## SOLUTION (6.19)

**Known:** A machine frame made of steel having known  $S_y$  and  $S_{sy}$  is loaded in a test fixture. The principal stresses at two critical points on the surface are known.

Find: Compute the test load at which the frame will experience initial yielding according to the

- (a) maximum-normal-stress theory
- (b) maximum-shear-stress theory
- (c) maximum-distortion-energy theory

Discuss the relative validity of each theory for this application. Compute the value of test load at which yielding would commence.

## **Schematic and Given Data:**





**Assumption:** The material is homogeneous.

# Analysis:<br>1. For tl

For the maximum-normal-stress theory, the  $\sigma_1$  -  $\sigma_2$  plot shows point a to be critical. Failure is predicted at

$$
Load = 4 kN \left| \frac{400 MPa}{200 MPa} \right| = 8 kN
$$

2. For maximum-shear-stress theory, the  $\sigma_1$  -  $\sigma_2$  plot shows point b to be critical. Failure is predicted at

Load = 4 kN  $\left| \frac{240 \text{ MPa}}{150 \text{ MPa}} \right|$  = 6.4 kN

3. For maximum-distortion-energy theory, the  $\sigma_1$  -  $\sigma_2$  plot shows point b to be critical. Failure is predicted at  $(275 \text{ MP})$ 

■

Load = 
$$
4 \text{ kN} \left| \frac{275 \text{ MPa}}{150 \text{ MPa}} \right| = 7.3 \text{ kN}
$$
  
More precisely, from Eq. (6.7),  
 $\sigma_e = [(150)^2 + (-100)^2 - (150) (-100)]^{1/2} = 218 \text{ MPa}$   
Thus, failure is predicted at

$$
Load = 4 kN \left| \frac{400 MPa}{218 MPa} \right| = 7.3 kN
$$

## **Comment:**

- 1. Maximum normal stress theory should not be used for this application since it gives good results only for brittle fractures.
- 2. Maximum shear stress theory may be used but is not very accurate.<br>3. Maximum distortion energy theory will give the best results for this
- 3. Maximum distortion energy theory will give the best results for this application.<br>4. Yielding is expected to begin at a load of 7.3 kN.
- Yielding is expected to begin at a load of 7.3 kN.

## SOLUTION (6.20)

**Known:** A machine component with given critical stresses is ductile, with yield strengths in tension and compression of 60 ksi.

Find: Determine the safety factor according to:

- (a) the maximum-normal-stress theory
- (b) the maximum-shear-stress theory
- (c) the maximum-distortion-energy theory

## **Schematic and Given Data:**



**Assumption:** The material is homogeneous.

- 1. From the above Mohr-circle,  $\tau_{\text{max}} = (20 + 15)/2 = 17.5$  ksi
- 2. (a) For the maximum-normal-stress theory:  $SF = 60/20 = 3.0$ 
	- (b) For the maximum-shear-stress theory:  $SF = 30/\tau_{max} = 30/17.5 = 1.72$
	- (c) For the maximum-distortion-energy theory:  $SF = 60/S'$ , where from Eq. (6.6)  $S' = [\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2]^{1/2}$  $=[(20)^{2} - (20)(-15) + (-15)^{2}]^{1/2} = 30.5$ thus,  $SF = 60/30.5 = 1.97$
- 3. The existence of a yield strength implies a ductile material for which:
	- maximum-distortion-energy theory is best
	- maximum-shear-stress theory may be acceptable
	- maximum-normal-stress theory is not appropriate

## SOLUTION (6.21)

**Known:** A machine component with given critical stresses is ductile, with yield strengths in tension and compression of 60 ksi.

Find: Determine the safety factor according to:

- (a) the maximum-normal-stress theory
- (b) the maximum-shear-stress theory
- (c) the maximum-distortion-energy theory

## **Schematic and Given Data:**



6-27

**Assumption:** The material is homogeneous.

## **Analysis:**

- From the above Mohr-circle,
- $\tau_{\text{max}} = (25 + 15)/2 = 20$  ksi
- 2. (a) For the maximum-normal-stress theory:  $SF = 60/25 = 2.4$ 
	- (b) For the maximum-shear-stress theory:  $SF = 30/\tau_{\text{max}} = 30/20 = 1.5$
	- (c) For the maximum-distortion-energy theory:  $SF = 60/S'$ , where from Eq. (6.6)  $S' = [\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2]^{1/2}$  $= [(25)^{2} - (25)(-15) + (-15)^{2}]^{1/2} = 35.0$ thus,  $SF = 60/35.0 = 1.71$
- 3. The existence of a yield strength implies a ductile material for which:
	- maximum-distortion-energy theory is best
	- maximum-shear-stress theory may be acceptable
	- maximum-normal-stress theory is not appropriate

## SOLUTION (6.22)

**Known:** Five states of biaxial stress are given.

**Find:** Based on using three different failure theories, list the five states in order of increased likelihood of causing failure.

