MECH 344/M

Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 9

SAMPLE PROBLEM 12.1D Helical Spring Design for Static Loading

A helical spring with squared and ground ends is required to exert a force of 60 lb at a length that cannot exceed 2.5 in., and 105 lb at a length that is 0.5 in. shorter. It must fit inside a 1.5-in.-diameter (hole). Loading is essentially static. Determine a satisfactory design, using oil-tempered ASTM 229 wire, without presetting.

SOLUTION

Known: A helical compression spring exerts a force of 60 lb at a length of 2.5 in. or less and 105 lb at a length that is 0.5 in. shorter.

Find: Determine a satisfactory spring geometry.

Schematic and Given Data: The force and deflection data given for the spring can be used to construct Figure 12.11.

Decisions:

1. As recommended in Section 12.4, choose a clash allowance which is 10 percent of maximum working deflection.
2. To avoid possible interference, provide the commonly recommended diametral clearance of about $0.1D$ between the spring and the 1.5-in. specified diameter.

Assumptions:

1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

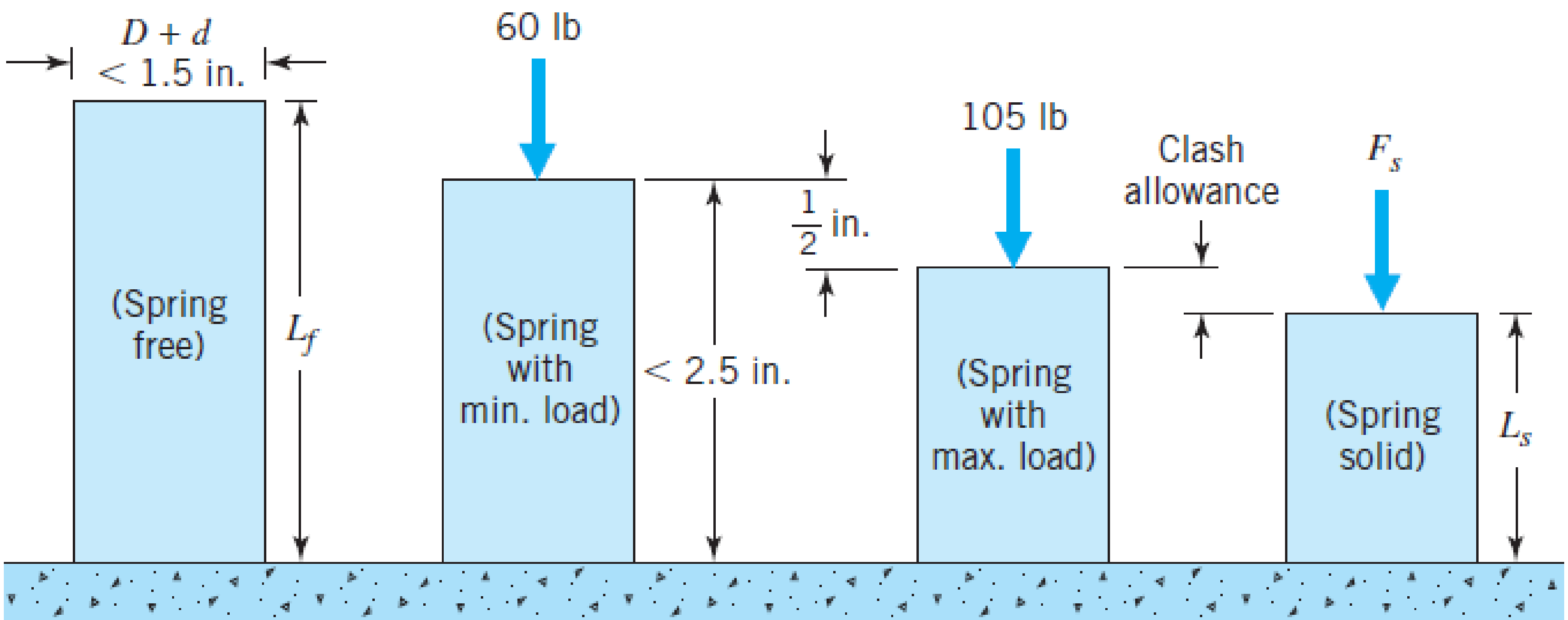


FIGURE 12.11

Helpful representation of information given in Sample Problem 12.1.

Design Analysis:

1. Figure 12.11 gives a convenient representation of the given information concerning spring geometry and loading. The required spring rate is

$$k = \frac{F}{\delta} = \frac{\Delta F}{\Delta \delta} = \frac{45 \text{ lb}}{0.5 \text{ in.}} = 90 \text{ lb/in.}$$

2. With a clash allowance which is 10 percent of maximum working deflection,

$$\text{Clash allowance} = 0.1 \frac{105 \text{ lb}}{90 \text{ lb/in.}} = 0.12 \text{ in.}$$

3. The force when solid (i.e., maximum force that must be resisted without "set") is therefore

$$F_{\text{solid}} = 105 + 90(0.12) = 116 \text{ lb}$$

4. We now proceed to determine a desirable combination of D and d that will satisfy the stress requirement (Eq. 12.6). In this problem the requirement that the spring fit inside a 1.5-in. hole permits a reasonable initial estimate of D—perhaps $D = 1.25$ in. As decided, $D + d$ must be less than 1.5 in. by a *diametral clearance* of about $0.1D$. Note that reasonable clearance is required because the outside diameter increases slightly as the spring is compressed. Since a small wire size should suffice for the loads involved, D would be expected to come out in the range of 1 to 1.25 in.

5. In order to solve Eq. 12.6 for d, we must also determine preliminary values of K_s and τ_{solid}, both of which are functions of d. Fortunately, neither quantity varies greatly over the ranges involved, so we should not be far off by estimating.

   a. $K_s = 1.05$. (Figure 12.4 shows little variation in K_s over the normal range of C between 6 and 12.)

   b. $\tau_{\text{solid}} = 101$ ksi. [For a "ballpark guess" of $d = 0.1$ in., Figure 12.7 shows S_u to be about 225 ksi. The corresponding maximum acceptable value of τ_{solid} (Eq. 12.9) is $0.45S_u$, or 101 ksi.]

$$\tau_s \leqq 0.45S_u \quad \text{(ferrous—without presetting)}$$

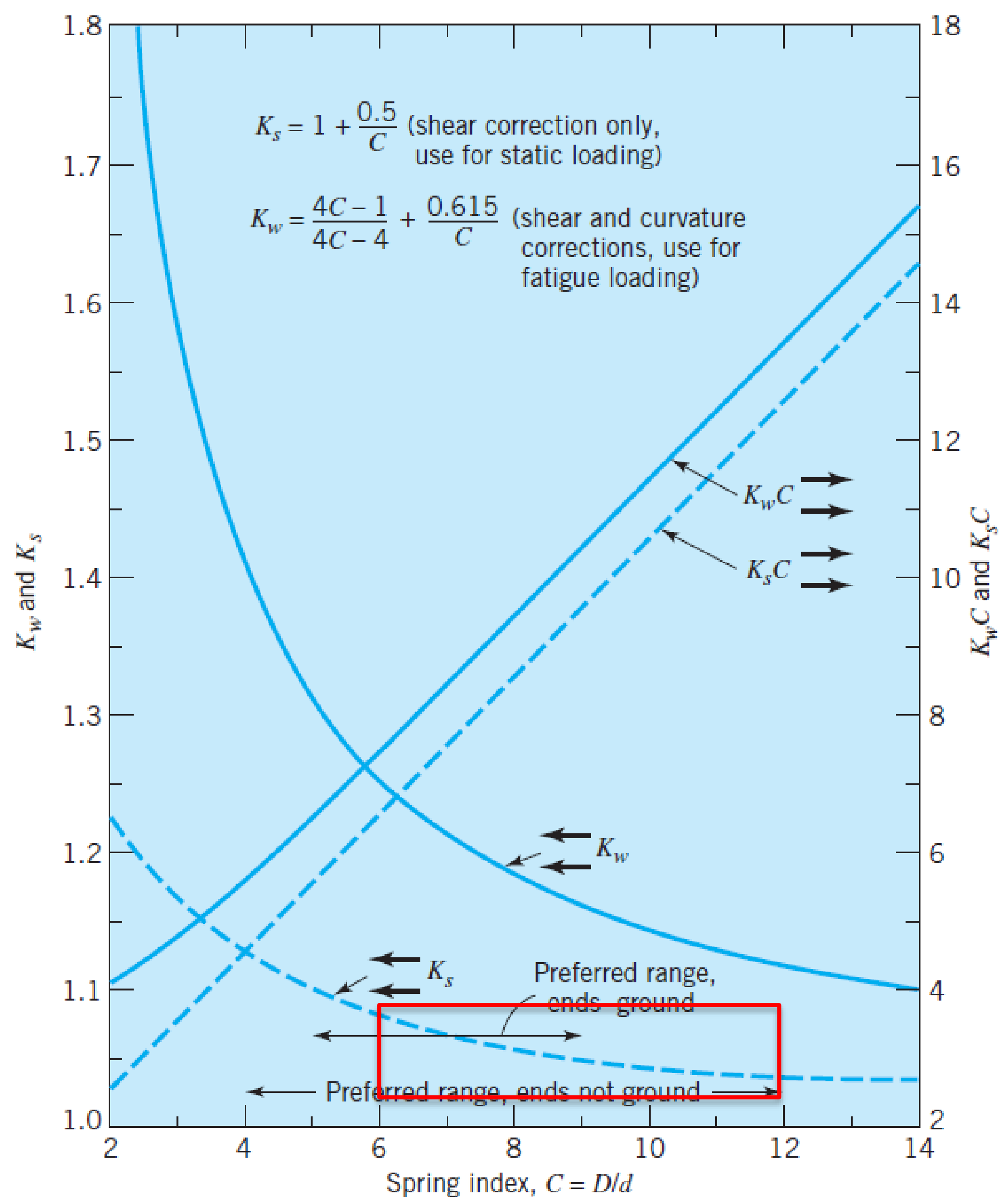


FIGURE 12.4

Stress correction factors for helical springs.

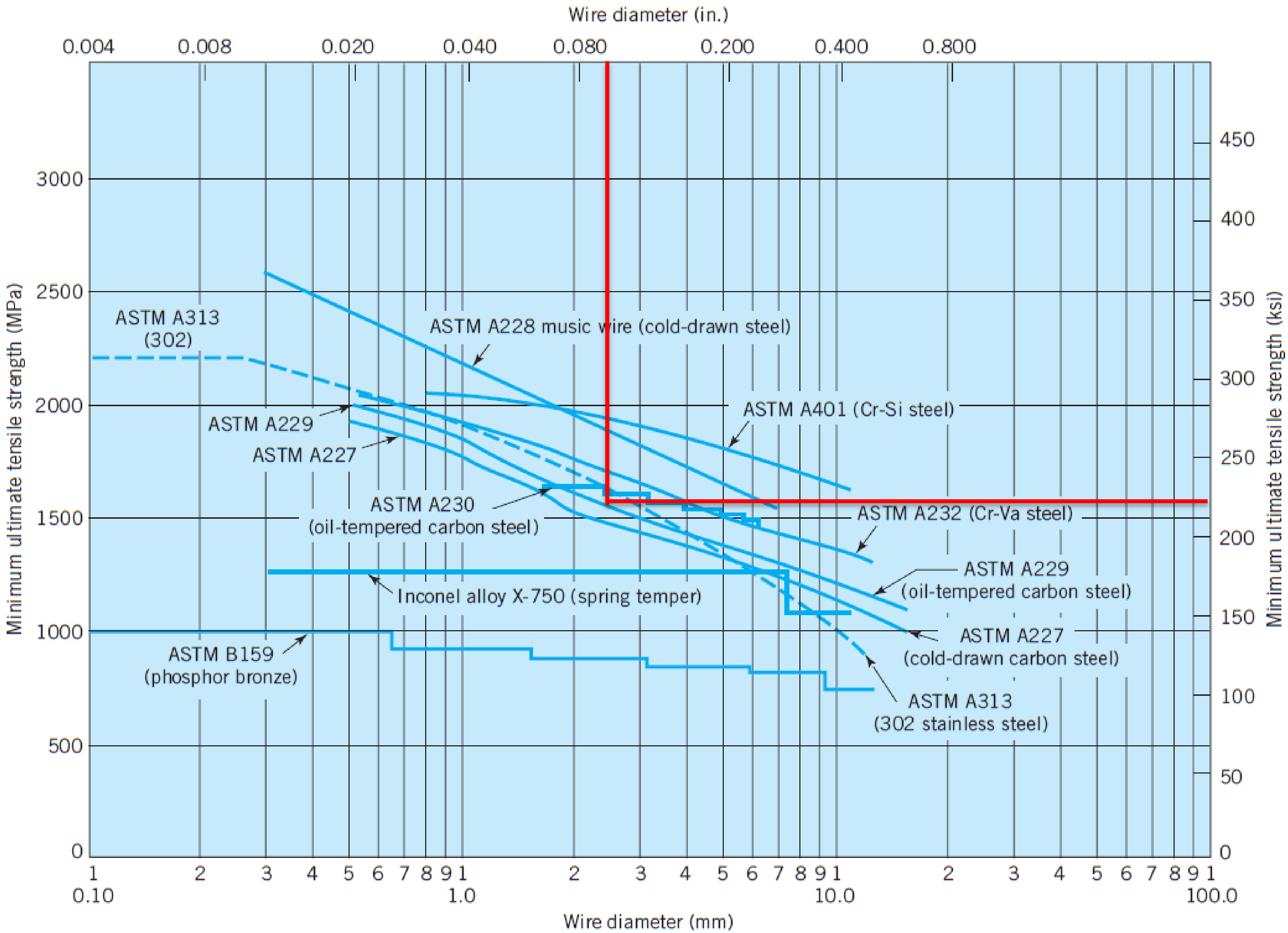


FIGURE 12.7

Tensile strengths of various spring wire materials and diameters, minimum values [2].

6. Substituting the preceding values into Eq. 12.6 gives

$$\tau_{\text{solid}} = \frac{8F_{\text{solid}}D}{\pi d^3}K_s$$

$$101{,}000 = \frac{8(116)(1.25)}{\pi d^3}(1.05)$$

or

$$d = 0.157 \text{ in.}$$

7. The estimates in steps 4 and 5 were deliberately made "rough" enough to give an unsatisfactory solution. A wire diameter of 0.157 in. has an ultimate strength of only about 210 ksi instead of the assumed 225 ksi. Furthermore, the preceding values of d and D provide a diametral clearance in a 1.5-in. hole of only 0.093, which is less than the desired value of $0.1D$. If we keep $d = 0.157$, and reduce D so that the wire is subjected to a little less torque (hence, a little less stress), this would also open up more diametral clearance. For a second trial, choose $d = 0.157$ in. and solve for the corresponding value of D. Both τ_{solid} and K_s will have different values than before, but this time they will be "correct" values for these quantities instead of estimates.

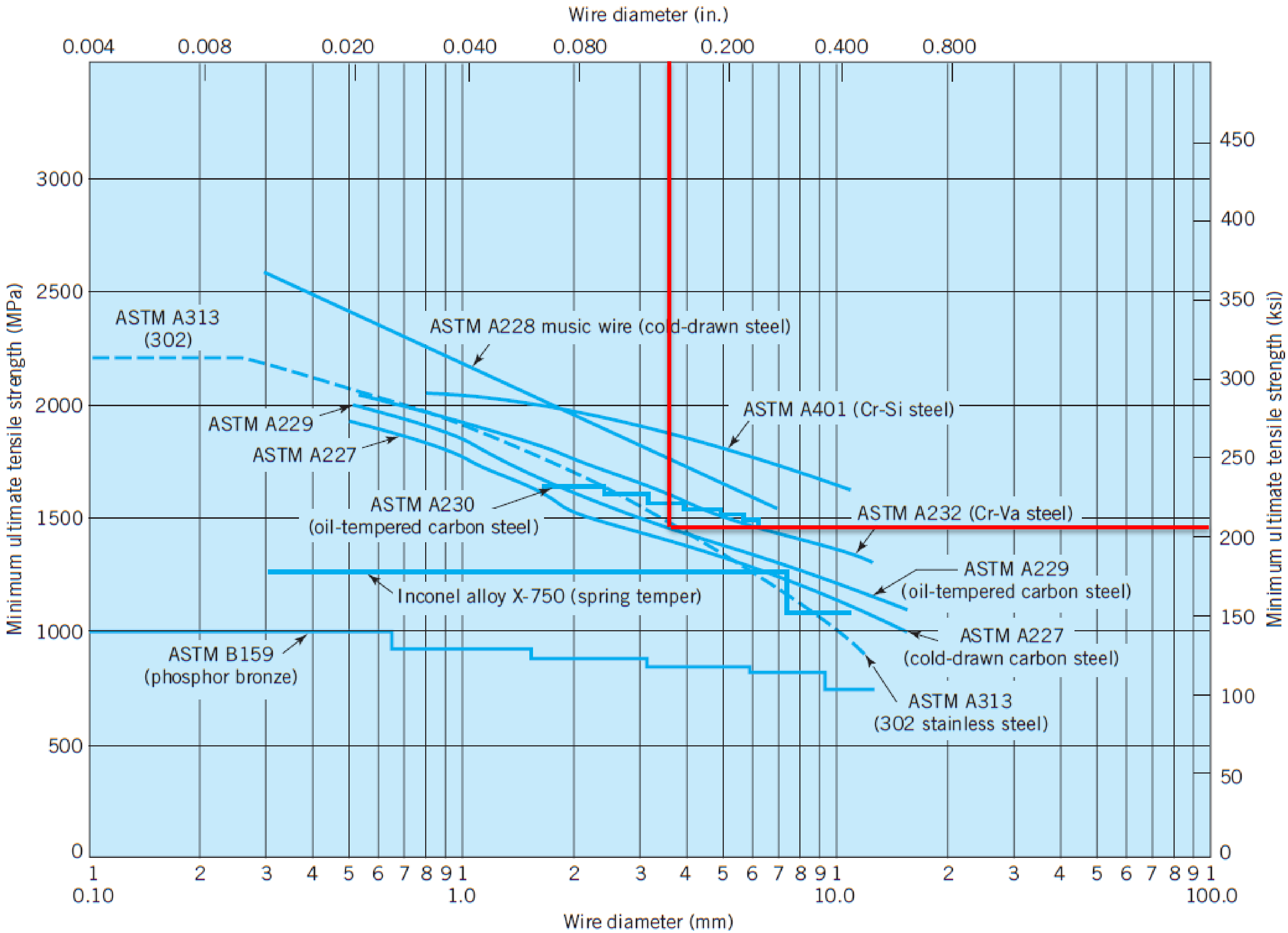


Figure 12.7

Tensile strengths of various spring wire materials and diameters, minimum values [2].

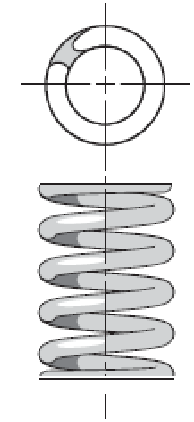
8. To avoid estimating K_s, use the *second* form of Eq. 12.6:

$$\tau = \frac{8FD}{\pi d^3}K_s = \frac{8F}{\pi d^2}CK_s \qquad \textbf{(12.6)}$$

$$\tau_{\text{solid}} = \frac{8F_{\text{solid}}}{\pi d^2}CK_s$$

$$0.45(210{,}000) = \frac{8(116)}{\pi(0.157)^2}CK_s$$

$$CK_s = 7.89$$



(*d*) Squared and ground ends
$L_s = N_t d$

From Figure 12.4, $C = 7.3$, and

$$D = Cd = 7.3(0.157) = 1.15 \text{ in.}$$

This combination of D and d not only conforms exactly to the desired stress criterion but also provides a little more than the minimum desired clearance in the 1.5-in. hole.

9. From Eq. 12.8,

$$k = \frac{d^4G}{8D^3N}, \qquad 90 = \frac{(0.157)^4(11.5 \times 10^6)}{8(1.15)^3N}$$

from which $N = 6.38$.

10. From Eq. 12.10, $N_t = N + 2 = 6.38 + 2 = 8.38$. From Figure 12.8, $L_s = N_t d = 8.38(0.157) = 1.32$ in.

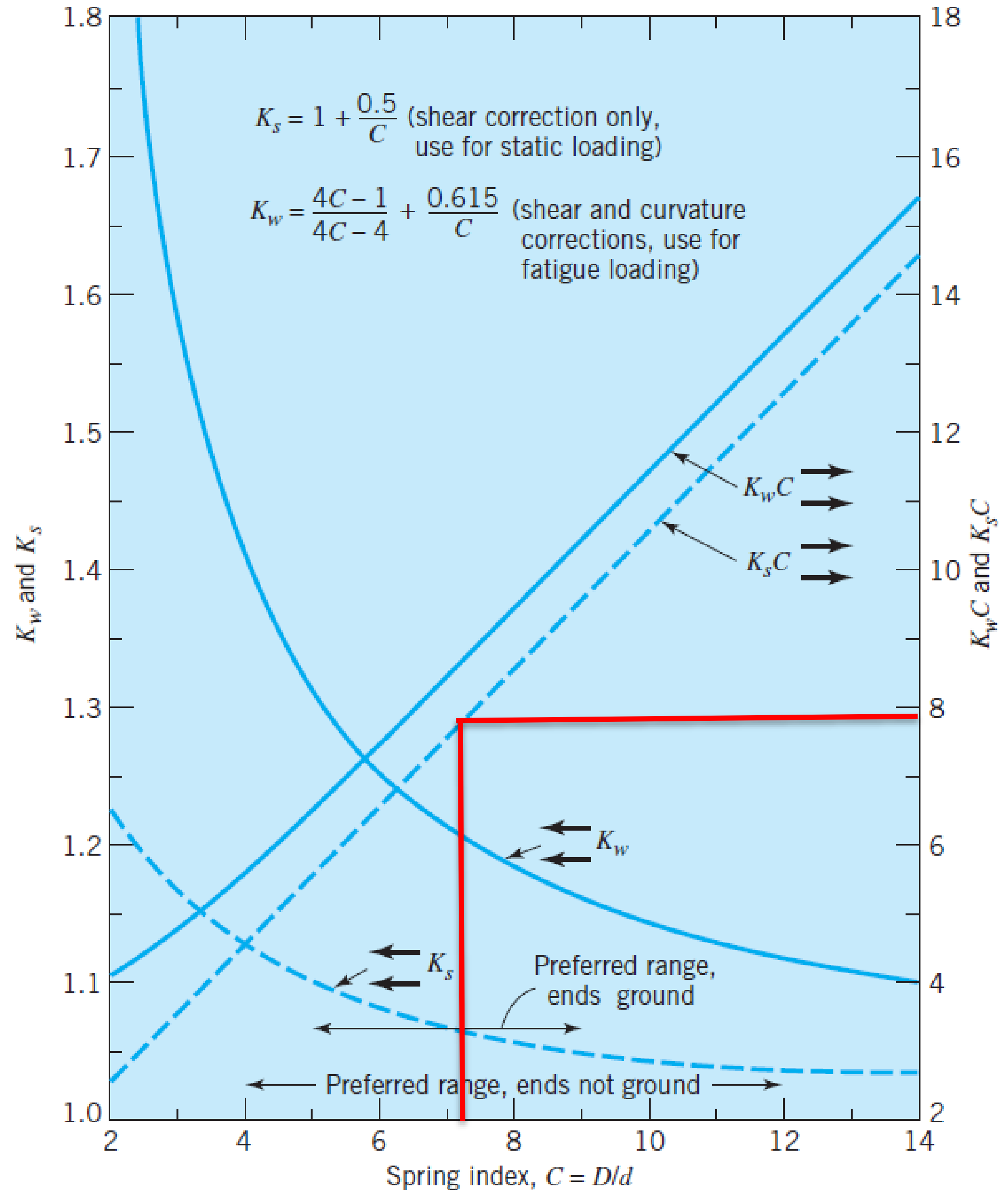


FIGURE 12.4

Stress correction factors for helical springs.

11. When force $F_{solid} = 116$ lb is released, the spring will elongate a distance of 116 lb/(90 lb/in.) = 1.29 in. Thus the free length of the spring, L_f, is $L_s + 1.29 = 1.32 + 1.29 = 2.61$ in. Furthermore, when loaded with 60 lb, the spring length will be [2.61 in. –60 lb/(90 lb/in.)] = 1.94 in. This more than satisfies the maximum length requirement of 2.5 in. at a 60-lb load.

