SOLUTION (4.18)

Known: A steel propeller shaft with a given diameter transmits a known power at a specified angular velocity.

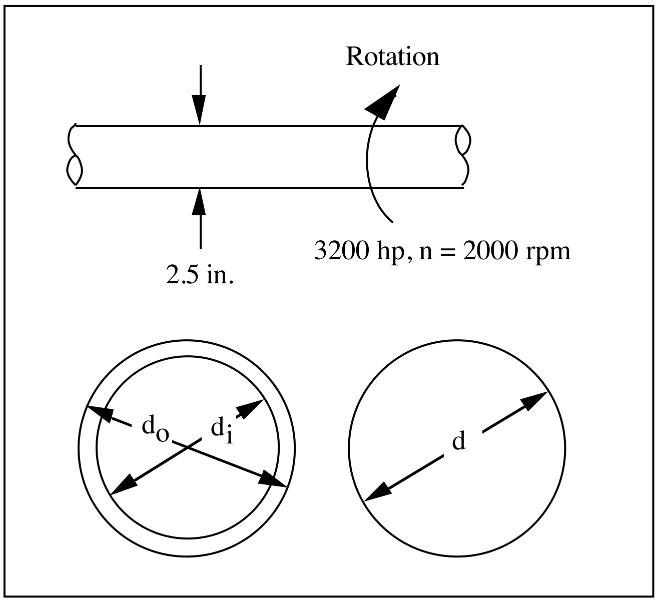
Find:

(a) Determine the nominal shear stress at the surface.

(b) Determine the outside diameter required to give the same outer surface stress if a hollow shaft of inside diameter 0.85 times the outside diameter is used.

(c) Compare the weights of the solid and hollow shafts.

Schematic and Given Data:



Assumptions:

1. Bending and axial loads are negligible.
2. The bar is straight and round.
3. The material is homogeneous, and perfectly elastic within the stress range involved.
4. The effect of stress raisers is negligible.

Analysis:

1. From Eq. (1.3), $T = \dfrac{5252\ \dot{W}}{n}$

$$T = \frac{5252(3200)}{2000} = 8403 \text{ lb ft} = 100{,}838 \text{ lb in.}$$

2. From Eq. (4.4), $\tau = \dfrac{16\ T}{\pi\ d^3}$

$$\tau = \frac{16\ (100{,}838)}{\pi\ (2.5)^3} = 32{,}868 \text{ psi} = 33 \text{ ksi}$$

■

3. For a hollow shaft,
$d_o^3 - (0.85)^4 d_o^3 = 2.5^3$
$0.478 d_o^3 = 15.625$
$d_o = 3.20$ in. ■

4. $\dfrac{\text{Wt. hollow}}{\text{Wt. solid}} = \dfrac{\text{Area hollow}}{\text{Area solid}} = \dfrac{3.20^2 - (0.85 \times 3.20)^2}{2.5^2} = 0.46$ ■

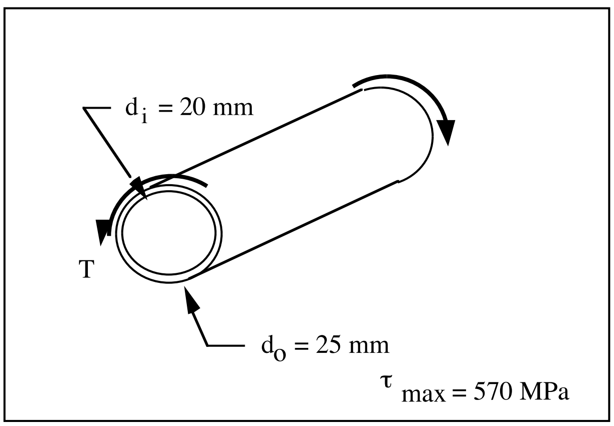
Comment: It is more economical to use a hollow shaft when pure shear stress is involved.

SOLUTION (4.19)

Known: The maximum shear stress is given for a hollow shaft of known geometry subjected to pure torsion.

Find: Determine the torque that produces the given maximum shear stress.

Schematic and Given Data:



Assumptions:
1. The shaft is straight.
2. The material is homogeneous and perfectly elastic.
3. There are no stress raisers.

Analysis:
1. From Eq. (4.3), $\tau = \dfrac{Tr}{J} = \dfrac{16Td_o}{\pi\left(d_o^4 - d_i^4\right)}$

2. $T = \dfrac{\pi(570)\left(25^4 - 20^4\right)}{16(25)} = 10.325 \times 10^6$ N•mm = 10,325 N•m ■

SOLUTION (4.20)

Known: The same value of torque is applied to both a solid square shaft ($b \times b$) and a solid round shaft of radius r.

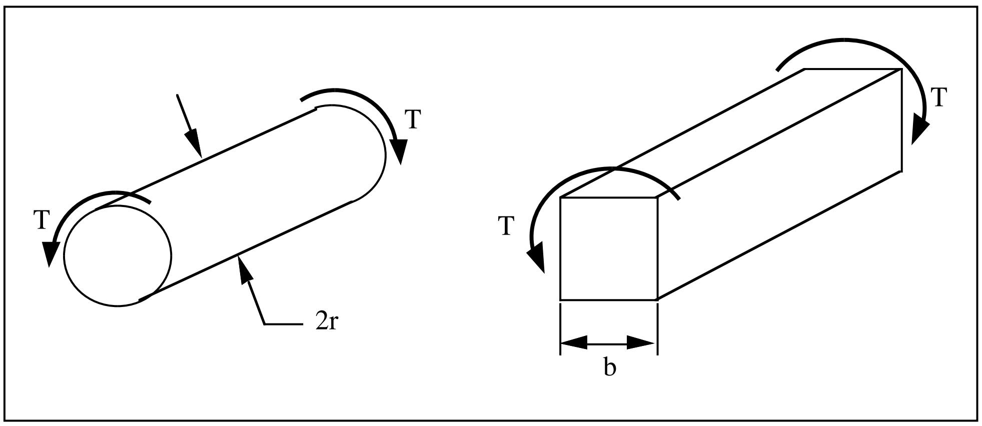
Find:

(a) Determine the ratio b/r for both square and round shafts so as to produce equal maximum shear values.

(b) Compare the weight of the two shafts for the square and round shafts.

(c) Compare the ratio of strength-to-weight for the square and round shafts.

Schematic and Given Data:



Assumptions:

1. The shafts are straight.
2. The material is homogeneous and perfectly elastic.
3. There are no stress raisers.
4. The shafts are made of the same material.

Analysis:

1. For equal stress, equate Eq. (4.4) with Eq. (4.5).

$$\frac{16\ T}{\pi(2r)^3} = \frac{4.8\ T}{b^3}$$

$$\frac{16\ b^3}{8\pi r^3} = 4.8$$

$$\left(\frac{b}{r}\right)^3 = 7.5398$$

$$\frac{b}{r} = 1.96$$

■

2. $$\frac{\text{Wt. square per unit length}}{\text{Wt. round per unit length}} = \frac{b^2}{\pi r^2} = \frac{b^2}{\pi\left(\frac{b}{1.96}\right)^2} = 1.22 \qquad \blacksquare$$

3. $$\frac{(\text{Strength/wt.})_{\text{square}}}{(\text{Strength/wt.})_{\text{round}}} = \frac{T/W_s}{T/W_r} = \frac{W_r}{W_s} = \frac{1}{1.22} = 0.82 \qquad \blacksquare$$

Comment: The round bar is more economical (higher strength to weight ratio) than the square bar for the same shear stress.

SOLUTION (4.21)

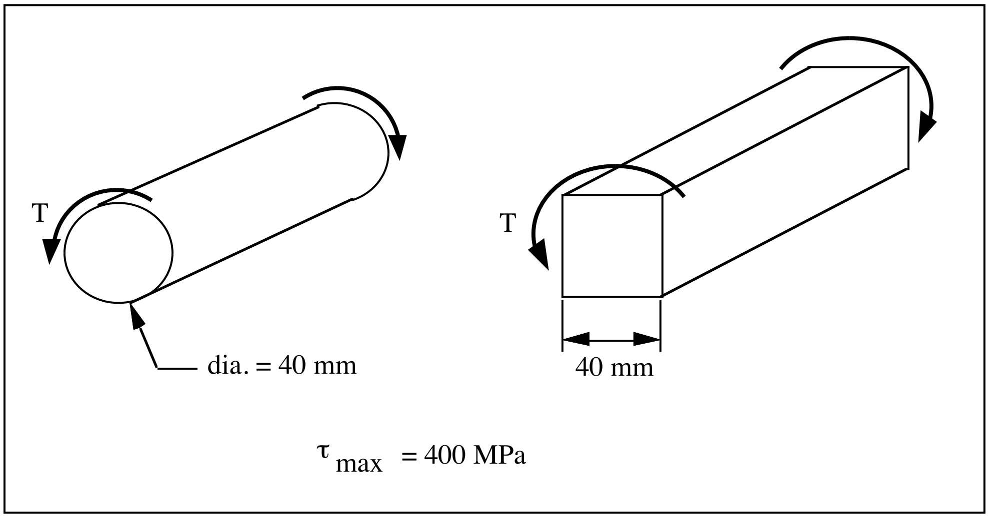
Known: The maximum shear stress produced in a shaft transmitting torque is given.

Find: Determine the torque:

(a) In a round shaft of 40 mm diameter.

(b) In a square shaft, 40 mm on a side.

Schematic and Given Data:



Assumptions:

1. The shafts are straight.
2. The material is homogeneous and perfectly elastic.
3. There are no stress raisers.

Analysis:

1. From Eq. (4.4), $\tau = \dfrac{16T}{\pi d^3}$

$$T = \frac{(400)(\pi)(40)^3}{16} = 5.03 \times 10^6 \text{ N·mm}$$

$$= 5030 \text{ N·m}$$ ■

2. From Eq. (4.5), $\tau = \dfrac{4.8T}{a^3}$

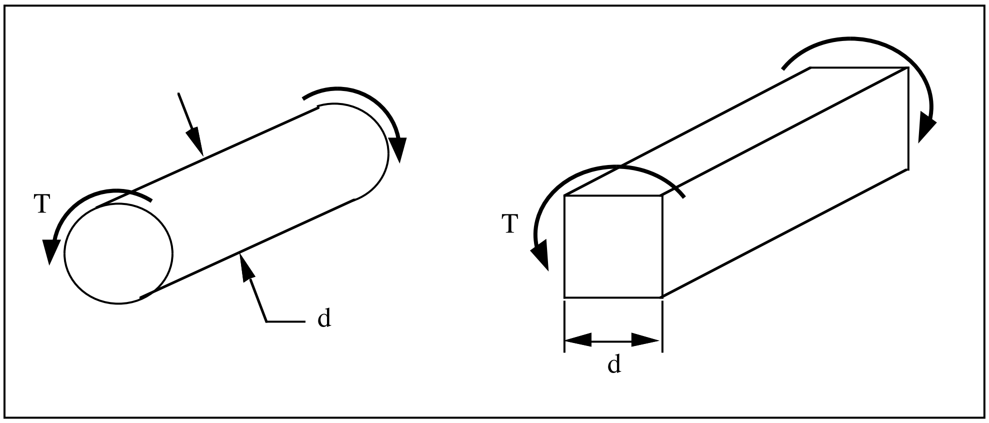
$$T = \frac{400(40)^3}{4.8} = 5.33 \times 10^6 \text{ N·mm} = 5330 \text{ N·m}$$ ■

SOLUTION (4.22)

Known: The geometries of a solid round shaft and a solid square shaft of the same size (circle diameter equal to side of square) are given.

Find: Compare the torque transmitting strength, the weight, and the ratio of strength to weight of the two shafts.

Schematic and Given Data:



Assumptions:

1. The shafts are straight.
2. The material is homogeneous and perfectly elastic.
3. There are no stress raisers.
4. The shafts are made of the same material.

Analysis:

1. For equal stress, equate Eq. (4.14) with Eq. (4.5).

$$\frac{16\, T_{round}}{\pi d^3} = \frac{4.8\, T_{square}}{d^3}$$

$$T_{square} = T_{round}\,\frac{16}{4.8\pi} = 1.061\, T_{round} \qquad \blacksquare$$

2. $$\frac{\text{Wt. square per unit length}}{\text{Wt. round per unit length}} = \frac{d^2}{\frac{\pi}{4} d^2} = \frac{4}{\pi} = 1.273 \qquad \blacksquare$$

3. $$\frac{(\text{Strength/wt.})_{square}}{(\text{Strength/wt.})_{round}} = \frac{1.061}{1.273} = 0.833 \qquad \blacksquare$$

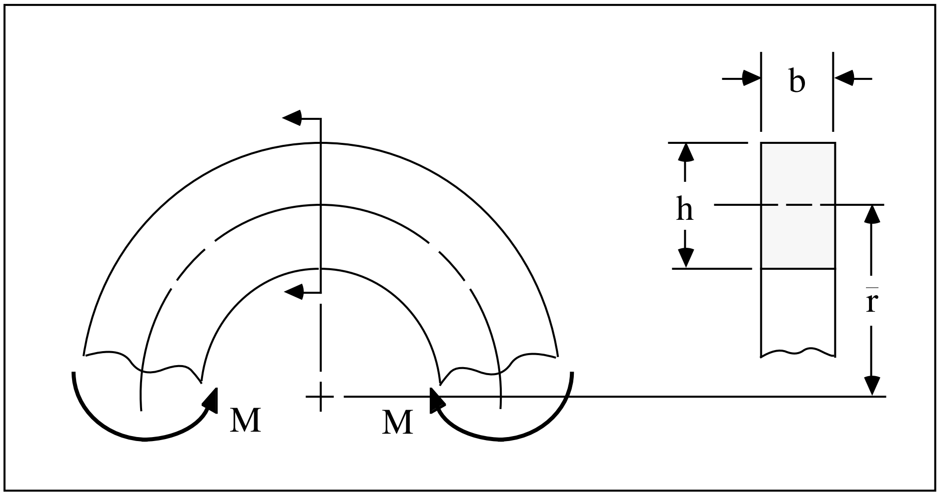
Comment: The round bar is more economical (higher strength to weight ratio) than the square bar for the same shear stress.

SOLUTION (4.33)

Known: A rectangular beam has an initial curvature, $\bar{r}$, equal to twice the section depth, h.

Find: Compare its extreme-fiber bending stresses with those of an otherwise identical straight beam.

Schematic and Given Data:



Assumption: The material is homogeneous and perfectly elastic.

