should be

$\tau_s \leqq 0.45S_u$ (ferrous—without presetting)

$\tau_s \leqq 0.35S_u$ (nonferrous and austenitic stainless—without presetting)

$\tau_s \leqq 0.65S_u$ (ferrous—with presetting)

$\tau_s \leqq 0.55S_u$ (nonferrous and austenitic stainless—with presetting)

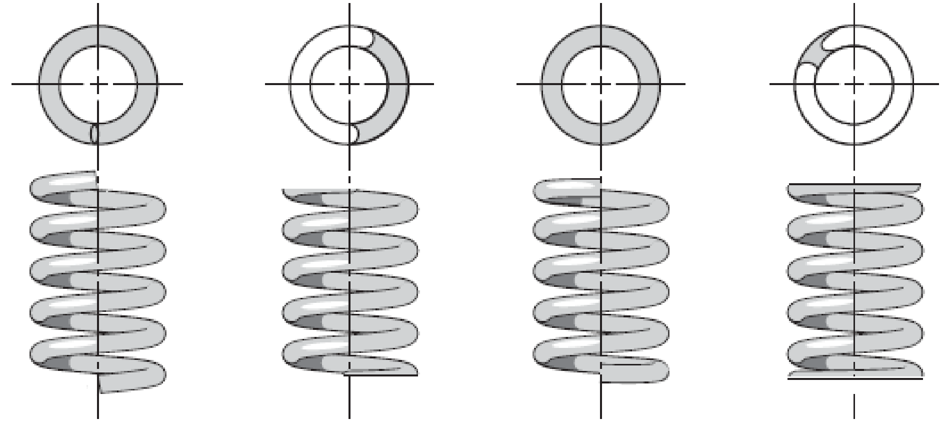
$$\tau = \frac{8FD}{\pi d^3}K_s = \frac{8F}{\pi d^2}CK_s \qquad \textbf{(12.6)}$$

12.5 End Designs of Helical Compression Springs

- 4 “standard” end designs in compression helical spring shown in figure with equations for their solid height, L_s . In all cases N_t = total number of turns, and N = number of active turns (the turns that contribute to the deflection).
- In all ordinary cases involving end plates contacting the springs on their end surfaces

$$N_t \approx N + 2 \qquad (12.10)$$

- The special springs have loading permitted in both tension or compression (b, c, and d) also you can control the active number of turns



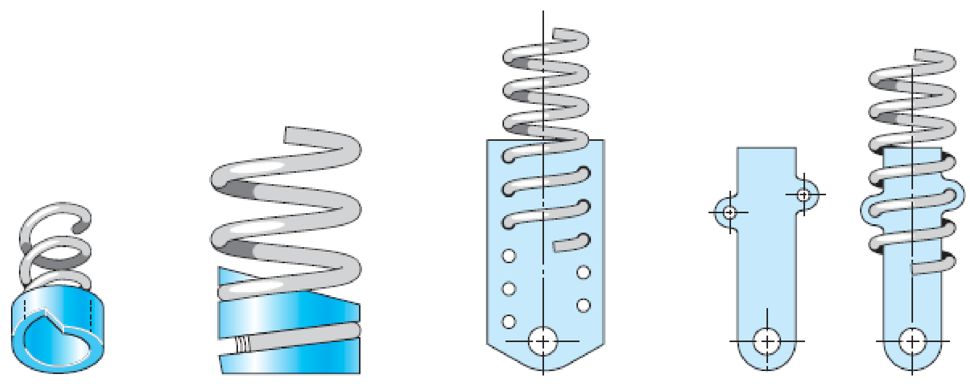
(*a*) Plain ends $L_s = (N_t + 1)d$

(*b*) Plain ends, ground $L_s = N_t d$

(*c*) Squared ends $L_s = (N_t + 1)d$

(*d*) Squared and ground ends $L_s = N_t d$

FIGURE 12.8
Compression spring ends and corresponding spring solid-height equations. (Note: Square ends are wound with a zero helix angle.)



(*a*) Contoured and plate

(*b*)

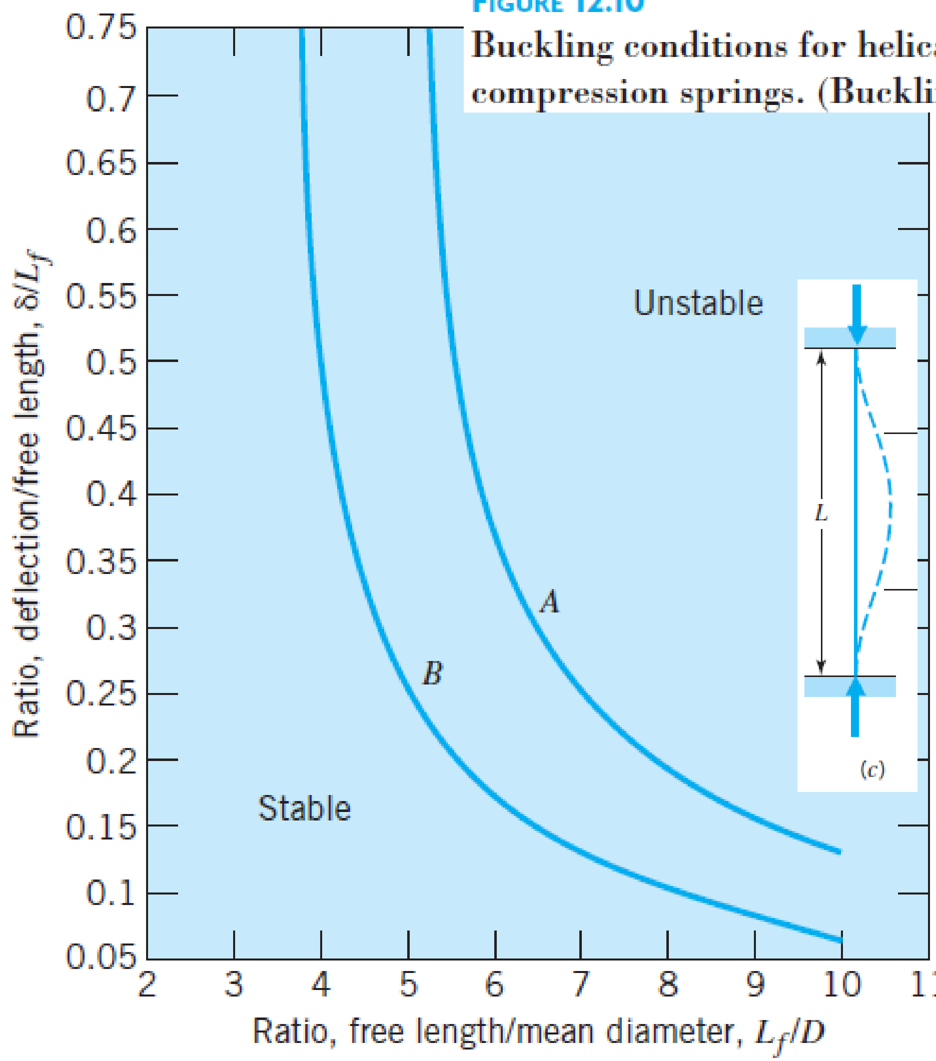
(*c*)

(*d*)

12.6 Buckling Analysis of Helical Compression Springs

- Coil springs loaded in compression act like columns and must be considered for possible buckling—particularly for large ratios of free length to mean diameter.
- Figure 12.10 gives the results for two of the end
  - Curve A (end plates constrained and parallel) represents the most common condition.
- If buckling happens, the preferred solution is to redesign the spring.
- Otherwise, the spring can be supported by placing it either inside or outside a cylinder that provides a small clearance.
- Friction and wear on the spring may have to be considered.

FIGURE 12.10
Buckling conditions for helical compression springs. (Buckling



A- end plates are constrained parallel
(buckling pattern as in Fig. 5.27*c*)
B- one end plate is free to tip
(buckling pattern as in Fig. 5.27*b*)

12.7 Design Procedure for Helical Compression Springs—Static Loading

- The two most basic requirements of a coil spring design are an acceptable stress level and the desired spring rate.
- To minimize weight, size, and cost, we usually design springs to the highest stress level that will not result in significant long term "set."
- Stress is usually considered before spring rate, in designing a spring, because stress involves D and d, but not N.
- In general, the stress requirement can be satisfied by many combinations of D and d, and the objective is to find one of these that best suits the requirements of the particular problem.
- With D and d at least tentatively selected, N is then determined on the basis of the required spring rate.
- Finally, the free length of the spring is determined by what length will give the desired clash allowance.
- If the resulting design is prone to buckling, or if the spring does not fit into the available space, another combination of D and d may be indicated.
- If the spring comes out too large or too heavy, a stronger material must be considered.

SAMPLE PROBLEM 12.1D Helical Spring Design for Static Loading

A helical spring with squared and ground ends is required to exert a force of 60 lb at a length that cannot exceed 2.5 in., and 105 lb at a length that is 0.5 in. shorter. It must fit inside a 1.5-in.-diameter (hole). Loading is essentially static. Determine a satisfactory design, using oil-tempered ASTM 229 wire, without presetting.

SOLUTION

Known: A helical compression spring exerts a force of 60 lb at a length of 2.5 in. or less and 105 lb at a length that is 0.5 in. shorter.

Find: Determine a satisfactory spring geometry.

Schematic and Given Data: The force and deflection data given for the spring can be used to construct Figure 12.11.

Decisions:

1. As recommended in Section 12.4, choose a clash allowance which is 10 percent of maximum working deflection.
2. To avoid possible interference, provide the commonly recommended diametral clearance of about $0.1D$ between the spring and the 1.5-in. specified diameter.

Assumptions:

1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

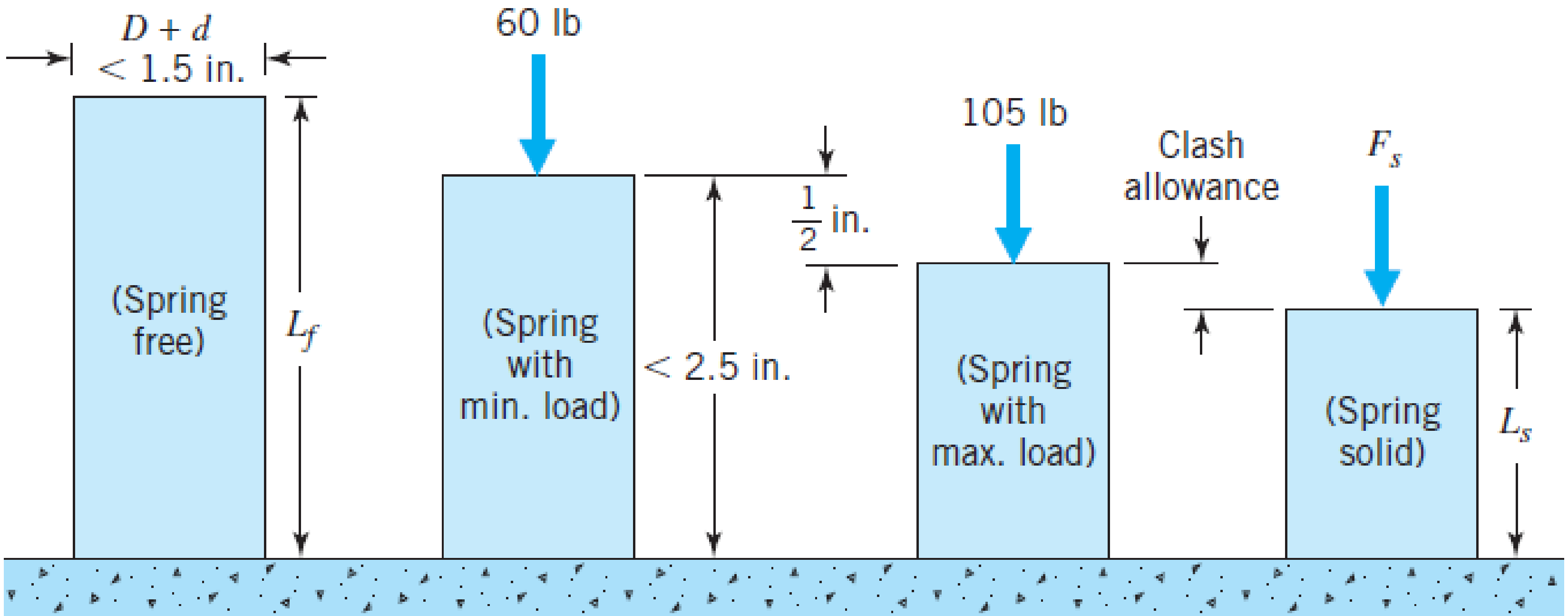


Figure 12.11

Helpful representation of information given in Sample Problem 12.1.