12. Buckling is checked for the worst case of deflection approaching the solid deflection (i.e., $\delta = \delta_s = 1.29$ in.),

$$\frac{\delta_s}{L_f} = \frac{1.29}{2.61} = 0.49$$

$$\frac{L_f}{D} = \frac{2.61}{1.15} = 2.27$$

$$d = 0.157 \text{ in.}$$

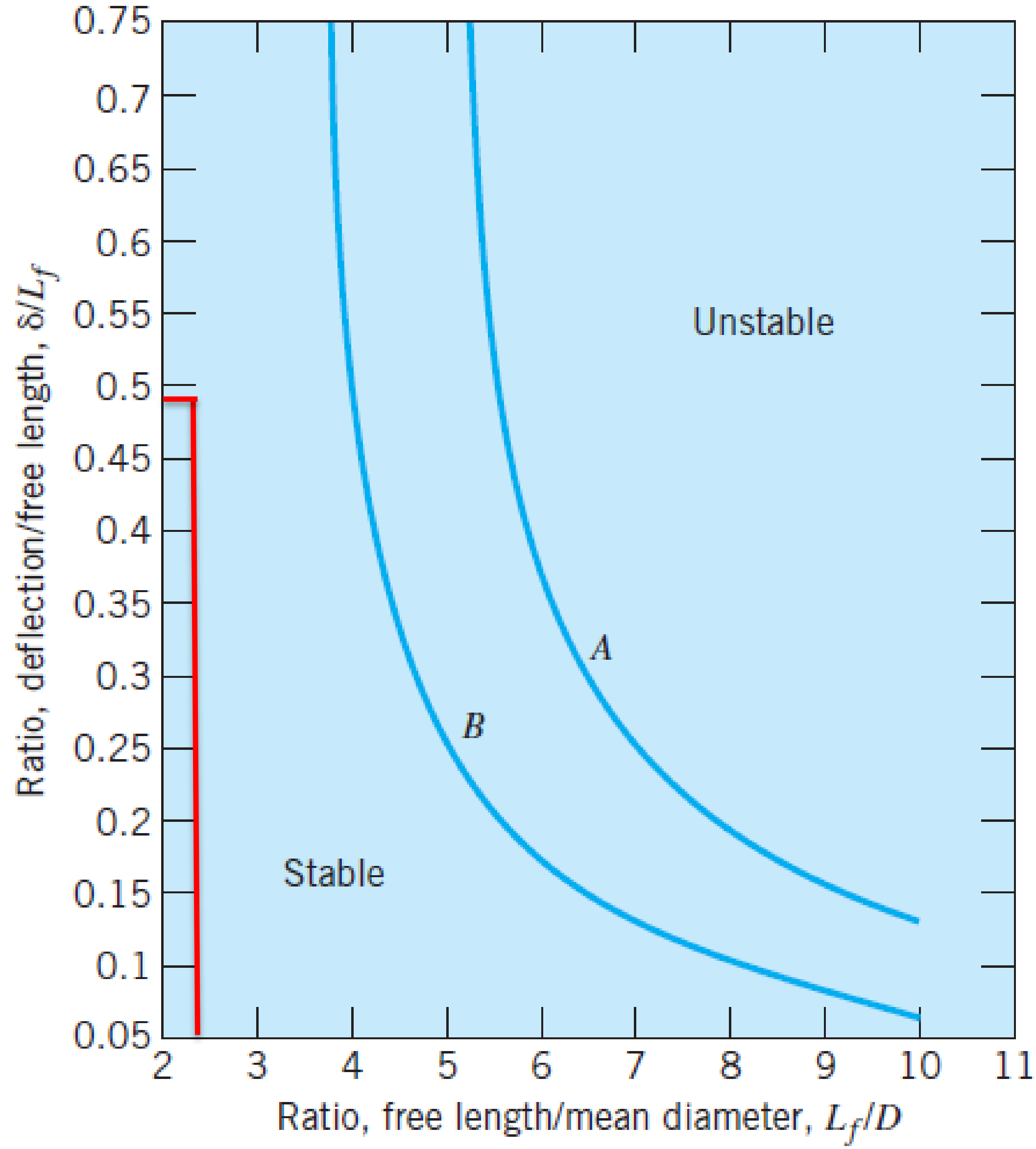
$$D = 1.15 \text{ in.}$$

$$N = 6.38$$

$$L_f = 2.61 \text{ in.}$$

Reference to Figure 12.10 indicates that this spring is far outside the buckling region, even if one end plate is free to tip.

13. The above solution satisfies the stress and spring rate requirements, while more than satisfying the buckling criterion and spatial limitations. (It is obvious that the requirements could also be satisfied with spring designs using a little thicker or a little thinner wire or even a wire of a little less tensile strength.) Hence, one apparently satisfactory answer to the problem is



A- end plates are constrained parallel
(buckling pattern as in Fig. 5.27*c*)
B- one end plate is free to tip
(buckling pattern as in Fig. 5.27*b*)

Comments:

1. The preceding information would permit a technician to draw or to make the spring.
2. The problem is not really finished, however, without dealing with the vital matter of *tolerances*. For example, small variations in d result in large variations in stress and deflection. Imposing extremely tight tolerances can add a substantial unnecessary cost. It is best to advise the spring manufacturer of any *critical* dimensions; for example, in this problem it might be important to hold all springs to 90 ± 4 lb/in. spring rate, and to the *same* length, ±0.002 in., when loaded with 60 lb. Fairly loose tolerances should be allowed on all other dimensions. The manufacturer will then be able to use wire stock of *slightly* varying diameter by adjusting other dimensions as necessary in order to comply with the critical specifications.

12.7 Design Procedure for Helical Compression Springs—Static Loading

- It may be helpful to note that there are, in general, three types of problems in selecting a satisfactory combination of D and d to satisfy the stress requirement.

1. Spatial restrictions place a limit on D, as when the spring must fit inside a hole or over a rod. This situation was illustrated by Sample Problem 12.1.
2. The wire size is fixed, as, for example, standardizing on one size of wire for several similar springs. This situation is also illustrated by Sample Problem 12.1, if steps 4, 5, 6, and 7 are omitted, and d = 0.157 in. is given.
3. No spatial restrictions are imposed, and any wire size may be selected. This completely general situation can theoretically be satisfied with an almost infinite range of D and d, but the extremes within this range would not be economical.

- Reference to Figure 12.4 suggests that good proportions generally require values of D/d in the range of 6 to 12 (but grinding the ends is difficult if D/d exceeds about 9).
- Hence, a good procedure would be to select an appropriate value of C and then use the second form of Eq. 12.6 to solve for d. This requires an estimate of S_u in order to determine the allowable value of τ_{solid}.
- If the resulting value of d is not consistent with the estimated value of S_u, a second trial will be necessary, as was the case in the sample problem.

12.8 Design of Helical Compression Springs for Fatigue Loading

- Figure 12.12 shows a generalized S–N curve, for reversed torsional loading of round steel wire strength S_u, dia < 10 mm, C_s of 1
- A corresponding constant-life fatigue diagram is plotted in Figure 12.13. Since compression coil springs are always loaded in fluctuating compression (and tensile coil springs in fluctuating tension), these springs do not normally experience a stress reversal.
- In the extreme case, the load drops to zero and is then reapplied in the same direction. Thus, as shown in Figure 12.13, the region of interest lies between $\tau_a/\tau_m = 0$ and $\tau_a/\tau_m = 1$, where τ_a/τ_m is the ratio of alternating shear stress to mean shear stress.

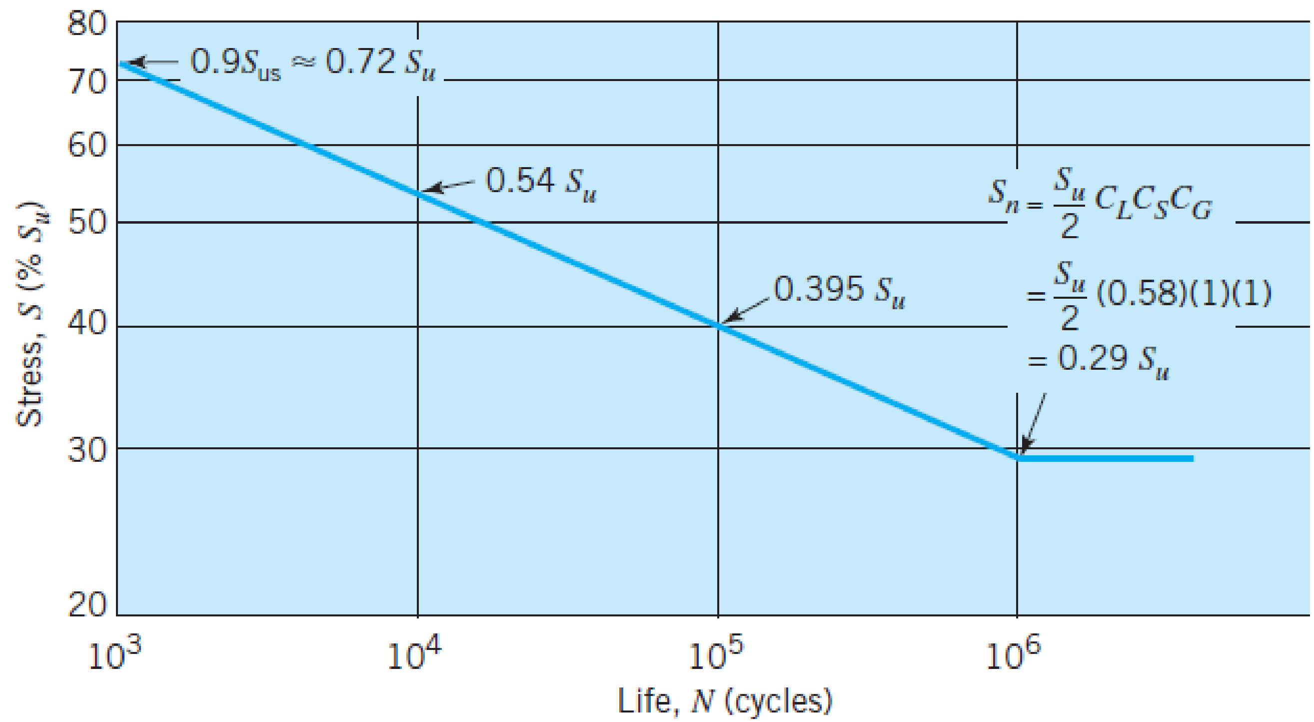


FIGURE 12.12
Estimated S–N curve for round steel spring wire, $d \leq 10$ mm, $C_S = 1$ (shot-peened) reversed, torsional loading.

12.8 Design of Helical Compression Springs for Fatigue Loading

- It is customary when working with coil springs to re plot the information in Figure 12.13 in the form used in Figure 12.14. This alternative form of constant-life fatigue diagram contains only the "region of interest" shown in Figure 12.13. Note, for example, that point P of Figure 12.13 corresponds to $\tau_m = 0.215S_u$, $\tau_a = 0.215S_u$, whereas in Figure 12.14 point P plots as $\tau_{min} = 0$, $\tau_{max} = 0.43S_u$.

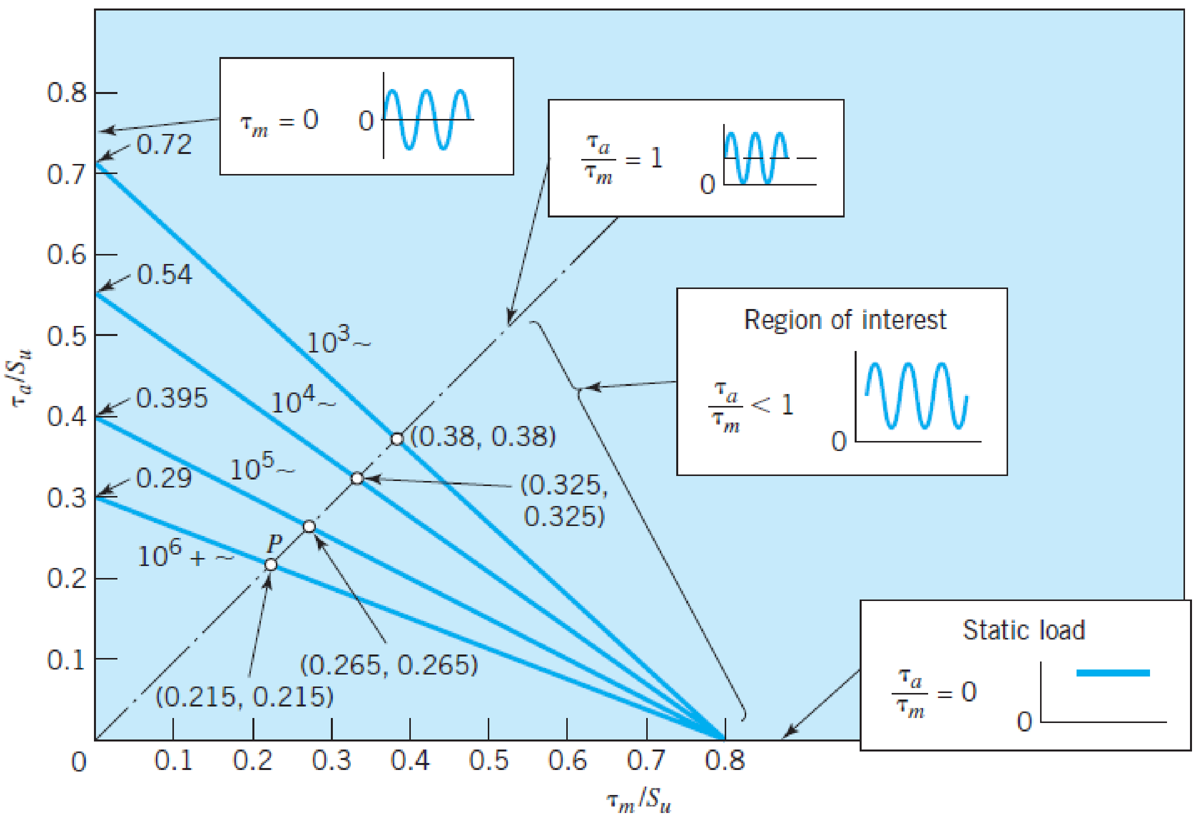


Figure 12.13
Constant-life fatigue diagram corresponding to Figure 12.12. Recall that $S_{us} \approx 0.8\ S_u$.

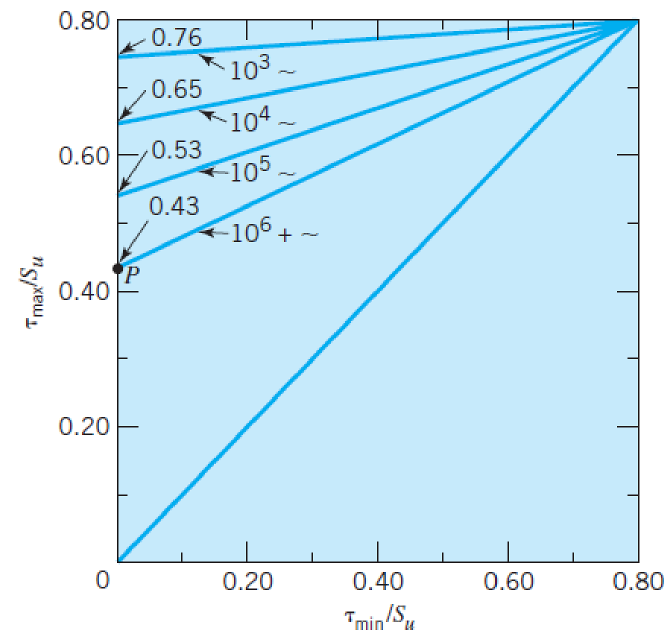


Figure 12.14
Alternative form of constant-life fatigue diagram (replot of "region of interest" of Figure 12.13).

12.8 Design of Helical Compression Springs for Fatigue Loading

- Figure 12.14 is based on actual torsional fatigue tests, with the specimens loaded in a zero-to-maximum fluctuation ($\tau_a/\tau_m = 1$).

- Figure 12.15 shows S–N curves based on 0-to-maxtress fluctuation. The top curve is drawn to agree with the values determined in Figure 12.13. The lower curves in Figure 12.15 are 0-to-max torsional S–N curves based on experimental data and suggested for design. These reflect production spring wire surface finish, rather than $C_s = 1$, as in the top curve.

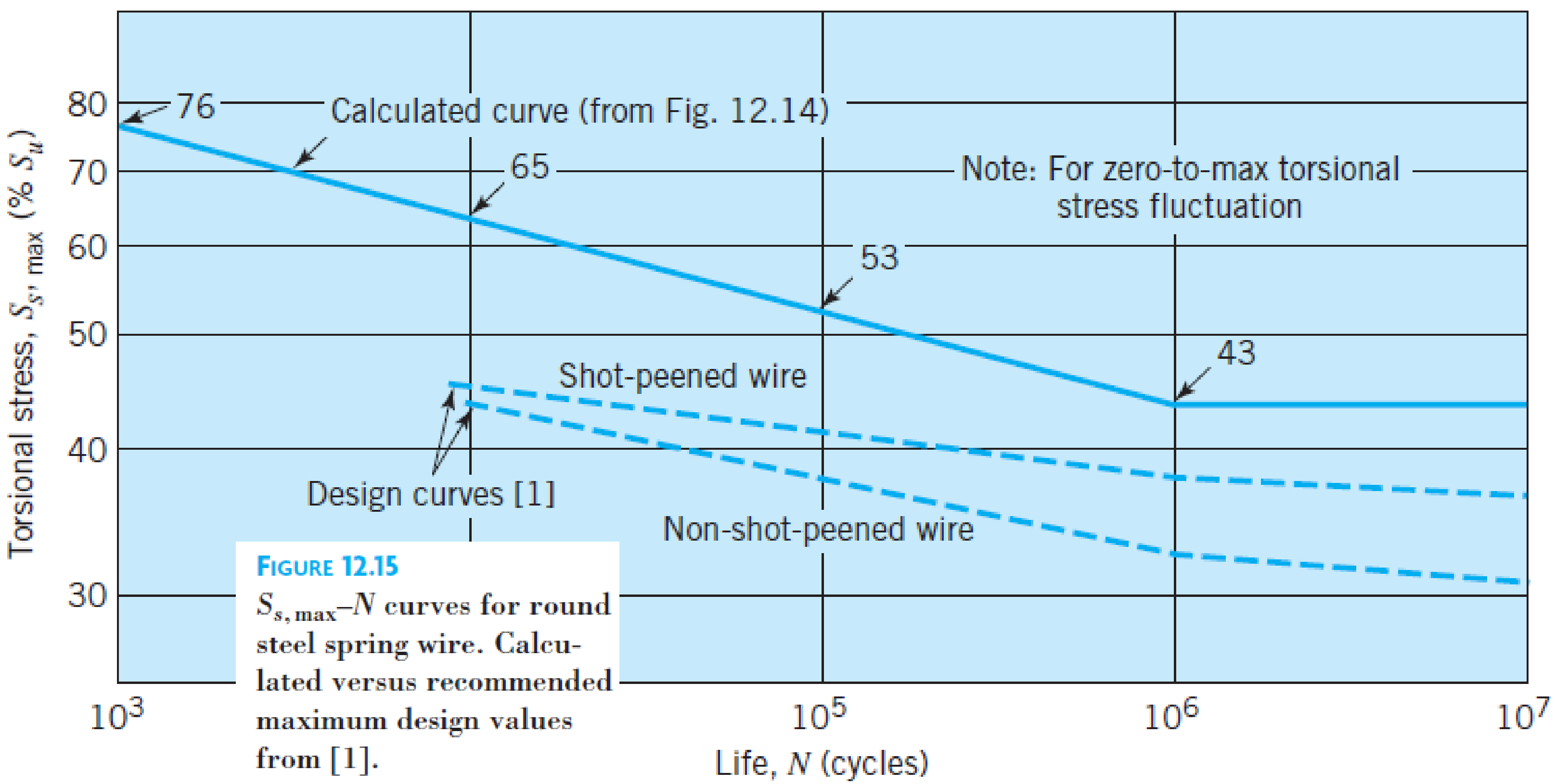


FIGURE 12.15 $S_{s,\max}$–N curves for round steel spring wire. Calculated versus recommended maximum design values from [1].

12.8 Design of Helical Compression Springs for Fatigue Loading

- Figure 12.16 is an independently obtained empirical constant-life fatigue diagram pertaining to most grades of engine valve spring wire. It represents actual test data. Design values should be somewhat lower.
- In the design of helical (or torsion bar) springs for fatigue loading, two previously mentioned manufacturing operations are particularly effective: shot peening and presetting.
- Recall that presetting always introduces surface residual stresses opposite those caused by subsequent load applications in the same direction as the presetting load.

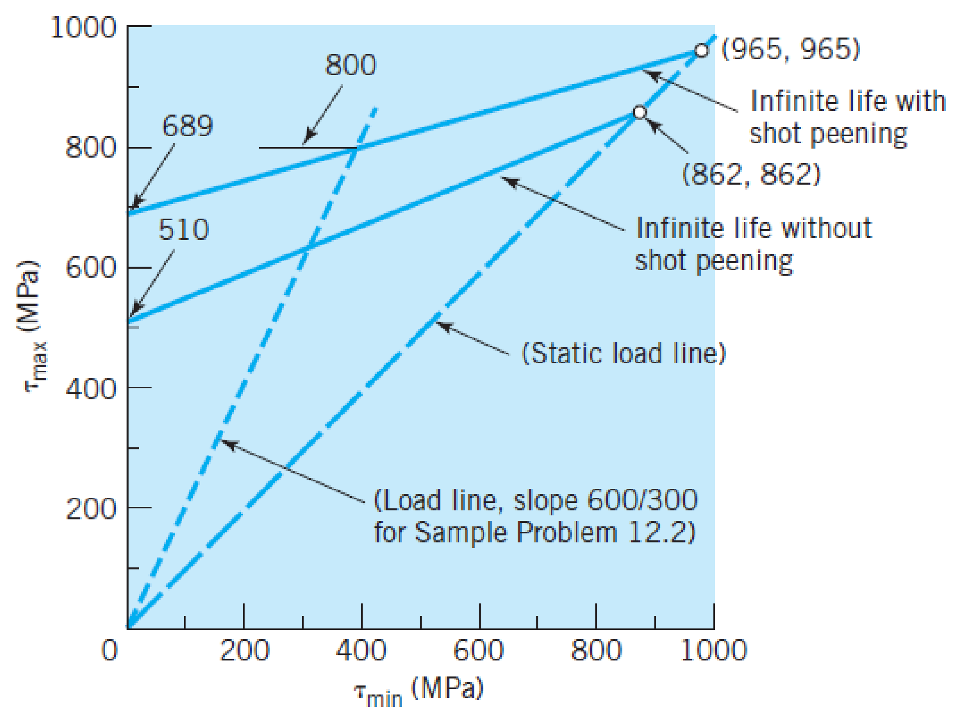


FIGURE 12.16
Infinite-life fatigue diagram. Pretempered carbon or alloy steel high-duty spring wire, $d \leq 5$ mm (0.2 in.).

12.8 Design of Helical Compression Springs for Fatigue Loading

- The corresponding coil spring (or torsion bar) torsional stress fluctuations with and without presetting are as shown in Figure 12.17.
- the theoretical maximum residual stress that can be introduced by presetting is $S_{sy}/3$.
- The practical maximum value is somewhat less. The fatigue improvement represented by the fluctuation with presetting in Figure 12.17 is readily apparent when the stress fluctuations are represented in Figures 12.13, 12.14, and 12.16.
- Maximum fatigue strengthening can be obtained by using both shot peening and presetting.

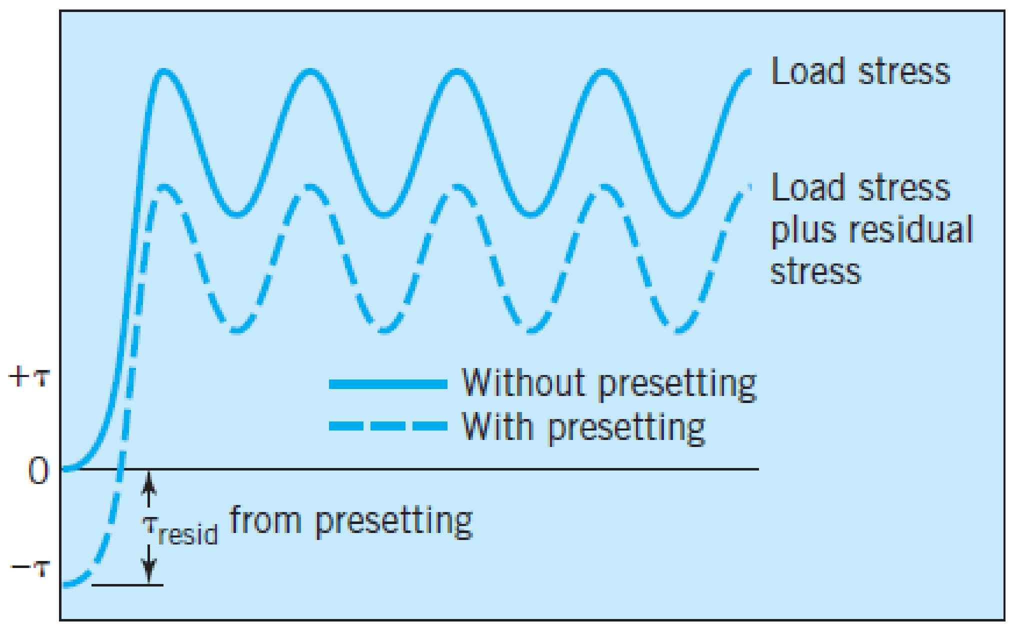


FIGURE 12.17
Stress fluctuation in a helical (or torsion bar) spring with and without presetting.

12.8 Design of Helical Compression Springs for Fatigue Loading

- Springs used in high-speed machinery must have f_n >> machine frequency.
- a conventional engine valve spring goes through one cycle of shortening and elongating every two engine revolutions. At 5000 engine rpm, the spring has an f of 2500 cpm, and the thirteenth harmonic 32,500 cpm, or 542 Hz.
- When a helical spring is compressed and then suddenly released, it vibrates longitudinally at its f_n until the energy is dissipated by damping, this phenomenon is called spring surge and causes local stresses approximating those for "spring solid." Spring surge also decreases the ability of the spring
- The natural frequency of spring surge (which should be made higher than the highest significant harmonic of the motion involved—typically about the thirteenth) is

$$f_n \propto \sqrt{k/m} \quad \text{or} \quad f_n \propto \frac{d}{D^2 N}\sqrt{G/\rho}$$

- For steel springs f_n in Hz is
- Spring design with high f_n requires operating at high stresses

$$f_n = \frac{13{,}900d}{ND^2} \quad (d \text{ and } D \text{ in inches}) \tag{12.11}$$

$$f_n = \frac{353{,}000d}{ND^2} \quad (d \text{ and } D \text{ in millimeters}) \tag{12.11a}$$

- This minimizes the required mass of the spring, thereby maximizing its f_n, which is proportional to. $1/\sqrt{m}$.