## **Schematic and Given Data:**







- 1. For the maximum-normal-stress theory  $SF = S_y/\sigma_{max}$ , and all stress states have the same safety factor.
- 2. For the maximum-shear-stress theory,  $SF = S_{sy}/\tau_{max}$ , and the order of decreased safety factor would be  $1 \& 4, 2, 3 \& 5.$
- 3. For the distortion energy theory,  $SF = S_y/\sigma_e$  where from Eq. (6.6),  $\sigma_e = (\sigma_1^2 + \sigma_2^2)$  $-\sigma_1\sigma_2$ <sup>1/2</sup>. The order of decreased safety factor is 4, 1, 2, 3 & 5.

**Comments:** For  $S_y = 80$ , the accompanying table lists the safety factors.



## Table of Safety Factors

### SOLUTION (6.23)

**Known:** The solutions to problems (a) 4.46, (b) 4.49, (c) 4.51, (d) 4.53, (e) 4.55, (f) 4.56, (g) 4.58, (h) 4.62, and (i) 4.67 are given.

**Find:** Determine the tensile strength a ductile material must have in order to provide a safety factor of 2 with respect to initial yielding at the locations investigated in the above listed problems. Determine the answer using both the maximum-shear-stress theory and the maximum-distortion-energy theory.

**Assumption:** The materials are homogeneous.

#### **Analysis:**



### SOLUTION (6.24)

**Known:** The solutions to problems (a) 4.46, (b) 4.49, (c) 4.51, (d) 4.53, (e) 4.55, (f) 4.56, (g) 4.58, (h) 4.62, and (i) 4.67 are given.

**Find:** Determine the tensile strength a ductile material must have in order to provide a safety factor of 1.5 with respect to initial yielding at the location(s) investigated in the above listed problems. Determine the answer using both the maximum-shear-stress theory and the maximum-distortion-energy theory.

**Assumption:** The materials are homogeneous.



## SOLUTION (6.25)

**Known:** The solutions to problems (a) 4.46, (b) 4.49, (c) 4.51, (d) 4.53, (e) 4.55, (f) 4.56, (g) 4.58, (h) 4.62, and (i) 4.67 are given.

**Find:** Use the modified Mohr theory to determine the ultimate tensile strength that would be required of a brittle material in order to provide a safety factor of  $\overline{4}$  to a member subjected to the same state(s) of stress as the above listed problems. If overloaded to failure, what would be the orientation of the brittle crack in each case?

### **Assumptions:**

- 1. The materials are homogeneous.
- 2. The ultimate compressive strength is 3.5 times the ultimate tensile strength.

## **Analysis:**



## SOLUTION (6.26)

**Known:** The solutions to problems (a) 4.46, (b) 4.49, (c) 4.51, (d) 4.53, (e) 4.55, (f) 4.56, (g) 4.58, (h) 4.62, and (i) 4.67 are given.

**Find:** Use the modified Mohr theory to determine the ultimate tensile strength that would be required of a brittle material in order to provide a safety factor of 3.5 to a member subjected to the same state(s) of stress as the above listed problems. If overloaded to failure, what would be the orientation of the brittle crack in each case?

### **Assumptions:**

- 1. The materials are homogeneous.
- 2. The ultimate compressive strength is 3.5 times the ultimate tensile strength.



## SOLUTION (6.27)

**Known:** The surface of a steel machine member is subjected to known principal stresses.

**Find:** Determine the tensile yield strength required to provide a safety factor of 2 with respect to initial yielding.

## **Schematic and Given Data:**



**Assumption:** The material is homogeneous

- 1. Maximum-shear-stress theory: For σ<sub>1</sub> = 200 MPa, σ<sub>2</sub> = 100 MPa, σ<sub>3</sub> = 0  $\tau_{max} = (0 + 200)/2 = 100$  MPa. Thus, for  $SF = 2$ ,  $S_y = 400$  MPa is required.
- 2. Maximum-distortion-energy theory: From Eq. (6.6),

 $σ<sub>e</sub> = (σ<sub>1</sub>2 + σ<sub>2</sub>2 - σ<sub>1</sub>σ<sub>2</sub>)<sup>1/2</sup>$  $=[200^{2} + 100^{2} - (200)(100)]^{1/2} = 173.2$  MPa. Thus, for  $SF = 2$ ,  $S_y = 346.4$  MPa is required.

## SOLUTION (6.28)

**Known:** The surface of a steel machine member is subjected to known principal stresses.

**Find:** Determine the tensile yield strength required to provide a safety factor of 2 with respect to initial yielding.