Design Analysis:

1. Figure 12.11 gives a convenient representation of the given information concerning spring geometry and loading. The required spring rate is

$$k = \frac{F}{\delta} = \frac{\Delta F}{\Delta \delta} = \frac{45 \text{ lb}}{0.5 \text{ in.}} = 90 \text{ lb/in.}$$

2. With a clash allowance which is 10 percent of maximum working deflection,

$$\text{Clash allowance} = 0.1 \frac{105 \text{ lb}}{90 \text{ lb/in.}} = 0.12 \text{ in.}$$

3. The force when solid (i.e., maximum force that must be resisted without "set") is therefore

$$F_{\text{solid}} = 105 + 90(0.12) = 116 \text{ lb}$$

4. We now proceed to determine a desirable combination of D and d that will satisfy the stress requirement (Eq. 12.6). In this problem the requirement that the spring fit inside a 1.5-in. hole permits a reasonable initial estimate of D—perhaps $D = 1.25$ in. As decided, $D + d$ must be less than 1.5 in. by a *diametral clearance* of about $0.1D$. Note that reasonable clearance is required because the outside diameter increases slightly as the spring is compressed. Since a small wire size should suffice for the loads involved, D would be expected to come out in the range of 1 to 1.25 in.

5. In order to solve Eq. 12.6 for d, we must also determine preliminary values of K_s and τ_{solid}, both of which are functions of d. Fortunately, neither quantity varies greatly over the ranges involved, so we should not be far off by estimating.

   a. $K_s = 1.05$. (Figure 12.4 shows little variation in K_s over the normal range of C between 6 and 12.)

   b. $\tau_{solid} = 101$ ksi. [For a "ballpark guess" of $d = 0.1$ in., Figure 12.7 shows S_u to be about 225 ksi. The corresponding maximum acceptable value of τ_{solid} (Eq. 12.9) is $0.45S_u$, or 101 ksi.]

$$\tau_s \leqq 0.45S_u \quad \text{(ferrous—without presetting)}$$

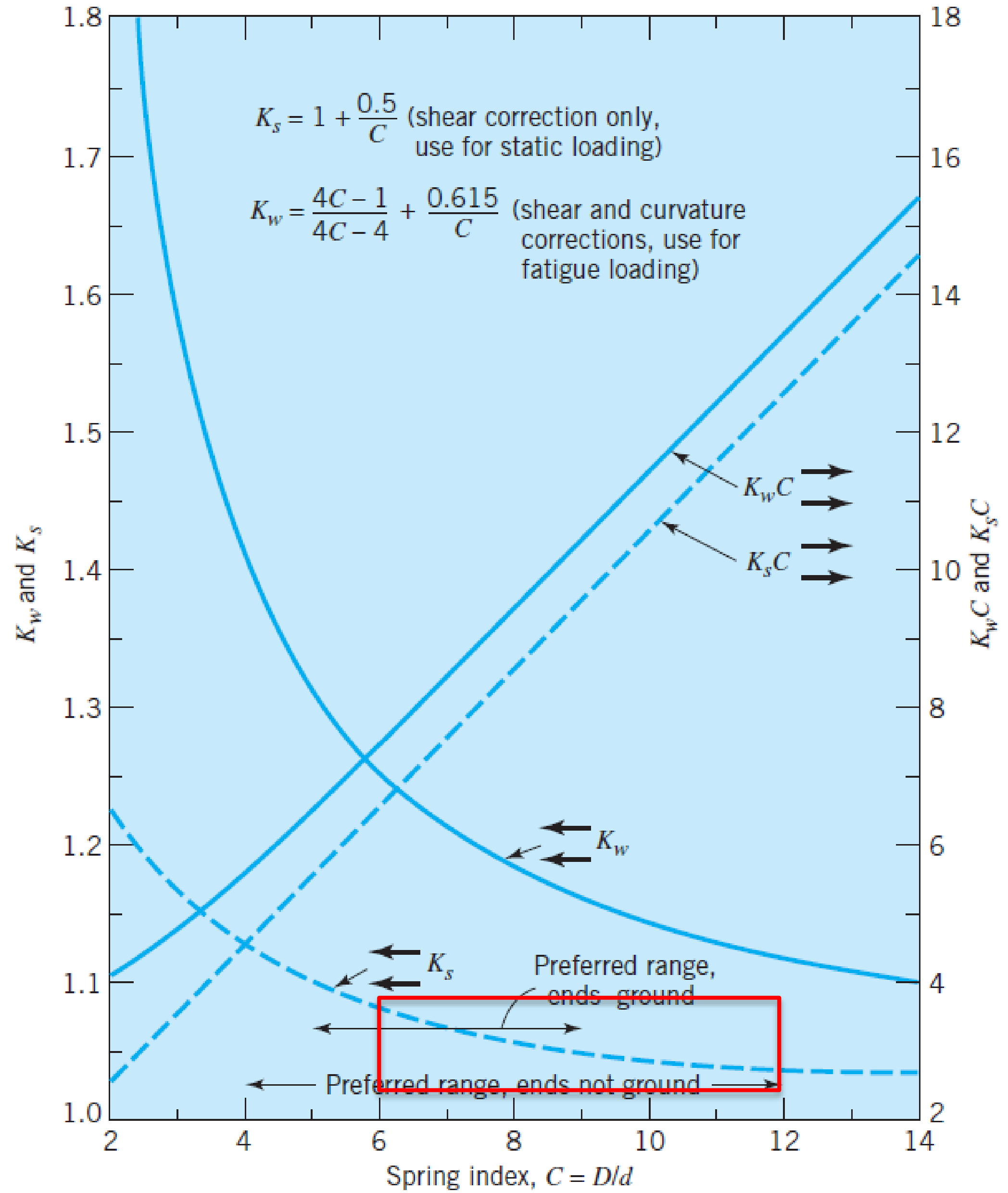


FIGURE 12.4

Stress correction factors for helical springs.

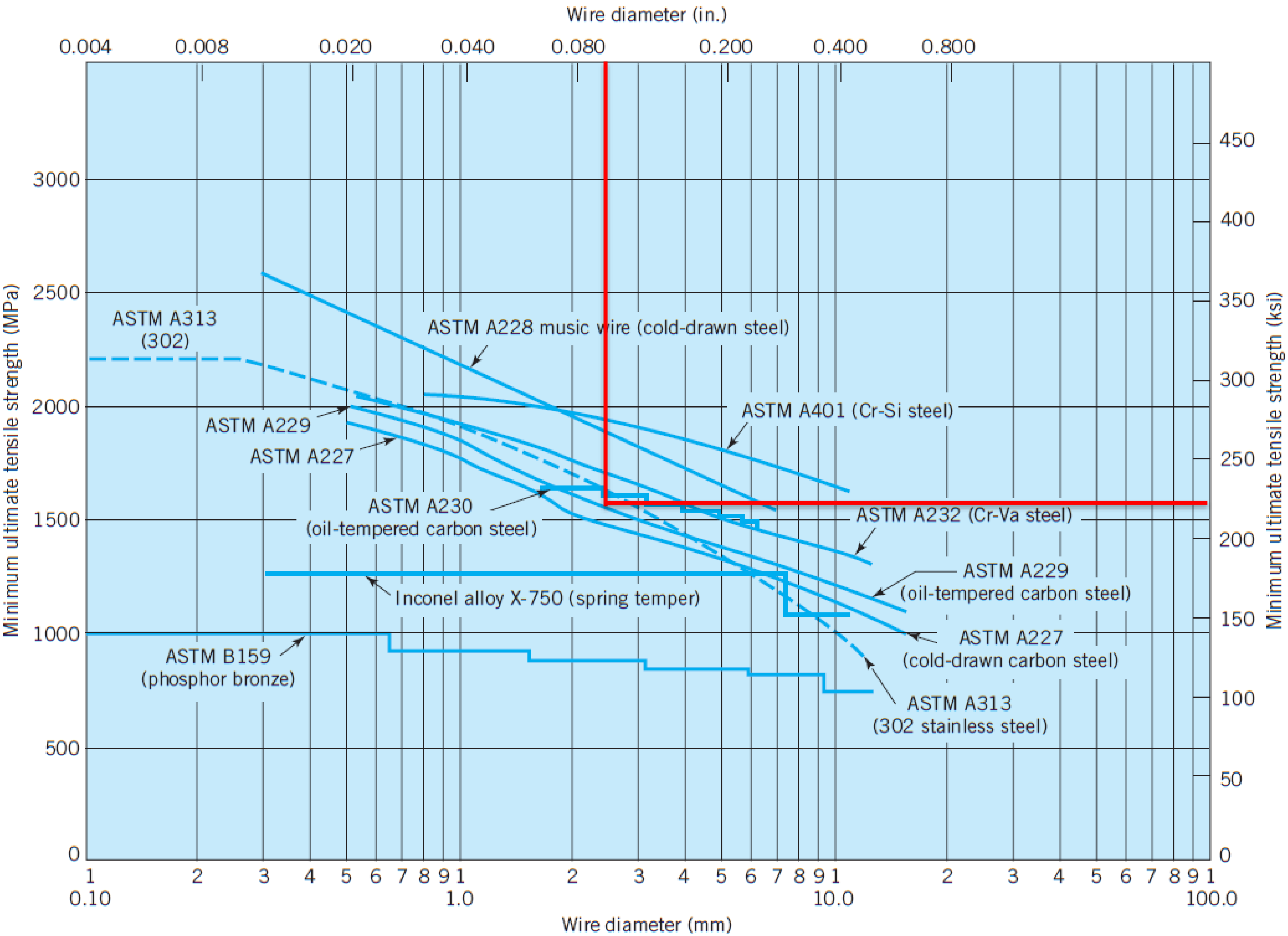


FIGURE 12.7

Tensile strengths of various spring wire materials and diameters, minimum values [2].

6. Substituting the preceding values into Eq. 12.6 gives

$$\tau_{\text{solid}} = \frac{8F_{\text{solid}}D}{\pi d^3}K_s$$

$$101{,}000 = \frac{8(116)(1.25)}{\pi d^3}(1.05)$$

or

$$d = 0.157 \text{ in.}$$

7. The estimates in steps 4 and 5 were deliberately made "rough" enough to give an unsatisfactory solution. A wire diameter of 0.157 in. has an ultimate strength of only about 210 ksi instead of the assumed 225 ksi. Furthermore, the preceding values of d and D provide a diametral clearance in a 1.5-in. hole of only 0.093, which is less than the desired value of $0.1D$. If we keep $d = 0.157$, and reduce D so that the wire is subjected to a little less torque (hence, a little less stress), this would also open up more diametral clearance. For a second trial, choose $d = 0.157$ in. and solve for the corresponding value of D. Both τ_{solid} and K_s will have different values than before, but this time they will be "correct" values for these quantities instead of estimates.