SAMPLE PROBLEM 12.2D Helical Spring Design for Fatigue Loading

A camshaft rotates 650 rpm, causing a follower to raise and lower once per revolution (Figure 12.18). The follower is to be held against the cam by a helical compression spring with a force that varies between 300 and 600 N as the spring length varies over a range of 25 mm. Ends are to be squared and ground. The material is to be shot-peened chrome–vanadium steel valve spring wire, ASTM A232, with fatigue strength properties as represented in Figure 12.16. Presetting is to be used. Determine a suitable combination of d, D, N, and L_f. Include in the solution a check for possible buckling and spring surge.

SOLUTION

Known: A helical compression spring operates with a force that varies between given minimum and maximum values as the spring length varies over a known range.

Find: Determine a suitable spring geometry.

Schematic and Given Data:

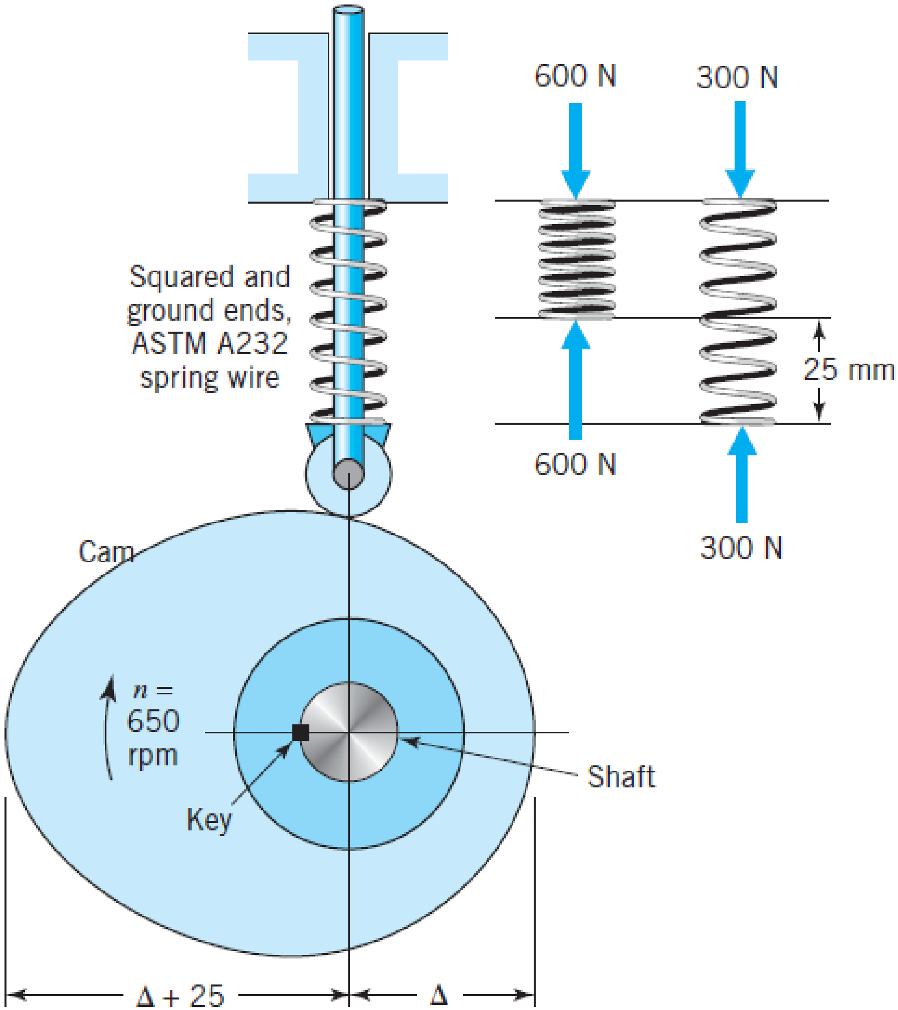


FIGURE 12.18
Diagram for Sample Problem 12.2.

Decisions:

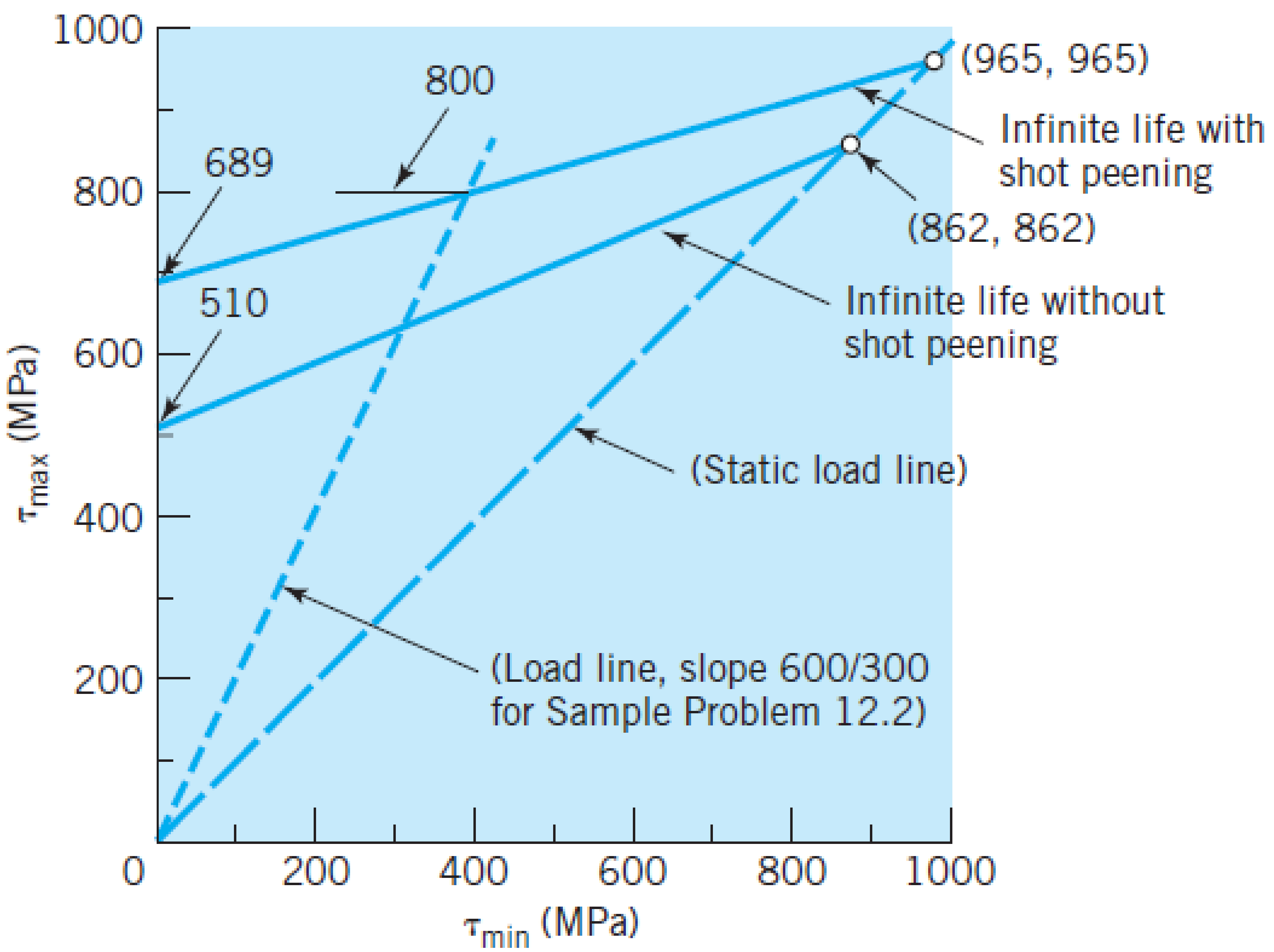
1. To minimize possible spring surge problems, design the spring so that stresses are as large as reasonable.
2. Select the smallest reasonable safety factor to minimize spring weight. (Minimizing spring weight allows us to maximize natural frequency.)
3. Select a spring proportion, $C = 10$. (This proportion is good from the standpoint of the Wahl factor, but costs for the spring may be higher because the ends must be ground.)
4. As recommended in Section 12.4, choose a clash allowance that is 10 percent of the maximum working deflection.

Assumptions:

1. The end plates are in contact with the spring ends.
2. The spring force acts along the spring axis.

Design Analysis:

1. Since, at 650 rpm, a million stress cycles are accumulated in 26 operating hours, infinite fatigue life is required. Stresses should be as high as reasonable to minimize possible spring surge problems. Regardless of the spring design, the ratio τ_{max}/τ_{min} will be the same as the ratio of maximum and minimum loads—that is, 600/300. A line of this slope is drawn on Figure 12.16, giving an intersection at $\tau_{max} = 800$ MPa.
2. Since Figure 12.16 represents actual test data, this value of τ_{max} makes no allowance for possible spring surge or a safety factor. The amplitude of possible surge can be limited by providing a minimal clash allowance—say, 10 percent of the maximum working deflection. Spring weight can be minimized, thus allowing the maximum natural frequency, by selecting the smallest reasonable safety factor—say, 1.1. (The use of presetting will provide some additional safety factor.) Thus a design value for τ_{max} might be chosen as 800 MPa divided by 1.1 (allowance for possible surge) and divided again by 1.1 (safety factor), or 661 MPa.



1000
800
600
400
200
0
200
400
600
800
1000
τ_{max} (MPa)
τ_{min} (MPa)
800
689
510
(965, 965)
Infinite life with shot peening
(862, 862)
Infinite life without shot peening
(Static load line)
(Load line, slope 600/300 for Sample Problem 12.2)

$$\tau = \frac{8FD}{\pi d^3}K_w = \frac{8F}{\pi d^2}CK_w \quad \textbf{(12.5)}$$

3. In the absence of any restrictions on d, for either the outer diameter or the inner diameter, let us arbitrarily select a spring *proportion* of, say, $C = 10$. This proportion is good from the standpoint of the Wahl factor, but the spring may cost an extra amount because the ends must be ground. Then, from Eq. 12.5,

$$d = \sqrt{\frac{8F_{\max}CK_w}{\pi\tau_{\max}}} = \sqrt{\frac{8(600)(10)(1.14)}{\pi(661)}} = 5.13 \text{ mm}$$

4. In the absence of any reason to stay with an odd value of d, it might be preferable to round off to $d = 5.0$ mm. Then, going back to Eq. 12.5 and solving for the value of C that gives a stress of 661 MPa (with load of 600 N) together with $d = 5.0$ mm, we have

$$CK_w = \frac{\pi\tau_{\max}d^2}{8F_{\max}} = \frac{\pi(661)(5)^2}{8(600)} = 10.82$$

From Figure 12.4, $C = 9.4$, $D = Cd = 47.0$ mm.

$$K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

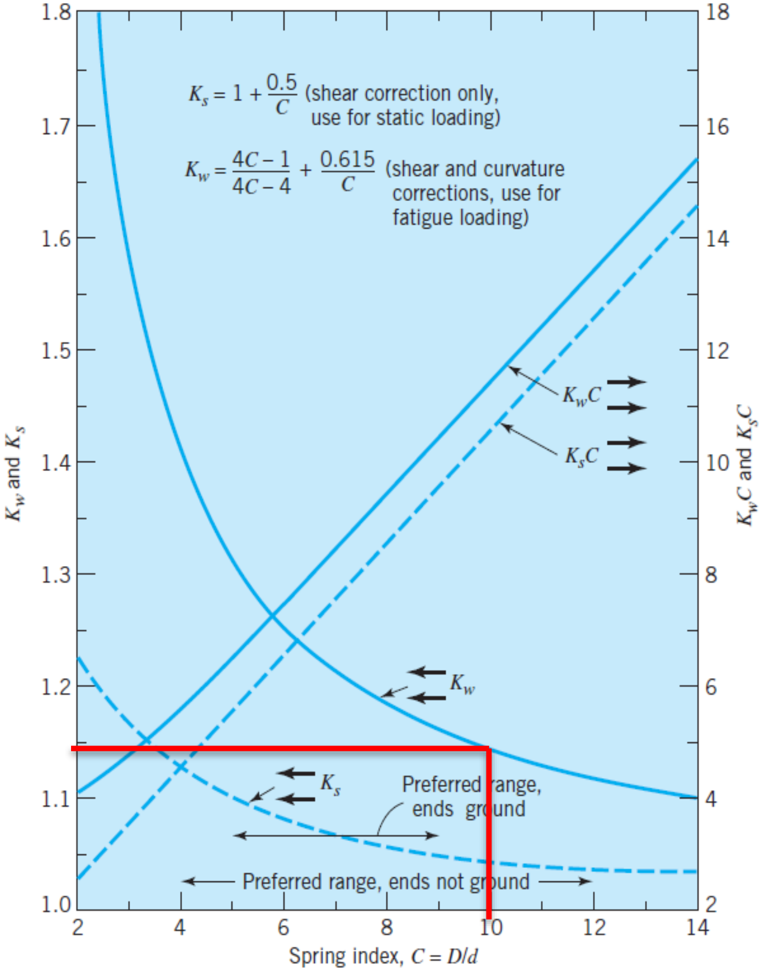


FIGURE 12.4
Stress correction factors for helical springs.

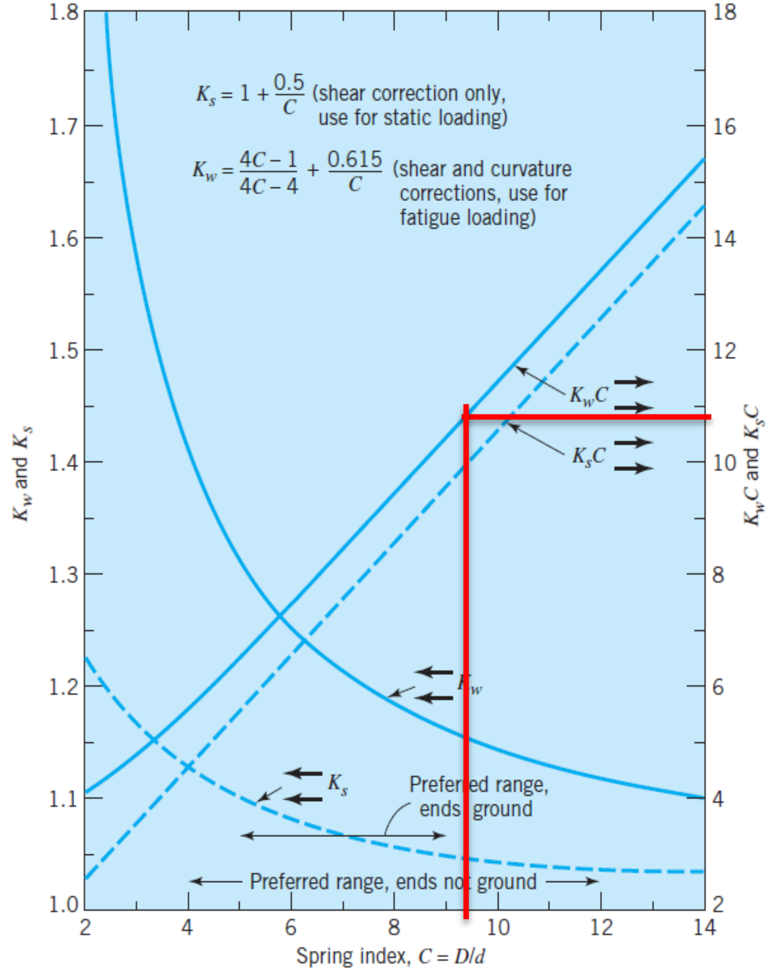


FIGURE 12.4
Stress correction factors for helical springs.

5. $k = 300\ \text{N}/25\ \text{mm} = 12\ \text{N/mm}$.

6. From Eq. 12.8,

$$N = \frac{dG}{8C^3k} = \frac{5(79{,}000)}{8(9.4)^3(12)} = 4.95$$

7. From Figure 12.8, $L_s = N_t d = (N + 2)d = (4.95 + 2)(5) = 34.75$ mm.

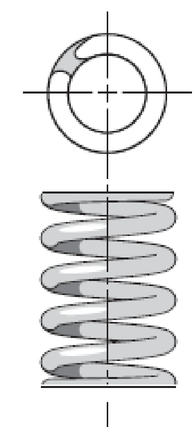
$$L_f = L_s + F_{\text{solid}}/k$$

With 10 percent clash allowance, $F_{\text{solid}} = 1.1F_{\max} = 660$ N. Then,

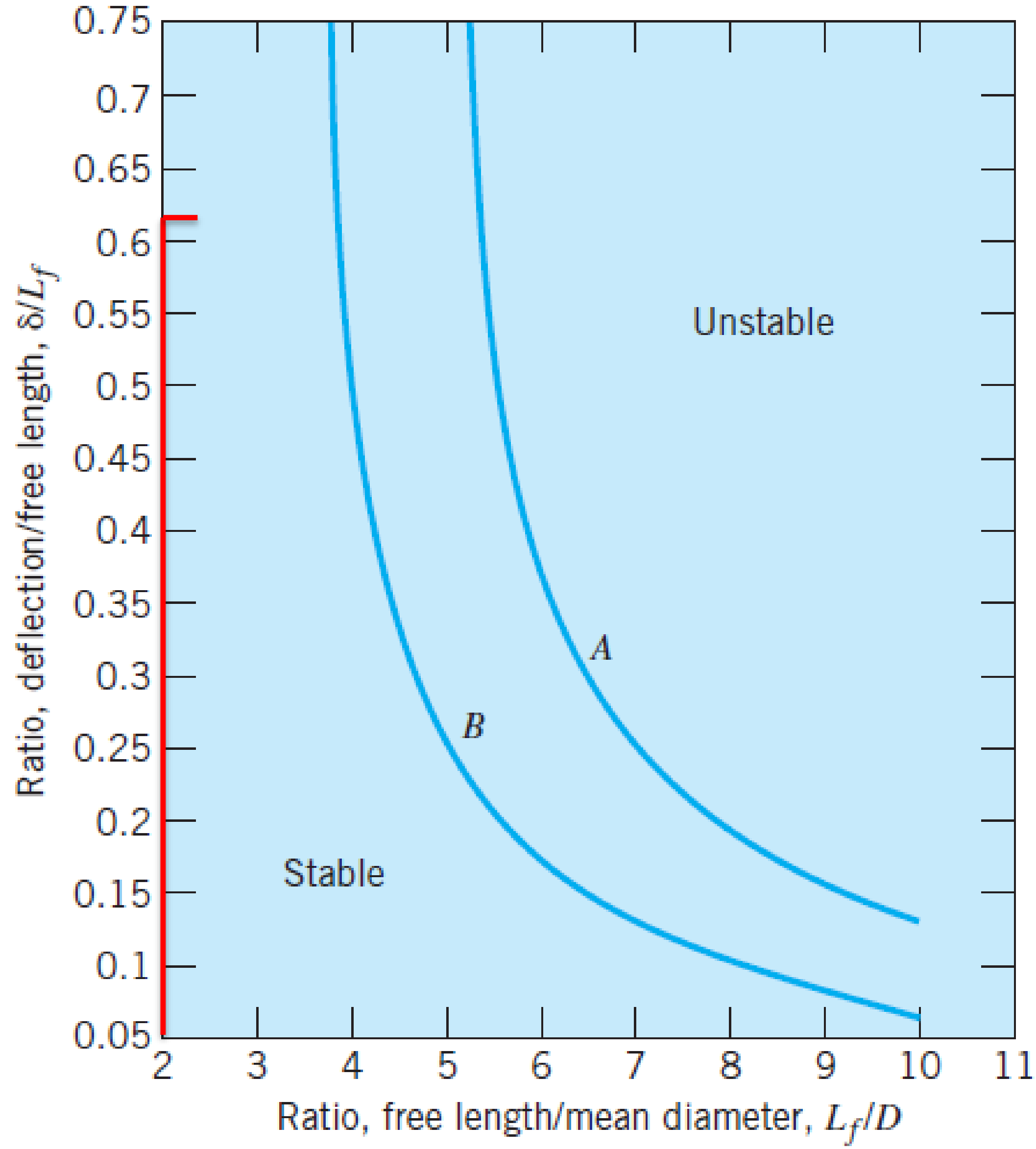
$$L_f = 34.75 + 660/12 = 89.75\ \text{mm}$$

8. Check for buckling to determine if the spring contacts the rod (for extreme case of $\delta = \delta_s$):

$$\left.\begin{aligned} \frac{L_f}{D} &= \frac{89.75}{47} = 1.91 \\ \frac{\delta_s}{L_f} &= \frac{\dfrac{660}{12}}{89.75} = 0.61 \end{aligned}\right\} \text{Far removed from buckling per Figure 12.10}$$



(*d*) Squared and ground ends
$L_s = N_t d$



A- end plates are constrained parallel
(buckling pattern as in Fig. 5.27*c*)
B- one end plate is free to tip
(buckling pattern as in Fig. 5.27*b*)

9. From Eq. 12.11a, the natural frequency is

$$f_n = \frac{353{,}000d}{ND^2} = \frac{353{,}000(5)}{(4.95)(47)^2} = 161.4\text{ Hz}$$

10. To summarize the results,

$$\begin{aligned} d &= 5\text{ mm} \\ D &= 47.0\text{ mm} \\ N &= 4.95 \\ L_f &= 89.75\text{ mm} \end{aligned}$$

Comments:

1. For the spring to be in resonance with the fundamental surge frequency $f_n = 161.4$ Hz, the camshaft would have to rotate at $(161.4)(60) = 9684$ rpm. For the thirteenth harmonic to be in resonance, the shaft must rotate $9684/13 = 745$ rpm. Rotation at 650 rpm should not result in spring surge (unless the cam contour is highly unusual, producing significant harmonics above the thirteenth).
2. No buckling or spring surge should occur (but allowance for possible repeated transient surge was made by appropriate selection of clash allowance and design stress).

Sample Problem 12.3D Helical Spring Fatigue Design

Repeat Sample Problem 12.2, except this time design the spring to use 5-mm wire of the same material but with the strength properties indicated in Figures 12.7 and 12.15.

Solution

Known: A helical compression spring of wire diameter $d = 5$ mm operates with a known fluctuating force that varies the spring length through a range of 25 mm.

Find: Determine a satisfactory spring geometry.

Schematic and Given Data: The schematic and given data are the same as in Sample Problem 12.2 except that the strength properties are those indicated in Figures 12.7 and 12.15 rather than in Figure 12.16.

Find: Determine a suitable spring geometry.

Schematic and Given Data:

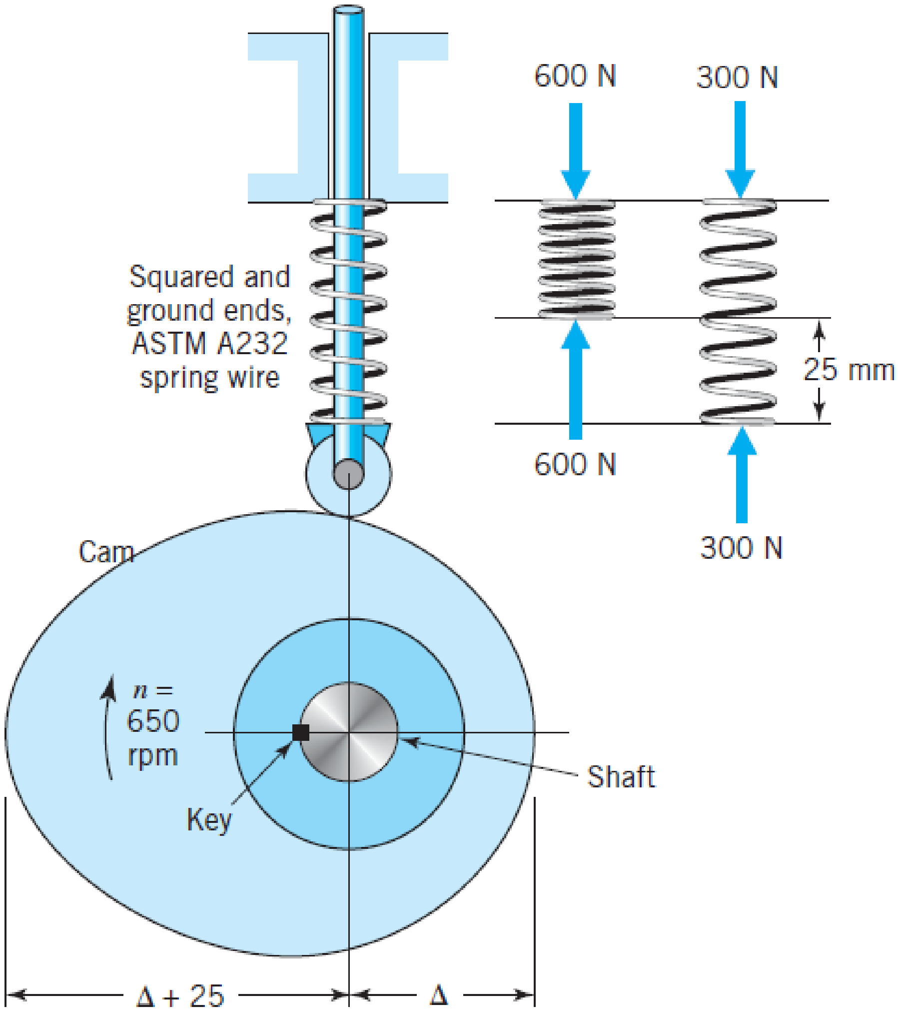


FIGURE 12.18

Diagram for Sample Problem 12.2.