Analysis:

1. For a straight beam, from Eq. (4.7) where $Z = \frac{bh^2}{6}$ (From Appendix B-1).

$$\sigma_i = -\frac{6M}{bh^2} \;;\; \sigma_o = +\frac{6M}{bh^2}$$

2. For a curved beam, from Eq. (4.10)

$$e = \bar{r} - \frac{A}{\int dA/\rho} = 2h - \frac{bh}{b\int_{r_i}^{r_o} d\rho/\rho} = 2h - \frac{h}{\ln\left(\frac{r_o}{r_i}\right)} = 2h - \frac{h}{\ln\left(\frac{2.5h}{1.5h}\right)}$$

$$= 2h - \frac{h}{\ln(5/3)} = .042385\ h$$

3. From Eq. (4.9),

$$\sigma_i = \frac{Mc_i}{eAr_i} = \frac{M(0.5h - .042385h)}{(.042385h)(bh)(1.5h)} = \frac{7.1978M}{bh^2}$$

But for direction of "M" shown, $\sigma_i = \frac{-7.20M}{bh^2}$

$$\sigma_o = \frac{M(0.5h + .042385h)}{(.042385h)(bh)(2.5h)} = \frac{-5.119M}{bh^2}$$

But for direction shown of M: $\sigma_o = \frac{+5.20M}{bh^2}$

4. From Eq. (4.11), with $Z = bh^2/6$:

   $K_i = \frac{7.1978}{6} = 1.20$ and $K_o = \frac{5.119}{6} = 0.85$

   which is consistent with Fig. 4.11.

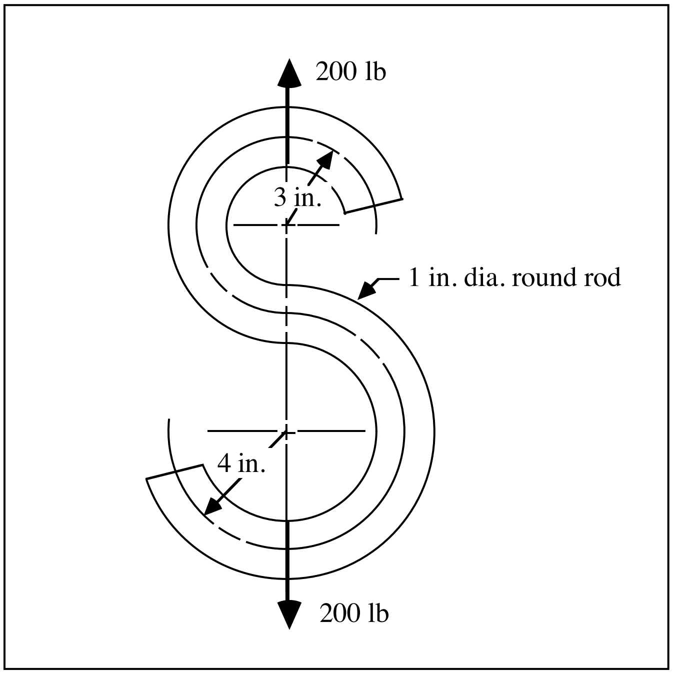
5. The inner and outer fiber bending stresses for the curved beam are 120% and 85% of the straight beam stresses. ■

SOLUTION (4.34)

Known: A known force is exerted on an S-hook.

Find: Determine the location and magnitude of the maximum tensile stress.

Schematic and Given Data:



Assumption: Material is homogeneous and perfectly elastic.

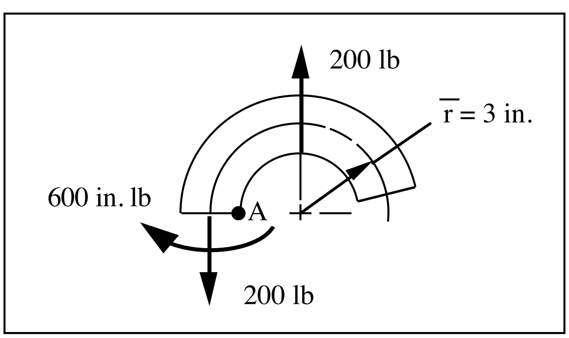
Analysis:

1. At point A, the tensile stress due to bending is

$$\sigma = \frac{32M}{\pi d^3} K_t \quad \text{[From Eq. (4.11)]}$$

The tensile stress due to tension is

$$\sigma = \frac{P}{A} = \frac{4P}{\pi d^2} \quad \text{[From Eq. (4.1)]}$$



Thus, the combined tensile stress is

$$\sigma = \frac{32M}{\pi d^3} K_t + \frac{4P}{\pi d^2}$$

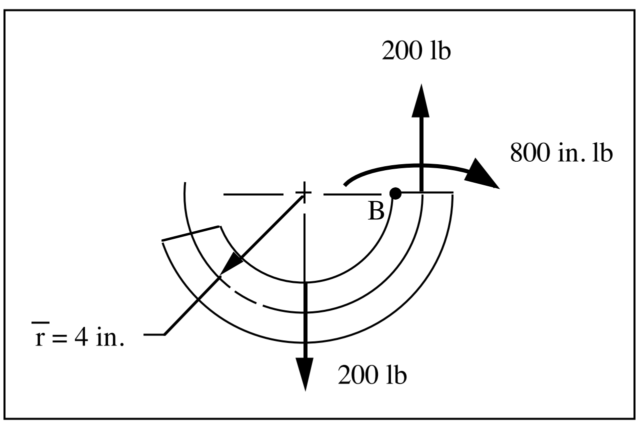
2. From Fig. 4.11, for $\frac{\bar{r}}{c} = \frac{3}{0.5} = 6,\ K_t = 1.14.$

3. $\sigma = (1.14)\,\frac{32(600)}{\pi(1)^3} + \frac{4(200)}{\pi(1)^2} = 6{,}967 + 255$
   $= 7{,}222$ psi
   At point A, $\sigma = 7.2$ ksi.
4. At point B, from Fig. 4.11, for

   $$\frac{\bar{r}}{c} = \frac{4}{0.5} = 8,\ K_t = 1.10$$

   $$\sigma = (1.10)\,\frac{32(800)}{\pi(1)^3} + \frac{4(200)}{\pi(1)^2}$$

   $= 8{,}964 + 255 = 9{,}219$ psi.



At point B, $\sigma = 9.2$ ksi. This point corresponds to the location of the maximum tensile stress. ■

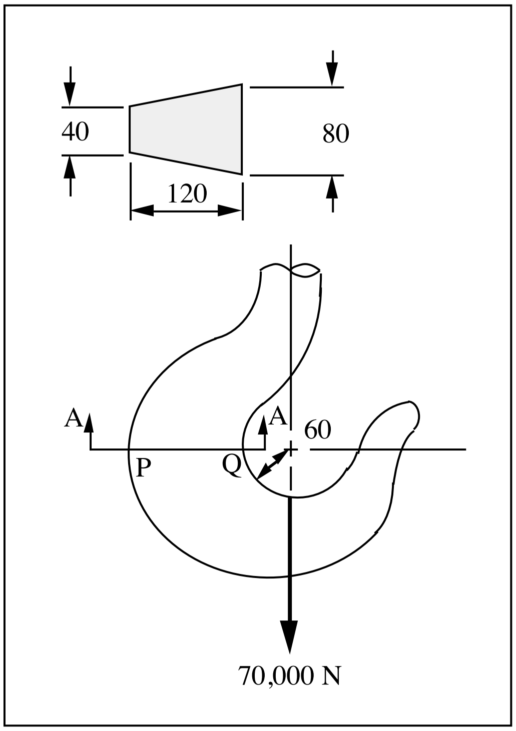
Comment: The inner fiber is stressed more than the outer fiber because the stresses due to the direct tension and bending are of the same sign, and hence, add up to give a large resultant stress.

SOLUTION (4.36)

Known: The critical section of a crane hook is considered to be trapezoidal with dimensions as shown.

Find: Determine the resultant stress (bending plus direct tension) at points P and Q.

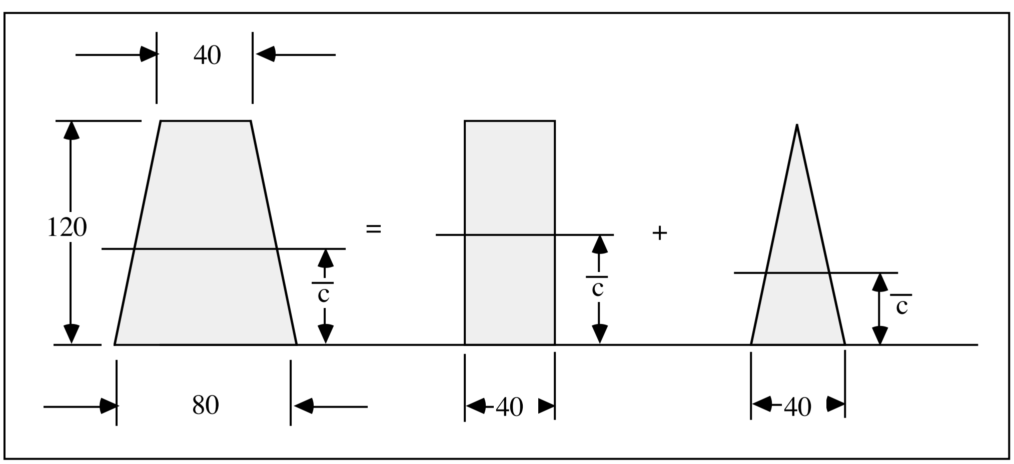
Schematic and Given Data:



Assumption: Material is homogeneous and perfectly elastic.

Analysis:

1.



$\bar{c}$ (rectangle) = 60, $\bar{c}$ (triangle) = 40

$$\bar{c}\text{ (trapezoid)} = \frac{A\bar{c}\text{ (rectangle)} + A\bar{c}\text{(triangle)}}{A\text{(trapezoid)}}$$

$$= \frac{4800(60) + 2400(40)}{60(120)} = 53.33$$

2. I of trapezoid about its centroidal axis = I of rectangle + I of triangle, each about the centroidal axis of <u>trapezoid</u>. Using the parallel axis theorem and equations from Appendix B-1:

$$I\text{(trapezoid)} = \frac{40(120)^3}{12} + 4800(60 - 53.33)^2$$

$$+ \frac{40(120)^3}{36} + 2400(40 - 53.33)^2$$

$$= 5.76 \times 10^6 + .213 \times 10^6 + 1.92 \times 10^6 + .426 \times 10^6$$

$$= 8.32 \times 10^6 \text{ mm}^4$$

3. $\frac{\bar{r}}{\bar{c}}$(trapezoid) $= \frac{60 + 53.33}{53.33} = 2.13$. From Fig. 4.11, $K_i = 1.52$, $K_o = 0.73$.

4. The tensile stress due to tension is

$$\sigma = \frac{P}{A} \quad \text{[From Eq. 4.1)]}$$

and tensile stress due to bending is

$$\sigma = K\frac{Mc}{I} \quad \text{[From Eq. (4.11)].}$$

5. At P, the resultant stress is

$$\sigma = \frac{P}{A} - K_o \frac{Mc}{I}$$

$$= \frac{70{,}000\text{N}}{7200 \text{ mm}^2} - (0.73) \frac{[70{,}000(60 + 53.33)\text{N}\cdot\text{mm}](66.67 \text{ mm})}{8.32 \times 10^6 \text{ mm}^4}$$

$$= 9.72 \text{ MPa} - 46.41 \text{ MPa} = -36.69 \text{ MPa}$$ ■

6. At Q, the resultant stress is

$$\sigma = \frac{P}{A} + K_i \frac{Mc}{I}$$

$$= 9.72 \text{ MPa} + (1.52)\,\frac{[70{,}000(113.33)\text{N·mm}](53.33 \text{ mm})}{8.32 \times 10^6 \text{ mm}^4}$$

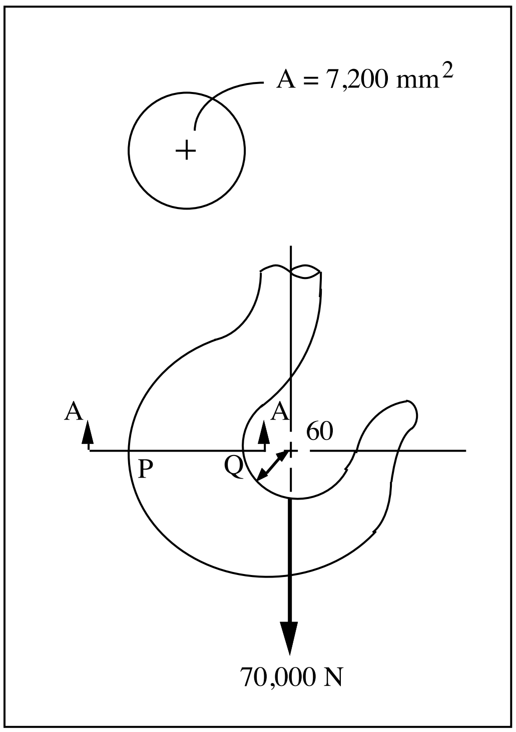
$$= 9.72 \text{ MPa} + 77.29 \text{ MPa} = 87.01 \text{ MPa} \qquad \blacksquare$$

SOLUTION (4.37)

Known: The critical section of a crane hook is considered to be circular with areas as shown.

Find: Determine the resultant stress (bending plus direct tension) at points P and Q.

Schematic and Given Data:



Assumption: The material is homogeneous and perfectly elastic.