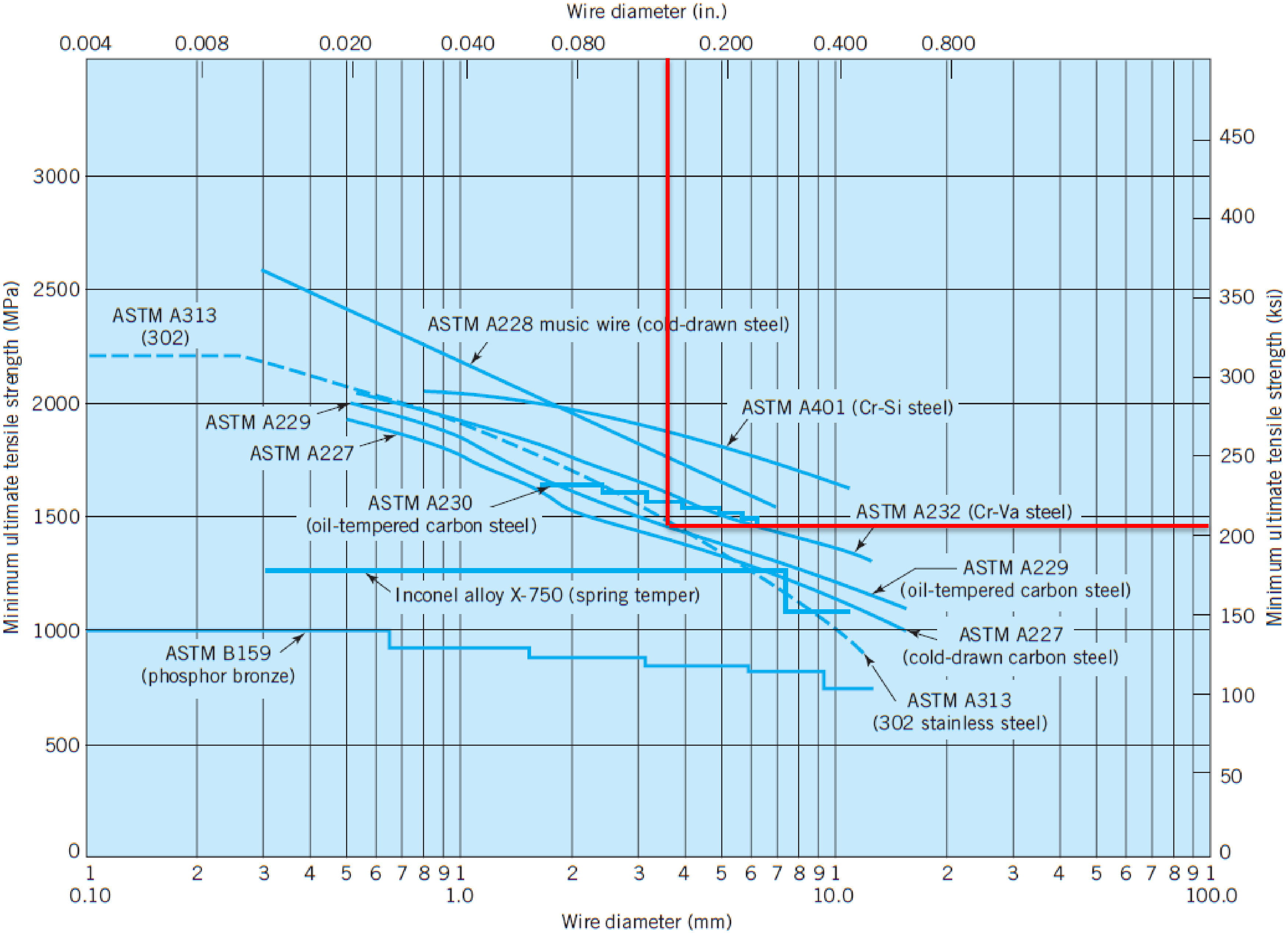


FIGURE 12.7

Tensile strengths of various spring wire materials and diameters, minimum values [2].

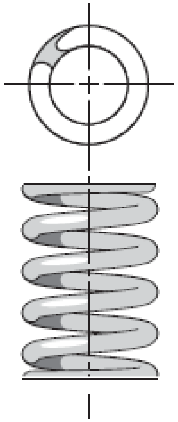
8. To avoid estimating K_s, use the *second* form of Eq. 12.6:

$$\tau = \frac{8FD}{\pi d^3}K_s = \frac{8F}{\pi d^2}CK_s \quad (12.6)$$

$$\tau_{solid} = \frac{8F_{solid}}{\pi d^2}CK_s$$

$$0.45(210{,}000) = \frac{8(116)}{\pi(0.157)^2}CK_s$$

$$CK_s = 7.89$$



(*d*) Squared and ground ends $L_s = N_t d$

From Figure 12.4, $C = 7.3$, and

$$D = Cd = 7.3(0.157) = 1.15 \text{ in.}$$

This combination of D and d not only conforms exactly to the desired stress criterion but also provides a little more than the minimum desired clearance in the 1.5-in. hole.

9. From Eq. 12.8,

$$k = \frac{d^4G}{8D^3N}, \qquad 90 = \frac{(0.157)^4(11.5 \times 10^6)}{8(1.15)^3N}$$

from which $N = 6.38$.

10. From Eq. 12.10, $N_t = N + 2 = 6.38 + 2 = 8.38$. From Figure 12.8, $L_s = N_t d = 8.38(0.157) = 1.32$ in.

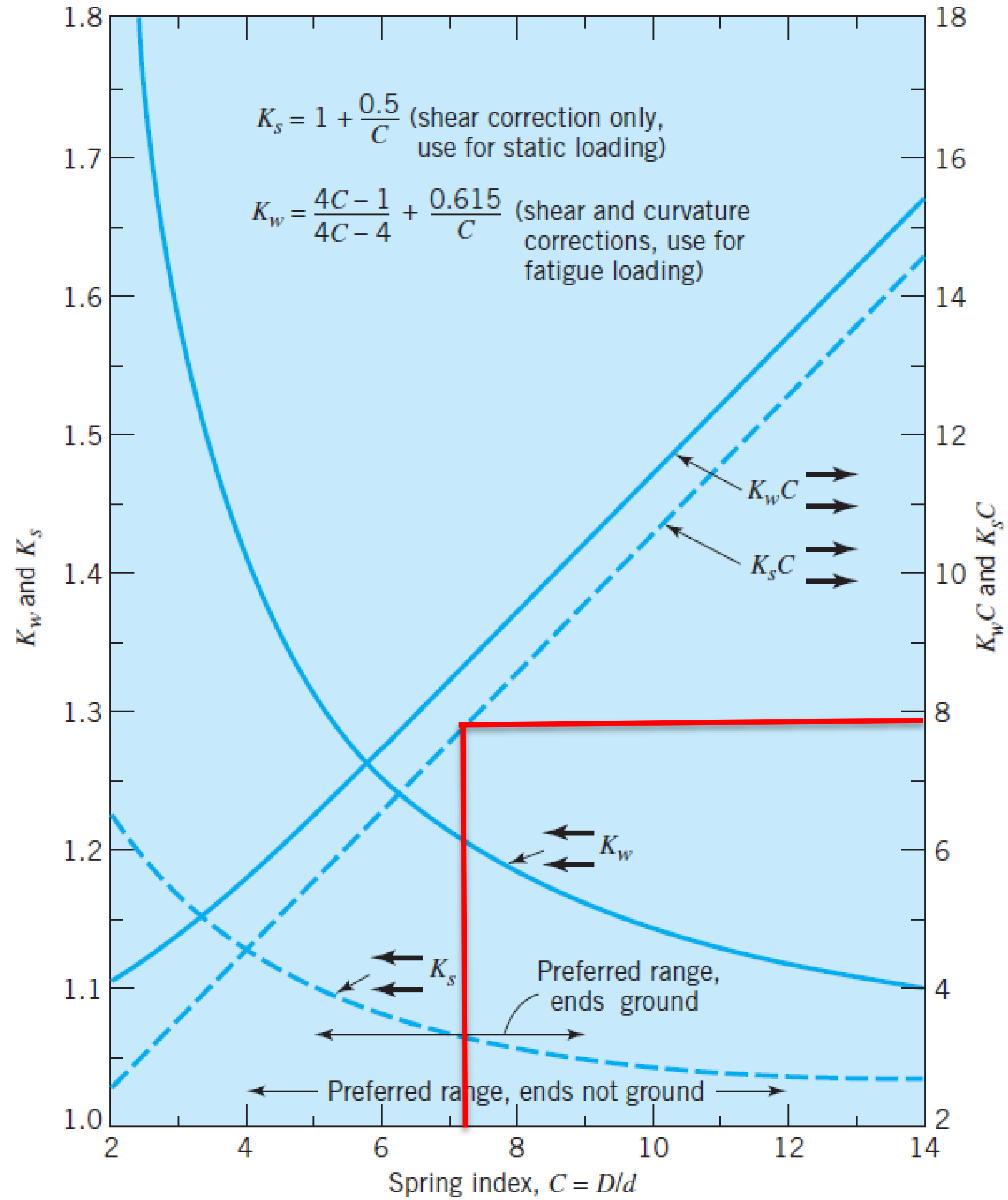


FIGURE 12.4

Stress correction factors for helical springs.

11. When force F_{solid} = 116 lb is released, the spring will elongate a distance of 116 lb/(90 lb/in.) = 1.29 in. Thus the free length of the spring, L_f, is L_s + 1.29 = 1.32 + 1.29 = 2.61 in. Furthermore, when loaded with 60 lb, the spring length will be [2.61 in. –60 lb/(90 lb/in.)] = 1.94 in. This more than satisfies the maximum length requirement of 2.5 in. at a 60-lb load.

12. Buckling is checked for the worst case of deflection approaching the solid deflection (i.e., $\delta = \delta_s$ = 1.29 in.),

$$\frac{\delta_s}{L_f} = \frac{1.29}{2.61} = 0.49 \qquad d = 0.157 \text{ in.}$$

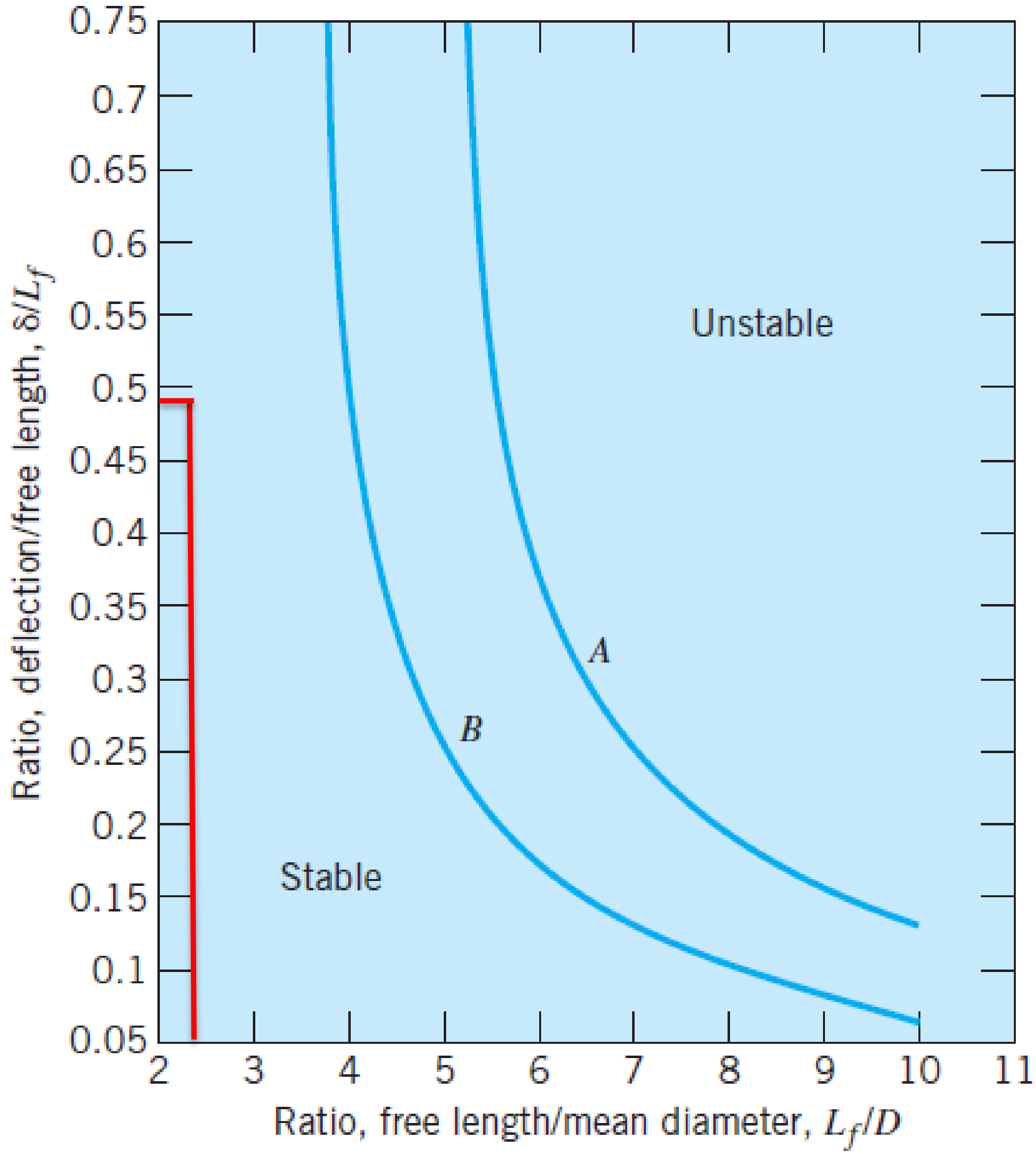
$$\qquad D = 1.15 \text{ in.}$$

$$\frac{L_f}{D} = \frac{2.61}{1.15} = 2.27 \qquad N = 6.38$$

$$\qquad L_f = 2.61 \text{ in.}$$

Reference to Figure 12.10 indicates that this spring is far outside the buckling region, even if one end plate is free to tip.