Decisions/Assumptions: Same as in Sample Problem 12.2.

Design Analysis:

$$\tau_s \leqq 0.65 S_u \quad \text{(ferrous—with presetting)} \tag{12.9}$$

1. From Figure 12.7, $S_u = 1500$ MPa for the given material and wire size.
2. From Figure 12.15, the maximum recommended design stress for infinite life and zero-to-maximum stress fluctuation (shot-peened wire) is $0.36S_u = 540$ MPa.
3. From Eq. 12.9 the effective torsional yield strength associated with 2 percent long-term set is $0.65\ S_u = 975$ MPa. Approximating S_{us} as $0.8S_u = 1200$ MPa, an estimated torsional fatigue strength curve for infinite life is plotted in Figure 12.19.

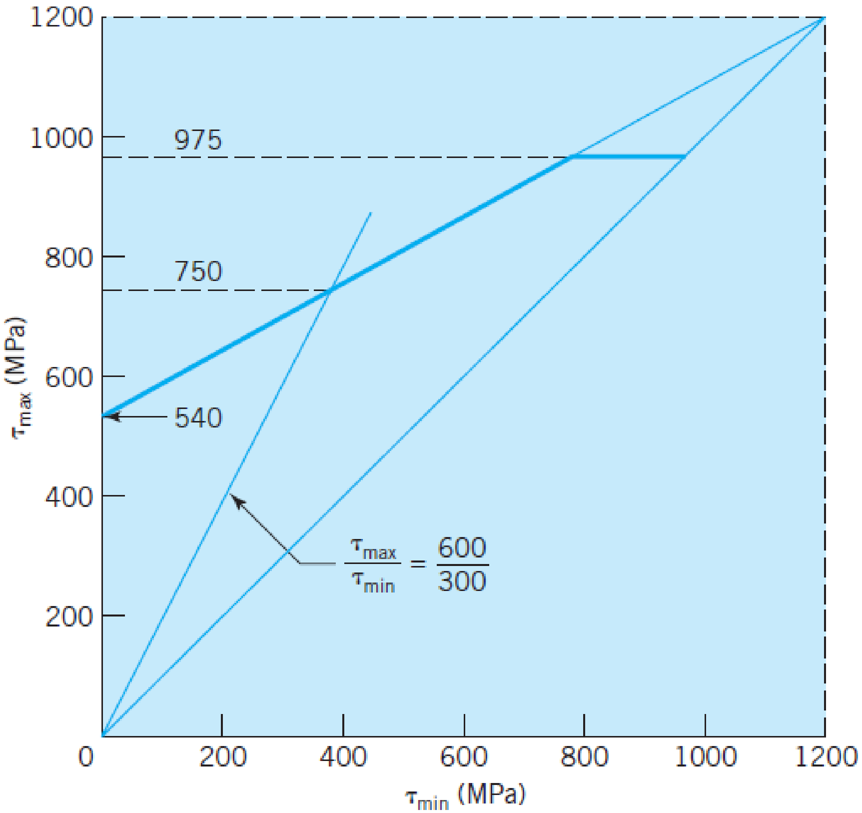


FIGURE 12.19
Fatigue diagram for Sample Problem 12.3.

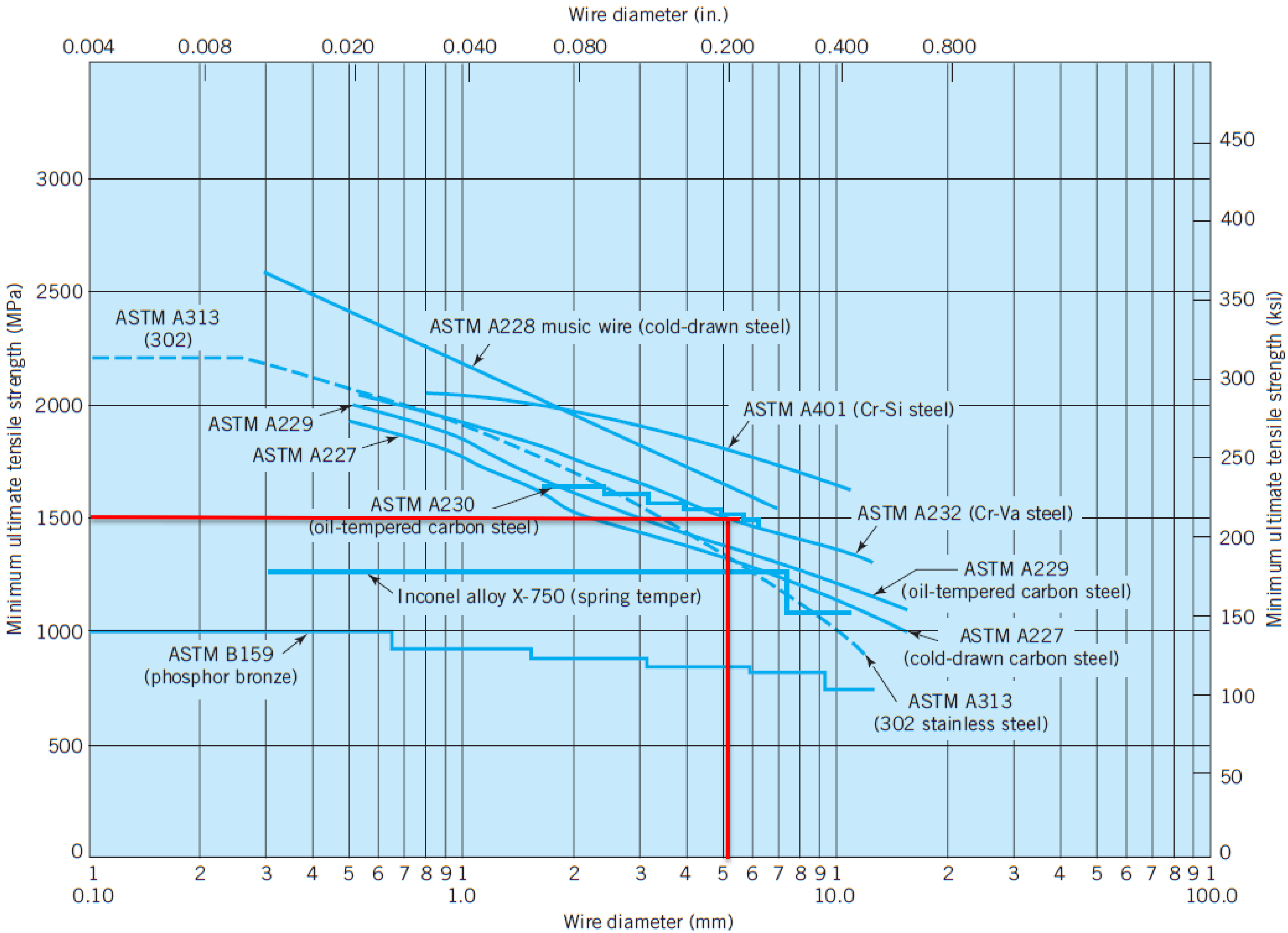


FIGURE 12.7

Tensile strengths of various spring wire materials and diameters, minimum values [2].

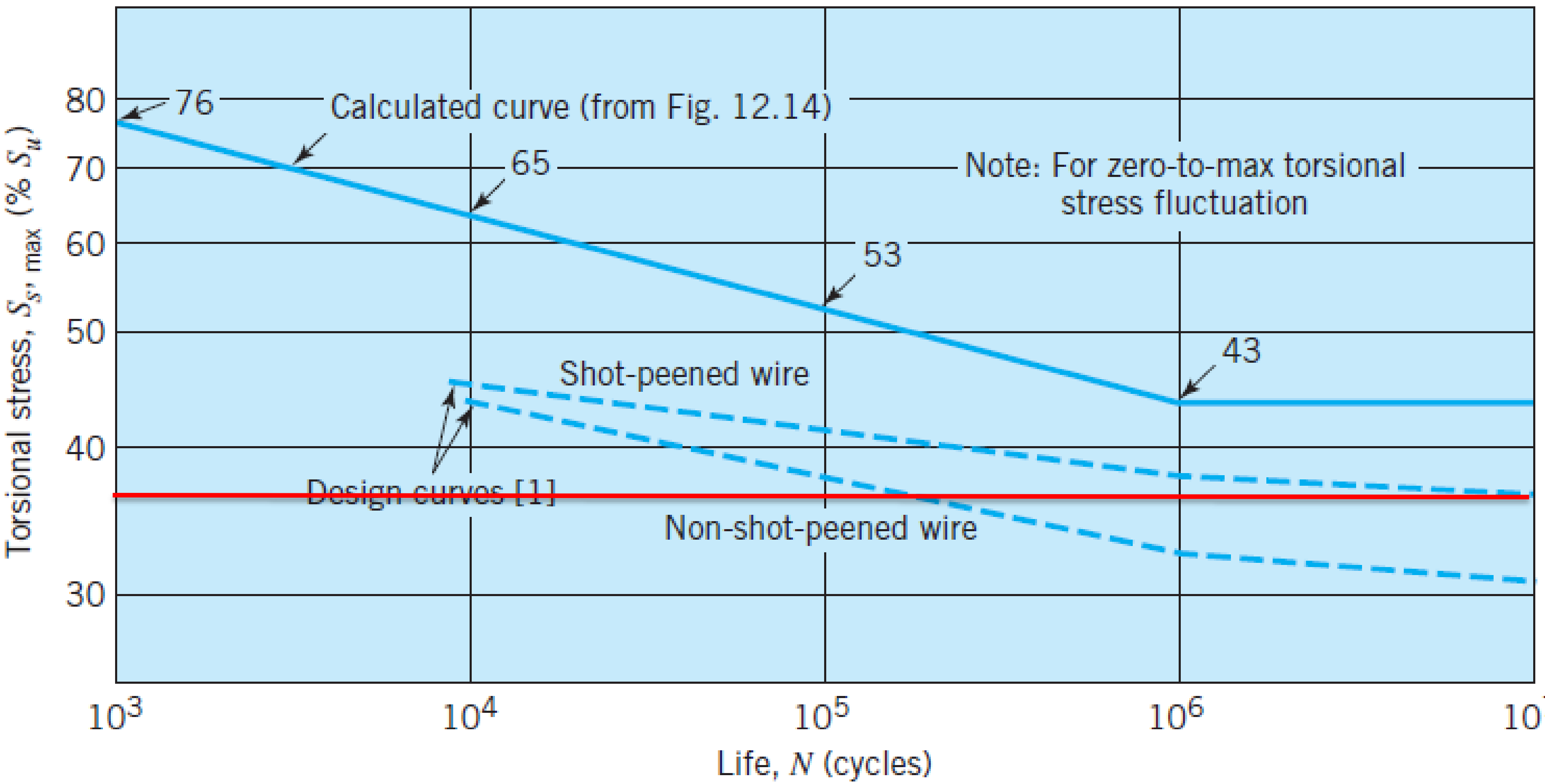


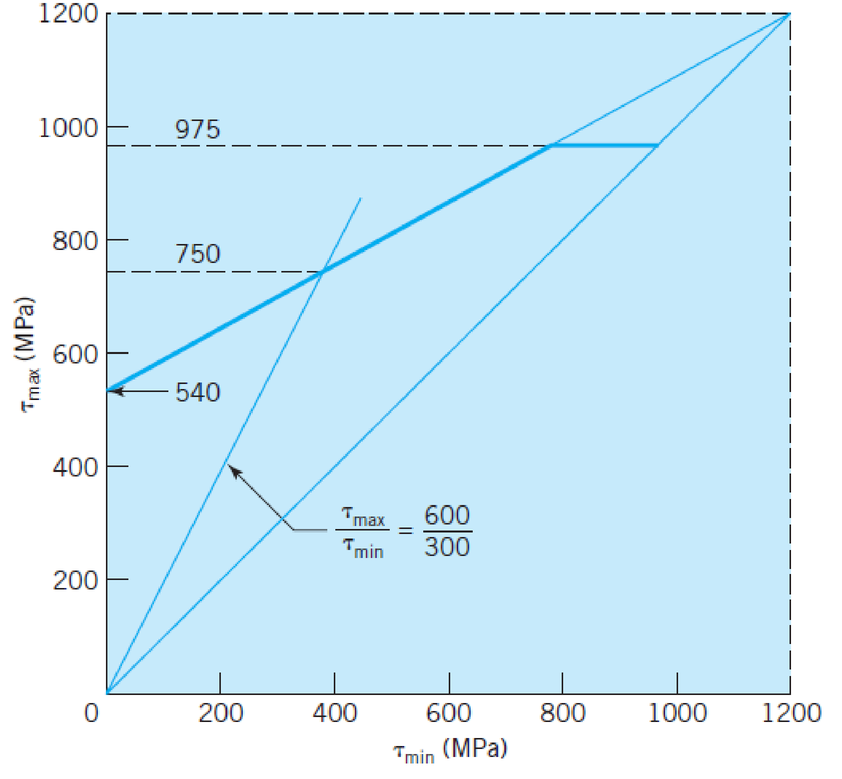
FIGURE 12.15

$S_{s,\max}$–N curves for round steel spring wire. Calculated versus recommended maximum design values from [1].

4. For $\tau_{max}/\tau_{min} = 600/300$, Figure 12.19 shows the limiting value of τ_{max} to be 750 MPa. Because Figure 12.15 represents *maximum* recommended values, it might be prudent to reduce this slightly. An additional "safety factor" of 1.13 would give a final design value of $\tau_{max} = 661$ MPa, exactly as in Sample Problem 12.2.

5. Using this design stress makes the balance of the solution identical to that given for Sample Problem 12.2.

Comment: It is often desirable to use more than one approach in solving engineering problems (as in going through both Sample Problems 12.2 and 12.3), and the reader should be aware that the results will not always agree as well as they did in



MECH 344/M
Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 9

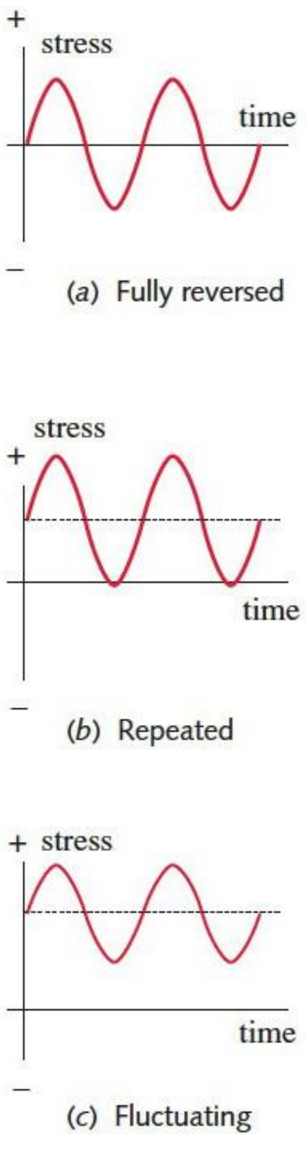
Contents of today's lecture

17

Shafts and Associated Parts

17.1 Introduction

- Shaft usually refers to long round member that rotates and transmits power. Associated parts such as gears, pulleys, and cams are attached to the shaft using pins, keys, splines, snap rings, etc.
- Shafts can be non round, or can be stationary. Can be subjected to various combinations of axial, bending, and torsional loads - both static or fluctuating.
- Typically, a rotating shaft transmitting power is subjected to a constant torque (producing a mean torsional stress) together with a completely reversed bending load (producing an alternating bending stress).
  - torsion due to the transmitted torque (R,F)
  - bending form transverse load gears (FR, F)
- Loading could be fully reversed, repeated or fluctuating
- Most of concern is the fatigue load that usually produces the failure of the shaft
- The stress concentration geometries bear high responsibility for failure by fatigue



IGURE 10-1

me-Varying Stresses

Stresses

- Two types of stress: mean and alternating for two types of load:

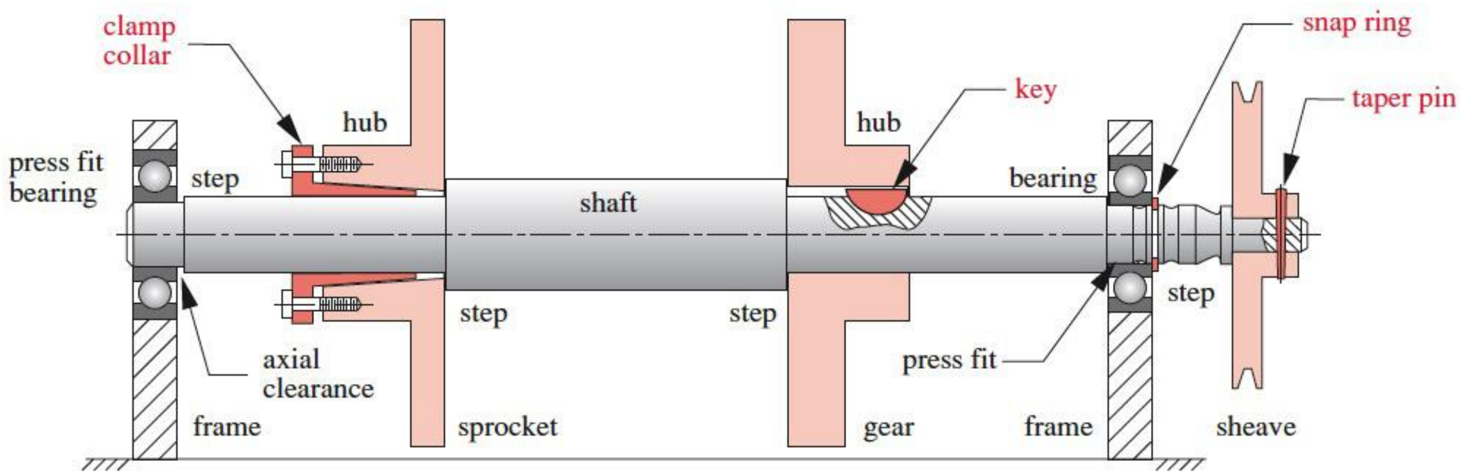
$$\sigma_a = k_f \frac{M_a c}{I} \qquad \tau_m = k_{fsm} \frac{T_m r}{J}$$

$$c = r = \frac{d}{2} \qquad J = \frac{\pi d^4}{32}$$

$$\sigma_a - k_f \frac{32M_a}{\pi d^3} \qquad \tau_m = k_{fsm} \frac{16T_m}{\pi d^3}$$

K_f is $K_{f(b)}$ stress concentration in bending and
K_{fsm} is $K_{f(t)}$ stress concentration in torsion

17.1 Introduction



- $K_{f(b)}$ and $K_{f(t)}$ stress concentration factors due to
  - Sudden diameter change, keyways, slots, holes, etc.
- Shafts must be designed to limit deflections – else hamper gear performance and cause objectionable noise.
- The angular deflection can affect non–self-aligning bearings.
- Torsional deflection can affect the accuracy of a cam- or gear-driven mechanism.
- The greater the flexibility—either lateral or torsional—the lower the corresponding critical speed.

17.2 Provision for Shaft Bearings

- Rotating shafts carrying gears, pulleys or cams must be supported by bearings.
- If 3 or more bearings are used, precise alignment must be maintained
- Shaft axial positioning and provision for carrying thrust loads normally require that one and only one bearing take thrust in each direction.
- If thrust load is shared among 2 or more plain thrust bearings, there must be sufficient axial clearance to ensure against "binding" under operating conditions.
- Production tolerances may be such that only one bearing will carry the thrust until after initial "wearing-in."

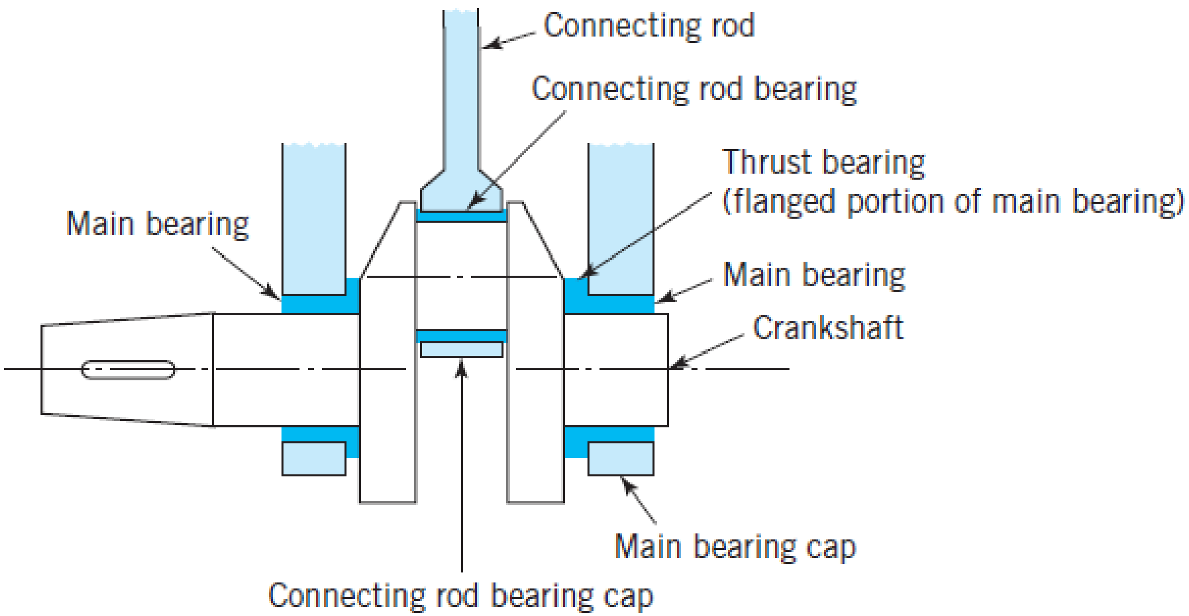
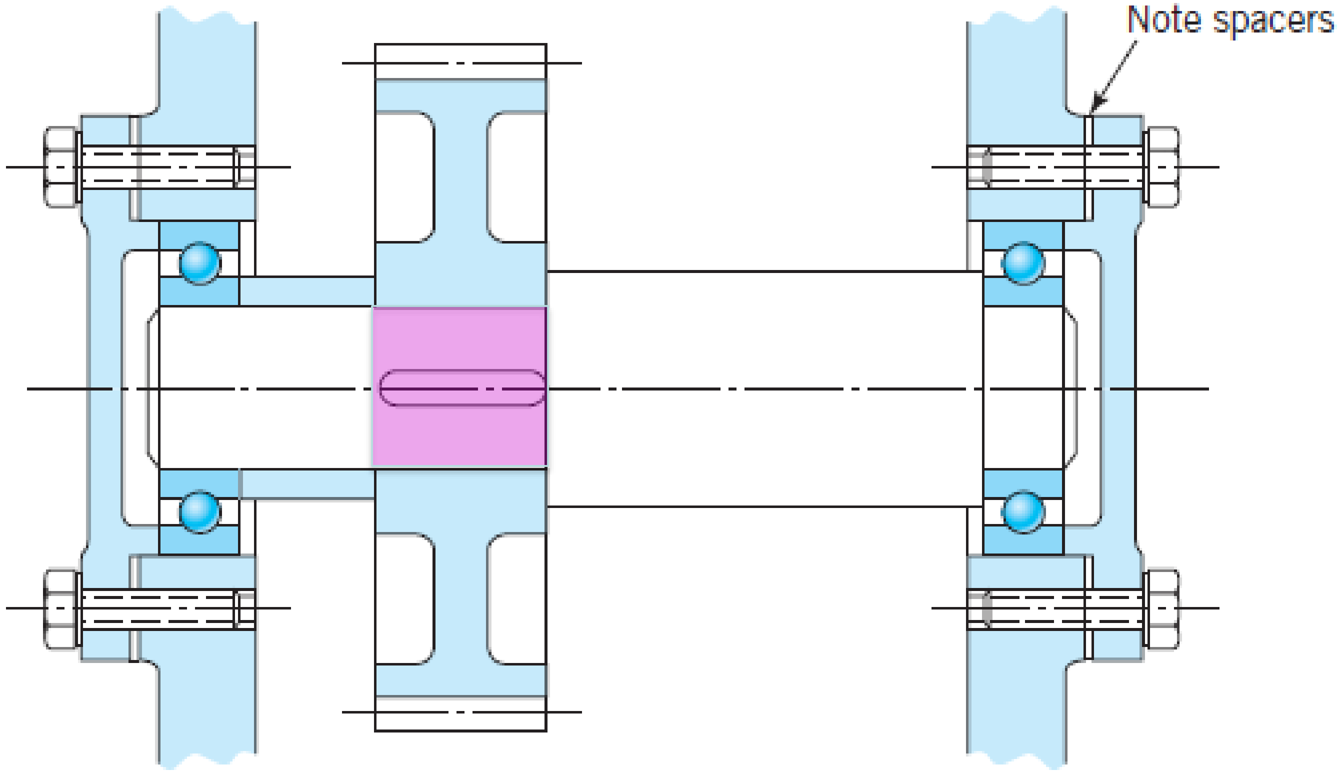


FIGURE 13.1
Crankshaft journal and thrust bearings. The crankshaft is supported by two *main bearings* and attaches to the connecting rod by the *connecting rod bearing*. All three are *journal* (or *sleeve*) *bearings*. Integral flanges on the main bearing inserts (commonly called merely *bearings*) serve as thrust bearings, which restrain axial motion of the shaft.