Analysis:

1. $\bar{c} = \dfrac{d}{2} = \dfrac{\sqrt{4A/\pi}}{2} = \sqrt{A/\pi} = \sqrt{7200/\pi} = 47.87 = 48 \text{ mm}$

2. $I = \dfrac{\pi d^4}{64} = \dfrac{\pi(96)^4}{64} = 4.17 \times 10^6 \text{ mm}^4$

3. $\dfrac{\bar{r}}{\bar{c}}\text{(circle)} = \dfrac{60 + 48}{48} = 2.25.$ From Fig. 4.11, $K_i = 1.50$, $K_o = 0.75$

4. The tensile stress due to tension is

$$\sigma = \frac{P}{A} \quad \text{[From Eq. 4.1)]}$$

and tensile stress due to bending is

$$\sigma = K\frac{Mc}{I} \quad \text{[From Eq. (4.11)].}$$

5. At P, the resultant stress is

$$\sigma = \frac{P}{A} - K_o \frac{Mc}{I}$$

$$= \frac{70{,}000\text{N}}{7200 \text{ mm}^2} - (0.75) \frac{[70{,}000(60 + 48)\text{N·mm}](48 \text{ mm})}{4.17 \times 10^6 \text{ mm}^4}$$

$$= 9.72 \text{ MPa} - 65.27 \text{ MPa} = -55.55 \text{ MPa}$$ ■

6. At Q, the resultant stress is

$$\sigma = \frac{P}{A} + K_i \frac{Mc}{I}$$

$$= 9.72 \text{ MPa} + (1.50) \frac{[70{,}000(108)\text{N·mm}](48 \text{ mm})}{4.17 \times 10^6 \text{ mm}^4}$$

$$= 9.72 \text{ MPa} + 130.53 \text{ MPa} = 140.25 \text{ MPa}$$ ■

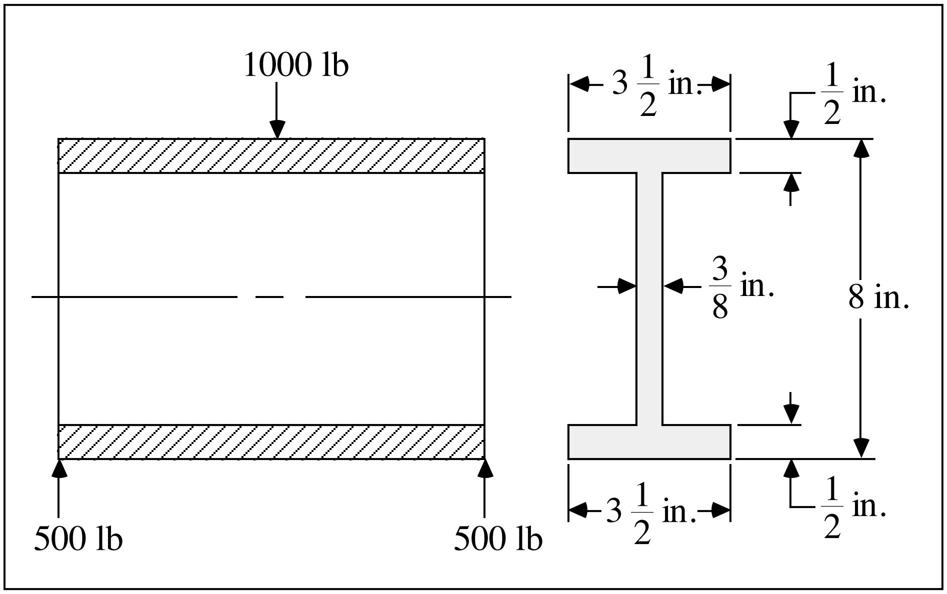
Comment: By comparing the resultant stresses in Solutions 4.21 and 4.22, it is evident that for the same area of cross section the trapezoidal section is stronger and hence, more economical than a circular cross section crane hook. This is the reason, for the use of trapezoidal shaped cross sections in crane hooks for practical applications.

SOLUTION (4.44)

Known: An I-beam with given dimensions is simply supported at each end and subjected to a know load at the center.

Find: Compute the maximum transverse shear stress. Compare the answer with the approximation obtained by dividing the shear load by the area of the web, with the web considered to extend for the full 8-in. depth.

Schematic and Given Data:



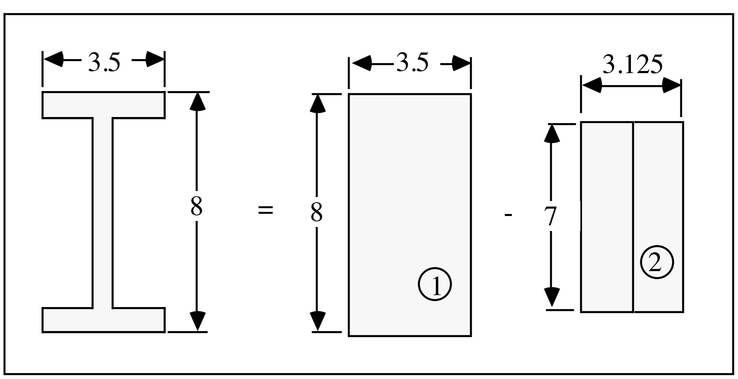
Assumption: The beam material is homogeneous and perfectly elastic.

Analysis:

1. τ_{max} exists at the neutral bending axis,

$$\tau = \frac{V}{Ib}\int ydA \qquad \text{[Eq. (4.12)]}$$

2.



From the above figure,

$$I = I_1 - I_2 = \frac{3.5(8)^3}{12} - \frac{3.125(7)^3}{12} = 60.01 \text{ in.}^4$$

3. $$\tau_{max} = \frac{500}{(60.01)\,(0.375)}\left[\int_0^{3.5} y(0.375dy) + \int_{3.5}^{4} y(3.5dy)\right]$$

$$= 22.22\left[0.375\left(\frac{3.5^2}{2}\right) + 3.5\left(\frac{4^2}{2} - \frac{3.5^2}{2}\right)\right] = 197 \text{ psi} \quad \blacksquare$$

4. To check,

$$\tau_{max} \approx \frac{500}{(\frac{3}{8})(8)} = 167 \text{ psi} \quad \blacksquare$$

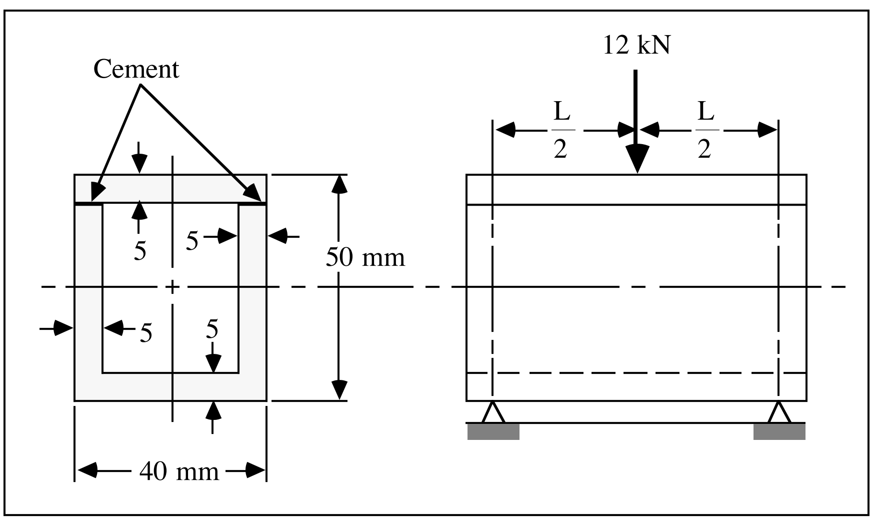
Comment: The rough check is 15% low in this case.

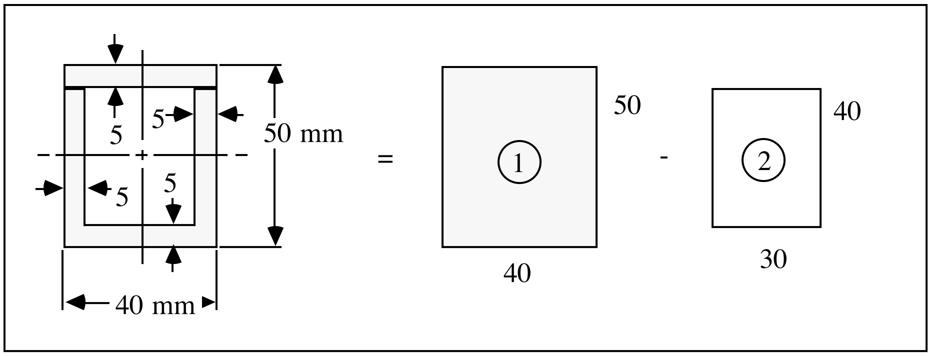
SOLUTION (4.45)

Known: A box section beam, where the top plate of the section is cemented in place, is loaded with a specified force.

Find: Determine the shear stress acting on the cemented joint.

Schematic and Given Data:





Assumption: The beam material is homogeneous and perfectly elastic.

Analysis:

1. From Eq. (4.12), $\tau = \dfrac{V}{Ib}\displaystyle\int_{y=20}^{y=25} y\,dA$

2. From the above figure, $I = I_1 - I_2$

$$= \frac{(40)(50)^3}{12} - \frac{(30)(40)^3}{12} = 256{,}667 \text{ mm}^4$$

3. $\tau = \dfrac{6000}{(256{,}667)(10)}\displaystyle\int_{20}^{25} y(40dy) = \dfrac{6000(40)}{2{,}566{,}670}\left.\dfrac{y^2}{2}\right]_{20}^{25}$

$= 10.5$ MPa ■

SOLUTION (4.46)

Known: A shaft between self-aligning bearings A and B is loaded through belt forces applied to a central sheave.

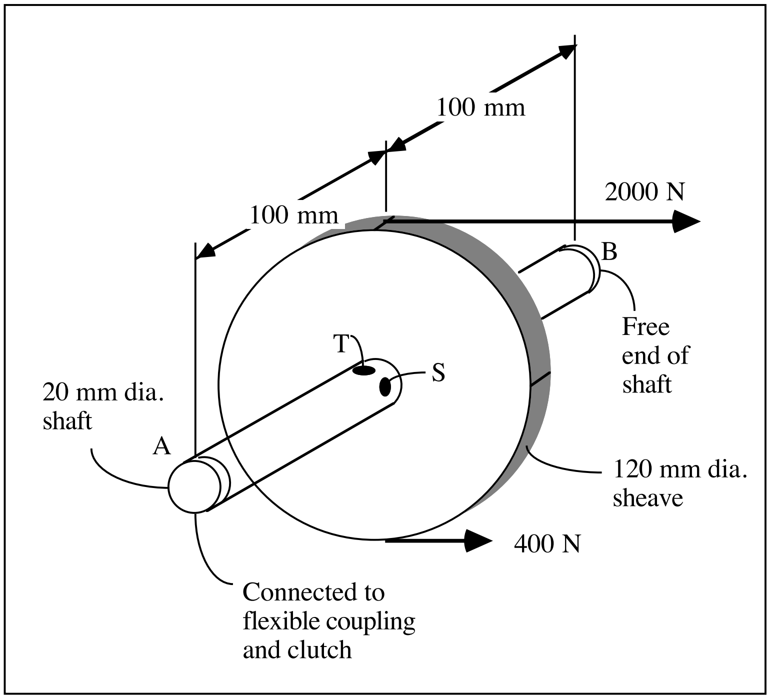
Find:

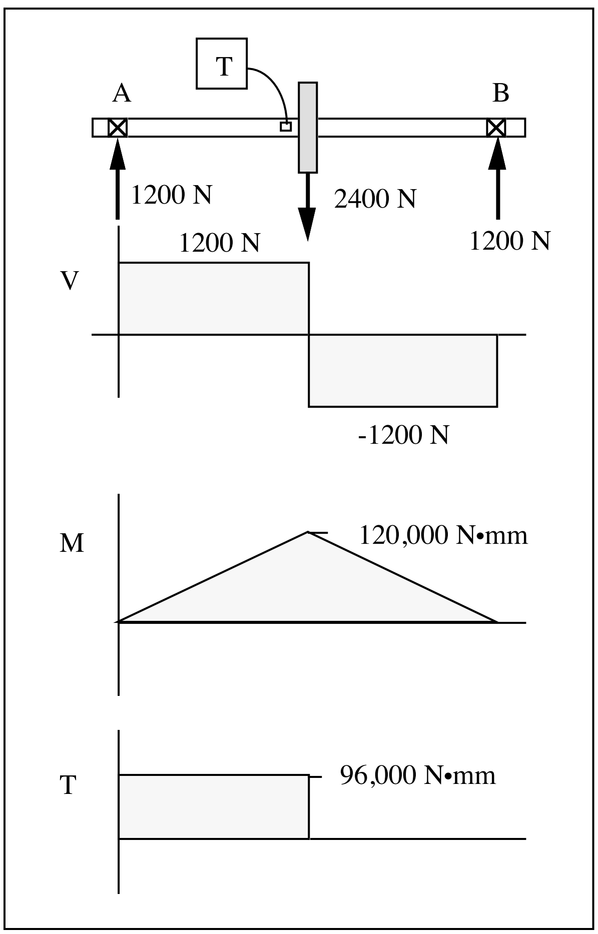
(a) Determine and make a sketch showing the stresses acting on the top and side elements, T and S.

(b) Represent the states of stress at T and S with three-dimensional Mohr circles.

(c) At location S, show the orientation and stresses acting on a principal element, and on a maximum shear element.

Schematic and Given Data:





Assumptions:

1. The weights of the shaft and sheave are negligible.

2. The shaft is straight.
3. The effect of stress concentrations is negligible.
4. The shaft material is homogeneous and perfectly elastic.