13. The above solution satisfies the stress and spring rate requirements, while more than satisfying the buckling criterion and spatial limitations. (It is obvious that the requirements could also be satisfied with spring designs using a little thicker or a little thinner wire or even a wire of a little less tensile strength.) Hence, one apparently satisfactory answer to the problem is



A- end plates are constrained parallel
(buckling pattern as in Fig. 5.27*c*)
B- one end plate is free to tip
(buckling pattern as in Fig. 5.27*b*)

Comments:

1. The preceding information would permit a technician to draw or to make the spring.
2. The problem is not really finished, however, without dealing with the vital matter of *tolerances*. For example, small variations in d result in large variations in stress and deflection. Imposing extremely tight tolerances can add a substantial unnecessary cost. It is best to advise the spring manufacturer of any *critical* dimensions; for example, in this problem it might be important to hold all springs to 90 ± 4 lb/in. spring rate, and to the *same* length, ±0.002 in., when loaded with 60 lb. Fairly loose tolerances should be allowed on all other dimensions. The manufacturer will then be able to use wire stock of *slightly* varying diameter by adjusting other dimensions as necessary in order to comply with the critical specifications.

12.7 Design Procedure for Helical Compression Springs—Static Loading

- It may be helpful to note that there are, in general, three types of problems in selecting a satisfactory combination of D and d to satisfy the stress requirement.

1. Spatial restrictions place a limit on D, as when the spring must fit inside a hole or over a rod. This situation was illustrated by Sample Problem 12.1.
2. The wire size is fixed, as, for example, standardizing on one size of wire for several similar springs. This situation is also illustrated by Sample Problem 12.1, if steps 4, 5, 6, and 7 are omitted, and d = 0.157 in. is given.
3. No spatial restrictions are imposed, and any wire size may be selected. This completely general situation can theoretically be satisfied with an almost infinite range of D and d, but the extremes within this range would not be economical.

- Reference to Figure 12.4 suggests that good proportions generally require values of D/d in the range of 6 to 12 (but grinding the ends is difficult if D/d exceeds about 9).
- Hence, a good procedure would be to select an appropriate value of C and then use the second form of Eq. 12.6 to solve for d. This requires an estimate of S_u in order to determine the allowable value of τ_{solid}.
- If the resulting value of d is not consistent with the estimated value of S_u, a second trial will be necessary, as was the case in the sample problem.

12.8 Design of Helical Compression Springs for Fatigue Loading

- Figure 12.12 shows a generalized S–N curve, for reversed torsional loading of round steel wire strength S_u, dia < 10 mm, C_s of 1
- A corresponding constant-life fatigue diagram is plotted in Figure 12.13. Since compression coil springs are always loaded in fluctuating compression (and tensile coil springs in fluctuating tension), these springs do not normally experience a stress reversal.
- In the extreme case, the load drops to zero and is then reapplied in the same direction. Thus, as shown in Figure 12.13, the region of interest lies between $\tau_a/\tau_m = 0$ and $\tau_a/\tau_m = 1$, where τ_a/τ_m is the ratio of alternating shear stress to mean shear stress.

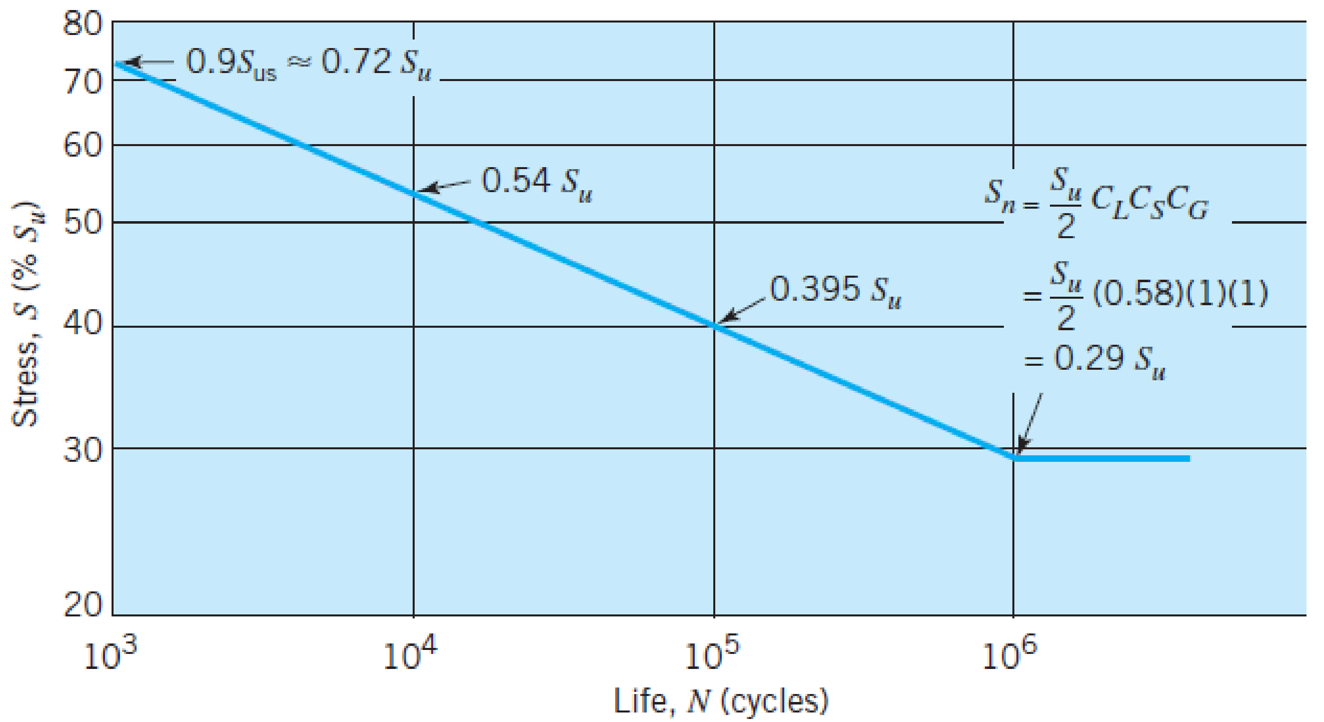


FIGURE 12.12
Estimated S–N curve for round steel spring wire, $d \leq 10$ mm, $C_S = 1$ (shot-peened) reversed, torsional loading.

12.8 Design of Helical Compression Springs for Fatigue Loading

- It is customary when working with coil springs to re plot the information in Figure 12.13 in the form used in Figure 12.14. This alternative form of constant-life fatigue diagram contains only the "region of interest" shown in Figure 12.13. Note, for example, that point P of Figure 12.13 corresponds to $\tau_m = 0.215S_u$, $\tau_a = 0.215S_u$, whereas in Figure 12.14 point P plots as $\tau_{min} = 0$, $\tau_{max} = 0.43S_u$.

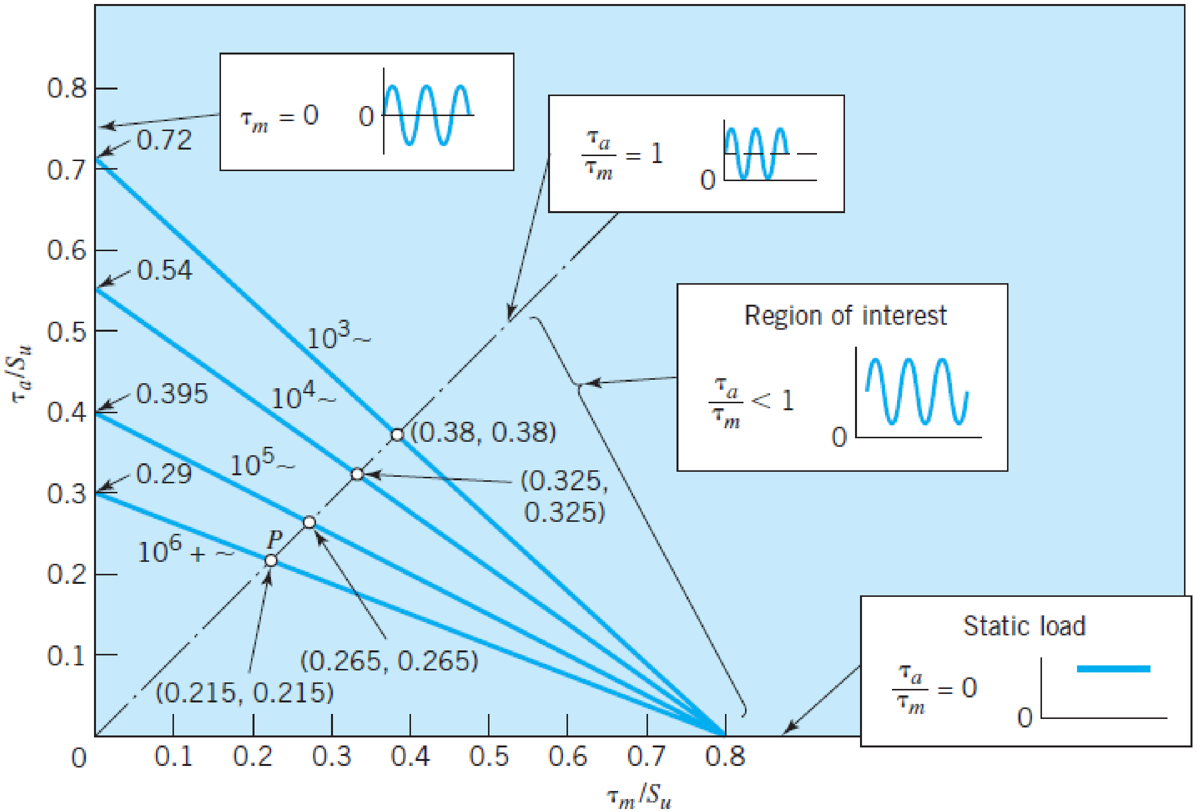


FIGURE 12.13
Constant-life fatigue diagram corresponding to Figure 12.12. Recall that $S_{us} \approx 0.8\ S_u$.

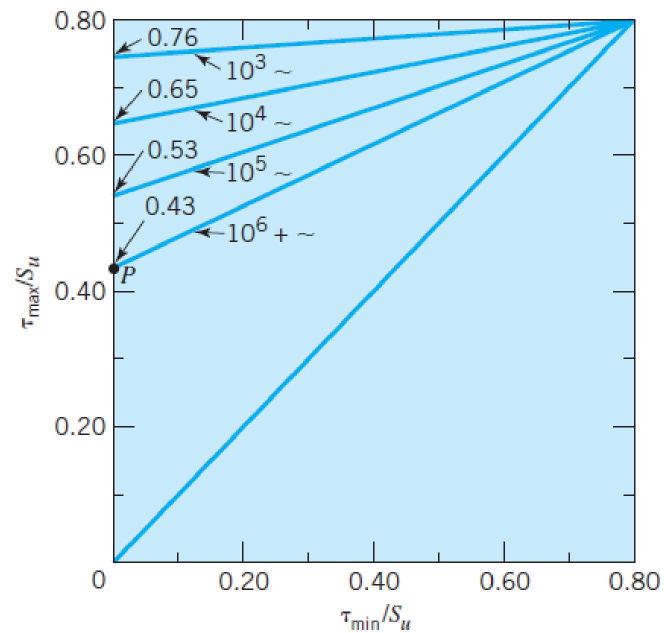


FIGURE 12.14
Alternative form of constant-life fatigue diagram (replot of "region of interest" of Figure 12.13).