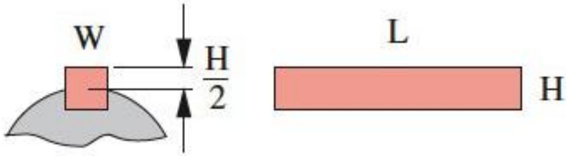
17.3 Mounting Parts onto Rotating Shafts

- Gears and cams are made integral with the shaft or are made separately and then mounted
- Hub is the portion of the mounted member in contact with shaft.
- Hub is mounted in different ways

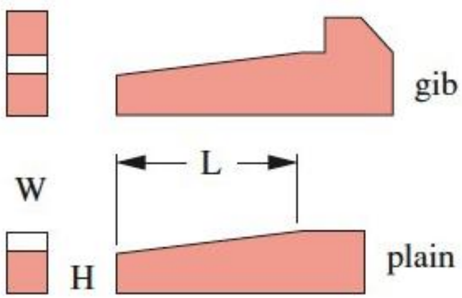


17.3 Mounting Parts onto Rotating Shafts

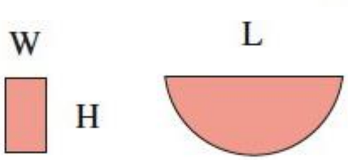
- One method is through key for torque transmission.
- The grooves in the shaft are called keyways in hub keyseats.



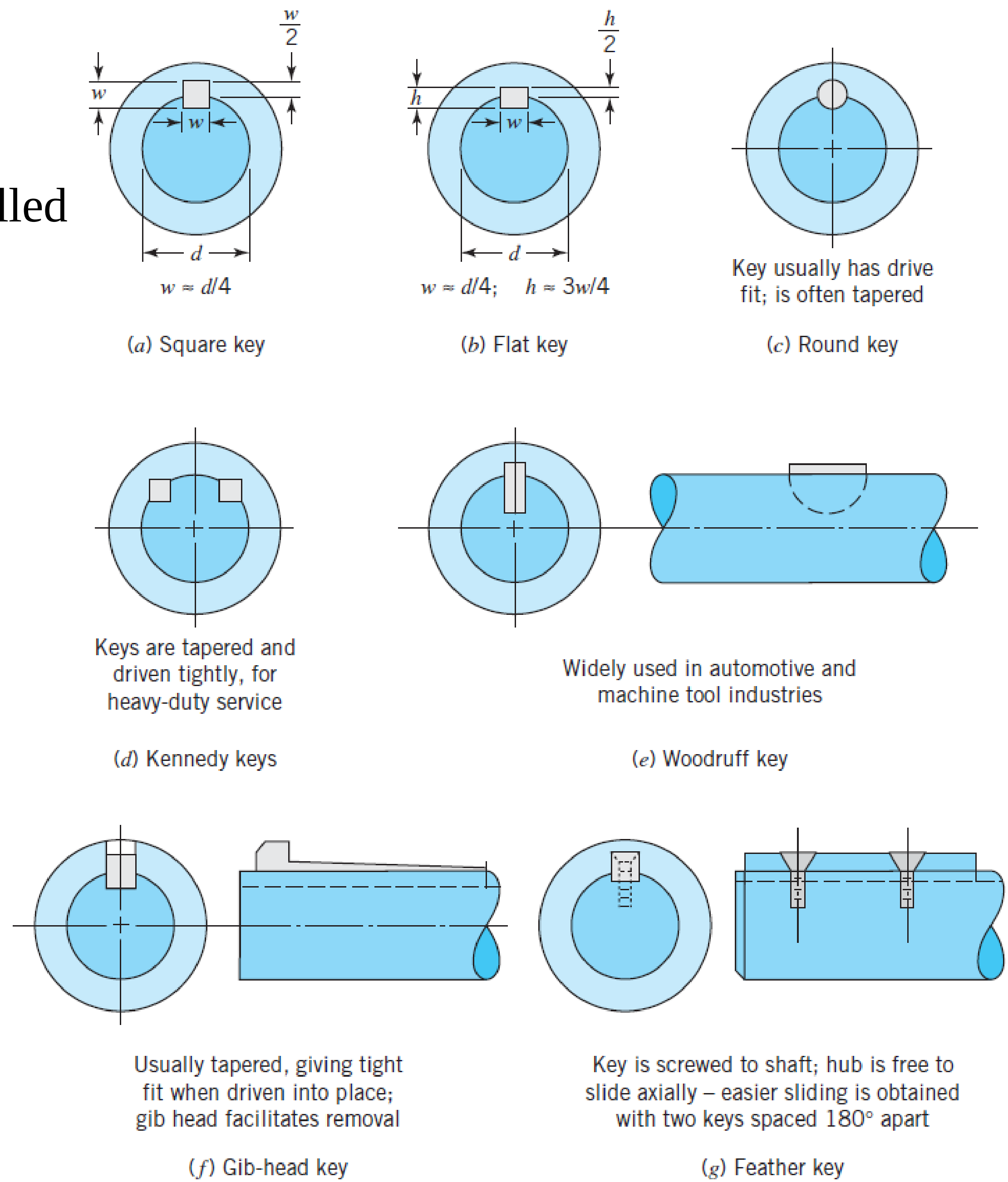
(a) Parallel key



(b) Tapered keys

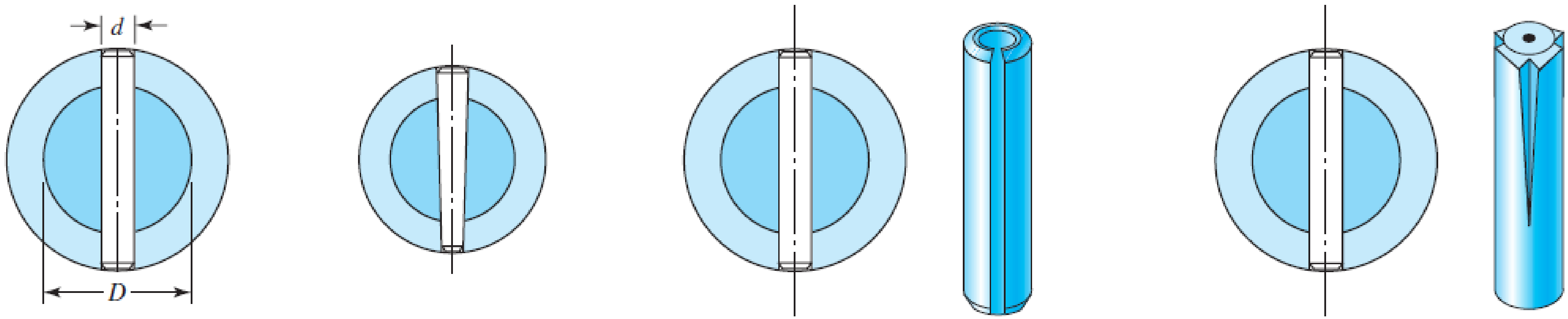


(c) Woodruff key



17.3 Mounting Parts onto Rotating Shafts

- A simpler attachment for transmitting relatively light loads is provided by pins, which provide a relatively inexpensive means of transmitting both axial and circumferential loads.



(*a*) Straight round pin (*b*) Tapered round pin (*c*) Split tubular spring pin

Grooves are produced by rolling, and provide spring action to retain pin

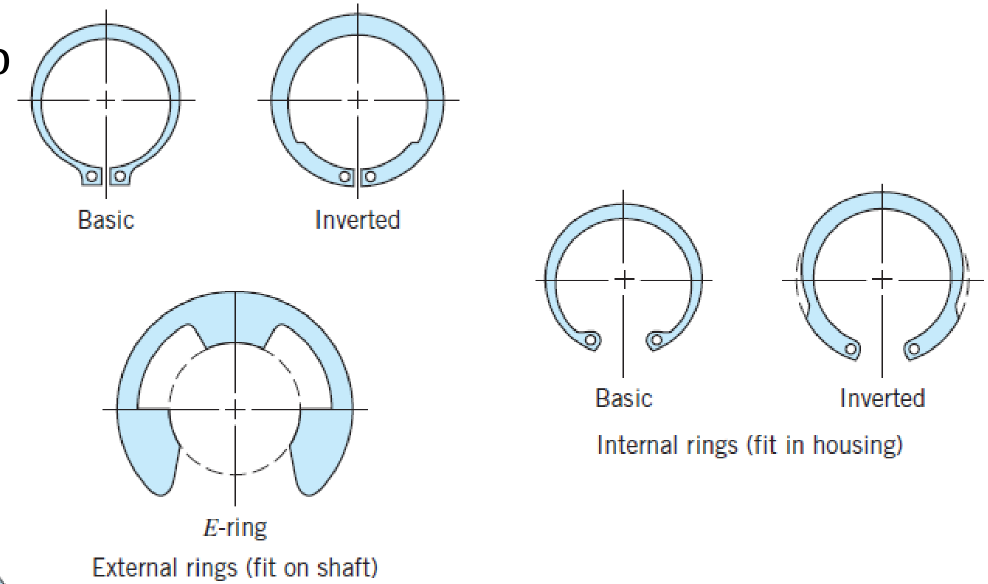
(*d*) Grooved pin

17.3 Mounting Parts onto Rotating Shafts

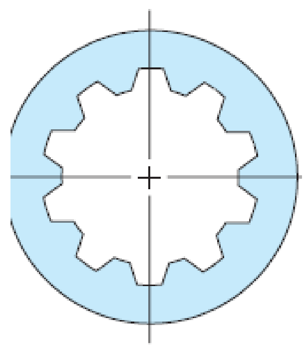
- Radially tapped holes in the hub help setscrews to prevent relative motion.
- The screw diameter is typically about one-fourth the shaft diameter. Two screws are commonly used, spaced 90° apart.
- Setscrews are inexpensive and adequate for relatively light service.
- Although special designs that provide increased protection against loosening in service are available, setscrews should not be counted on in applications for which loosening would impose a safety hazard.
- Setscrews may be used together with keys. Typically, one screw bearing on the key and another bearing directly on the shaft are used to prevent axial motion.
- An excellent and inexpensive method of axially positioning and retaining hubs and bearings onto shafts is by retaining rings, commonly called snap rings.
- Shaft snap rings require grooves that weaken the shaft, but this is no disadvantage if they are located where stresses are low
- "push-on" retaining rings do not require grooves - low-cost, compact means of assembling parts; but they do not provide the positive, precision positioning.

17.3 Mounting Parts onto Rotating Shafts

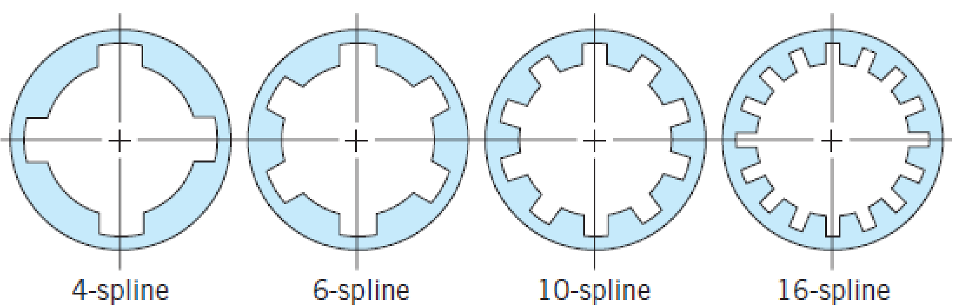
- Mating splines cut in the shaft and hub provide the strongest connection for transmitting torque



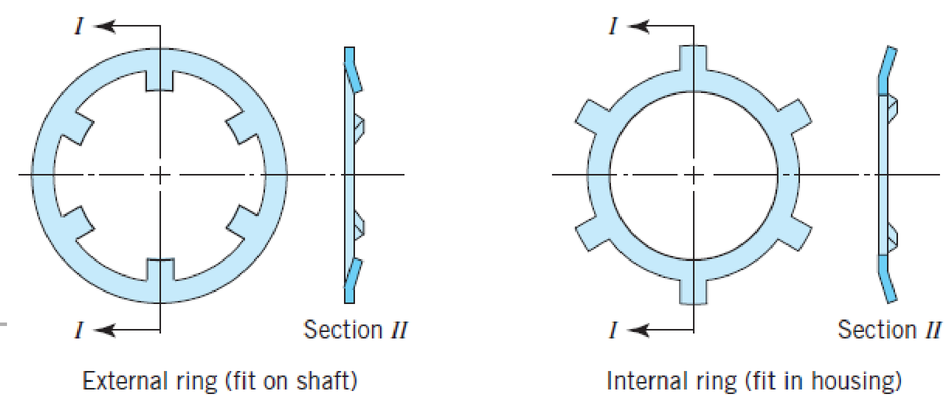
(a) Conventional type, fitting in grooves



(b) Involute



(a) Straight-sided



(b) Push-on type – no grooves required

Teeth deflect when installed to "bite in" and resist removal (less positive than conventional type)

17.3 Mounting Parts onto Rotating Shafts

- Perhaps the simplest of all hub-to-shaft attachments is interference fit
- Done with force or thermal expansion

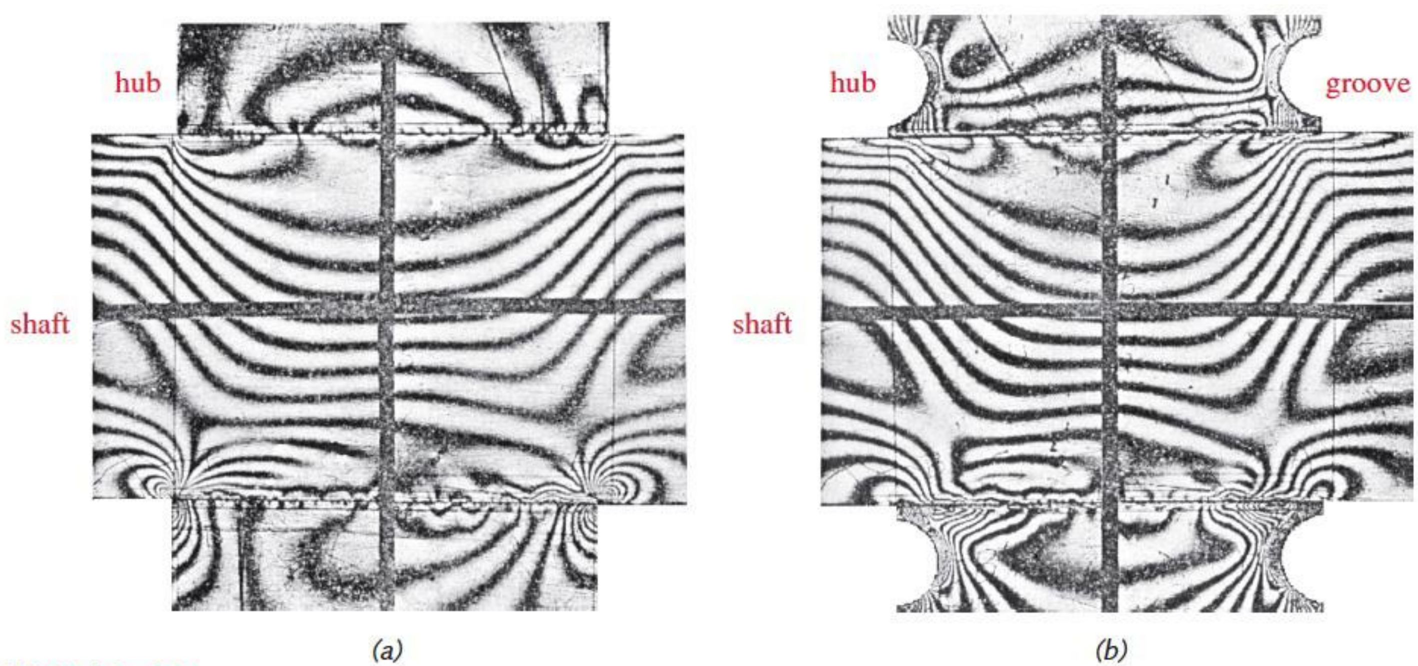


FIGURE 10-19

Photoelastic Stress Analysis of (*a*) A Plain Press-fit Assembly and (*b*) A Grooved-Hub Press-fit Assembly Source: R. E. Peterson and A. M. Wahl, "Fatigue of Shafts at Fitted Members, with a Related Photoelastic Analysis," *ASME J. App. Mech.*, vol. 57, p. A1, 1935.

17.4 Rotating-Shaft Dynamics

- Rotating shafts, - running at high speeds, must be designed to avoid operation at critical speeds. Sufficient lateral rigidity so that lowest $\omega_c >>$ the operating range.
- When torsional fluctuations are encountered (camshafts, crankshafts) - torsional nat freq of the shaft must be well removed from the input torsional freq - provide sufficient torsional stiffness to make the lowest torsional natural frequency significantly above that of the highest torsional disturbing frequency.
- Practicalities of manufacturing and operation - center of mass of a rotating system $\neq$ center of rotation. So as speed is increases, centrifugal force tends to bow the shaft. The more the bow the more unequal the centers become.
- Below the lowest (or fundamental) critical speed of rotation, the centrifugal and shaft elastic forces balance at a finite shaft deflection.
- At ω_c, equilibrium requires an infinite displacement of the mass center. While damping, from the shaft bearings, reduces the displacement, this might still break the shaft
- Rotation sufficiently above the critical speed results in a satisfactory equilibrium position by moving the mass center toward the center of rotation.

Shaft Whirl

- In some high-speed turbines, satisfactory operation is provided by quickly going through the ω_c, without allowing sufficient time for an equilibrium deflection to be reached, and then running well above the critical speed.
- ω_c is numerically the same as the lateral natural frequency of vibration, which is induced when rotation is stopped and the shaft center is displaced laterally, then suddenly released. For all except the simple “ideal” case of a massless shaft supporting a single concentrated mass, additional critical speeds at higher frequencies are also present.
- Equations for the lowest or fundamental critical speed are in equations 17.1 through 17.3.

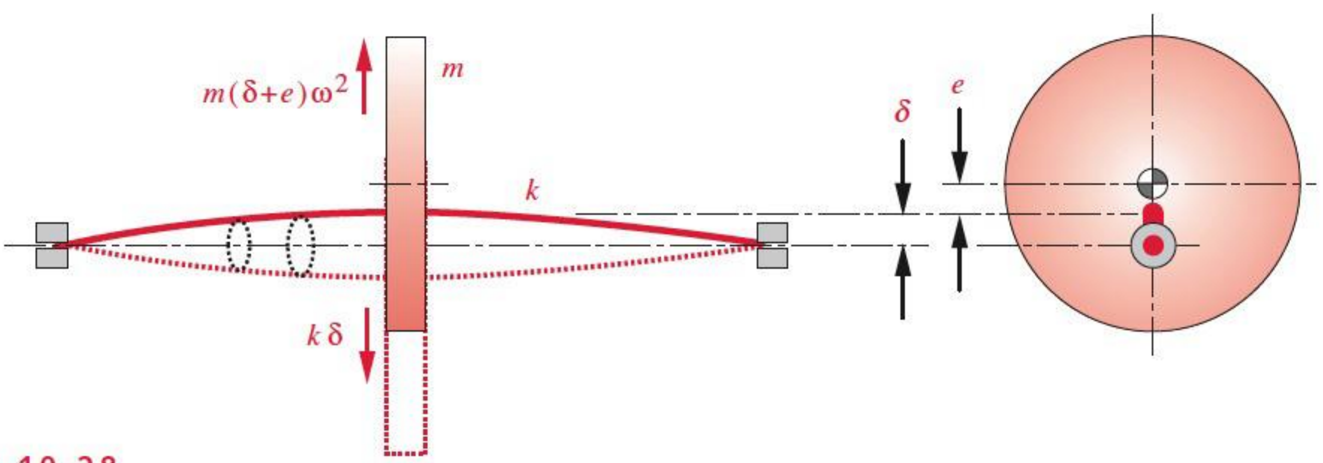
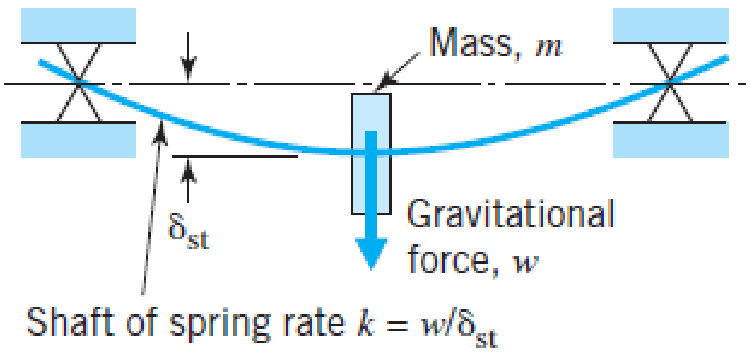


FIGURE 10-28

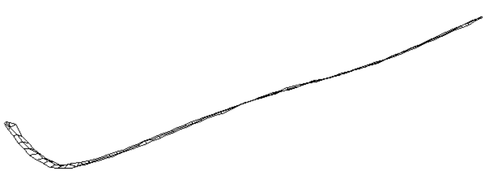
Shaft Whirl (amplitude greatly exaggerated)

Configuration	Critical Speed Equation
(a) Single mass	$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{w}} = \sqrt{\frac{g}{\delta_{st}}} \qquad \textbf{(17.1)}$$ $$n_c = \frac{30}{\pi}\sqrt{\frac{k}{m}} = \frac{30}{\pi}\sqrt{\frac{kg}{w}} = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}} \qquad \textbf{(17.1a)}$$

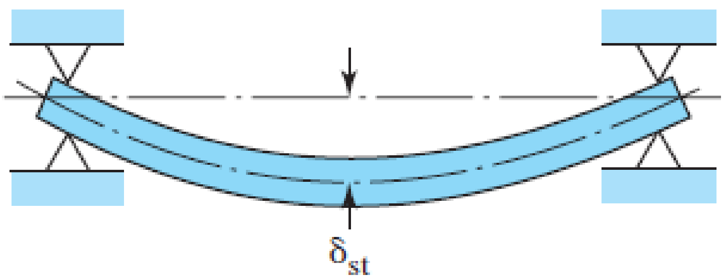


- Equation 17.1 calculates ω_n, and n_c, for a single mass with deflection δ_{st} using a spring model.

Quantity	Symbol	SI	British Gravitational
Mass	m	kg	lb · s^2/in. (slug)
Gravitational force	w	N	lb
Static deflection	δ_{st}	m	in.
Shaft spring rate	k	N/m	lb/in.
Acceleration of gravity	g	m/s^2	in./s^2
Natural frequency	ω_n	rad/s	rad/s
Critical speed	n_c	rpm	rpm



(c) Shaft mass only



$$\omega_n \approx \sqrt{\frac{5g}{4\delta_{st}}} \tag{17.3}$$

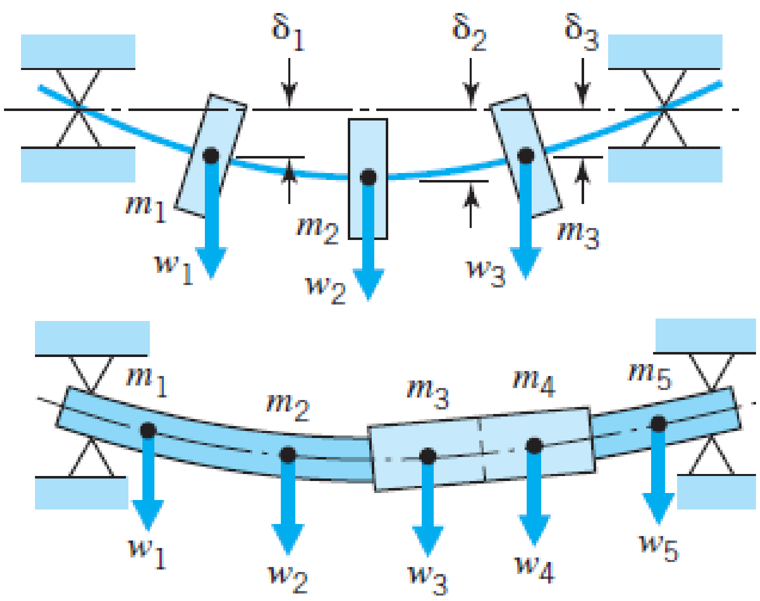
$$\delta_{st} = \frac{5wL^4}{384EI}$$

- Equation 17.3 is for calculation of ω_n for the shaft mass only.
- Eq. 17.3 shows that for the shaft mass only distributed that $n_c \cong (5/4)^{1/2}$ whereas if this distributed shaft mass is discretized into one mass acting at the center of the shaft, $n_c \cong (1)^{1/2}$ as in 17.1.

$$\sqrt{\frac{g}{\delta_{st}}}$$

Quantity	Symbol	SI	British Gravitational
Mass	m	kg	lb · s²/in. (slug)
Gravitational force	w	N	lb
Static deflection	δ_{st}	m	in.
Shaft spring rate	k	N/m	lb/in.
Acceleration of gravity	g	m/s²	in./s²
Natural frequency	ω_n	rad/s	rad/s
Critical speed	n_c	rpm	rpm

(b) Multiple masses



$$\left.\begin{aligned} n_c &\approx \frac{30}{\pi}\sqrt{\frac{g(w_1\delta_1 + w_2\delta_2 + \cdots)}{w_1\delta_1^2 + w_2\delta_2^2 + \cdots}} \\ n_c &\approx \frac{30}{\pi}\sqrt{\frac{g\sum w\delta}{\sum w\delta^2}} \end{aligned}\right\} \quad \textbf{(17.2)}$$

- Equation 17.2 determines n_c with multiple masses where the deflection for each mass is known.