Analysis:

1. For torsion, Eq. (4.4),

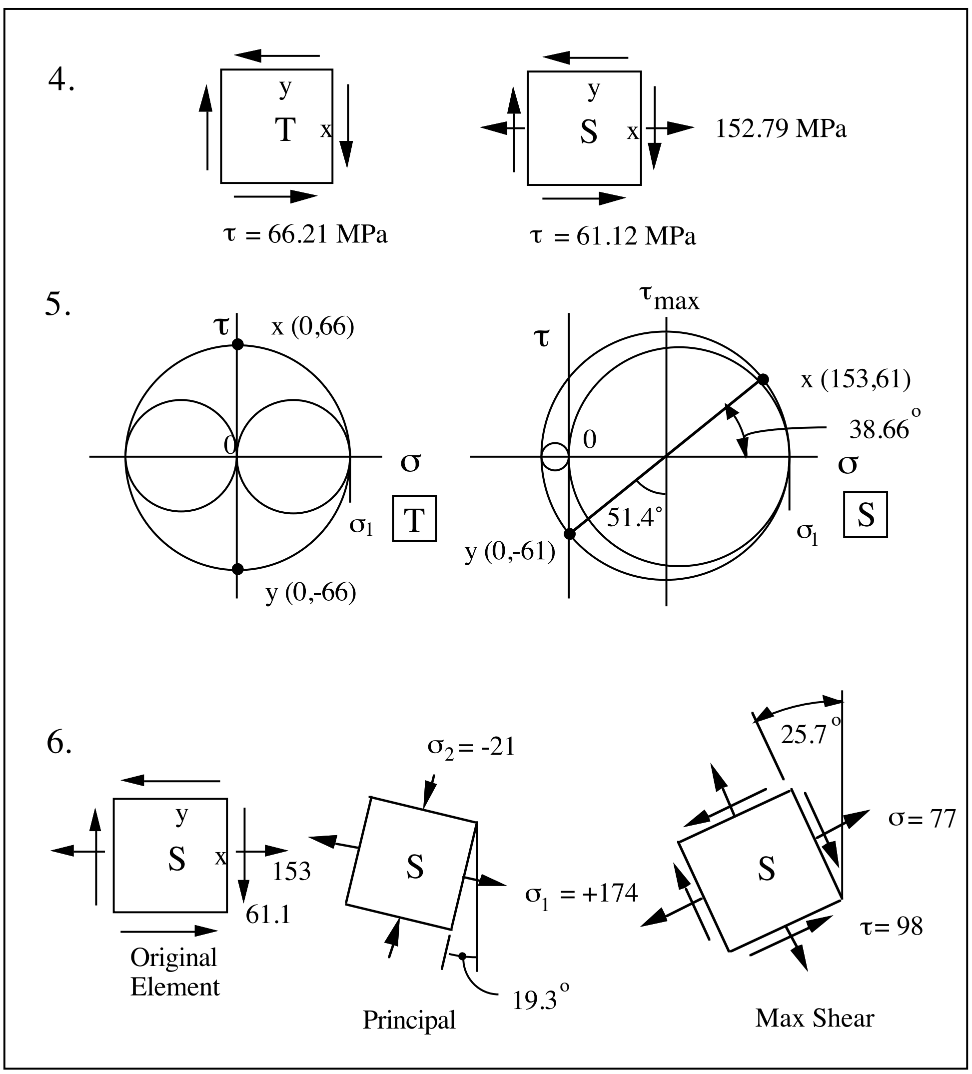
$$\tau = \frac{16T}{\pi d^3} = \frac{16(1600)(60)}{\pi(20)^3} = 61.12 \text{ MPa}$$

2. For bending, Eq. (4.8),

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(1200)(100)}{\pi(20)^3} = 152.79 \text{ MPa}$$

3. For transverse shear, Eq. (4.13),

$$\tau = \frac{4}{3}\frac{V}{A} = \frac{4(1200)}{3(\pi)(10)^2} = 5.09 \text{ MPa}$$



SOLUTION (4.49)

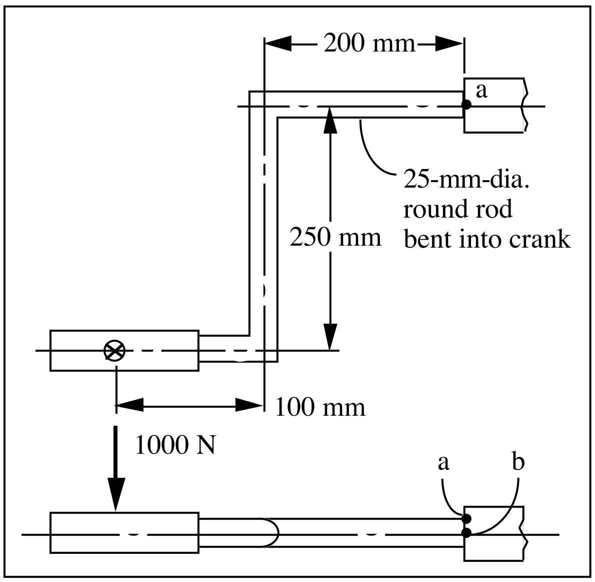
Known: A static vertical load is applied to the handle of a hand crank.

Find:

(a) Copy the drawing and mark on it the location at highest bending stress. Make a three-dimensional Mohr-circle representation of the stresses at this point.

(b) Mark on the drawing the location at highest combined torsional and transverse shear stress. Make a three-dimensional Mohr-circle representation of the stresses at this point.

Schematic and Given Data:



Assumptions:

1. The weight of the hand crank is negligible.
2. The effect of the stress concentration is negligible.
3. The crank material is homogeneous and perfectly elastic.

Analysis:

1. For bending, Eq. (4.8)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(300\text{ mm})(1000\text{ N})}{\pi(25\text{ mm})^3} = 195.6\text{ MPa}$$

2. For torsion, Eq. (4.4)

$$\tau = \frac{16T}{\pi d^3} = \frac{16(250\text{ mm})(1000\text{ N})}{\pi(25\text{ mm})^3} = 81.5\text{ MPa}$$

3. For transverse shear, Eq. (4.13)

$$\tau_{tr} = \frac{4V}{3A} = \frac{4(1000\ \text{N})}{3\pi(12.5\ \text{mm})^2} = 2.7\ \text{MPa}$$

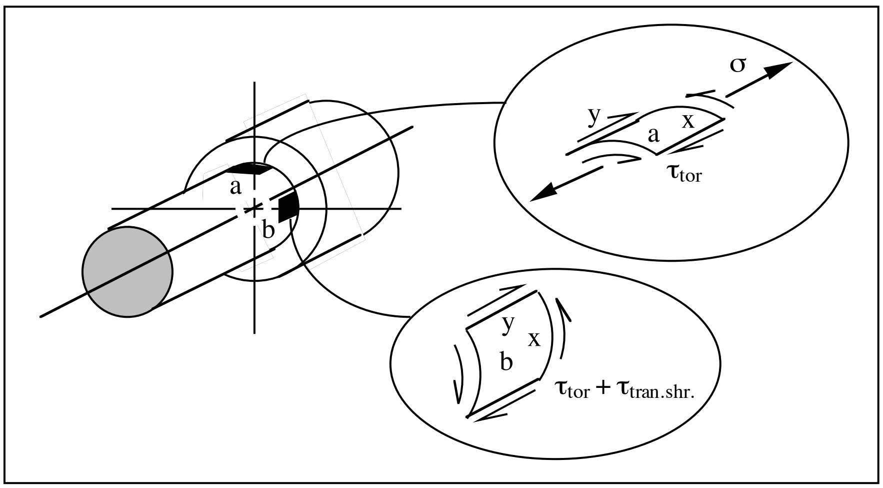
4. From the Mohr circle for "a",

τ_{max} = 128 MPa, σ_{max} = 225 MPa.

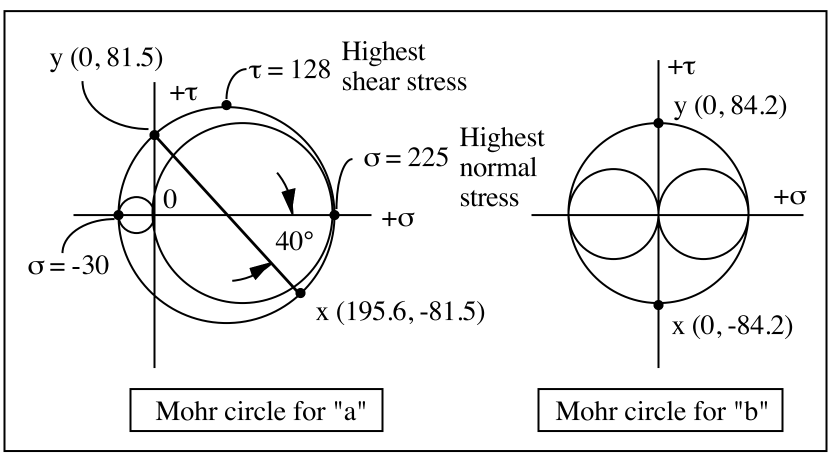
From the Mohr circle for "b",

τ_{max} = 84.2 MPa, σ_{max} = 84.2 MPa.

5.



6.

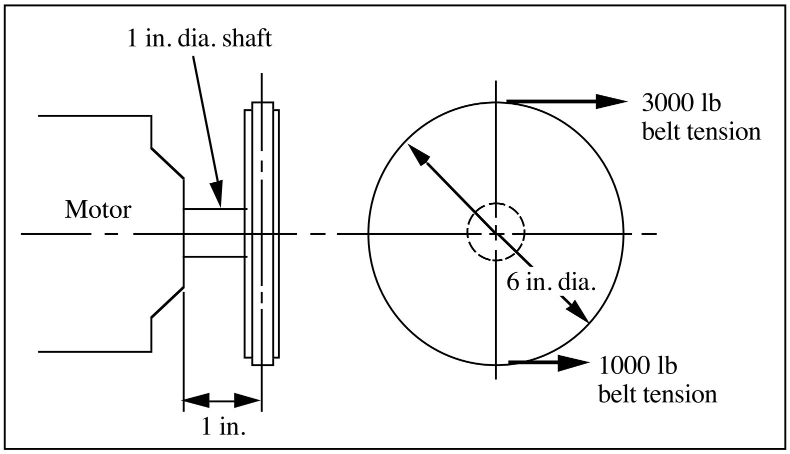


SOLUTION (4.51)

Known: An electric motor is loaded by a belt drive.

Find: Copy the drawing and show on both views the location or locations on the shaft of the highest stress. Make a complete Mohr-circle representation of the stress at this location.

Schematic and Given Data:



Assumptions:

1. The weight of the structure is negligible.
2. The effect of stress concentration is negligible.
3. The shaft material is homogeneous and perfectly elastic.

Analysis:

1. For torsion, Eq. (4.4)

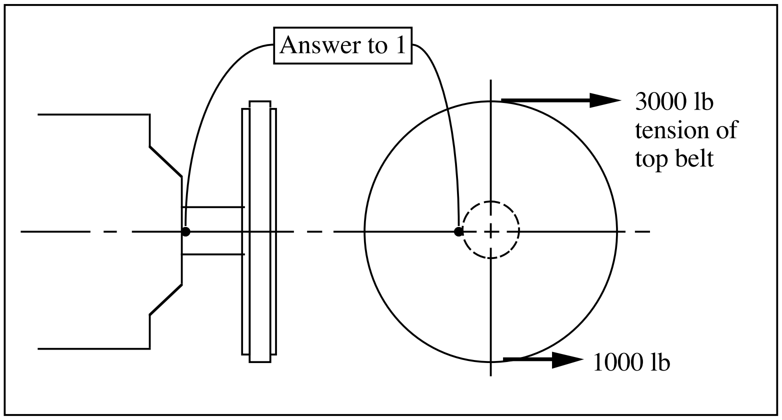
$$\tau = \frac{16T}{\pi d^3} = \frac{16(2000\ \text{lb})(3\ \text{in.})}{\pi(1\ \text{in.})^3} = 31\ \text{ksi}$$

2. For bending, Eq. (4.8)

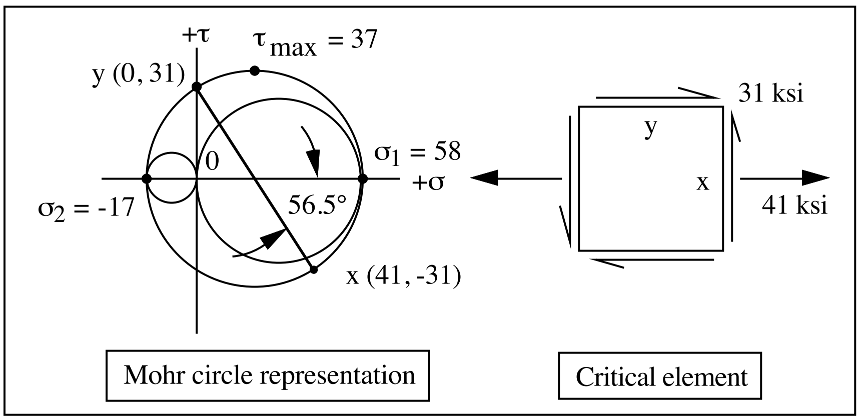
$$\sigma = \frac{32M}{\pi d^3} = \frac{32(4000\ \text{lb})(1\ \text{in.})}{\pi(1\ \text{in.})^3} = 41\ \text{ksi}$$

3. The Mohr circle representation is given above.

4.



5.

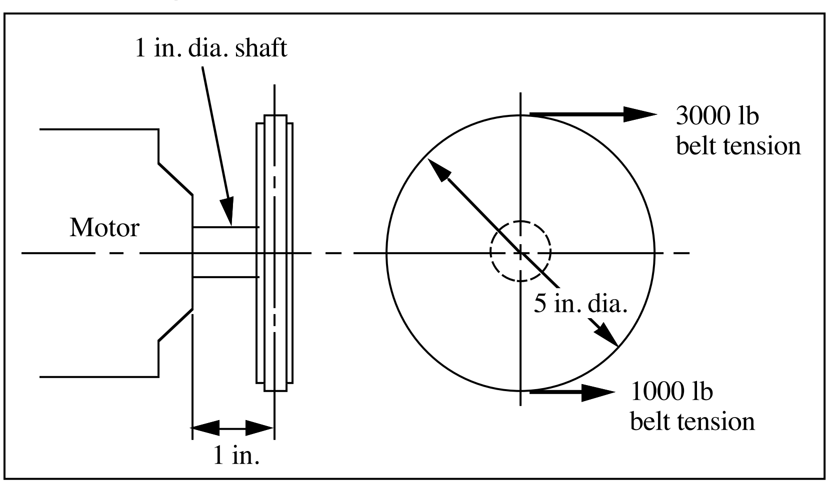


SOLUTION (4.52)

Known: An electric motor is loaded by a belt drive.

Find: Copy the drawing and show on both views the location or locations of the highest stress on the shaft. Make a complete Mohr-circle representation of the stress at this location.

Schematic and Given Data:



Assumptions:

1. The weight of the structure is negligible.
2. The effect of stress concentration is negligible.
3. The shaft material is homogeneous and perfectly elastic.