12.8 Design of Helical Compression Springs for Fatigue Loading

- Figure 12.14 is based on actual torsional fatigue tests, with the specimens loaded in a zero-to-maximum fluctuation ($\tau_a/\tau_m = 1$).
- Figure 12.15 shows S–N curves based on 0-to-maxtress fluctuation. The top curve is drawn to agree with the values determined in Figure 12.13. The lower curves in Figure 12.15 are 0-to-max torsional S–N curves based on experimental data and suggested for design. These reflect production spring wire surface finish, rather than $C_s = 1$, as in the top curve.

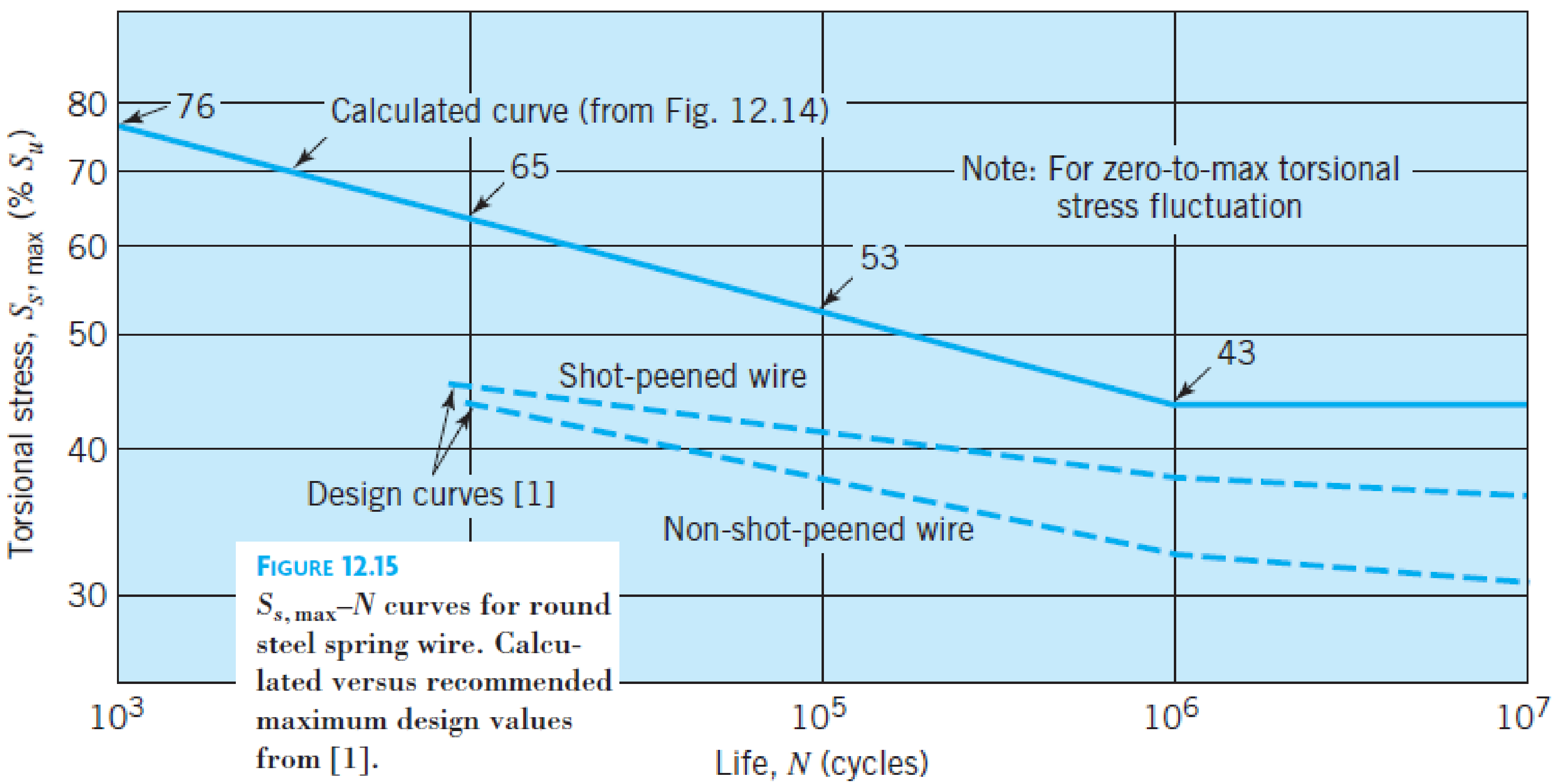


FIGURE 12.15
$S_{s,\max}$–N curves for round steel spring wire. Calculated versus recommended maximum design values from [1].

12.8 Design of Helical Compression Springs for Fatigue Loading

- Figure 12.16 is an independently obtained empirical constant-life fatigue diagram pertaining to most grades of engine valve spring wire. It represents actual test data. Design values should be somewhat lower.
- In the design of helical (or torsion bar) springs for fatigue loading, two previously mentioned manufacturing operations are particularly effective: shot peening and presetting.
- Recall that presetting always introduces surface residual stresses opposite those caused by subsequent load applications in the same direction as the presetting load.

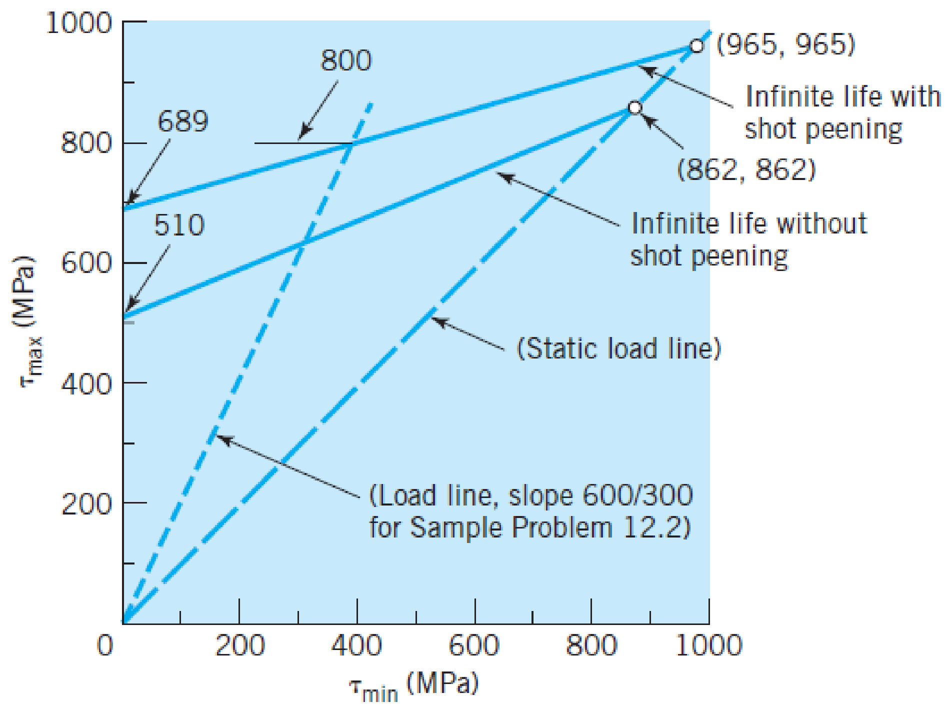


FIGURE 12.16
Infinite-life fatigue diagram. Pretempered carbon or alloy steel high-duty spring wire, $d \leq 5$ mm (0.2 in.).

12.8 Design of Helical Compression Springs for Fatigue Loading

- The corresponding coil spring (or torsion bar) torsional stress fluctuations with and without presetting are as shown in Figure 12.17.
- the theoretical maximum residual stress that can be introduced by presetting is $S_{sy}/3$.
- The practical maximum value is somewhat less. The fatigue improvement represented by the fluctuation with presetting in Figure 12.17 is readily apparent when the stress fluctuations are represented in Figures 12.13, 12.14, and 12.16.
- Maximum fatigue strengthening can be obtained by using both shot peening and presetting.

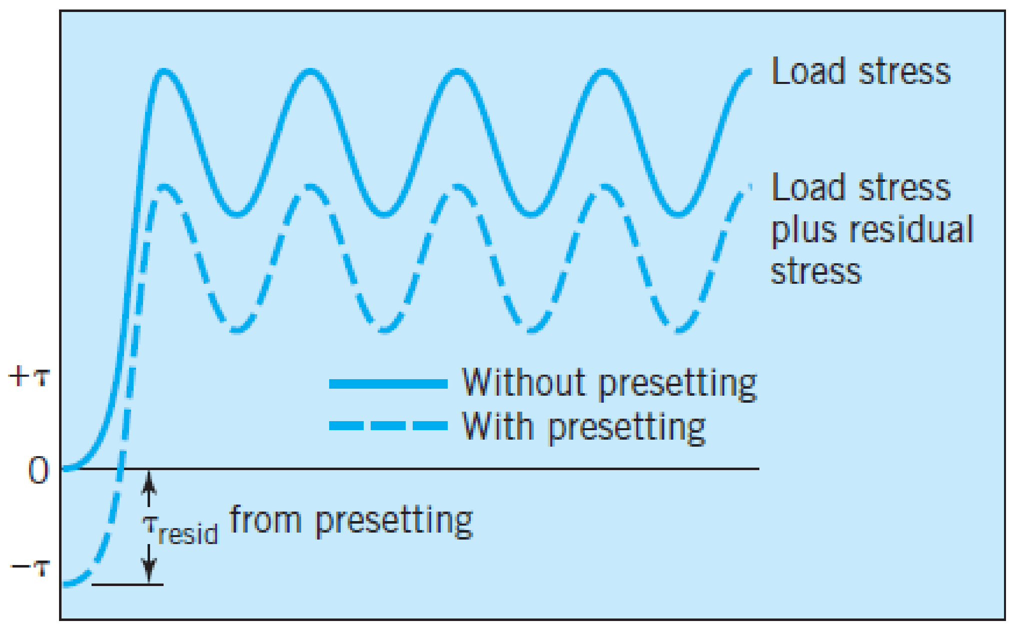


FIGURE 12.17
Stress fluctuation in a helical (or torsion bar) spring with and without presetting.

12.8 Design of Helical Compression Springs for Fatigue Loading

- Springs used in high-speed machinery must have f_n >> machine frequency.
- a conventional engine valve spring goes through one cycle of shortening and elongating every two engine revolutions. At 5000 engine rpm, the spring has an f of 2500 cpm, and the thirteenth harmonic 32,500 cpm, or 542 Hz.
- When a helical spring is compressed and then suddenly released, it vibrates longitudinally at its f_n until the energy is dissipated by damping, this phenomenon is called spring surge and causes local stresses approximating those for "spring solid." Spring surge also decreases the ability of the spring
- The natural frequency of spring surge (which should be made higher than the highest significant harmonic of the motion involved—typically about the thirteenth) is

$$f_n \propto \sqrt{k/m} \quad \text{or} \quad f_n \propto \frac{d}{D^2 N}\sqrt{G/\rho}$$

- For steel springs f_n in Hz is
- Spring design with high f_n requires operating at high stresses

$$f_n = \frac{13{,}900d}{ND^2} \quad (d \text{ and } D \text{ in inches}) \qquad \textbf{(12.11)}$$

$$f_n = \frac{353{,}000d}{ND^2} \quad (d \text{ and } D \text{ in millimeters}) \qquad \textbf{(12.11a)}$$

- This minimizes the required mass of the spring, thereby maximizing its f_n, which is proportional to. $1/\sqrt{m}$.