### **Schematic and Given Data:**



**Assumption:** The material is homogeneous.

- 1. Maximum-shear-stress theory: For  $σ_1 = 300$  MPa,  $σ_2 = 100$  MPa,  $σ_3 = 0$  $\tau_{\text{max}} = (0 + 300)/2 = 150 \text{ MPa}.$ Thus, for  $SF = 2$ ,  $S_y = 600$  MPa is required.
- 2. Maximum-distortion-energy theory: From Eq. (6.6),  $\sigma_e = (\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2)^{1/2}$  $= [3002 + 1002 - (300)(100)]^{1/2} = 265$  MPa. Thus, for  $SF = 2$ ,  $S_y = 530$  MPa is required.

## SOLUTION (6.29)

**Known:** A round steel bar with  $S_y = 500$  MPa is loaded simultaneously with known axial tension and torsion stresses.

Find: Determine the safety factor.

## **Schematic and Given Data:**



## **Assumptions:**

- 
- 1. The rod material is ductile.<br>2. The material is homogeneo
- 2. The material is homogeneous.<br>3. The distortion energy theory is The distortion energy theory is preferred (it is more accurate than the other theories).

- From Eq.  $(6.8)$ , the distortion energy stress is  $\sigma_e = (\sigma_x^2 + 3 \tau_{XY}^2)^{1/2} = (50^2 + 3(100)^2)^{1/2}$  $\sigma_e = 180.3 \text{ MPa}$
- 2. The safety factor is  $SF = S_y/\sigma_e = 500/180.3 = 2.77$

**Comment:** The maximum shear stress theory predicts a safety factor

$$
SF = \frac{S_{sy}}{\tau_{\text{max}}} = \frac{S_y / 2}{\sqrt{\tau_{xy}^2 + (\frac{\sigma_x - \sigma_y}{2})^2}} = \frac{250}{\sqrt{100^2 + 25^2}} = 2.43
$$

## SOLUTION (6.30)

**Known:** A round steel bar with  $S_y = 400$  MPa is loaded simultaneously with known axial tension and torsion stresses.

Find: Determine the safety factor.

## **Schematic and Given Data:**



## **Assumptions:**

- 1. The rod material is ductile.<br>2. The material is homogeneo
- 2. The material is homogeneous.<br>3. The distortion energy theory is
- The distortion energy theory is preferred (it is more accurate than the other theories).

- From Eq.  $(6.8)$ , the distortion energy stress is  $\sigma_e = (\sigma_x^2 + 3 \tau_{xy}^2)^{1/2} = (50^2 + 3(100)^2)^{1/2}$  $σ<sub>e</sub> = 180.3 MPa$
- 2. The safety factor is  $SF = S_v/\sigma_e = 400/180.3 = 2.22$

**Comment:** The maximum shear stress theory predicts a safety factor

$$
SF = \frac{S_{sy}}{\tau_{\text{max}}} = \frac{S_y / 2}{\sqrt{\tau_{xy}^2 + (\frac{\sigma_x - \sigma_y}{2})^2}} = \frac{200}{\sqrt{100^2 + 25^2}} = 1.94
$$

## SOLUTION (6.31)

**Known:** A round shaft of known strength and specified safety factor is loaded with a known torque.

**Find:** Determine the shaft diameter.

## **Schematic and Given Data:**



**Assumption:** The shaft material is homogeneous.

## **Analysis:**

(a) For the maximum-normal-stress theory,

$$
SF = 2 = \frac{S_y}{\sigma_{\text{max}}} = \frac{S_y}{\tau_{xy}} = \frac{60,000}{\frac{16(5000)}{\pi d^3}}
$$

Solving for d, gives  $d = 0.95$  in.

(b) For the maximum-shear-stress theory,

$$
SF = 2 = \frac{S_{sy}}{\tau_{max}} = \frac{S_y/2}{\tau_{xy}} = \frac{30,000}{\frac{16(5000)}{\pi d^3}}
$$

Solving for d, gives  $d = 1.19$  in.

(c) From Eq. (6.8),  $\sigma_e = \sqrt{0 + 3\tau^2} = \sqrt{3}\tau$ . For the distortion-energy theory,

$$
SF = 2 = \frac{S_y}{\sigma_e} = \frac{60,000}{\frac{\sqrt{3} \ 16(5000)}{\pi d^3}}
$$

Solving for d, gives  $d = 1.14$  in.

#### **Comments:**

- 1. If the shaft is a ductile material, the distortion-energy theory is the most accurate, followed by the maximum-shear-stress theory . If the shaft were a brittle material, then the normal-stress-theory would be the most appropriate of the three theories used in the analysis.
- 2. A steel shaft with  $S_y = 60,000$  psi would have an elongation in 2 in. of approximately 20%, and hence would be a ductile material. Indeed, most steel shafts are ductile.
- 3. Good test data pertaining to actual material and torsion loading would be recommended to improve the failure theory prediction.