Analysis:

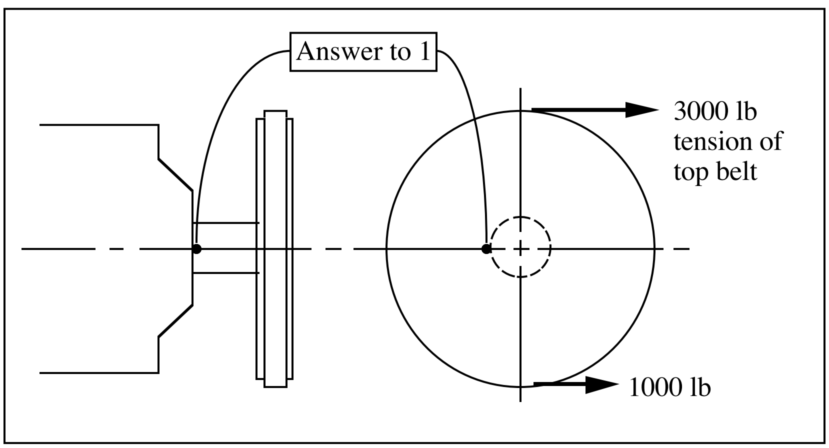
1. For torsion, Eq. (4.4)

$$\tau = \frac{16T}{\pi d^3} = \frac{16(2000\ \text{lb})(2.5\ \text{in.})}{\pi(1\ \text{in.})^3} = 25\ \text{ksi}$$

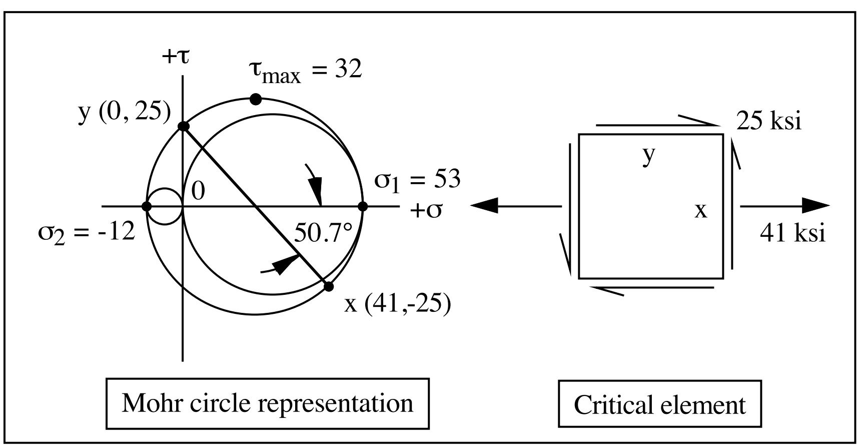
2. For bending, Eq. (4.8)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(4000\ \text{lb})(1\ \text{in.})}{\pi(1\ \text{in.})^3} = 41\ \text{ksi}$$

3. The Mohr circle representation is given below.
4.



5.

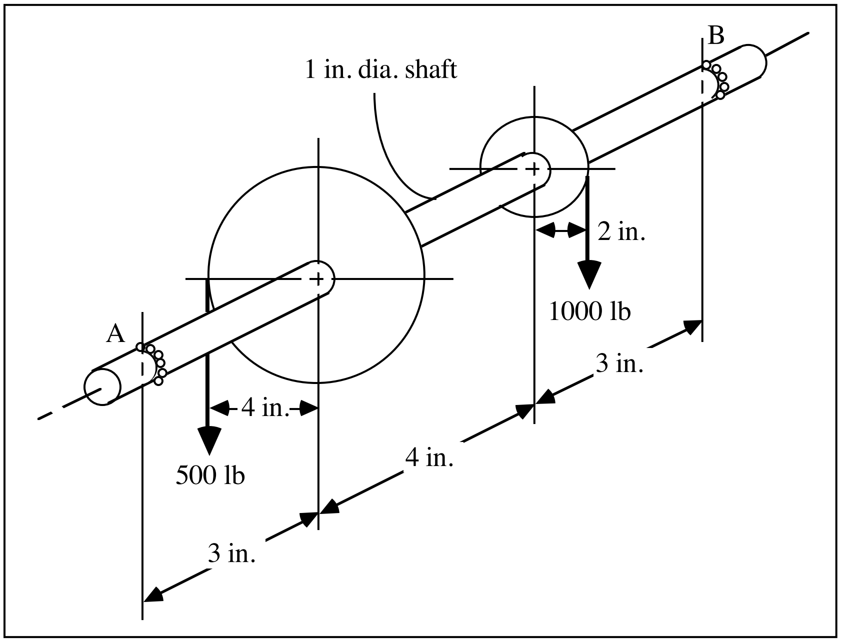


SOLUTION (4.53)

Known: A solid round shaft with a known diameter is supported by self-aligning bearings at A and B. Two chain sprockets that are transmitting a load are attached to the shaft.

Find: Identify the specific shaft location subjected to the most severe state of stress, and make a Mohr-circle representation of this stress state.

Schematic and Given Data:

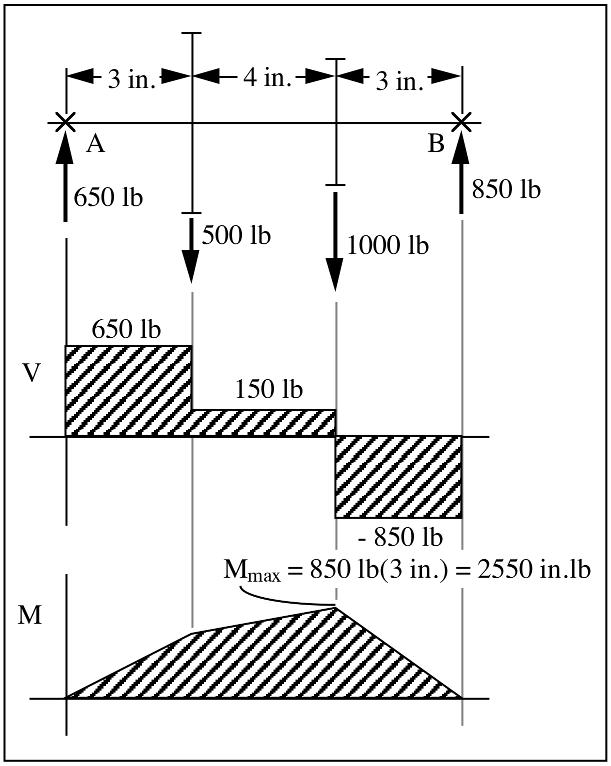


Assumptions:

1. The loads are static.
2. Stress concentrations can be ignored.
3. The shaft is straight.
4. The shaft material is homogeneous and perfectly elastic.

Analysis:

1.



$[\Sigma M_A = 0]$

$1000(7) + 500(3) = F_B(10)$

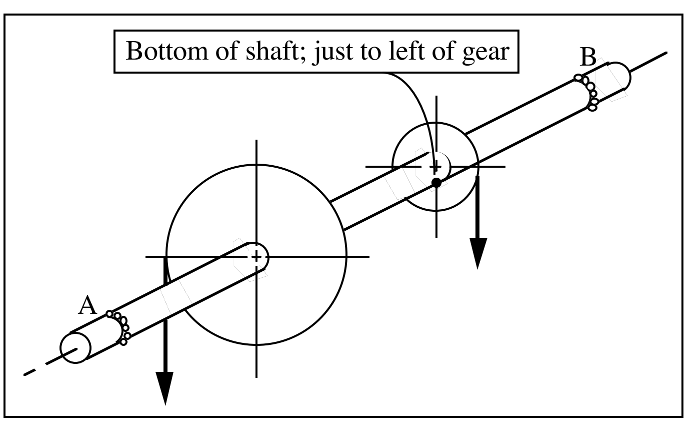
$F_B = 8500/10 = 850$ lb

$[\Sigma F_v = 0]$

$F_A = 500 + 1000 - 850$

$= 650$ lb

2.



The location subjected to the most severe state of stress is at the bottom of the shaft, just to the left of the smaller gear.

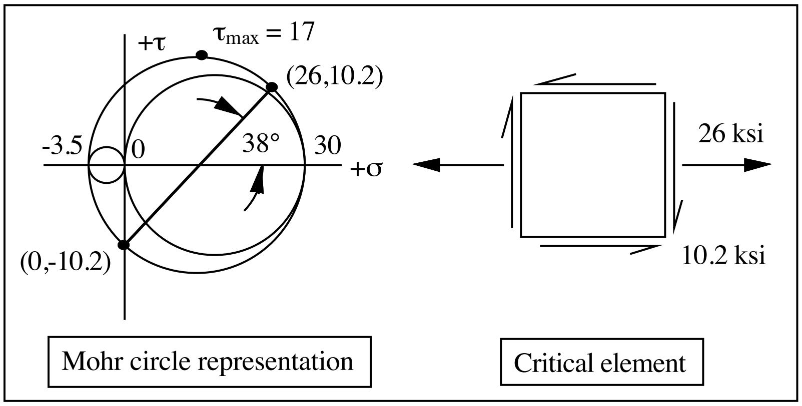
3. For bending, Eq. (4.8)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(2550)}{\pi(1)^3} = 26 \text{ ksi}$$

For torsion, Eq. (4.4)

$$\tau = \frac{16T}{\pi d^3} = \frac{16(2000)}{\pi(1)^3} = 10.2 \text{ ksi}$$

4.

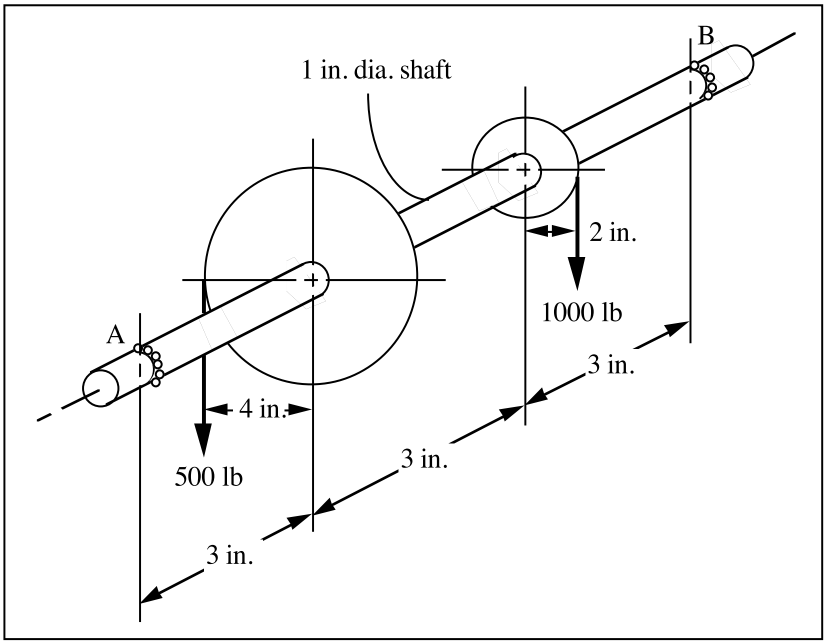


SOLUTION (4.54)

Known: A solid round shaft with a known diameter is supported by self-aligning bearings at A and B. Two chain sprockets that are transmitting a load are attached to the shaft.

Find: Identify the specific shaft location subjected to the most severe state of stress, and make a Mohr-circle representation of this stress state.

Schematic and Given Data:

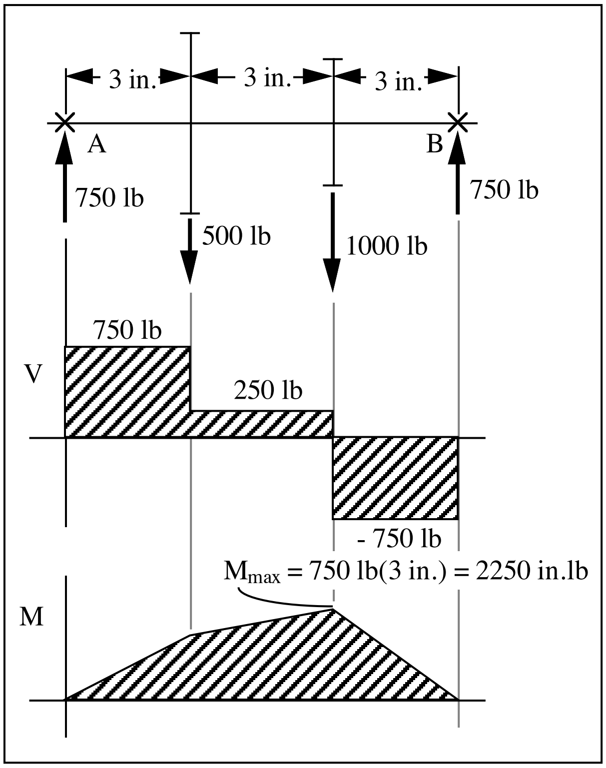


Assumptions:

1. The loads are static.
2. Stress concentrations can be ignored.
3. The shaft is straight.
4. The shaft material is homogeneous and perfectly elastic.

Analysis:

1.



$\left[\Sigma M_A = 0\right]$

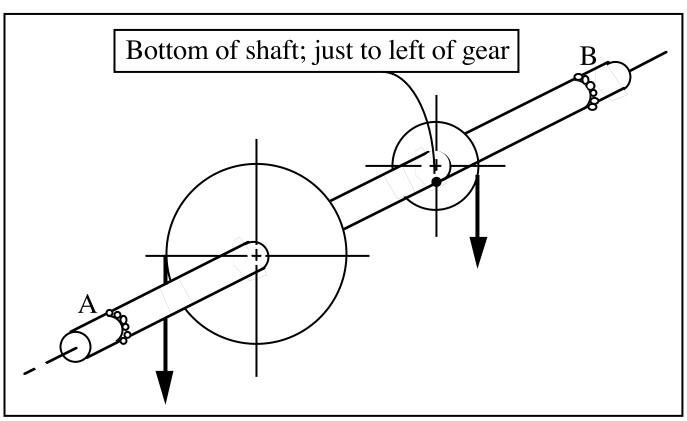
$$1000(6) + 500(3) = F_B(10)$$

$$F_B = 7500/10 = 750 \text{ lb}$$

$\left|\Sigma F_v = 0\right|$

$$F_A = 500 + 1000 - 750 = 750 \text{ lb}$$

2.



The location subjected to the most severe state of stress is at the bottom of the shaft, just to the left of the smaller gear.

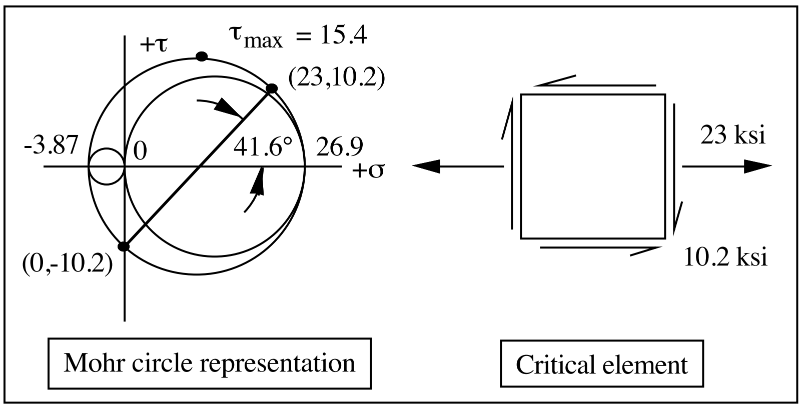
3. For bending, Eq. (4.8)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(2250)}{\pi(1)^3} = 22.92 = 23 \text{ ksi}$$

For torsion, Eq. (4.4)

$$\tau = \frac{16T}{\pi d^3} = \frac{16(2000)}{\pi(1)^3} = 10.2 \text{ ksi}$$

4.