Quantity	Symbol	SI	British Gravitational
Mass	m	kg	lb · s²/in. (slug)
Gravitational force	w	N	lb
Static deflection	δ_{st}	m	in.
Shaft spring rate	k	N/m	lb/in.
Acceleration of gravity	g	m/s^2	$in./s^2$
Natural frequency	ω_n	rad/s	rad/s
Critical speed	n_c	rpm	rpm

17.4 Rotating-Shaft Dynamics

- With multiple masses, in Eq. 17.2, the deflection at each mass can be calculated by superposition.
- Deflection δ_1 at m_1 is sum of the deflections at m1 caused by each mass acting alone; e.g., $\delta_1 = \delta_{11} + \delta_{12} + \ldots + \delta_{1i}$
- Dunkerley's equation (which under estimates the n_c) is used to calculate the n_c of the system, using n_i, of the shaft with only mass m_i on the shaft
- The critical speed can be estimated
- Using 17.4 a or b

$$\frac{1}{n_c^2} = \sum \frac{1}{n_i^2} = \frac{1}{n_1^2} + \frac{1}{n_2^2} + \frac{1}{n_3^2} \cdots \qquad \textbf{(17.4a)}$$

$$\frac{1}{\omega_c^2} = \sum \frac{1}{\omega_i^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} \cdots \qquad \textbf{(17.4b)}$$

- When the critical speed of the shaft is unacceptable, the shaft diameter can be modified using

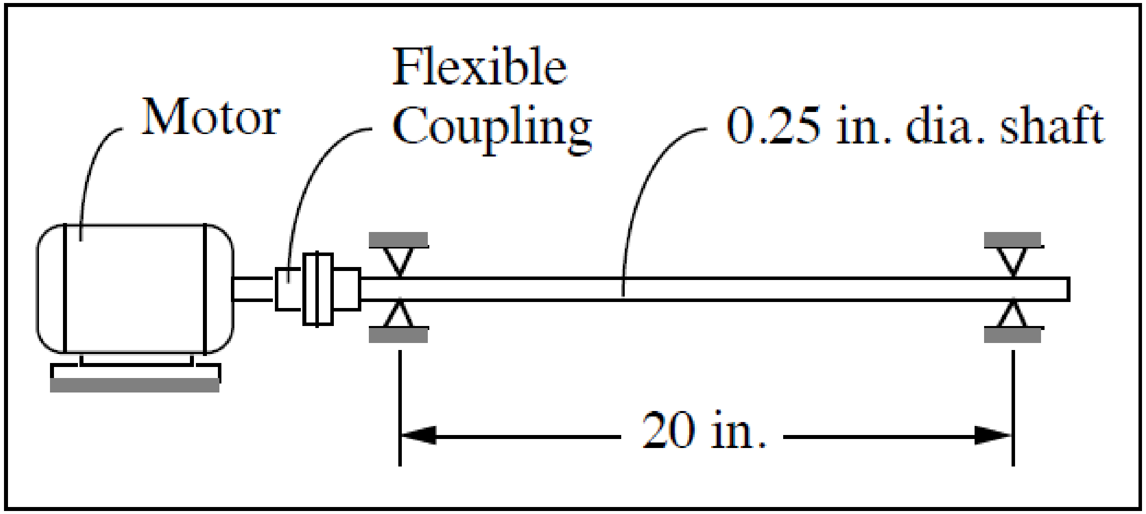
$$d_{\text{new}} = d_{\text{old}} \sqrt{\frac{(n_c)_{\text{new}}}{(n_c)_{\text{old}}}} \qquad \textbf{(17.5)}$$

17.4 Rotating-Shaft Dynamics

Known: A simply supported steel shaft is connected to an electric motor with a flexible coupling.

Find: Determine the value of the critical speed of rotation for the shaft.

Schematic and Given Data:



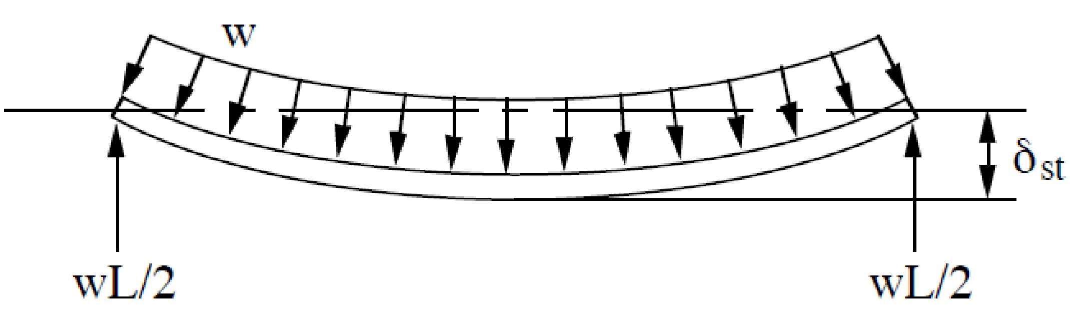
Assumptions:

1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.

17.4 Rotating-Shaft Dynamics

Analysis:

1. For the simply supported uniform load case:



$$w = A\rho = \frac{\pi d^2}{4}\rho \quad \text{where } \rho = 0.28\ \frac{\text{lb}}{\text{in.}^3} \text{ for steel}$$

$$w = \frac{\pi(0.25)^2}{4}(0.28) = 0.0137\ \frac{\text{lb}}{\text{in.}}$$

2. From Appendix D-2,

$$\delta_{st} = \frac{5wL^4}{384EI} \text{ for a uniform load distribution}$$

where $E = 30 \times 10^6$ psi (Appendix C-1)

17.4 Rotating-Shaft Dynamics

$$I = \frac{\pi d^4}{64} = \frac{\pi(0.25)^4}{64} = 1.92 \times 10^{-4} \text{ in.}^4 \text{ (Appendix B-1)}$$

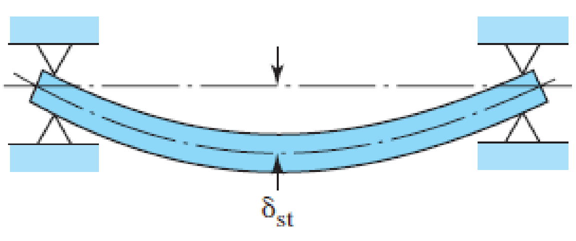
$$\delta_{st} = \frac{5wL^4}{384EI} \qquad \delta_{st} = \frac{5(0.0137)(20)^4}{384(30 \times 10^6)(1.92 \times 10^{-4})} = 4.98 \times 10^{-3} \text{ in.}$$

3. Using Fig. 17.5(c), to find the shaft critical speed

$$\omega_n \approx \sqrt{\frac{5g}{4\delta_{st}}} = \sqrt{\frac{5(32.2 \frac{\text{ft}}{\text{s}^2})(12 \frac{\text{in.}}{\text{ft}})}{4(4.98 \times 10^{-3} \text{ in.})}}$$

$$\omega_n \approx 311 \text{ Rad/s}$$

(c) Shaft mass only



$$\omega_n \approx \sqrt{\frac{5g}{4\delta_{st}}} \tag{17.3}$$

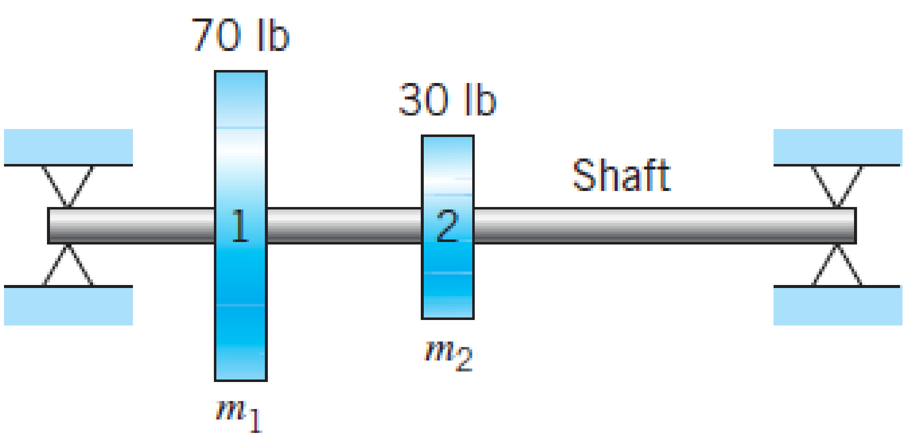
$$\delta_{st} = \frac{5wL^4}{384EI}$$

Sample Problem 17.1 Critical Speed of Shafts

Figure 17.6 shows two masses fixed on a solid shaft which is supported on both ends with bearings. The masses, m_1 and m_2, weigh 70 lb and 30 lb respectively. The shaft deflections have been calculated and the influence coefficients found to be

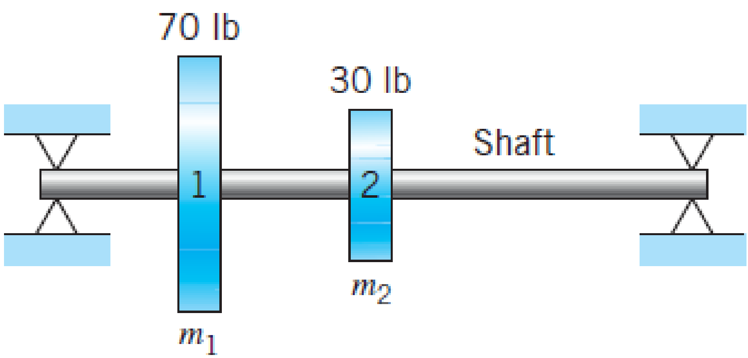
$$a_{11} = 3.4 \times 10^{-6} \text{ in./lb}$$
$$a_{22} = 20.4 \times 10^{-6} \text{ in./lb}$$
$$a_{21} = a_{12} = 6.8 \times 10^{-6} \text{ in./lb}$$

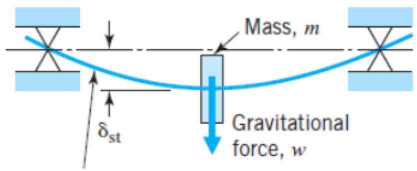
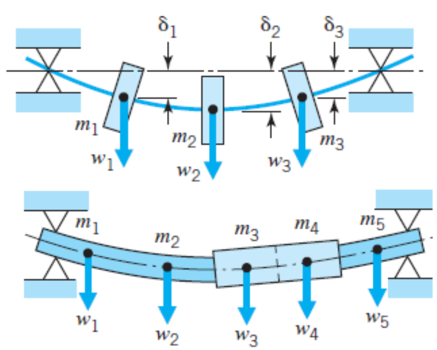
Note that a_{11} is the shaft deflection at position 1 which results from a 1 lb mass located at position 1. The coefficient a_{12} is the shaft deflection at position 1 resulting from a 1 lb mass located at position 2.



Known: A solid shaft has two masses attached along its length and is supported on both ends by bearings. The shaft masses are known along with the deflection influence coefficients for the mass positions. The shaft mass can be ignored in this analysis.

Find: Determine the critical shaft speed using the Dunkerley equation and the Rayleigh equation.



<table>
<tr><th>Configuration</th><th>Critical Speed Equation</th><th></th></tr>
<tr><td>(a) Single mass

</td><td>$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{w}} = \sqrt{\frac{g}{\delta_{st}}}$
$n_c = \frac{30}{\pi}\sqrt{\frac{k}{m}} = \frac{30}{\pi}\sqrt{\frac{kg}{w}} = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}}$</td><td>(17.1)
(17.1a)</td></tr>
<tr><td>(b) Multiple masses

</td><td>$n_c \approx \frac{30}{\pi}\sqrt{\frac{g(w_1\delta_1 + w_2\delta_2 + \cdots)}{w_1\delta_1^2 + w_2\delta_2^2 + \cdots}}$
$n_c \approx \frac{30}{\pi}\sqrt{\frac{g\sum w\delta}{\sum w\delta^2}}$</td><td>(17.2)</td></tr>
</table>

Assumptions:

1. The shaft mass is negligible and does not contribute to shaft deflection.
2. The shaft remains linearly elastic.
3. The shaft is simply supported at each end.

Analysis:

1. The shaft deflections at each position due to each mass individually are:

$$\delta_{11} = w_1 a_{11} = (70\text{ lb})(3.4 \times 10^{-6}\text{ in./lb}) = 2.38 \times 10^{-4}\text{ in.}$$
$$\delta_{22} = w_2 a_{22} = (30\text{ lb})(20.4 \times 10^{-6}\text{ in./lb}) = 6.12 \times 10^{-4}\text{ in.}$$

2. The total shaft deflection for each mass position is

$$\delta_1 = w_1 a_{11} + w_2 a_{12} = 70(3.4 \times 10^{-6}) + 30(6.8 \times 10^{-6}) = 4.42 \times 10^{-4}\text{ in.}$$
$$\delta_2 = w_2 a_{22} + w_1 a_{12} = 30(20.4 \times 10^{-6}) + 70(6.8 \times 10^{-6}) = 1.088 \times 10^{-3}\text{ in.}$$

3. From Eq. 17.1a,

$$\omega_1 = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{g}{\delta_{11}}} = \sqrt{\frac{386\,\frac{\text{in.}}{\text{s}^2}}{2.38 \times 10^{-4}\text{ in.}}} = 1273.5\,\frac{\text{rad}}{\text{s}}$$

$$\omega_2 = \sqrt{\frac{g}{\delta_{22}}} = \sqrt{\frac{386\,\frac{\text{in.}}{\text{s}^2}}{6.12 \times 10^{-4}\text{ in.}}} = 794.2\,\frac{\text{rad}}{\text{s}}$$

4. Using the Dunkerley equation, Eq. 17.4,

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} = \frac{1}{(1273.5)^2} + \frac{1}{(794.2)^2}$$

yields

$$\omega_c = 673.9\,\frac{\text{rad}}{\text{s}}$$

5. Using the Rayleigh equation, Eq. 17.2,

$$\omega_c = \sqrt{\frac{g(w_1\delta_1 + w_2\delta_2)}{(w_1\delta_1^2 + w_2\delta_2^2)}}$$

$$= \sqrt{\frac{386\,\frac{\text{in.}}{\text{s}^2}\,[(70\text{ lb})(4.42 \times 10^{-4}\text{ in.}) + (30\text{ lb})(1.088 \times 10^{-3}\text{ in.})]}{(70\text{ lb})(4.42 \times 10^{-4}\text{ in.})^2 + (30\text{ lb})(1.088 \times 10^{-3}\text{ in.})^2}}$$

$$= \sqrt{\frac{386\,\frac{\text{in.}}{\text{s}^2}\,(6.36 \times 10^{-2}\text{ lb in.})}{4.92 \times 10^{-5}\text{ lb in.}}} = 706.3\,\frac{\text{rad}}{\text{s}}$$

Comment: Dunkerley's equation and Rayleigh's equation produce different answers for critical speeds. The Dunkerley equation underestimates and the Rayleigh equation overestimates critical speed. The true value is probably bounded by these two solutions.

- As shown in Sample Problem, shaft critical speeds can be estimated by calculating static deflections at several points.

- Available computer programs enable calculation of static deflections and shaft critical speeds.

Shaft Design

- Both stress and deflections need to be considered as both could be critical

- Usually, shafts are designed based on stress considerations and then the deflections are calculated when the geometry is completed

- Natural frequencies in both torsion and bending are also evaluated to avoid resonance after the geometry is established

- If the shaft operates close to the resonance frequency, fast failure is expected

17.5 Overall Shaft Design

1. Keep shafts as short as possible, with bearings close to the applied loads. This reduces deflections and bending moments and increases critical speeds.
2. Place necessary stress raisers away from highly stressed shaft regions if possible. If not possible, use generous radii and good surface finishes. Consider local surface-strengthening processes (as shot peening or cold rolling).
3. Use inexpensive steels for deflection-critical shafts, as all steels have essentially the same elastic modulus.
4. When weight is critical, consider hollow shafts. For example, propeller shafts on rear-wheel-drive cars are made of tubing in order to obtain the low weight–stiffness ratio needed to keep critical speeds above the operating range.

- The maximum allowable deflection of a shaft is usually determined by critical speed, gear, or bearing requirements.
- Critical speed requirements vary greatly with the specific application.
- Allowable shaft deflections for satisfactory gear and bearing performance vary with the gear or bearing design and with the application

17.5 Overall Shaft Design

1. Deflections should not cause mating gear teeth to separate more than about 0.13 mm (0.005 in.), nor should they cause the relative slope of the gear axes to change more than about 0.03°.
2. The shaft (journal) deflection across a plain bearing must be small compared to the thickness of the oil film. If the angular deflection of the shaft at the bearing is excessive, the shaft will bind unless the bearings are self-aligning.
3. The shaft angular deflection at a ball or roller bearing should generally not exceed 0.04° unless the bearing is self-aligning.

- Shaft deflections must be computed. Additionally, torsional deflections must be considered because of torsional natural frequency requirements and necessary limitations on torsional deflections.
- ASME std. for Design of Transmission Shafting gives non-conservative results, as it
  - assumes fully reversed bending moment load with zero mean moment load component and steady mean torque with zero alternating torque component.

17.5 Overall Shaft Design

- The more general approach is to use fatigue design of machine components.
    - Determination of the fatigue strength of a rotating shaft usually requires an analysis for the general case of biaxial loading, as summarized in Table 8.2, Figure 8.16,
- Early in the design of any given shaft, an estimate is usually made of whether strength or deflection will be the critical factor.
- A preliminary design is based on this criterion; then the remaining factor (deflection or strength) is checked.

SAMPLE PROBLEM 17.2D Snowmobile Track Drive Shaft

Figure 17.7*b* shows a drive shaft, supported in a snowmobile frame by bearings *A* and *B*, which is chain-driven by sprocket *C*. (The engine and transmission are above and forward of the shaft; hence, the 30° chain angle.) Track sprockets T_1 and T_2 drive the snowmobile track. Basic dimensions are given in Figure 17.7*b*. Determine an appropriate design for the shaft, based on a maximum engine output of 20 kW at a vehicle speed of 72 km/h. Because the track and chain do not impose stringent deflection requirements and the bearings can be self-aligning if necessary, the preliminary design should be based on fatigue strength.

SOLUTION

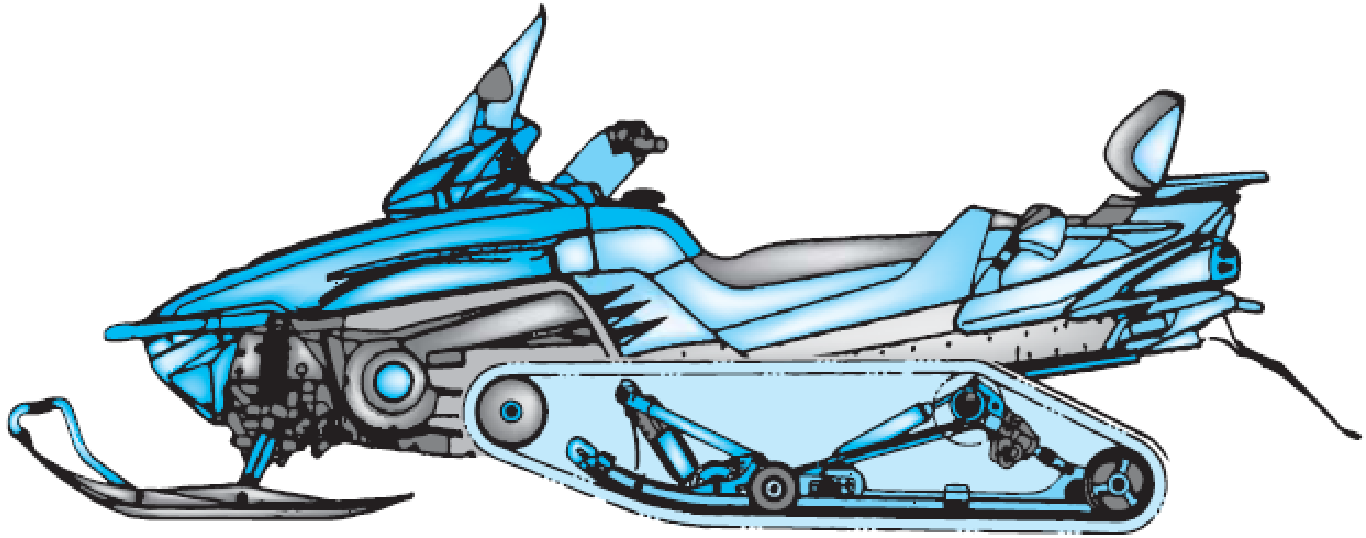
Known: A drive shaft is chain-driven by a sprocket and supported in a snow-mobile frame by two bearings. Basic dimensions of the shaft and locations of the bearings, drive sprocket, and sprockets for the snowmobile track are given. The power of the engine driving the chain and the speed of the vehicle are specified.

Find: Determine a design for the shaft based on fatigue strength considerations.

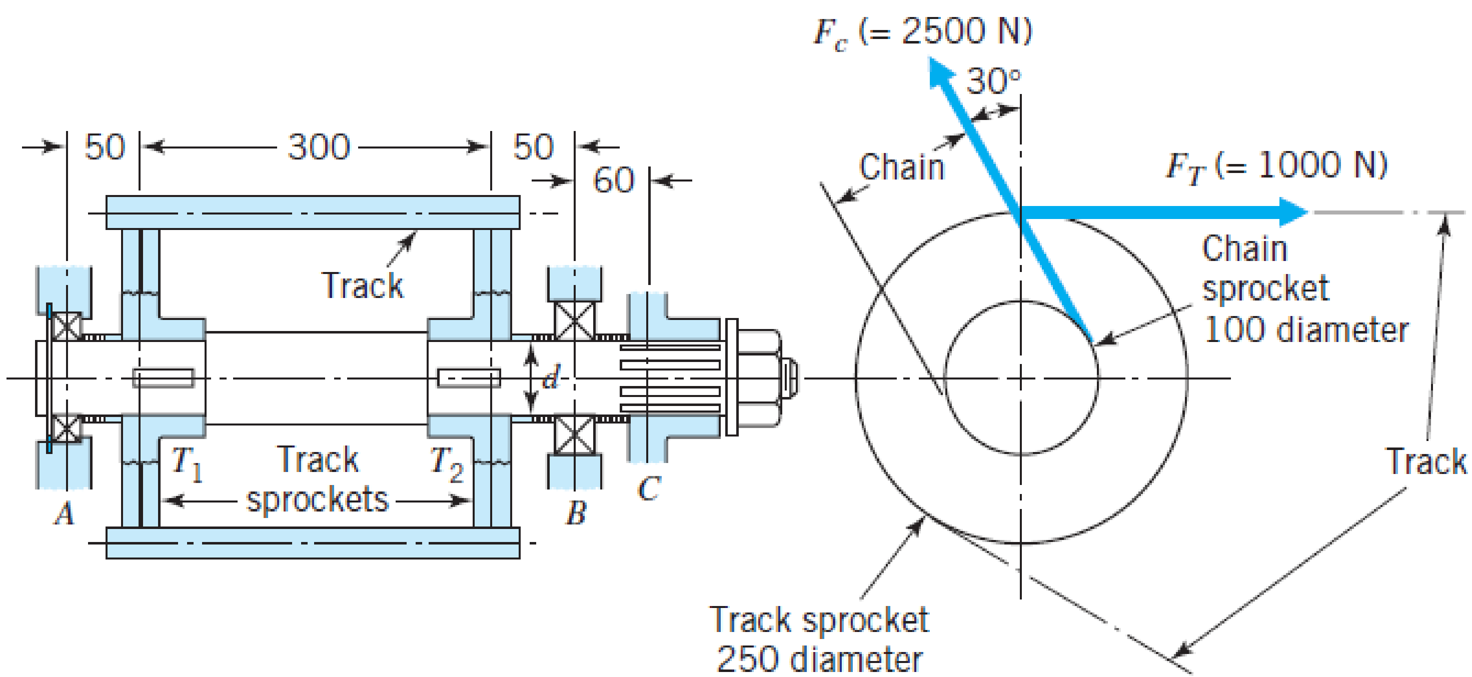
Schematic and Given Data: (See Figure 17.7)

Assumptions:

1. The full engine power reaches the snowmobile track.
2. Half of the track tension force is transmitted to each of the track sprockets; that is, there is an even division of torque.
3. Bearings *A* and *B* are self-aligning within the range of the angular shaft deflections induced.
4. The stress concentration at the bearing inner race (at *B*) is the same as that at the edges of the sprocket hub (at *S*).