SAMPLE PROBLEM 12.2D Helical Spring Design for Fatigue Loading

A camshaft rotates 650 rpm, causing a follower to raise and lower once per revolution (Figure 12.18). The follower is to be held against the cam by a helical compression spring with a force that varies between 300 and 600 N as the spring length varies over a range of 25 mm. Ends are to be squared and ground. The material is to be shot-peened chrome–vanadium steel valve spring wire, ASTM A232, with fatigue strength properties as represented in Figure 12.16. Presetting is to be used. Determine a suitable combination of d, D, N, and L_f. Include in the solution a check for possible buckling and spring surge.

SOLUTION

Known: A helical compression spring operates with a force that varies between given minimum and maximum values as the spring length varies over a known range.

Find: Determine a suitable spring geometry.

Schematic and Given Data:

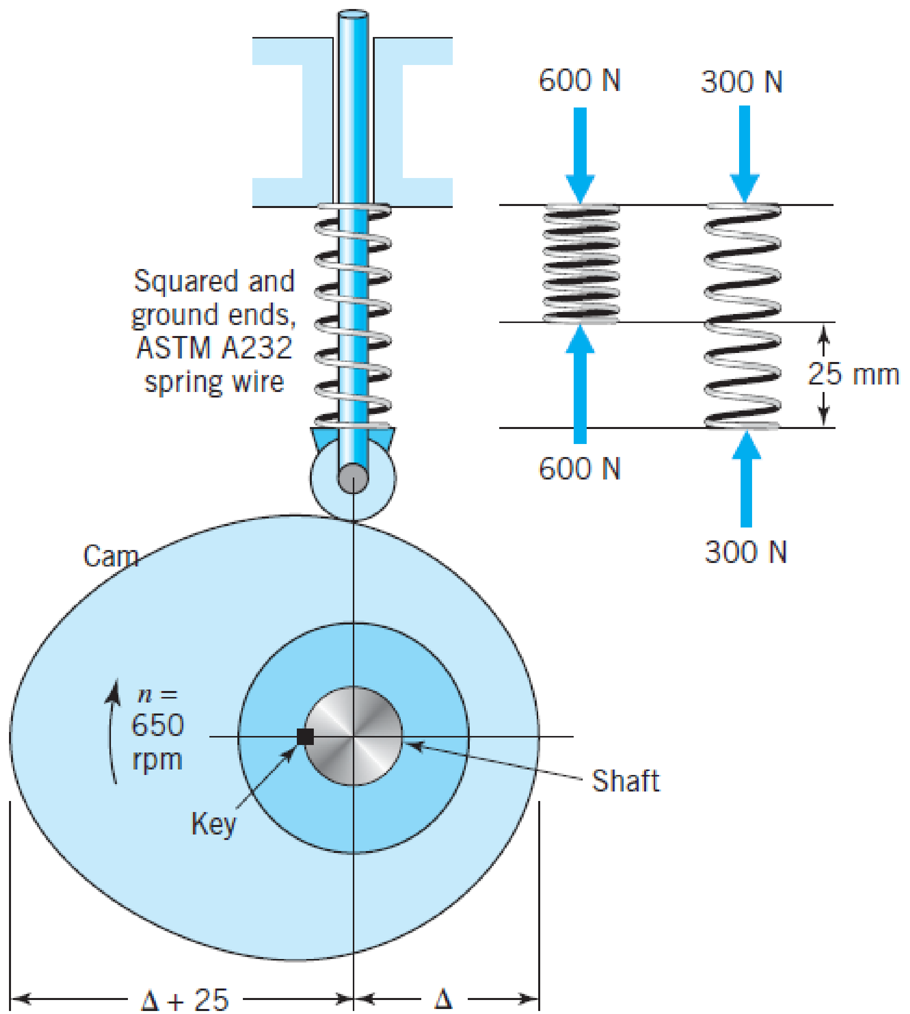


FIGURE 12.18

Diagram for Sample Problem 12.2.

Decisions:

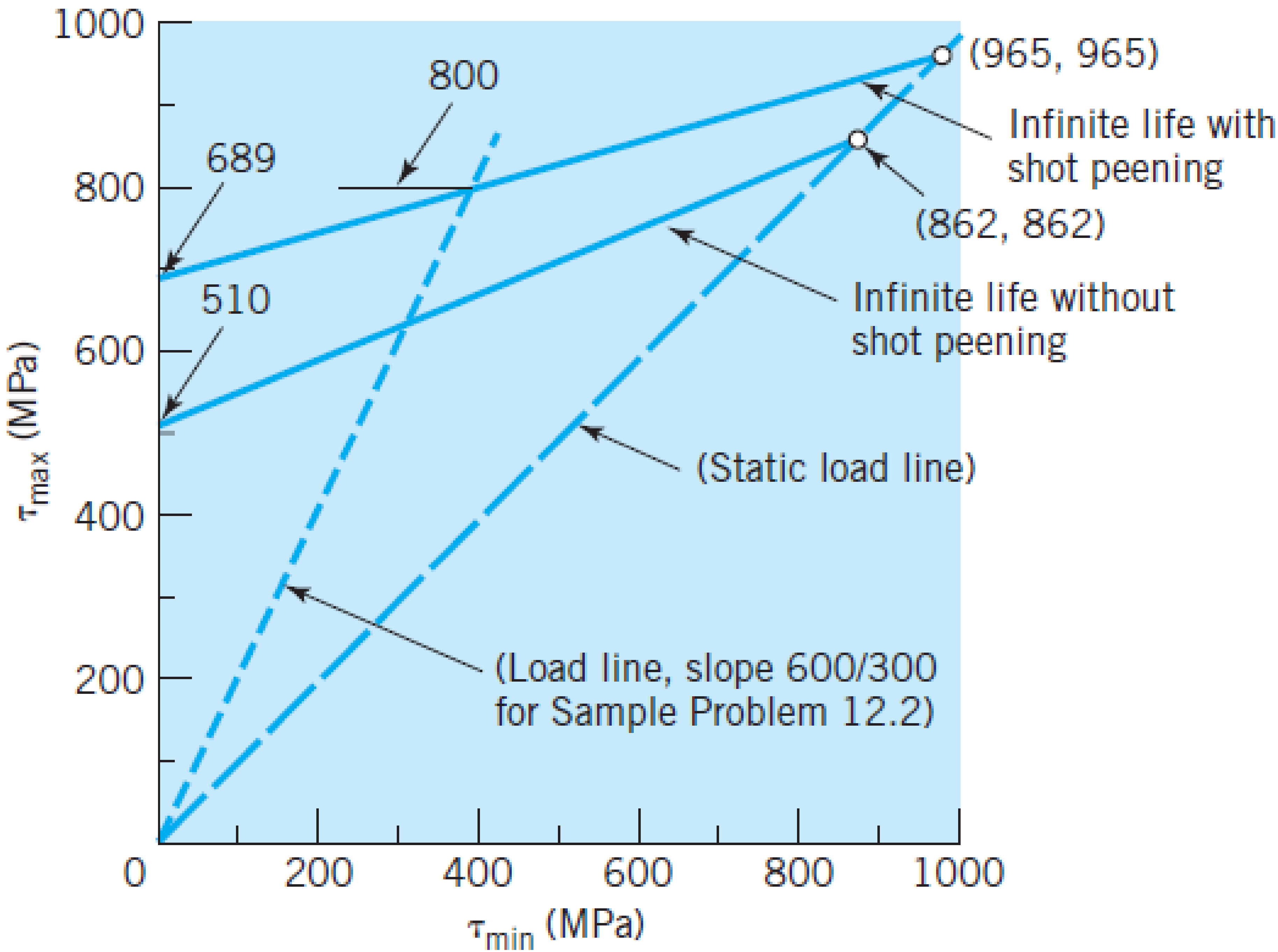
1. To minimize possible spring surge problems, design the spring so that stresses are as large as reasonable.
2. Select the smallest reasonable safety factor to minimize spring weight. (Minimizing spring weight allows us to maximize natural frequency.)
3. Select a spring proportion, $C = 10$. (This proportion is good from the standpoint of the Wahl factor, but costs for the spring may be higher because the ends must be ground.)
4. As recommended in Section 12.4, choose a clash allowance that is 10 percent of the maximum working deflection.

Assumptions:

1. The end plates are in contact with the spring ends.
2. The spring force acts along the spring axis.

Design Analysis:

1. Since, at 650 rpm, a million stress cycles are accumulated in 26 operating hours, infinite fatigue life is required. Stresses should be as high as reasonable to minimize possible spring surge problems. Regardless of the spring design, the ratio τ_{max}/τ_{min} will be the same as the ratio of maximum and minimum loads—that is, 600/300. A line of this slope is drawn on Figure 12.16, giving an intersection at $\tau_{max} = 800$ MPa.
2. Since Figure 12.16 represents actual test data, this value of τ_{max} makes no allowance for possible spring surge or a safety factor. The amplitude of possible surge can be limited by providing a minimal clash allowance—say, 10 percent of the maximum working deflection. Spring weight can be minimized, thus allowing the maximum natural frequency, by selecting the smallest reasonable safety factor—say, 1.1. (The use of presetting will provide some additional safety factor.) Thus a design value for τ_{max} might be chosen as 800 MPa divided by 1.1 (allowance for possible surge) and divided again by 1.1 (safety factor), or 661 MPa.


1000
800
600
400
200
0
200
400
600
800
1000
τ_{max} (MPa)
τ_{min} (MPa)
800
689
510
(965, 965)
Infinite life with shot peening
(862, 862)
Infinite life without shot peening
(Static load line)
(Load line, slope 600/300 for Sample Problem 12.2)

$$\tau = \frac{8FD}{\pi d^3} K_w = \frac{8F}{\pi d^2} C K_w \tag{12.5}$$

3. In the absence of any restrictions on d, for either the outer diameter or the inner diameter, let us arbitrarily select a spring *proportion* of, say, $C = 10$. This proportion is good from the standpoint of the Wahl factor, but the spring may cost an extra amount because the ends must be ground. Then, from Eq. 12.5,

$$d = \sqrt{\frac{8F_{\max} C K_w}{\pi \tau_{\max}}} = \sqrt{\frac{8(600)(10)(1.14)}{\pi(661)}} = 5.13 \text{ mm}$$

4. In the absence of any reason to stay with an odd value of d, it might be preferable to round off to $d = 5.0$ mm. Then, going back to Eq. 12.5 and solving for the value of C that gives a stress of 661 MPa (with load of 600 N) together with $d = 5.0$ mm, we have

$$C K_w = \frac{\pi \tau_{\max} d^2}{8F_{\max}} = \frac{\pi(661)(5)^2}{8(600)} = 10.82$$

From Figure 12.4, $C = 9.4$, $D = Cd = 47.0$ mm.