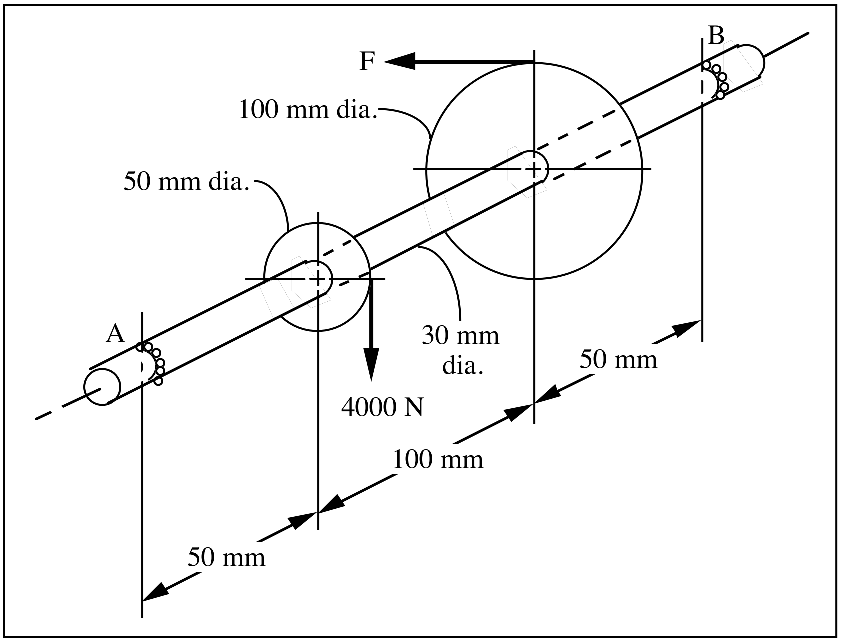


SOLUTION (4.55)

Known: A solid round shaft with a known diameter is supported by self-aligning bearings at A and B. Two chain sprockets that are transmitting a load are attached to the shaft.

Find: Identify the specific shaft location subjected to the most severe state of stress, and make a Mohr-circle representation of this stress state.

Schematic and Given Data:

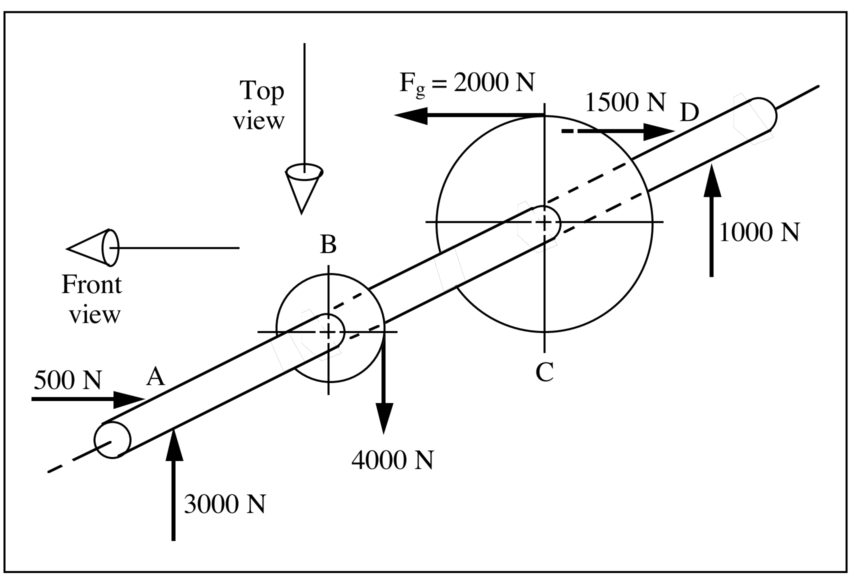


Assumptions:

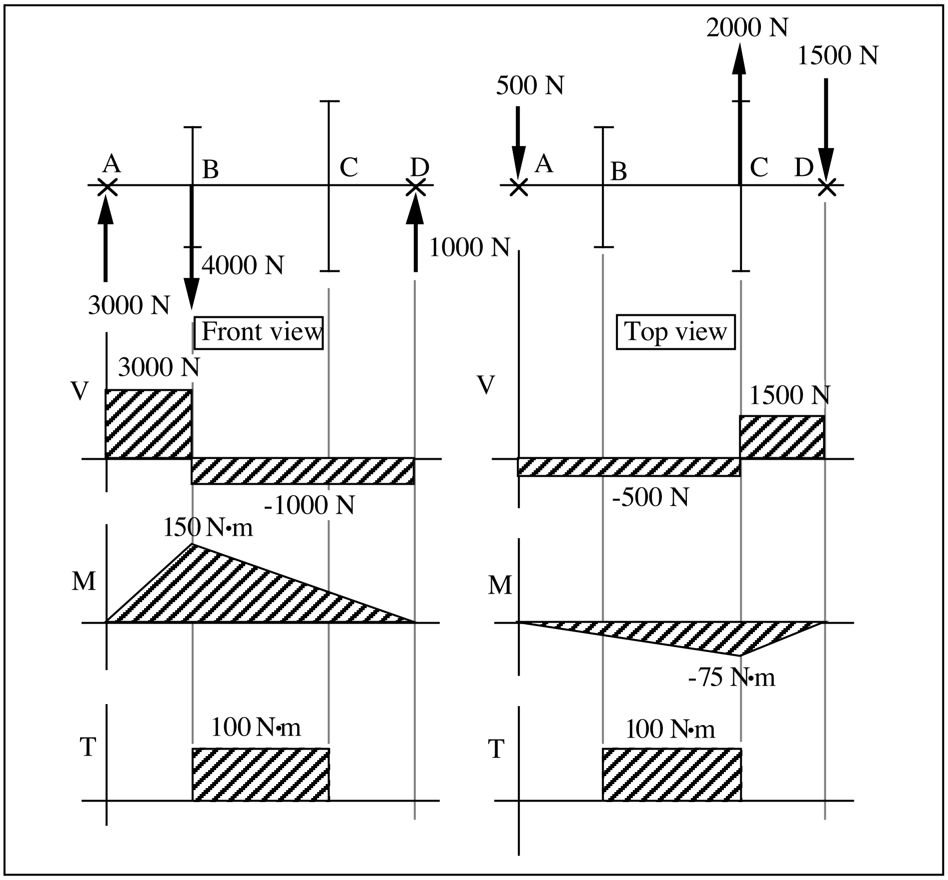
1. The loads are static.
2. Stress concentration can be ignored.
3. The shaft is straight.
4. The shaft material is homogeneous and perfectly elastic.

Analysis:

1.



2.



3. The most severe state of stress is at B

$$M = \sqrt{[(3000)(50)]^2 + [(500)(50)]^2} = 152{,}069 \text{ N·mm}$$

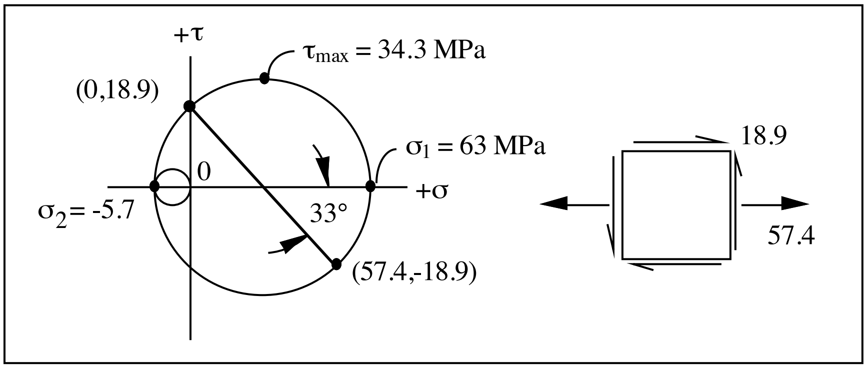
$$T = 4000\ (25) = 100{,}000 \text{ N·mm}$$

4. For bending, Eq. (4.8)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(152{,}069)}{\pi(30)^3} = 57.4 \text{ MPa}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(100{,}000)}{\pi(30)^3} = 18.9 \text{ MPa}$$

5.



Mohr circle representation of point B

SOLUTION (4.56)

Known: A small pressurized cylinder is attached at one end and loaded with a pipe wrench at the other. The stresses due to the internal pressure and the pipe wrench are known.

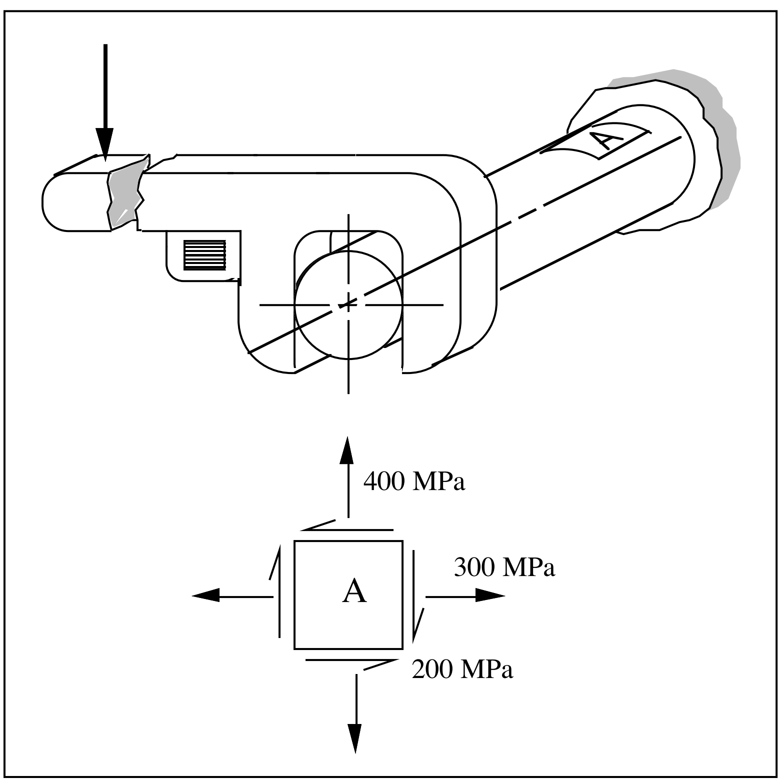
Find:

(a) Draw a Mohr-circle representation of the state of stress at point A.

(b) Determine the magnitude of the maximum shear stress at A.

(c) Sketch the orientation of a principal element, and show all stresses acting on it.

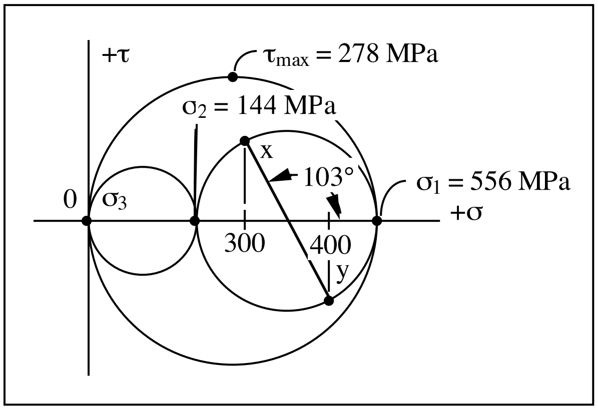
Schematic and Given Data:



Assumption: The "positive-clockwise" rule is used.

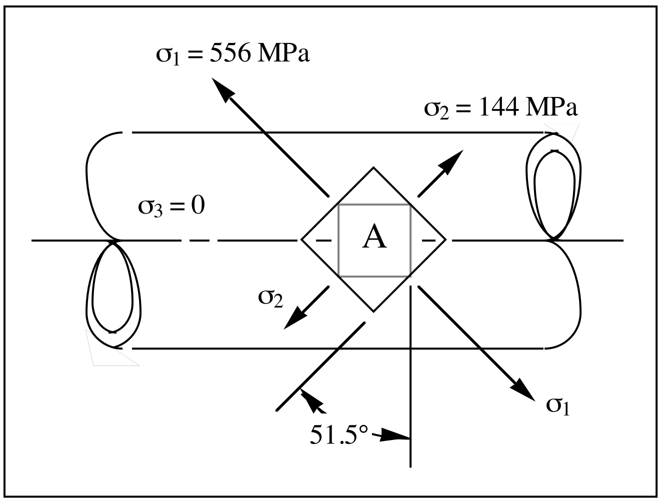
Analysis:

1.



2. The maximum shear stress at A is τ_{max} = 278 MPa ■

3.



SOLUTION (4.57)

Known: A small pressurized cylinder is attached at one end and loaded with a pipe wrench at the other. The stresses due to the internal pressure and the pipe wrench are known.

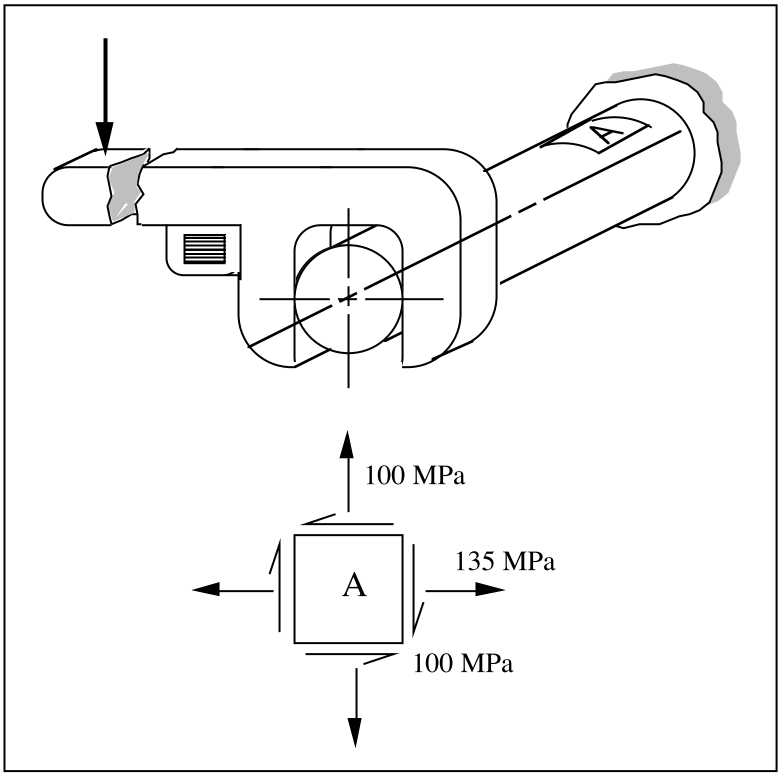
Find:

(a) Draw a Mohr-circle representation of the state of stress at point A.