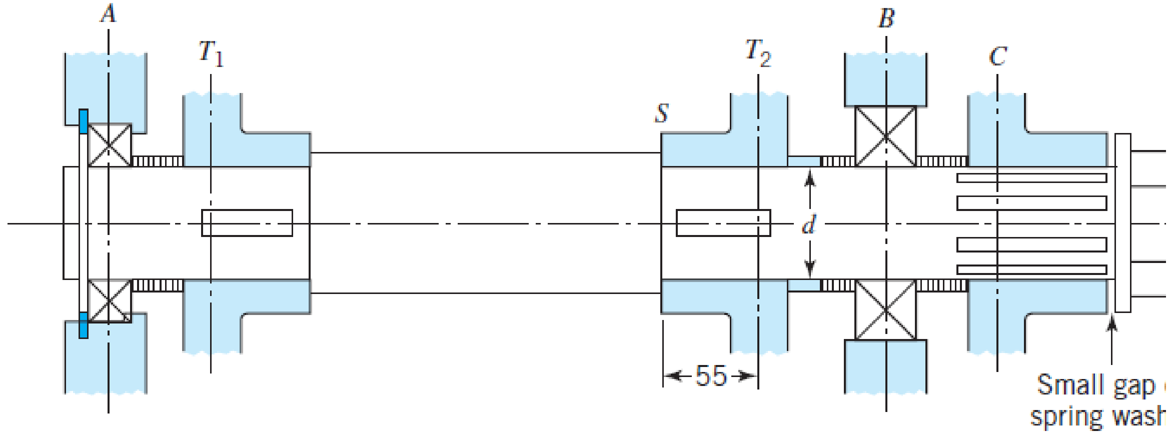
(*a*) Snowmobile



(*b*) General arrangement

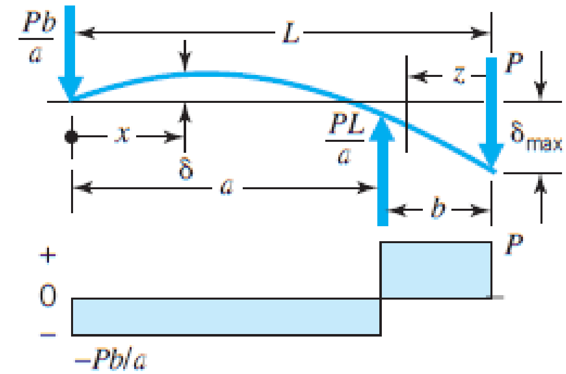
Decisions:

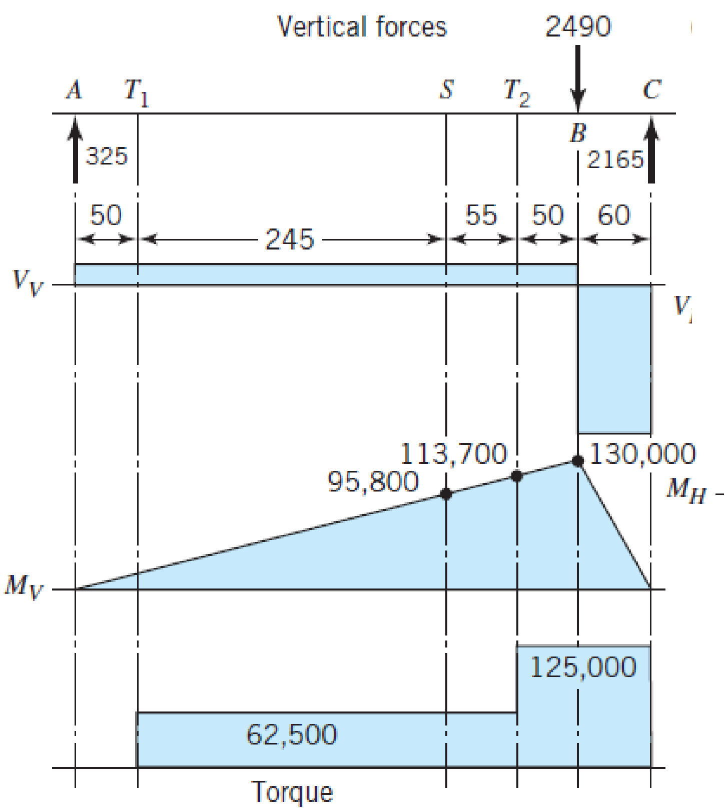
1. Figure 17.7*c* shows a proposed shaft layout. Note that the chain sprocket is mounted outboard to provide easy access to the chain for servicing, and that the retaining nut on the end tightens directly against the shaft. (If the nut were to tighten against the hub of sprocket *C*, the initial tightening load of the nut would impose a static tensile stress in the shaft between the nut and shoulder *S*. This would be undesirable from the standpoint of shaft fatigue strength.) Since bearing *B* will carry by far the greater load, provision is shown for bearing *A* to carry thrust in both directions. Note the snap rings in the housing and on the shaft to retain bearing *A*. Thrust loads will be small, because there are only cornering maneuvers, and this arrangement permits the use of a straight roller bearing at *B* if desired. Torque is shown transmitted from the chain sprocket by splines, and to the track sprockets by keys.
2. Because there is a large shaft bending moment in the vicinity of sprocket T_2, a tentative dimension locating the shaft shoulder is selected as shown in Figure 17.7*c*. (This dimension will be needed to compute loads at this point of stress concentration.)
3. On the basis of cost, tentatively select cold-drawn 1020 steel having $S_u = 530$ MPa, $S_y = 450$ MPa, and machined surfaces.
4. Select ratios of $D/d = 1.25$ and $r/d = 0.03$ at shoulder *S* to give conservatively high values of K_f.
5. A safety factor of 2.5 is chosen based on the information given in Section 6.12.
6. A standard-size bearing will be selected.



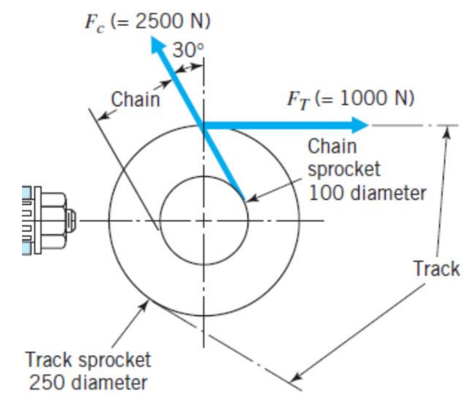
(c) Shaft layout

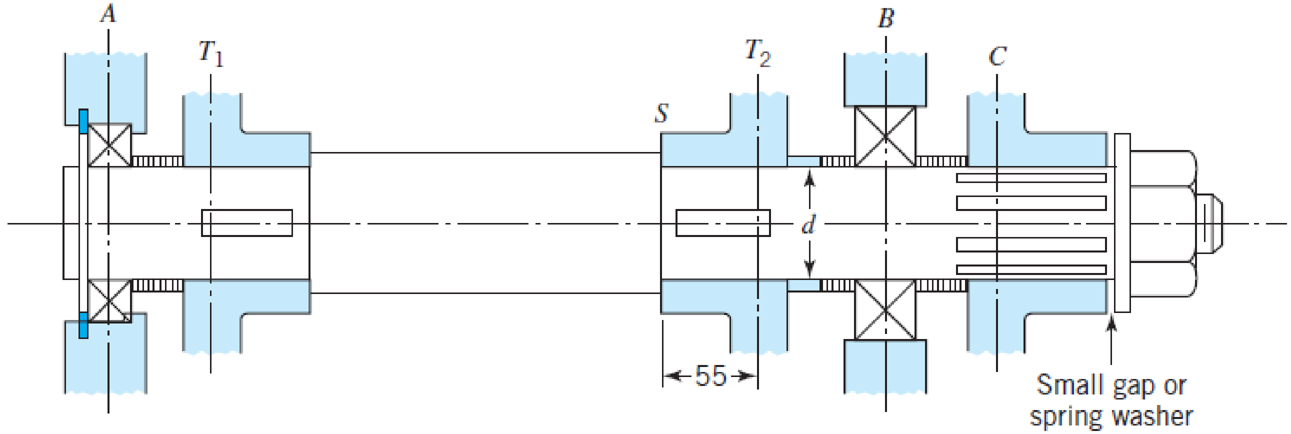
4. Overhung load





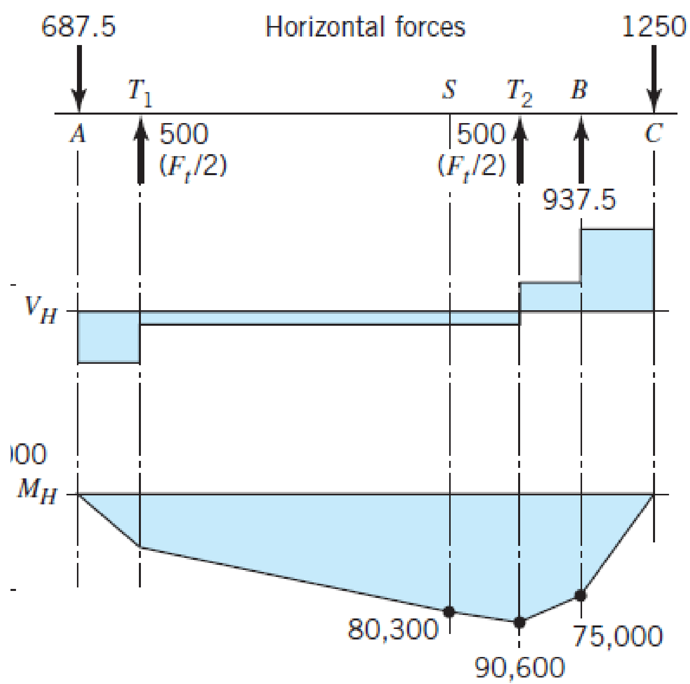
(d) Loading dia

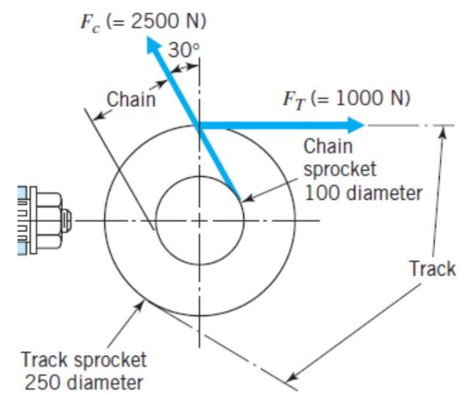




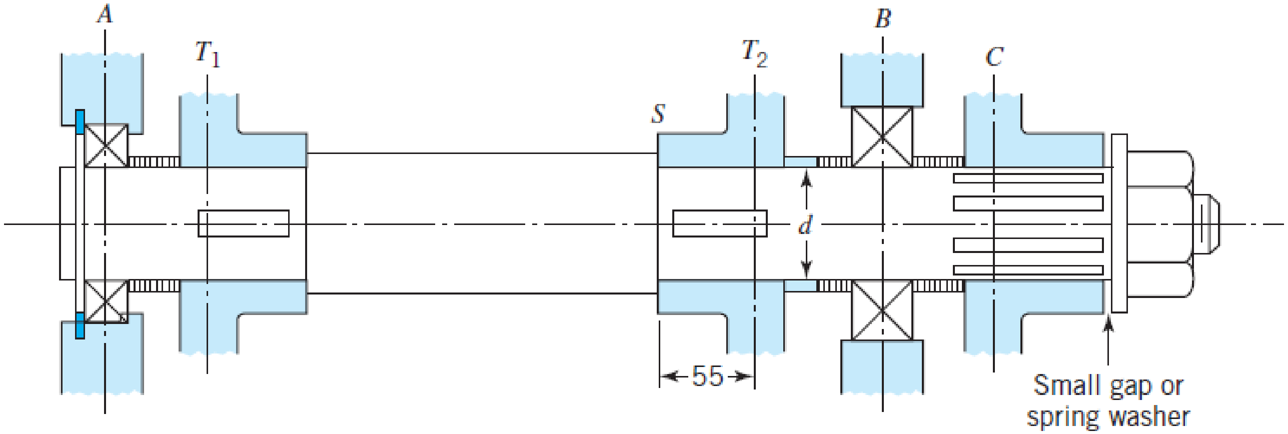
(c) Shaft layout

$R_A = \{1250\ (60) + 500\ (350) + 500\ (50)\}\ /400 = 687.5$



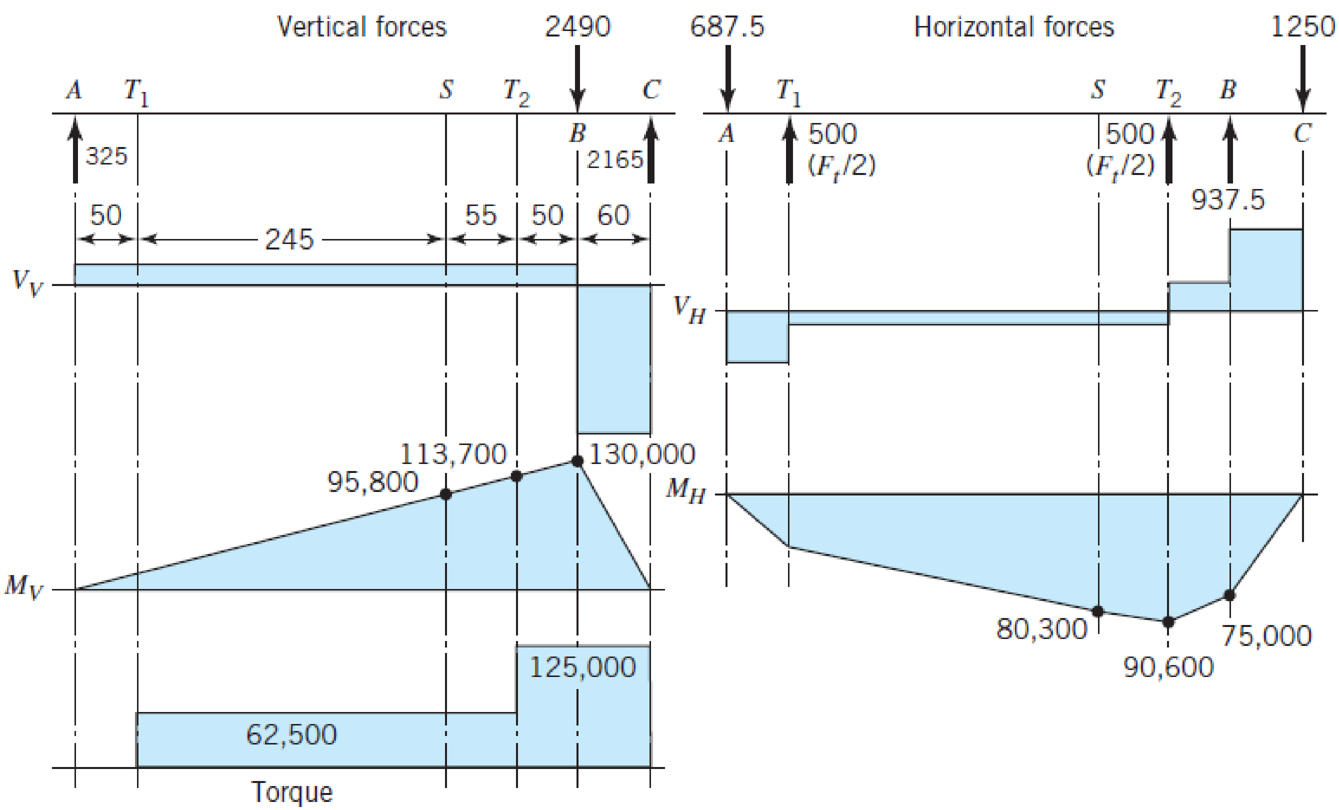


ng diagrams



(c) Shaft layout

Appendix d2 for overhung loads



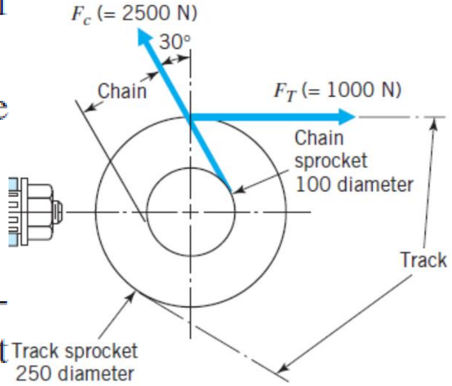
(d) Loading diagrams

Analysis:

1. Since as assumed the full engine power reaches the track, the track tension F_T can be computed as

$$F_T = \frac{\text{engine power}}{\text{vehicle velocity}} = \frac{20{,}000 \text{ w}}{20 \text{ m/s}} = 1000 \text{ N}$$

Half of this force as assumed is transmitted to each of the track sprockets, T_1 and T_2.



2. By setting the summation of moments about the shaft axis equal to zero, we determine the chain sprocket tension F_C to be

$$F_C = 1000 \text{ N} \,(125 \text{ mm}/50 \text{ mm}) = 2500 \text{ N}$$

3. Force, shear, and moment diagrams for the vertical and horizontal planes are determined in the usual manner (as in Figure 2.11) and drawn in Figure 17.7*d*. Note that the only vertical applied load is a component of the chain tension ($F_C \cos 30°$). Applied horizontal loads include the chain tension component ($F_C \sin 30°$) and also the two track sprocket forces (each of $F_T/2$). No moment loads exist at bearings *A* and *B*, for the two bearings were assumed to be self-aligning.

4. The torque diagram shows, as assumed, an even division of torque between the two track sprockets.

5. From an inspection of the loading diagrams and shaft layout, it is clear that the critical location for determining the value of *d* will either be at *S*, or near *B* or *C*. Failure could occur at *S* owing to the stress concentration. Failure at precisely *B* or *C* is unlikely because the shaft is reinforced by the bearing race and sprocket

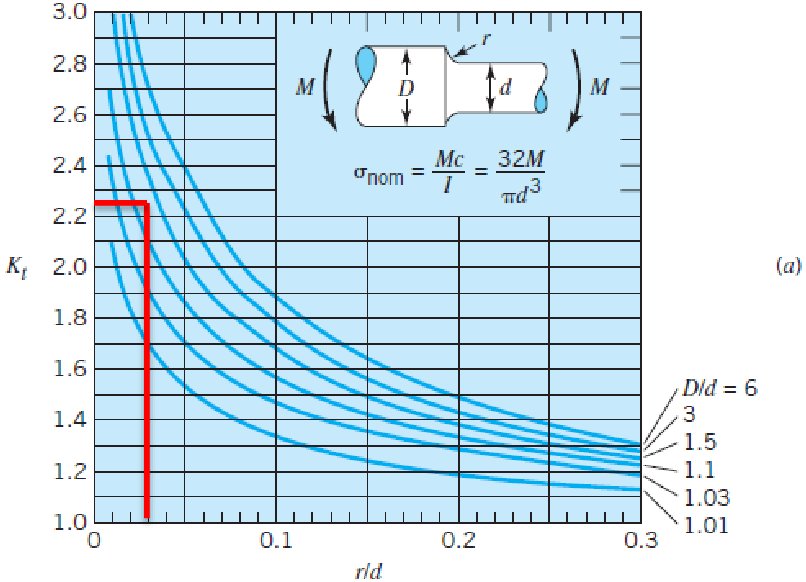
hub at these points; but at the edges of the sprocket hub and bearing inner race a stress concentration exists not unlike that at S, as assumed. Conservatively, therefore, shaft diameter d will be calculated on the basis of the loads at B (where the resultant bending moment is slightly higher than at T_2) and the estimated stress concentration factors at S.

6. For estimating K_f at shoulder S, information is needed on material, surface finish, and geometric proportions of the shoulder. The material selected was cold-drawn 1020 steel having $S_u = 530$ MPa, $S_y = 450$ MPa, and machined surfaces. The geometry of the shoulder is given by $D/d = 1.25$ and $r/d = 0.03$ as decided. From Figure 4.35, $K_t = 2.25$ and 1.8 for bending and torsional loads, respectively. From Figure 8.23 and an assumption that r will be about 1 mm, q is estimated at 0.7. Applying Eq. 8.2 gives values for K_f of 1.9 and 1.6 for bending and torsional loading.

7. Following the procedure specified for general biaxial loads in Figure 8.16, the equivalent alternating stress is due only to bending:

$$\sigma_{ea} = \sigma = \frac{32M}{\pi d^3} K_{f(b)} = \frac{32\sqrt{130{,}000^2 + 75{,}000^2}}{\pi d^3}(1.9) = \frac{2.9 \times 10^6}{d^3}$$

$$K_f = 1 + (K_t - 1)q$$



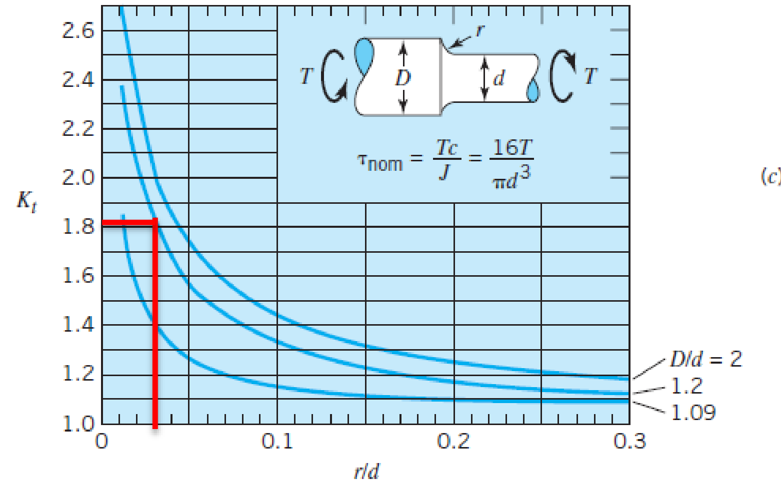
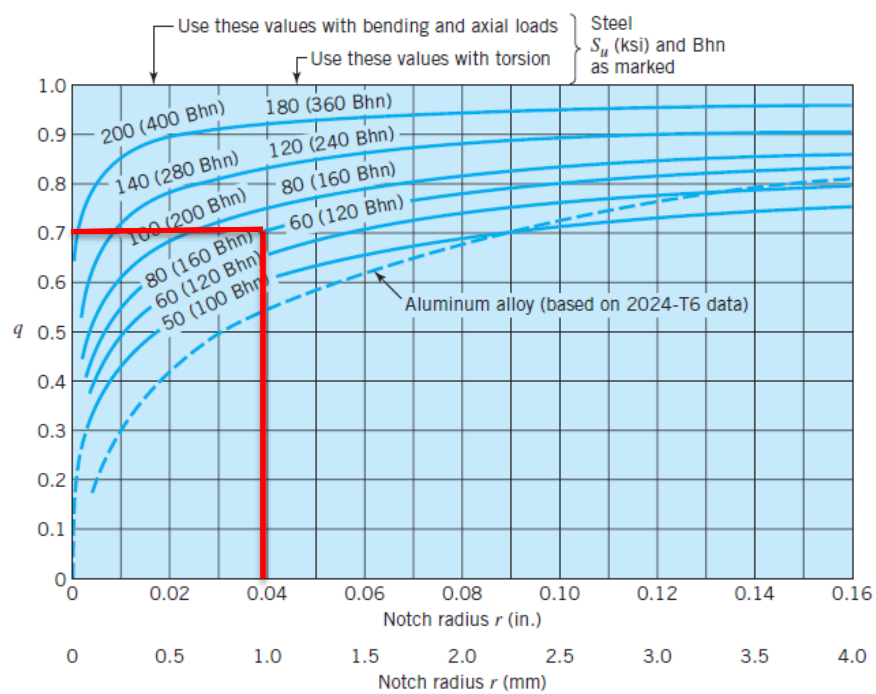


FIGURE 4.35
Shaft with fillet (*a*) bending; (*b*) axial load; (*c*) torsion [7].



The equivalent mean stress is due only to torsion:

$$\sigma_{em} = \tau = \frac{16T}{\pi d^3}K_{f(t)} = \frac{16(125{,}000)}{\pi d^3}(1.6) = \frac{1.0 \times 10^6}{d^3}$$

Thus, regardless of the value of d,

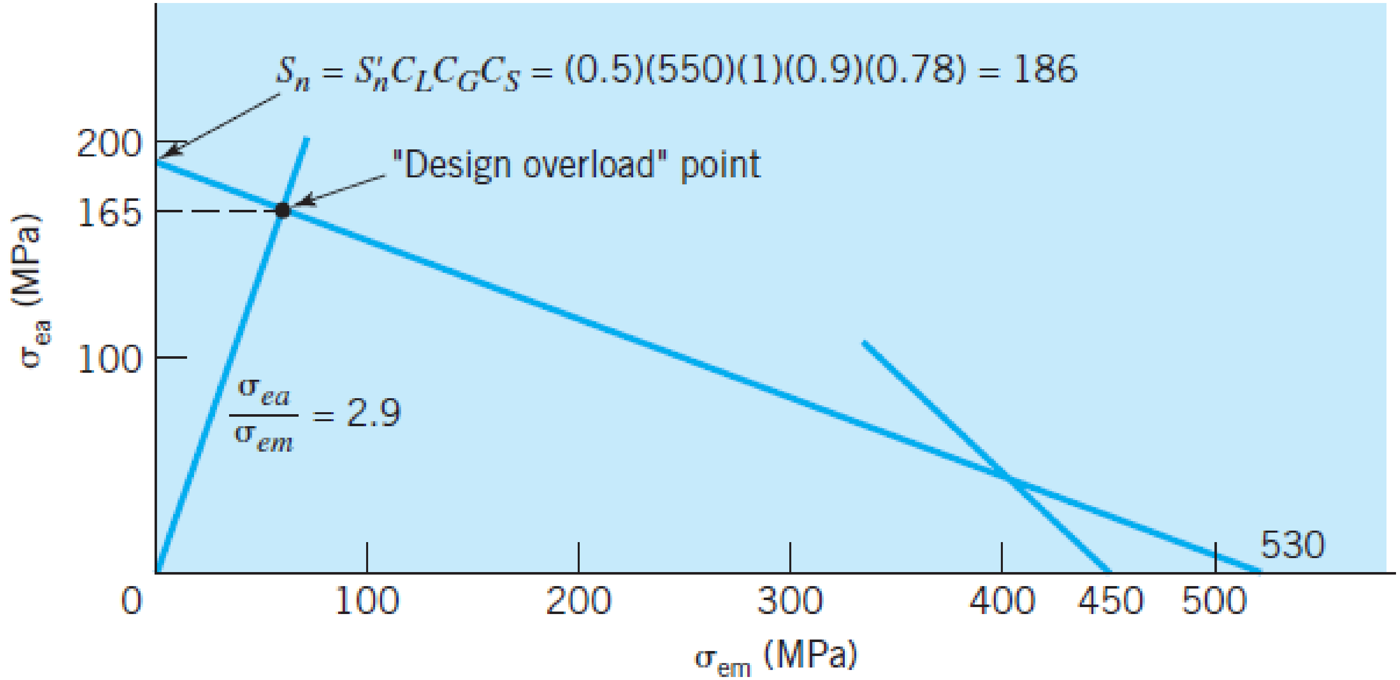
$$\sigma_{ea}/\sigma_{em} = 2.9$$

8. Figure 17.7*e* shows a fatigue strength diagram for this case, with "load line" at a slope of 2.9. The diagram indicates that for infinite life, σ_{ea} is limited to 165 MPa; but this is at the design overload that incorporates a safety factor of 2.5 (as decided). Hence,

$$\sigma_{ea} = \frac{2.9 \times 10^6}{d^3}(2.5) = 165 \text{ MPa}, \qquad \text{or} \quad d = 35.3 \text{ mm}$$

9. With the shaft layout shown, dimension d must, as decided, correspond to a standard bearing bore. Selection of $d = 35$ mm should be satisfactory; for a little more conservative choice, $d = 40$ mm might be preferred. In accordance with the choice of $r/d = 0.03$, the fillet radius must be at least $0.03d$ at S. A more generous radius would be preferred. Specify, say, $r = 2$ mm.