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

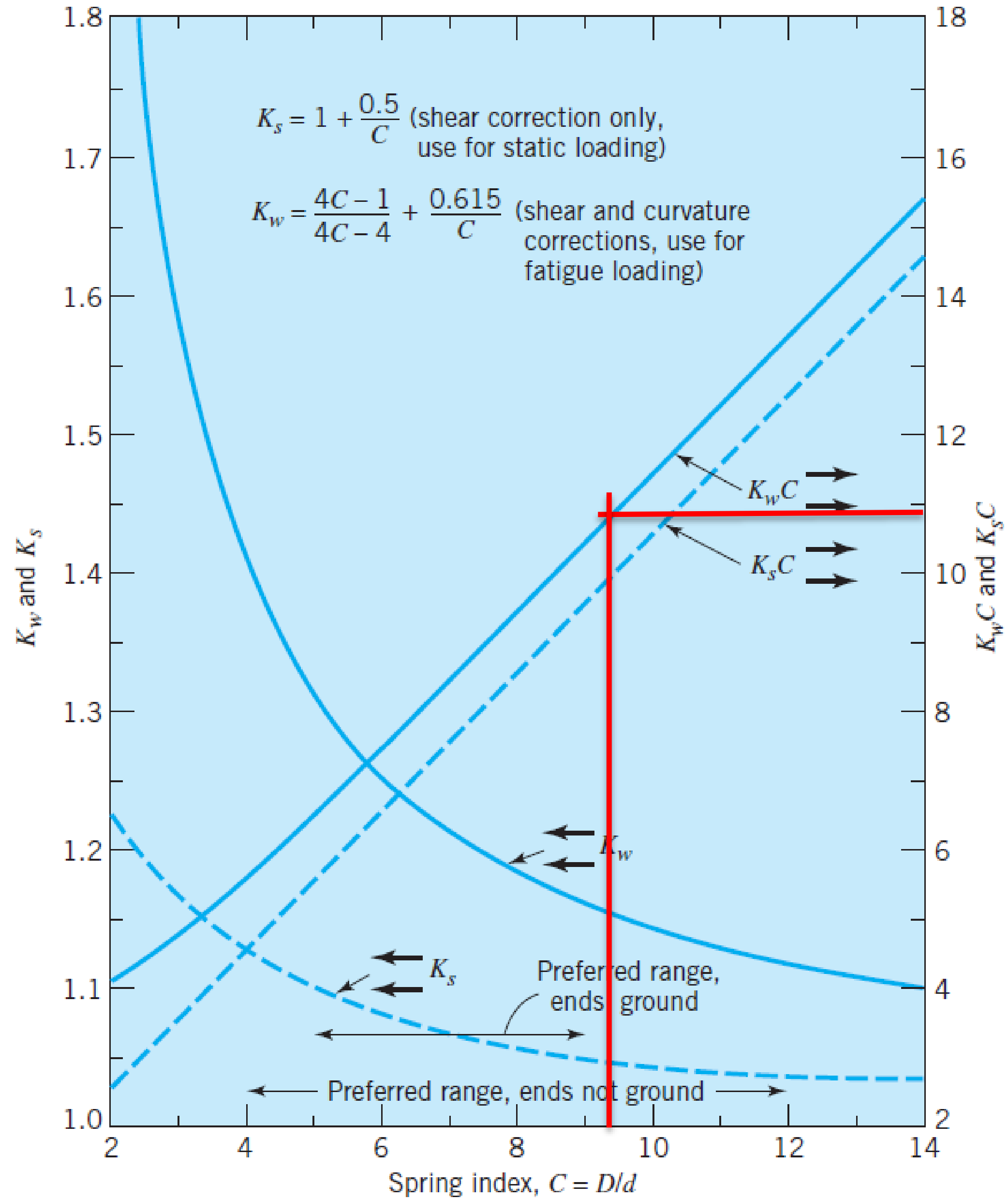


FIGURE 12.4

Stress correction factors for helical springs.

5. $k = 300 \text{ N}/25 \text{ mm} = 12 \text{ N/mm}$.

6. From Eq. 12.8,

$$N = \frac{dG}{8C^3k} = \frac{5(79{,}000)}{8(9.4)^3(12)} = 4.95$$

7. From Figure 12.8, $L_s = N_t d = (N + 2)d = (4.95 + 2)(5) = 34.75$ mm.

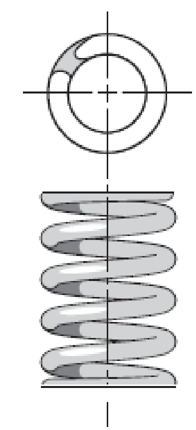
$$L_f = L_s + F_{\text{solid}}/k$$

With 10 percent clash allowance, $F_{\text{solid}} = 1.1F_{\max} = 660$ N. Then,

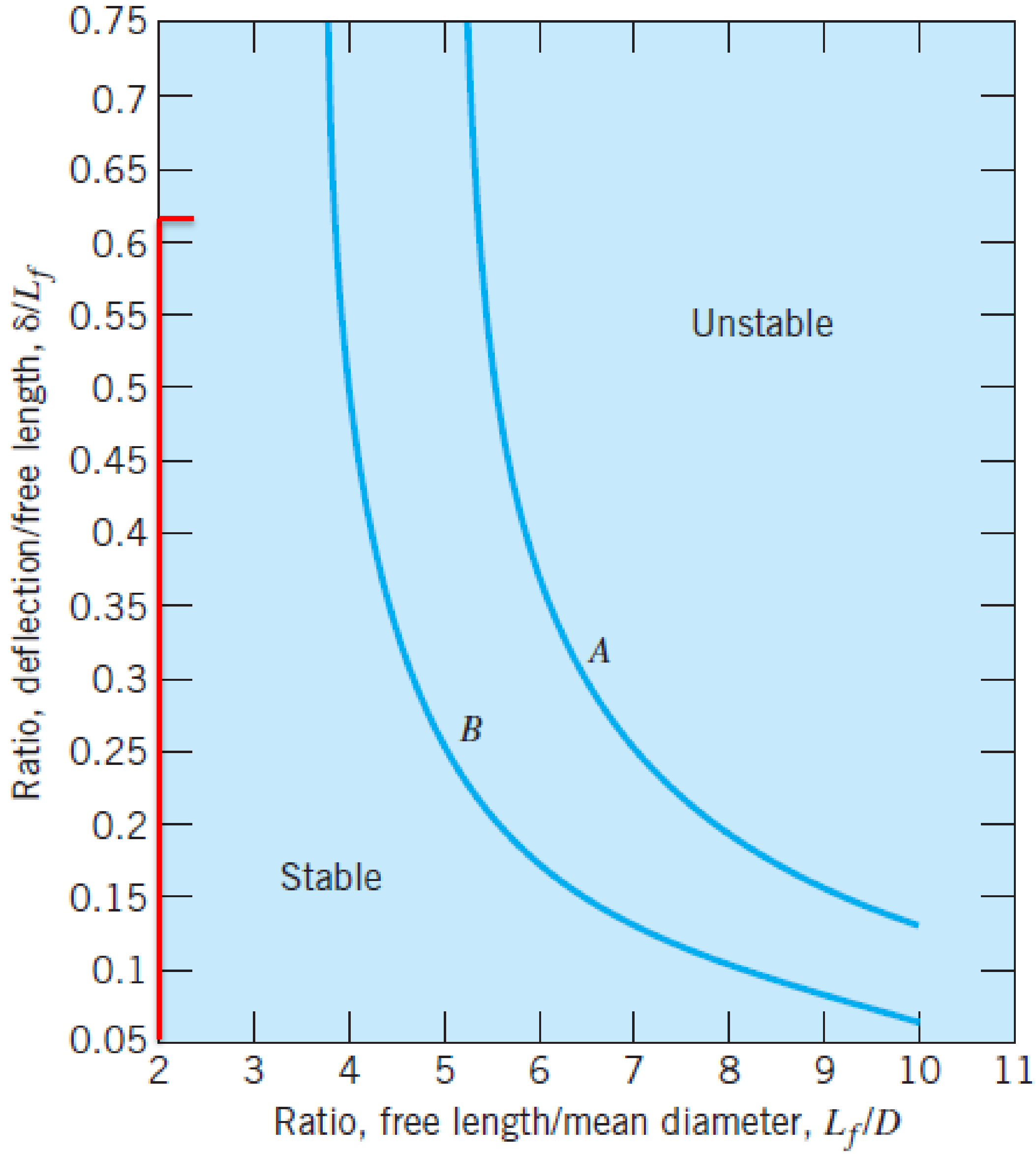
$$L_f = 34.75 + 660/12 = 89.75 \text{ mm}$$

8. Check for buckling to determine if the spring contacts the rod (for extreme case of $\delta = \delta_s$):

$$\left.\begin{aligned} \frac{L_f}{D} &= \frac{89.75}{47} = 1.91 \\ \frac{\delta_s}{L_f} &= \frac{\frac{660}{12}}{89.75} = 0.61 \end{aligned}\right\} \text{Far removed from buckling per Figure 12.10}$$



(*d*) Squared and ground ends
$L_s = N_t d$



A- end plates are constrained parallel
(buckling pattern as in Fig. 5.27*c*)

B- one end plate is free to tip
(buckling pattern as in Fig. 5.27*b*)

9. From Eq. 12.11a, the natural frequency is

$$f_n = \frac{353{,}000d}{ND^2} = \frac{353{,}000(5)}{(4.95)(47)^2} = 161.4 \text{ Hz}$$

10. To summarize the results,

$$d = 5 \text{ mm}$$
$$D = 47.0 \text{ mm}$$
$$N = 4.95$$
$$L_f = 89.75 \text{ mm}$$

Comments:

1. For the spring to be in resonance with the fundamental surge frequency $f_n = 161.4$ Hz, the camshaft would have to rotate at $(161.4)(60) = 9684$ rpm. For the thirteenth harmonic to be in resonance, the shaft must rotate $9684/13 = 745$ rpm. Rotation at 650 rpm should not result in spring surge (unless the cam contour is highly unusual, producing significant harmonics above the thirteenth).
2. No buckling or spring surge should occur (but allowance for possible repeated transient surge was made by appropriate selection of clash allowance and design stress).

SAMPLE PROBLEM 12.3D Helical Spring Fatigue Design

Repeat Sample Problem 12.2, except this time design the spring to use 5-mm wire of the same material but with the strength properties indicated in Figures 12.7 and 12.15.

SOLUTION

Known: A helical compression spring of wire diameter $d = 5$ mm operates with a known fluctuating force that varies the spring length through a range of 25 mm.

Find: Determine a satisfactory spring geometry.

Schematic and Given Data: The schematic and given data are the same as in Sample Problem 12.2 except that the strength properties are those indicated in Figures 12.7 and 12.15 rather than in Figure 12.16.

Find: Determine a suitable spring geometry.

Schematic and Given Data:

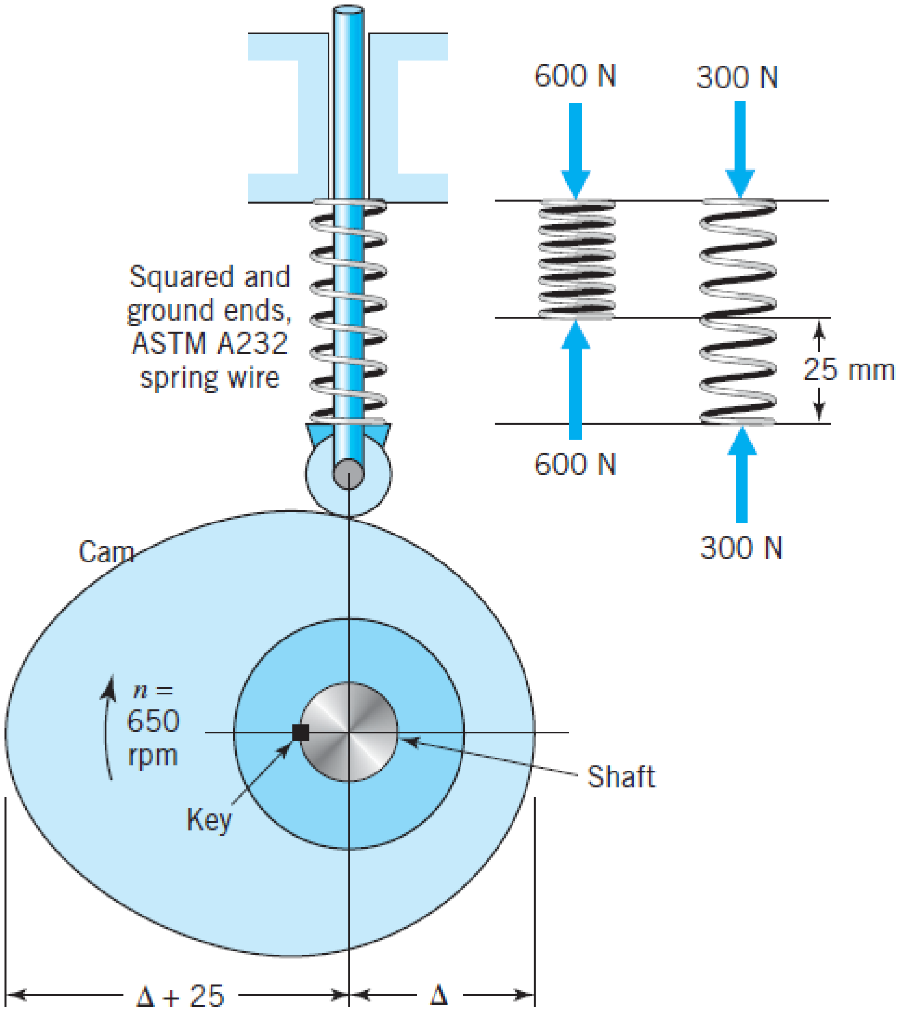


Figure 12.18
Diagram for Sample Problem 12.2.