(b) Determine the magnitude of the maximum shear stress at A.

(c) Sketch the orientation of a principal element, and show all stresses acting on it.

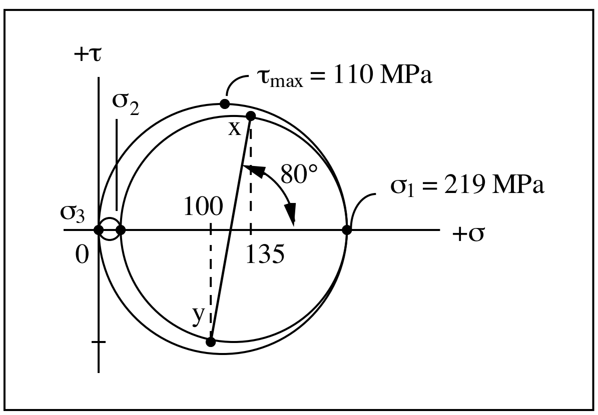
Schematic and Given Data:



Assumption: The "positive-clockwise" rule is used.

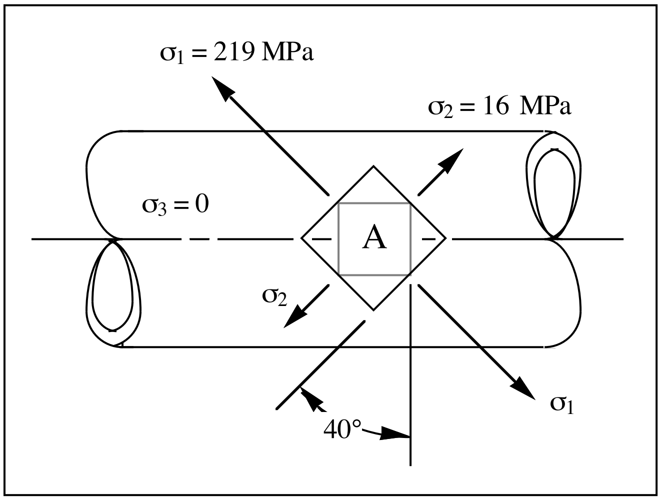
Analysis:

1.



2. The maximum shear stress at A is τ_{max} = 110 MPa ■

3.



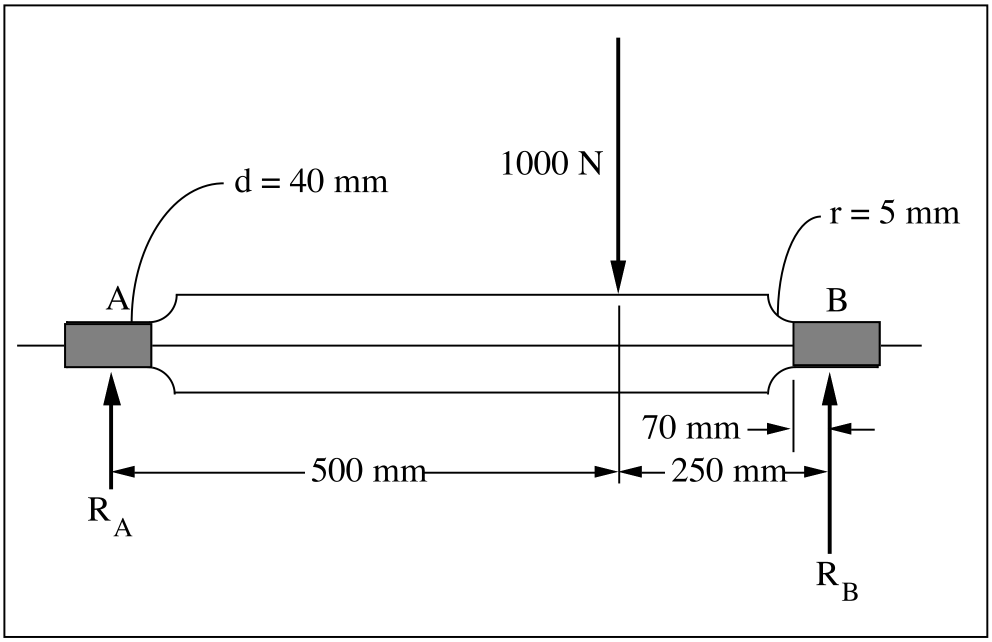
Comment: At the outer surface of an internally pressurized cylinder, the tangential stress is analytically twice the axial stress. That is, the axial stress for a thick walled cylinder is $\sigma_a = \dfrac{p_i r_i^2}{r_o^2 - r_i^2}$ and the tangential stress is $\sigma_t = \dfrac{2p_i r_i^2}{r_o^2 - r_i^2}$. Therefore, $\dfrac{\sigma_t}{\sigma_a} = 2$. We could speculate in this problem that a stress concentration existed which increased the axial stress from 50 MPa to 60 MPa.

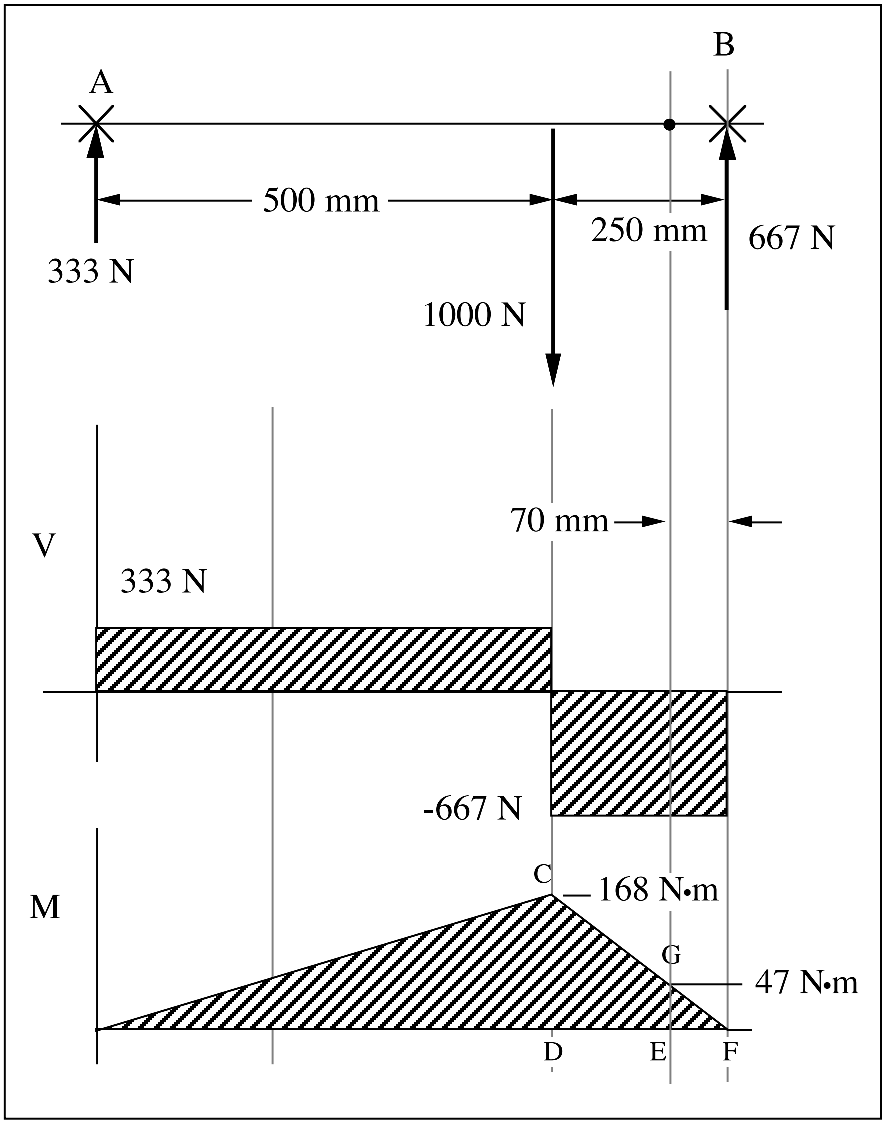
SOLUTION (4.77)

Known: A stepped shaft with known dimensions is supported by bearings and carries a known load.

Find: Determine the maximum stress at the shaft fillet.

Schematic and Given Data:





Assumptions:

1. The shaft remains straight .
2. The material is homogeneous and perfectly elastic.

Analysis:

1. $\Sigma M_B = 0$: Hence $R_A = 333$ N
   $\Sigma F_Y = 0$: $R_A + R_B = 1000$, Therefore $R_B = 667$ N
2. From similar triangle, ΔCDF and ΔGEF,
   GE = 47 N•m. The stress due to bending at the critical shaft fillet is equal to
   $\sigma_{nom} = \dfrac{32M}{\pi d^3} = \dfrac{32(47)}{\pi(0.04)^3} = 7.5$ MPa.
3. r/d for the critical shaft fillet = 5/40 = 0.125
   D/d = 80/40 = 2
   From Fig. (4.35a), $K_t = 1.65$
   Therefore, $\sigma_{max} = \sigma_{nom} K_t = 7.5(1.65) = 12.4$ MPa

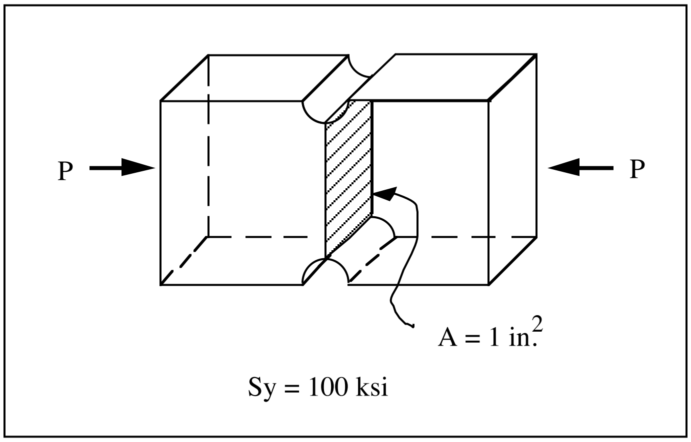
SOLUTION (4.88)

Known: Three notched tensile bars (see Fig. 4.39) have stress-concentrations of 1, 1.5 and 2.5 respectively. Each is made of ductile steel and have S_y = 100 ksi, a rectangular cross-section with a minimum area of 1 in.^{2}, and is initially free of residual stress.

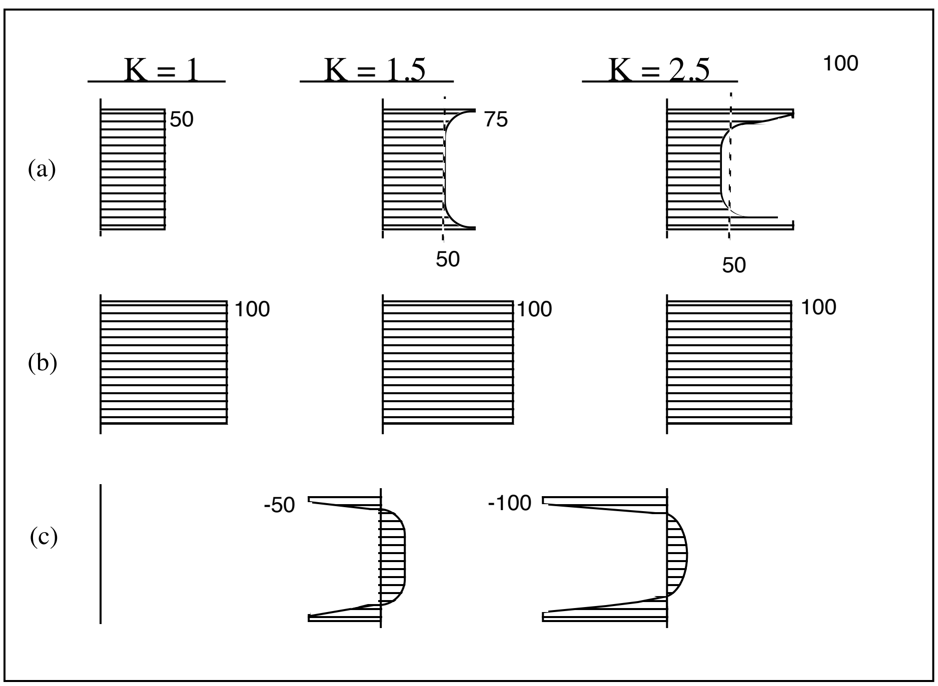
Find: Draw the shape of the stress-distribution curve for each case when

(a) A tensile load of 50,000 lb is applied.
(b) The load is increased to 100,000 lb.
(c) The load is removed.

Schematic and Given Data:



Analysis:



SOLUTION (4.89)

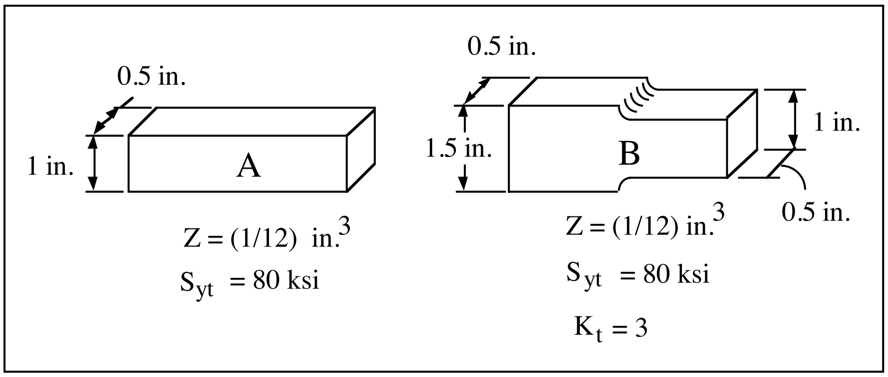
Known: Two rectangular steel beams having a known tensile yield strength are loaded in bending and Z = I/c is known. The dimensions and a stress concentration factor are given for both beams.

Find:

(a) For each beam, determine what moment, M, causes (1) initial yielding, and (2) complete yielding.

(b) Beam A is loaded to cause yielding to a depth of 1/4 in. Determine and plot the distribution of residual stresses which remain after the load is removed.

Schematic and Given Data:



Assumptions:

1. The idealized stress-strain curve is appropriate.
2. The beam is homogeneous.
3. There are no residual stresses initially.