Comment: The angular deflections at A and B should be checked to determine whether self-aligning bearings are necessary.



(*e*) Fatigue diagram

Assumptions:

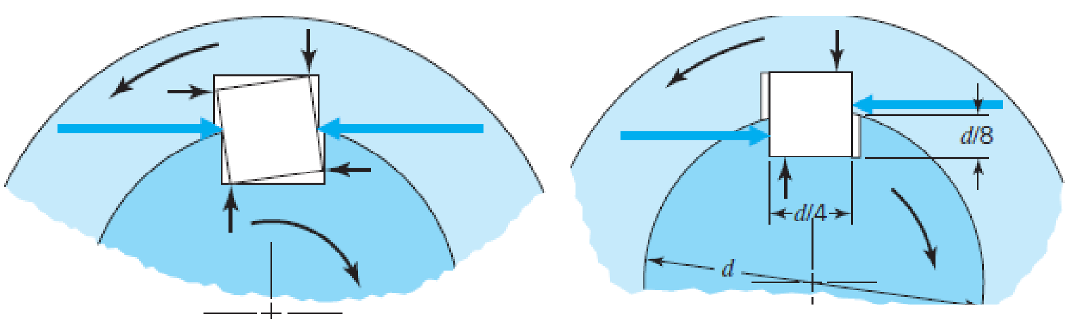
1. The full engine power reaches the snowmobile track.
2. Half of the track tension force is transmitted to each of the track sprockets; that is, there is an even division of torque.
3. Bearings *A* and *B* are self-aligning within the range of the angular shaft deflections induced.
4. The stress concentration at the bearing inner race (at *B*) is the same as that at the edges of the sprocket hub (at *S*).

17.5 Overall Shaft Design

- Shafts supporting helical or bevel gears are subject to mean loads that include tension or compression as well as torsion.
- Since axial stresses are functions of d^2 rather than d^3, a simple ratio between σ_{ea} and σ_{em} for all values of d does not exist for these cases.
- The most expedient procedure is first to ignore the axial stress when solving for d, and then to check the influence of the axial stress for the diameter obtained.
- A slight change in diameter may or may not be indicated.
- If the diameter selected must correspond to a standard size (as in Sample Problem 17.2D), it is likely that a consideration of the axial stress will not change the final choice.

17.6 Keys, Pins, and Splines

- Most common torque-transmitting shaft-to-hub connections are keys (square) of standard proportions (square) - key width $\cong$ 1/4th of shaft diaKeys are
- Usually made of cold-finished low-carbon steel (as SAE or AISI 1020), but heat treated alloy steels are used when greater strength is required.
- The loading of a key is a complex function of the clearances and elasticities.
- Figure a shows the loading of a loosely fitted square key. The primary loading is by the heavy horizontal force vectors; but these tend to rotate the key CCW until one or both pairs of its diagonally opposite corners make contact with the keyway sides, bottoms, or both
- Figure b shows a key that is tightly fitted at the top and bottom. The horizontal forces shown are commonly assumed to be uniformly distributed over the key surfaces and to be equal to shaft torque divided by shaft radius.



(a) Loosely fitted key

(b) Key tightly fitted at top and bottom

- Both assumptions are not correct, but in view of the complexities and uncertainties, they provide a reasonable basis for design and analysis

17.6 Keys, Pins, and Splines

- As an illustration of key sizing, let us estimate the length of key required in Figure b to transmit a torque equal to the elastic torque capacity of the shaft.
- Assume that the shaft and key are made of ductile materials having the same strength and (with the distortion energy theory), $S_{sy} = 0.58S_y$.
- The shaft torque capacity is $T = \frac{\pi d^3}{16}(0.58 S_y)$

$$\boxed{\tau_{max} = 16T/\pi d^3}$$

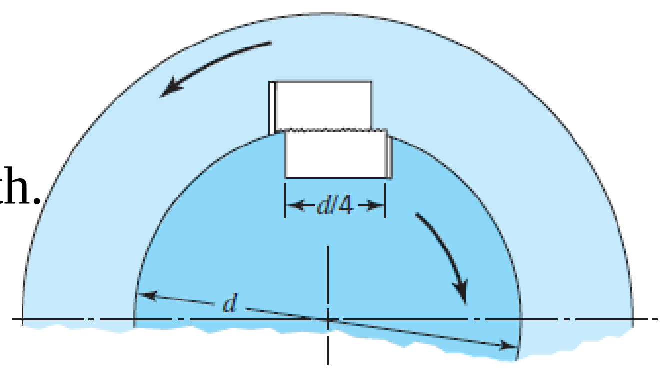
- The torque transmitted by compressive forces acting on the sides of the key is the product of limiting stress, contact area, and radius:

$$T = S_y \frac{Ld}{8}\frac{d}{2} = \frac{S_y L d^2}{16}$$

- The torque that can be transmitted by key shear (Figure c) is also the same:

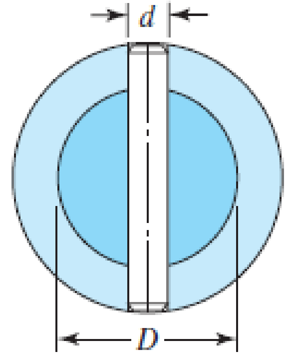
$$T = (0.58\ S_y)\frac{Ld}{4}\frac{d}{2} = \frac{0.58 S_y L d^2}{8}$$

- Equating a and b, L = 1.82d; a and c gives L = 1.57d.
- Balanced design requires the key L + 1.8d.
- Key designed for balanced compression and shear strength would require a key depth > than key width.
- Keys normally extend along the full width of the hub, and for good stability, hub widths are commonly 1.5d to 2d.



(c) Shear failure of a tightly fitted key

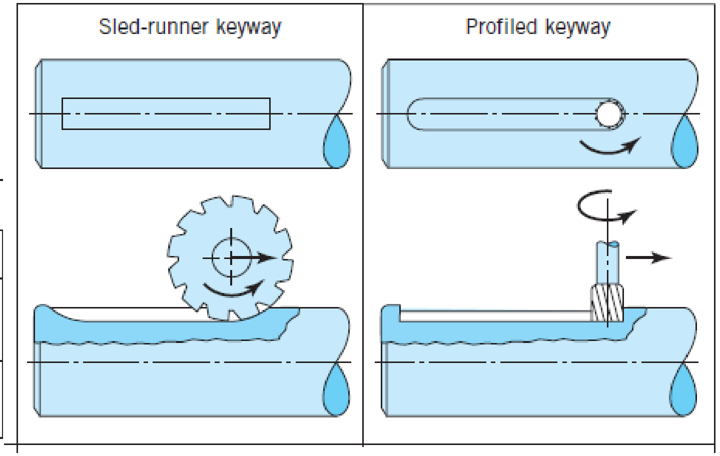
17.6 Keys, Pins, and Splines



(a) Straight round pin

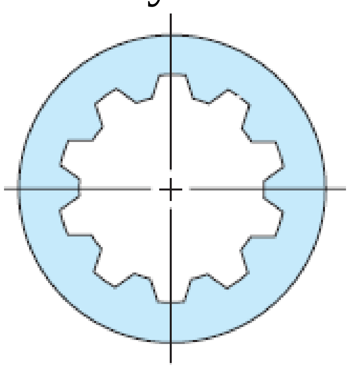
- If the shaft diar is based on deflection rather than strength, a shorter key may be adequate.
- If the dia is based on strength with shock or fatigue loading present, stress concentration at the keyway must be taken into account when estimating shaft strength.
- Figure shows 2 common keyway cutting methods & associated values of K_f.
- The torque capacity of a round pin connection is limited by the strength of the pin in double shear. For a solid pin of dia d and shear yield strength S_{sy} , the torque capacity based on pin yielding is $T = \pi d^2 D S_{sy}/4$
- So shear-loaded pins are made small and weak material to limit their capacity to the safe torque that can be carried by the shaft.
- Then the shear pin serves as a safety - If a propeller strikes an obstruction, the shear pin fails, preventing possible damage to drive train.

Steel	Fatigue stress concentration factor, K_f			
	Bending	Torsion	Bending	Torsion
Annealed (less than 200 Bhn)	1.3	1.3	1.6	1.3
Quenched and drawn (over 200 Bhn)	1.6	1.6	2.0	1.6

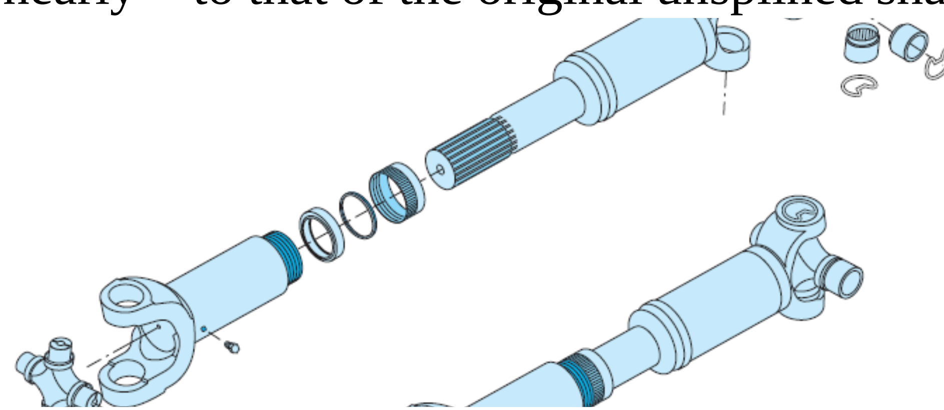


17.6 Keys, Pins, and Splines

- Splines act like multiple keys. They have either involute or straight-sided profiles, the former being the usual type in modern machinery.
- Involute splines usually have a 30° pressure angle, and one-half the depth of standard gear teeth (other standard are 37.5° and 45°).
- The fit between mating splines is characterized as sliding, close, or press.
- Figure 17.14 shows a sliding-fit spline that permits the length of an automotive propeller shaft to change slightly with rear-wheel jounce.
- Splines can be cut or rolled onto a shaft. The strength of a splined shaft is usually taken as the strength of a round shaft of dia = the minor dia of spline
- However, for rolled splines, the favorable effects of cold working and residual stresses may make the strength nearly = to that of the original unsplined shaft.



(b) Involute



17.7 Couplings and Universal Joints

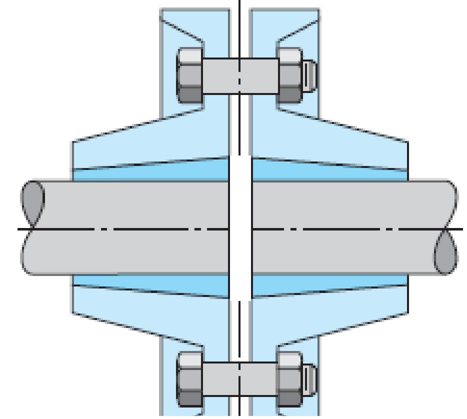
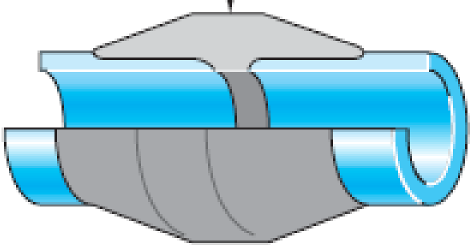


FIGURE 17.10
Rigid shaft coupling.

- Colinear shafts can be joined by rigid couplings - The coupling halves attached to the shaft ends with keys, or slotted tapered sleeves.
- The flanged portions at the outside diameter serve a safety function by shielding the bolt heads and nuts.
- For designing such a coupling, the force flow concept leads to consider, (1) the torque capacity of the key or wedged, (2) the strength of the relatively thin web portions that are drilled to accommodate the bolts, and (3) the strength of the bolts.
- Rigid couplings are limited to rare instances where shafts are colinear within extremely close tolerances and stay that way.
- If shafts are laterally or angularly misaligned, the installation of a rigid coupling forces them into alignment.
- This subjects the coupling, shafts, and shaft bearings to unnecessary loads that may lead to early failures.
- This can be eliminated by flexible couplings

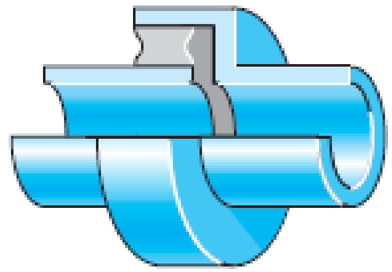
17.7 Couplings and Universal Joints

- Figure shows a few of the many designs that use a flexible material like rubber.
- These can be designed to provide elasticity and damping for control of torsional vibration & providing for misalignment.
- Other flexible couplings use all metallic components, and these tend to have greater torque capacity for a given size.

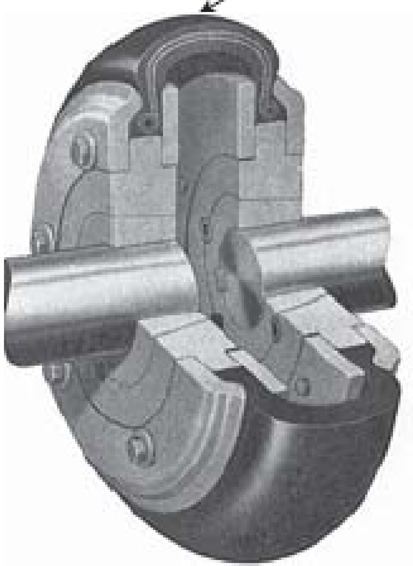


(*a*) Basic shear-type coupling

(*b*) Constant-stress, constant strain shear coupling



(*c*) Tube form shear coupling



(*d*) Heavy-duty coupling

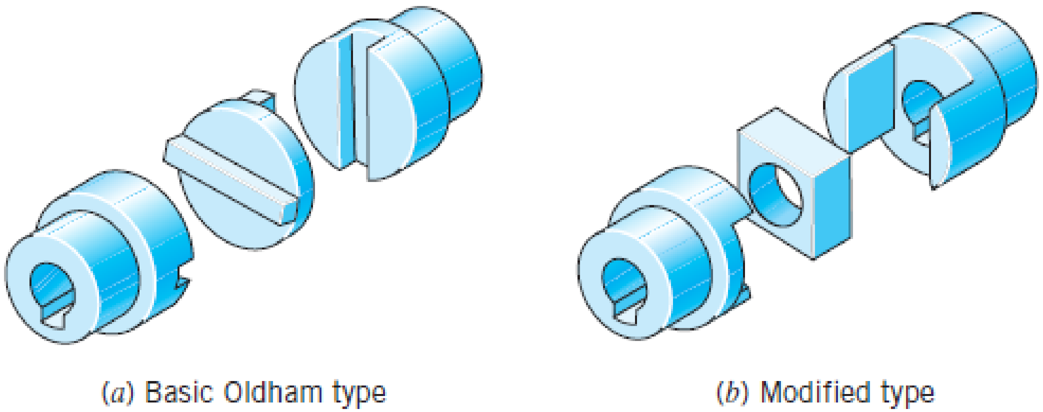
(*b*) Gear coupling

(*a*) Roller chain coupling

17.7 Couplings and Universal Joints

- One design of fairly ancient origin is the Oldham coupling, shown in Figure
- The sliding of the center block permits a substantial amount of shaft lateral offset, and built-in axial clearance permits some angular misalignment.

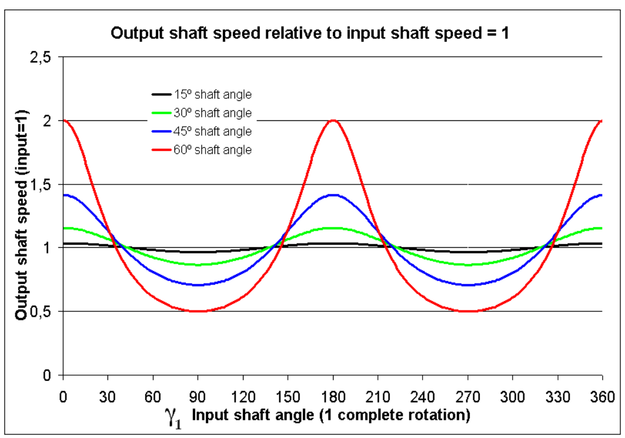
FIGURE 17.13
Oldham or slider block couplings. Both versions have a freely sliding center slider block that provides pairs of sliding surfaces at 90° orientation. The greater the shaft misalignment, the greater the sliding. Lubrication and wear must be considered.



- Universal joints permit substantial angular misalignment- Figure shows the common type (Cardan or Hooke's joint) used at the ends of rearwheel-drive auto.
- Plain bushings or needle bearings are used at the yoke-to-cross connections. If the input yoke rotates at constant angular velocity, the output yoke velocity will have a speed fluctuation at twice rotating speed, the magnitude of which increases with the misalignment angle.

17.7 Couplings and Universal Joints

- If two joints are used, with yokes aligned as shown in Figure, speed fluctuations across the two joints cancel to give uniform output yoke rotation if all three shafts are in the same plane, and if the misalignment angles at the two joints are equal.
- Other types of universal joints that transmit angular velocity uniformly across a single joint have been devised. These are known as CV universal joints.
- Common application is in front-wheel-drive auto, where drive shafts are short, and shaft angles (steering and wheel jounce) can be large.



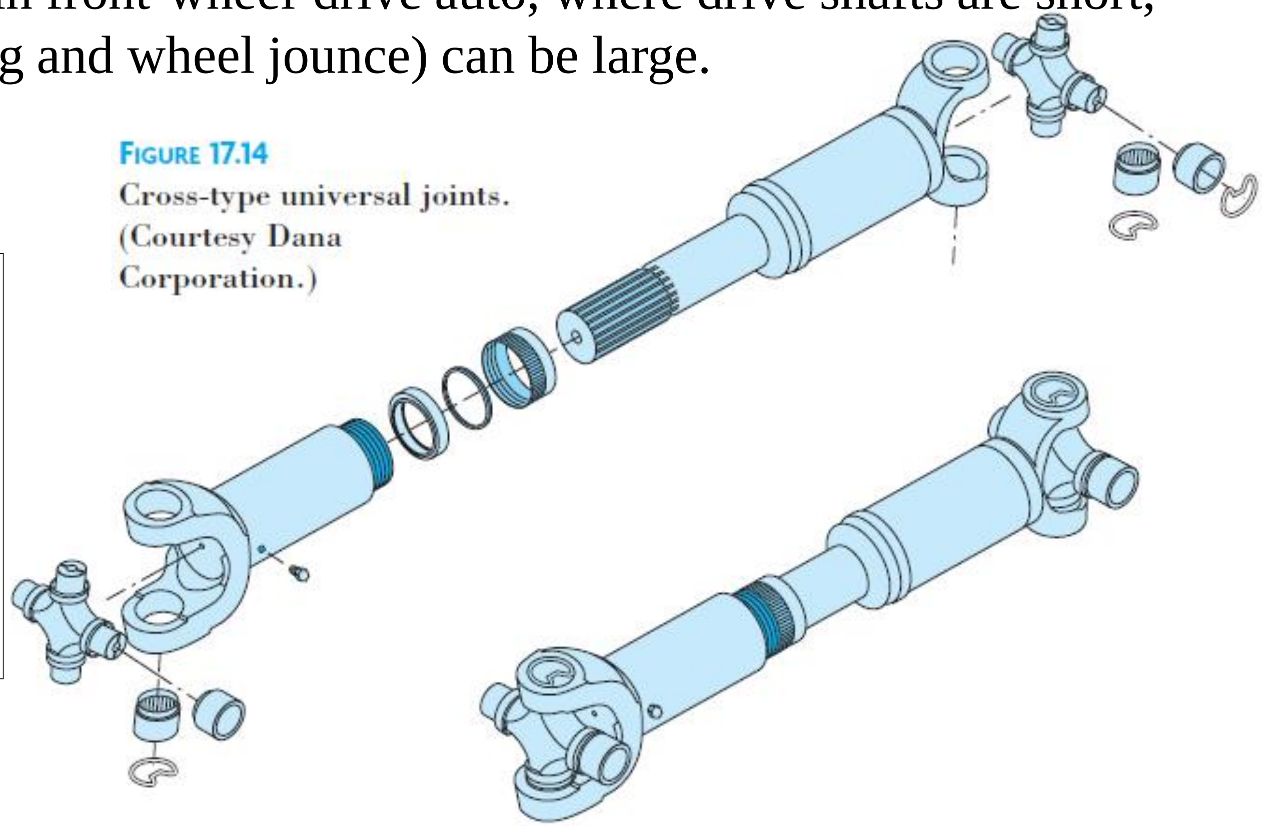


Figure 17.14
Cross-type universal joints. (Courtesy Dana Corporation.)

17.7 Couplings and Universal Joints

- To protect persons from rotating universal joints, couplings, and shafts, machine guards are generally required. Figure shows coupling guard for pump assembly.
- Coupling and shaft guards provide physical protection from the rotating components, and when properly designed can be opened or removed to service the connected equipment
- The guard can also protect couplings and shafts from external and environmental damage.

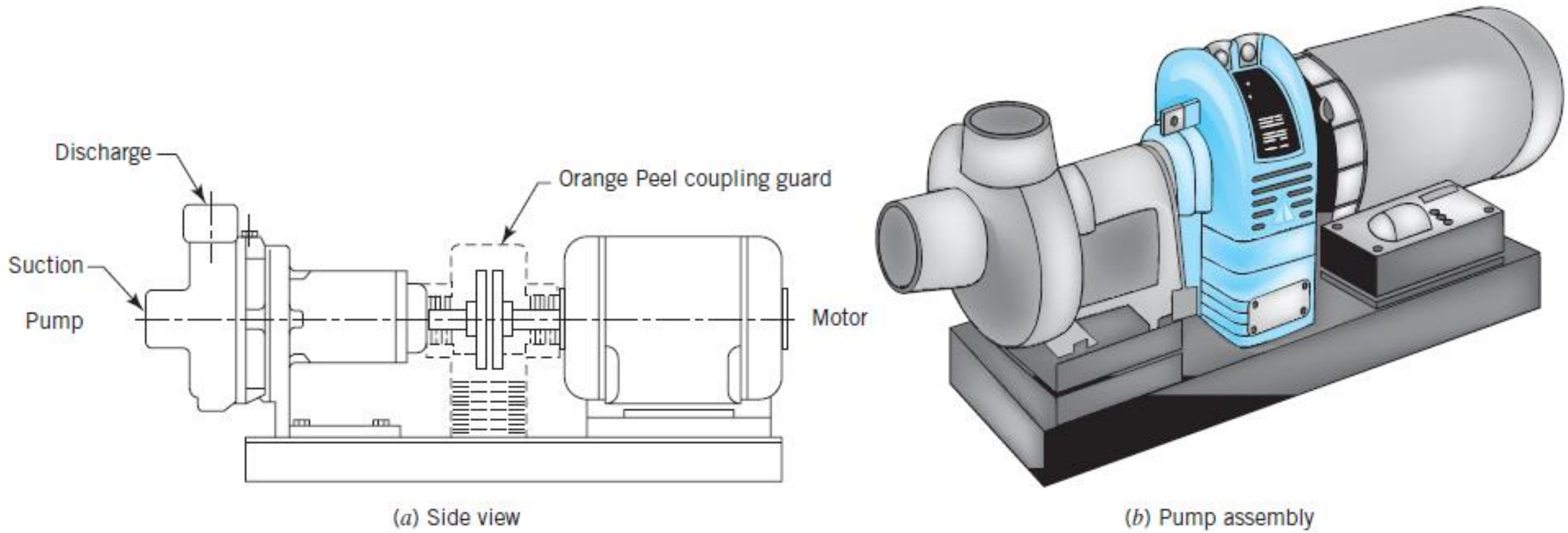


FIGURE 17.15
Orange Peel, LLC coupling guard for motor-pump assembly.