Decisions/Assumptions: Same as in Sample Problem 12.2.

Design Analysis:

$$\tau_s \leqq 0.65 S_u \quad \text{(ferrous—with presetting)} \tag{12.9}$$

1. From Figure 12.7, $S_u = 1500$ MPa for the given material and wire size.
2. From Figure 12.15, the maximum recommended design stress for infinite life and zero-to-maximum stress fluctuation (shot-peened wire) is $0.36S_u = 540$ MPa.
3. From Eq. 12.9 the effective torsional yield strength associated with 2 percent long-term set is $0.65\ S_u = 975$ MPa. Approximating S_{us} as $0.8S_u = 1200$ MPa, an estimated torsional fatigue strength curve for infinite life is plotted in Figure 12.19.

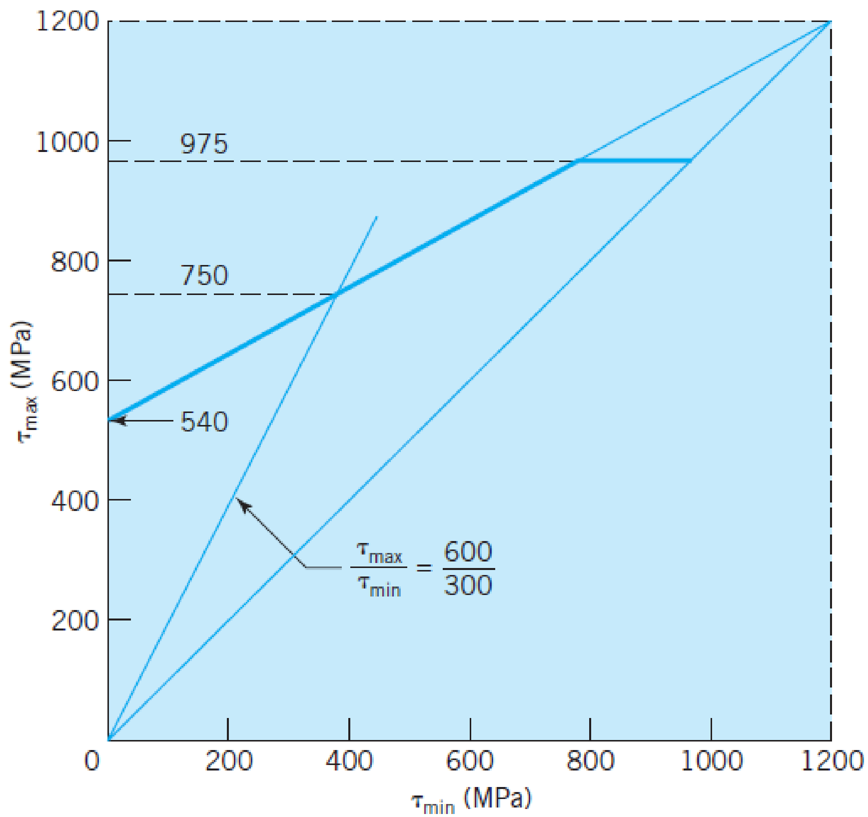


Figure 12.19
Fatigue diagram for Sample Problem 12.3.

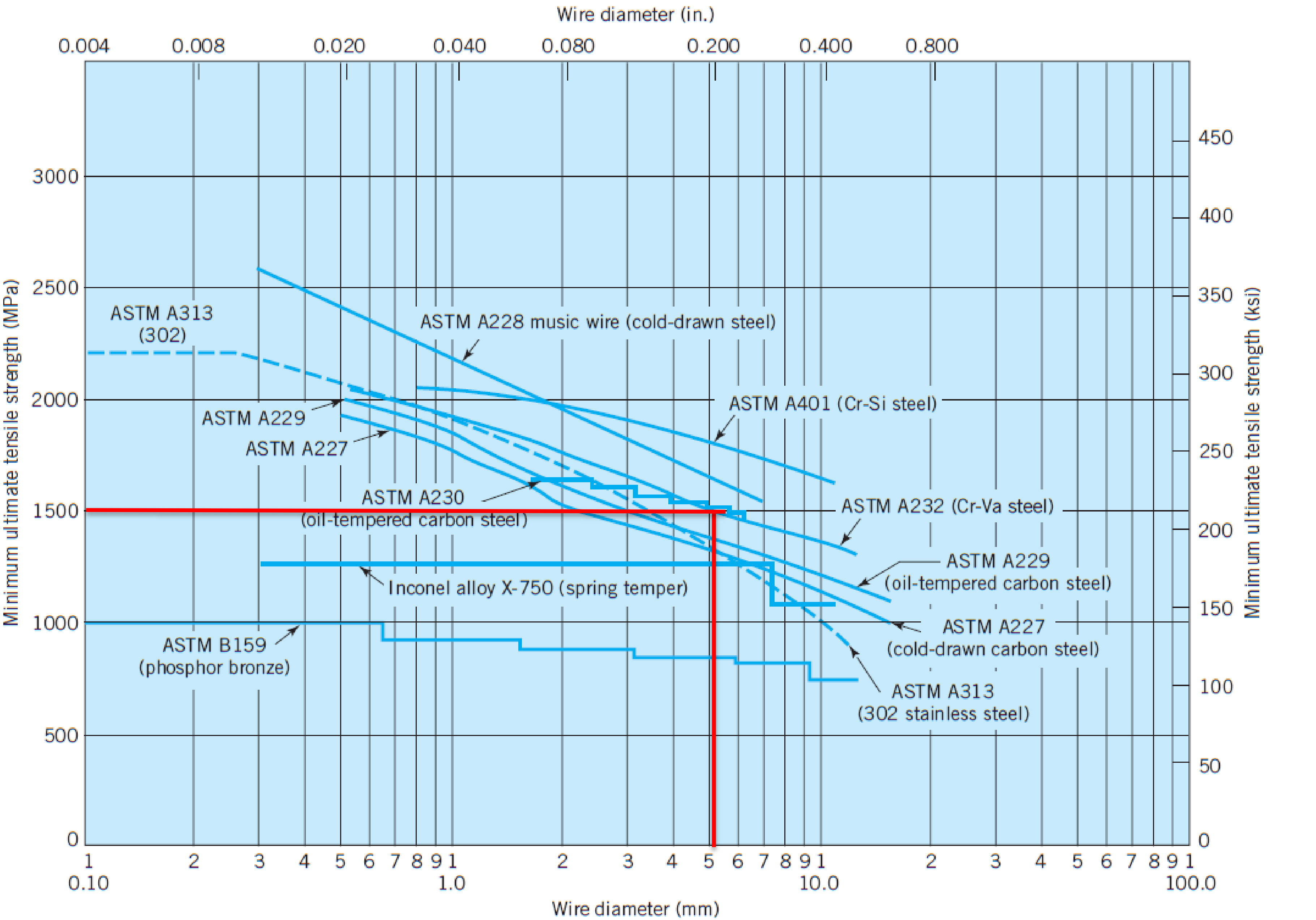


FIGURE 12.7

Tensile strengths of various spring wire materials and diameters, minimum values [2].

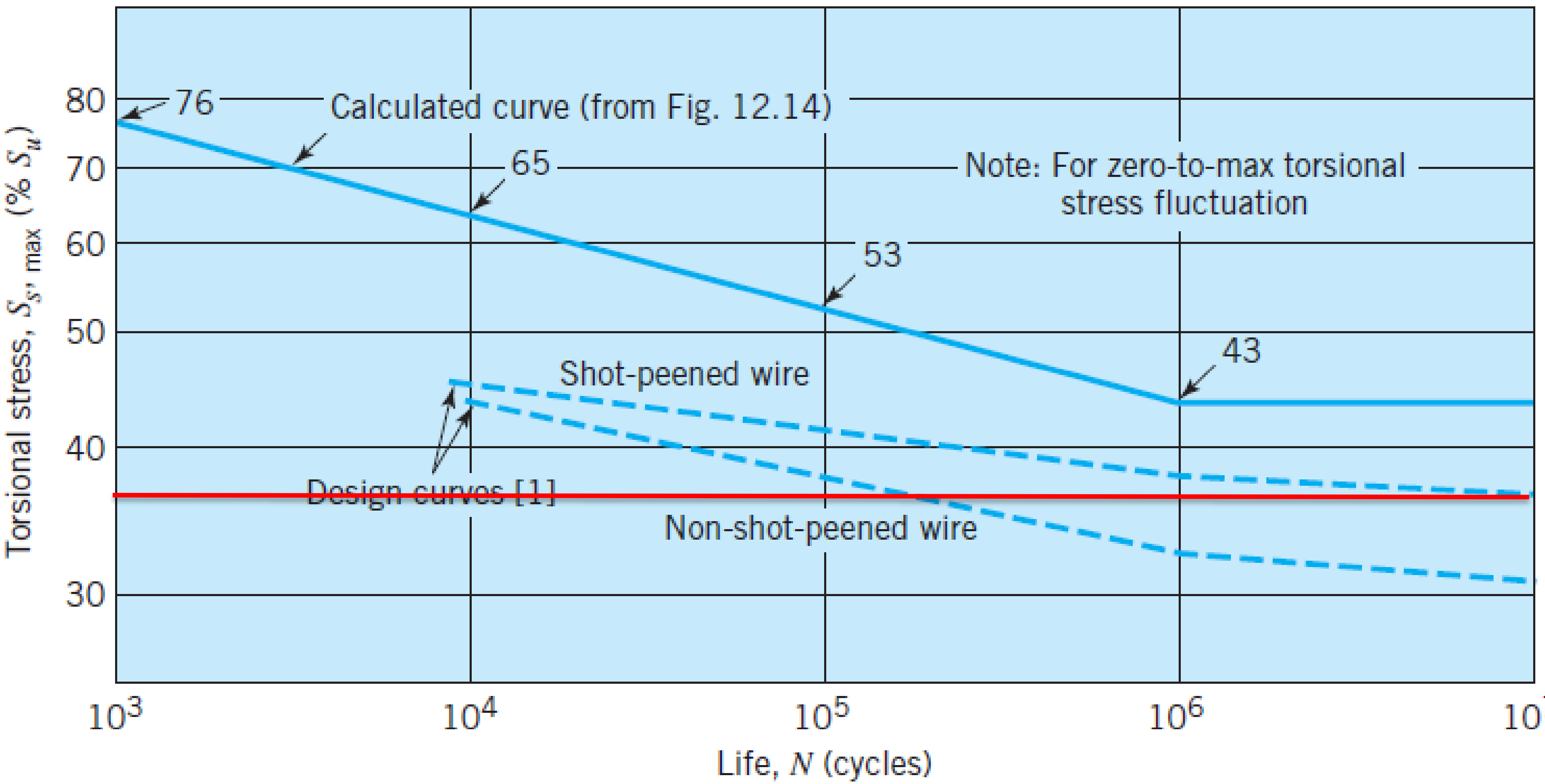


FIGURE 12.15

$S_{s,\max}$–N curves for round steel spring wire. Calculated versus recommended maximum design values from [1].

4. For $\tau_{max}/\tau_{min} = 600/300$, Figure 12.19 shows the limiting value of τ_{max} to be 750 MPa. Because Figure 12.15 represents *maximum* recommended values, it might be prudent to reduce this slightly. An additional "safety factor" of 1.13 would give a final design value of $\tau_{max} = 661$ MPa, exactly as in Sample Problem 12.2.

5. Using this design stress makes the balance of the solution identical to that given for Sample Problem 12.2.

Comment: It is often desirable to use more than one approach in solving engineering problems (as in going through both Sample Problems 12.2 and 12.3), and the reader should be aware that the results will not always agree as well as they did in