Analysis:

1. For initial yielding, using Eqs. (4.7) and (4.21),

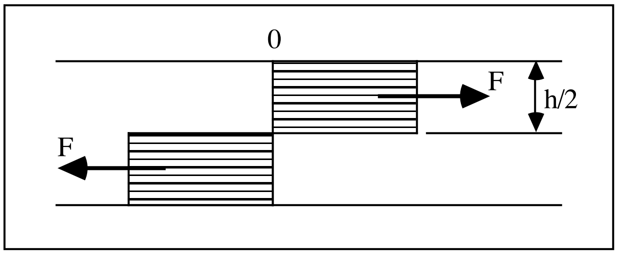
$$\text{Beam A: } \sigma = S_{yt} = \frac{M}{Z}$$

$$M = S_{yt}\,Z = 80{,}000\left(\frac{1}{12}\right) = 6667 \text{ in·lb} \qquad \blacksquare$$

$$\text{Beam B: } \sigma = S_{yt} = \frac{M}{Z}\,K_t$$

$$M = \frac{S_{yt}\,Z}{K_t} = \frac{80{,}000\left(\frac{1}{12}\right)}{3} = 2222 \text{ in·lb} \qquad \blacksquare$$

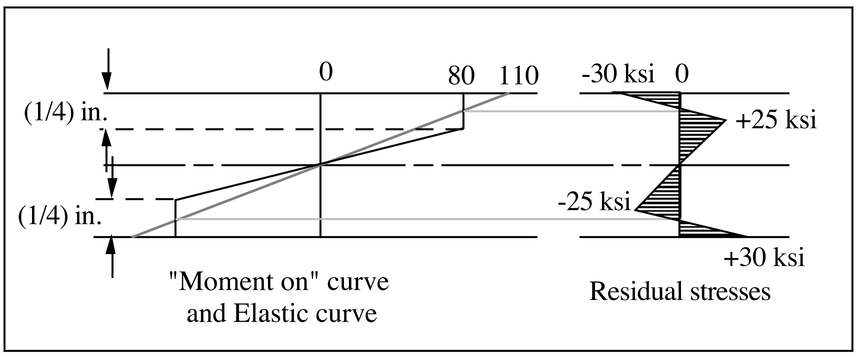
2. For complete yielding,



Thus, for both beams,

$$M = F\left(\frac{h}{2}\right) = \left[(S_{yt})\left(\frac{h}{2}\right)(b)\right]\left(\frac{h}{2}\right) = \left[(80{,}000)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right]\left(\frac{1}{2}\right) = 10{,}000 \text{ in.lb}$$ ■

3. For loading beam A to yield the beam to a depth of 1/4 in. and then releasing the load we have:



4. The moment to yield the beam to a depth of 1/4 in. is given by
M = (stress)(area)(mom. arm) + (avg. stress)(area)(mom. arm)

$$= 80{,}000\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right) + \left(\frac{80{,}000}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = 9167 \text{ in}\bullet\text{lb}$$

5. The elastic stress for M = 9167 in•lb is calculated as

$$\sigma_{elastic} = \frac{M}{Z} = \frac{9167}{\frac{1}{12}} = 110 \text{ ksi}$$

6. The residual stresses are shown on the right side in the above figure.

Comment: An overload causing yielding produced residual stresses that are favorable to future loads in the same direction and unfavorable to future loads in the opposite direction.

SOLUTION (4.90)

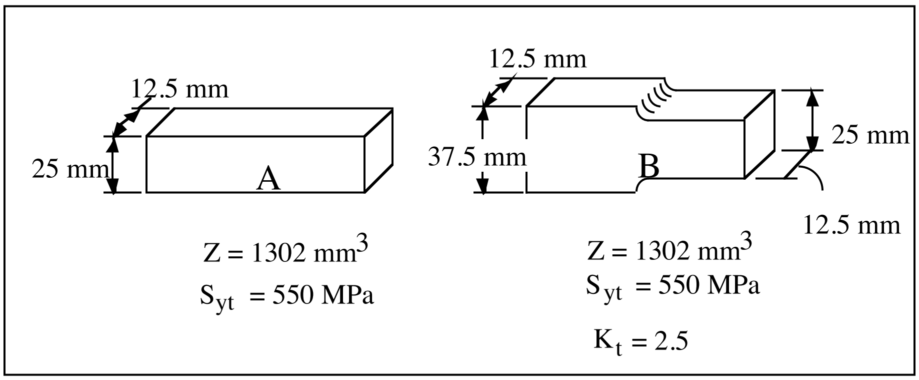
Known: Two rectangular steel beams having a known tensile yield strength are loaded in bending and Z = I/c is known. The dimensions and a stress concentration factor are given for both beams.

Find:

(a) For each beam, determine what moment, M, causes (1) initial yielding and (2) complete yielding.

(b) Beam A is loaded to cause yielding to a depth of 6.35 mm. Determine and plot the distribution of residual stresses that remain after the load is removed.

Schematic and Given Data:



Assumptions:

1. The idealized stress-strain curve is appropriate.
2. The beam is homogenous.
3. There are no residual stresses initially.

Analysis:

1. For initial yielding, using Eqs. (4.7) and (4.21),

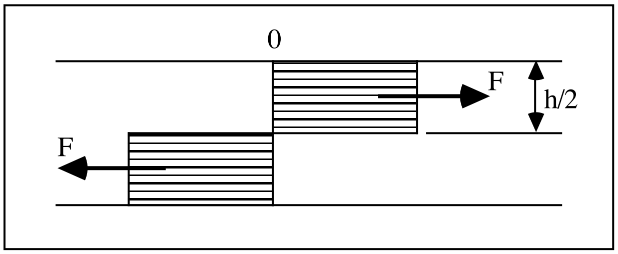
Beam A: $\sigma = S_{yt} = \dfrac{M}{Z}$

$$M = S_{yt} Z = (550 \text{ MPa})(1302 \text{ mm}^3) = 716.1 \text{ N·m} \quad ■$$

Beam B: $\sigma = S_{yt} = \dfrac{M}{Z} K_t$

$$M = \frac{S_{yt} Z}{K_t} = \frac{550 \text{ MPa}\left(1302 \text{ mm}^3\right)}{2.5} = 286.44 \text{ N·m} \quad ■$$

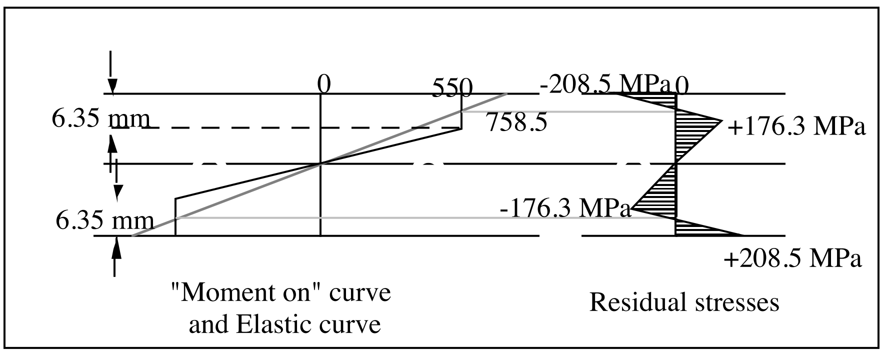
2. For complete yielding,



Thus, for both beams,

$$M = F\left(\frac{h}{2}\right) = \left[(S_{yt})\left(\frac{h}{2}\right)(b)\right]\left(\frac{h}{2}\right) = \left[(550\text{ MPa})\left(\frac{25}{2}\right)(12.5)\right]\left(\frac{25}{2}\right) = 1074.22\text{ N·m} \qquad ■$$

3. For loading beam A to yield the beam to a depth of 6.35 mm and then releasing the load we have:



4. The moment to yield the beam to a depth of 6.35 mm is given by
M = (stress)(area)(mom. arm) + (avg. stress)(area)(mom. arm)

$$= 550\text{ MPa}(6.35\text{ mm})(12.5\text{ mm})(18.65\text{ mm})$$

$$+\left(\frac{550\text{ MPa}}{2}\right)(6.15\text{ mm})(12.5\text{ mm})(8.20\text{ mm})$$

$M = 987.54$ N·m

5. The elastic stress for $M = 987.54$ N·m is calculated as

$$\sigma_{elastic} = \frac{M}{Z} = \frac{987.54\text{ N·m}}{1302\text{ mm}^3} = 758.5\text{ MPa}$$

6. The residual stresses are shown on the right side in the above figure.

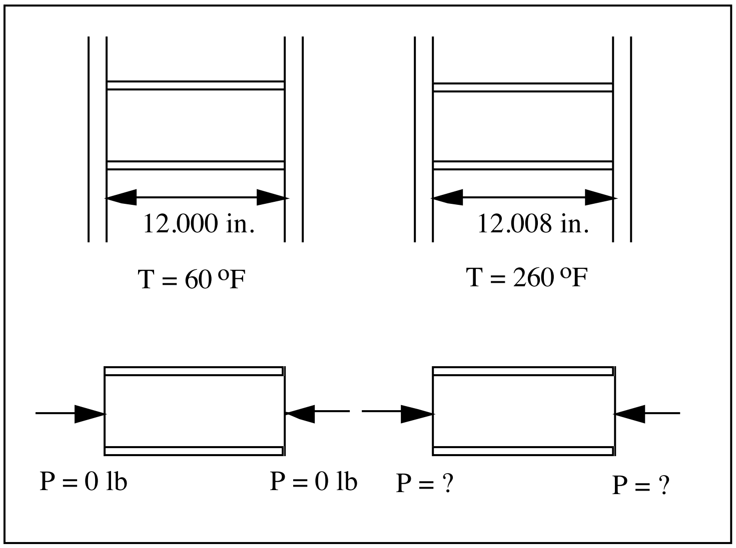
Comment: An overload causing yielding produced residual stresses that are favorable to future loads in the same direction and unfavorable to future loads in the opposite direction.

SOLUTION (4.91)

Known: A 12 in. length of aluminum tubing with a cross-sectional area of 1.5 in.^{2} expands 0.008 in. from a stress-free condition at 60°F when the tube is heated to a uniform 260°F.

Find: Determine the end loads on the aluminum tubing loads and the resultant compressive stresses.

Schematic and Given Data:



Assumptions:

1. The tube material is homogenous and isotropic.
2. The material stresses remain within the elastic range.
3. No local or column bending occurs.

Analysis:

1. For the unrestrained tube

$$\in = \alpha\Delta T = \left(12\times10^{-6}/^{\circ}F\right)\left(200\ ^{\circ}F\right) = 2.4x10^{-3}\ \frac{in.}{in.}$$

$$\Delta L = L\in = 12\ in.\left(2.4\times10^{-3}\right) = 0.0288\ in.$$

2. Since the measured expansion was only 0.008 in., the constraints must apply forces sufficient to produce a deflection of 0.0208 in. From the relationship

$$\delta = \frac{PL}{AE}$$

which is from elementary elastic theory, where δ = 0.0208 in., L = 12.000 in., A = 1.5 in.^{2}, and E = 10.4 × 10^6 ksi.
With substitution we have

$$0.0208 \text{ in.} = \frac{P(12.000 \text{ in.})}{(1.5 \text{ in.}^2)(10.4\times10^6 \text{ psi})}$$

yielding P = 27,040 lb

3. The resultant stress is, $\sigma = \frac{P}{A} = \frac{27{,}040 \text{ lb}}{1.5 \text{ in.}^2} = 18{,}027 \text{ psi}$ ■

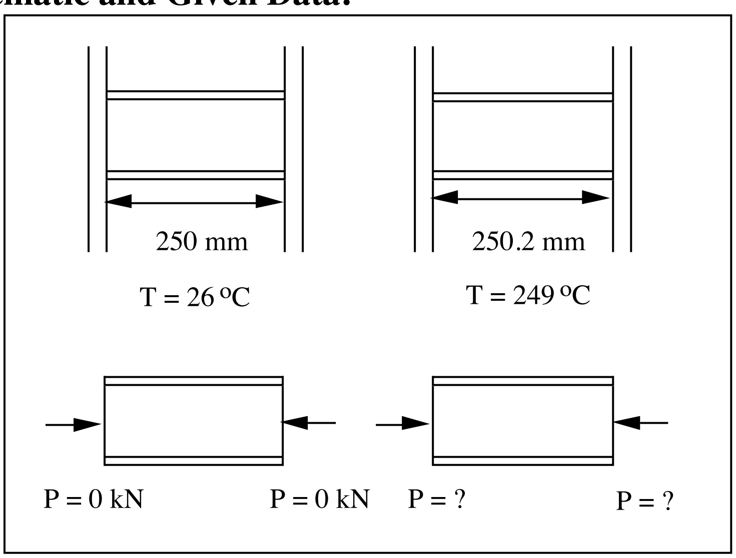
Comment: Since these answers are based on elastic relationships, they are valid only if the material has a yield strength of at least 18.03 ksi at 260°F.

SOLUTION (4.92)

Known: A 250 mm length of steel tubing with a cross-sectional area of 625 mm^2 expands longitudinally 0.20 mm from a stress-free condition at 26°C when the tube is heated to a uniform 249°C.

Find: Determine the end loads on the steel tubing and the resultant internal stresses.

Schematic and Given Data:



Assumptions:

1. The tube material is homogenous and isotropic.
2. The material stresses remain within the elastic range.

Analysis:

1. For the unrestrained tube

   $\in = \alpha \Delta T = \left(12\times10^{-6}\right)\left(249\text{-}26\right) = 2.68\times10^{-3}$

   $\Delta L = L\in = 250 \text{ mm} \left(2.68\times10^{-3}\right) = 0.669 \text{ mm}$

2. Since the measured expansion was only 0.20 mm, the constraints must apply forces sufficient to produce a deflection of 0.469 mm. From the relationship

   $$\delta = \frac{PL}{AE}$$

   which is from elementary elastic theory, where $\delta = 0.469$ mm, $L = 250$ mm, and

   $A = 625 \text{ mm}^2 = 625 \text{ mm}^2 \times \frac{1 \text{ m}}{1000 \text{ mm}} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 0.000625 \text{ m}$

   $E = 207\times10^{9} \text{ Pa} = 207\times10^{9} \frac{N}{m^2} = 207\times10^{3} \frac{N}{mm^2}$

   Therefore, $0.469 \text{ mm} = \dfrac{P\left(250 \text{ mm}\right)}{\left(625 \text{ mm}^2\right)\left(207\times10^{3} \dfrac{N}{mm^2}\right)}$

   and P = 242,707 N

3. The resultant stress is, $\sigma = \dfrac{242{,}707 \text{ N}}{625 \text{ mm}^2} = 388 \text{ MPa}$

■

Comment: Since these answers are based on elastic relationships, they are valid only if the material has a yield strength of at least 388 MPa at 249°C.