#### SOLUTION (6.32)

**Known:** A round shaft of known strength and specified safety factor is loaded with a known torque.

Find: Determine the shaft diameter.

## **Schematic and Given Data:**



**Assumption:** The shaft material is homogeneous.

## **Analysis:**

(a) For the maximum-normal-stress theory,

$$
SF = 2 = \frac{S_y}{\sigma_{\text{max}}} = \frac{S_y}{\tau_{xy}} = \frac{60,000}{\frac{16(6000)}{\pi d^3}}
$$

Solving for d, gives  $d = 1.01$  in.

(b) For the maximum-shear-stress theory,

$$
SF = 2 = \frac{S_{sy}}{\tau_{max}} = \frac{S_y/2}{\tau_{xy}} = \frac{30,000}{\frac{16(6000)}{\pi d^3}}
$$

Solving for d, gives  $d = 1.27$  in.

(c) From Eq. (6.8),  $\sigma_e = \sqrt{0 + 3\tau^2} = \sqrt{3}\tau$ . For the distortion-energy theory,

$$
SF = 2 = \frac{S_y}{\sigma_e} = \frac{60,000}{\frac{\sqrt{3} \ 16(6000)}{\pi d^3}}
$$

Solving for d, gives  $d = 1.21$  in.

### **Comments:**

- 1. If the shaft is a ductile material, the distortion-energy theory is the most accurate, followed by the maximum-shear-stress theory (most steel shafts are ductile). If the shaft were a brittle material, then the normal-stress-theory would be the most appropriate of the three theories used in the analysis.
- 2. Good test data pertaining to actual material and torsion loading would be recommended to improve the failure theory prediction.

## SOLUTION (6.33)

**Known:** A round steel bar of given strength is subjected to known tensile, torsional, bending and transverse shear stresses.

Find: (a) Draw a sketch showing the maximum normal and shear stress, and (b) determine the safety factor for yield failure.

## **Schematic and Given Data:**





**Assumption:** The location T (top) is subjected to torsion and bending tension, but no transverse shear. Location S (side) is on a neutral bending axis and is on the side where 4V/3A and Tc/J are additive.

### **Analysis:**

- 1. The Mohr circle plot shows "S" has the higher shear stress, and "T" the higher tensile stress. These locations are 90<sup>o</sup> apart.
- 2. The safety factor with respect to initial yielding according to the maximum-shearstress theory is

$$
SF = \frac{S_y/2}{\tau_{\text{max}}} = \frac{400}{370} = 1.08
$$

3. From Eq. (6.8),

$$
\sigma_{\rm e} = \sqrt{3} (370) = 641 \text{ MPa}
$$

4. The safety factor according to the distortion energy theory is

$$
SF = \frac{S_y}{\sigma_e} = \frac{800}{641} = 1.25
$$

## **Comments:**

1. The effect of tensile force P/A is not considered while calculating the stresses on location S. If it is considered the safety factors according to the maximum shear stress theory and distortion energy theory are 1.076 and 1.24 respectively.

■

2. The maximum shear stress theory is more conservative in predicting failure than the distortion energy theory.

### SOLUTION (6.34)

**Known:** A steel member has a specified safety factor and given stresses.

Find: Determine the tensile yield strength with respect to initial yielding according to: (a) the maximum-shear-stress theory,  $(b)$  the maximum-distortion-energy theory.

### **Schematic and Given Data:**



## **Analysis:**

- (a) For the maximum-shear-stress theory, with  $\sigma_1 = 100 \text{ MPa}$ ,  $\sigma_2 = 20 \text{ MPa}$ , and  $\sigma_3 = -80 \text{ MPa}$ , we have  $\tau_{\text{max}} = (100 + 80)/2 = 90 \text{ MPa}$  $S_y = (2.5)(2)(\tau_{max}) = 450 \text{ MPa}$
- (b) For the maximum-distortion-stress theory, using Eq.(6.5)

$$
\sigma_e=\frac{\sqrt{2}}{2}[(\sigma_2\text{-}\sigma_1)^2+(\sigma_3\text{-}\sigma_1)^2+(\sigma_3\text{-}\sigma_2)^2]^{1/2}=\frac{\sqrt{2}}{2} [(-80)^2+(-180)^2+(-100)^2]^{1/2}=156\text{ MPa}
$$

The yield strength is  $S_y = 2.5(\sigma_e) = 390$  MPa

## SOLUTION (6.35)

**Known:** A downhold oil tool has known biaxial static stresses, an ultimate tensile strength of 97,000 psi and a yield strength of 63,300 psi.

Find: Determine the safety factor according to:

- (a) the maximum-normal-stress theory
- (b) the maximum-shear-stress theory
- (c) the maximum-distortion-energy theory

## **Schematic and Given Data:**



## **Analysis:**

1. Maximum-normal-stress theory For  $\sigma_1 = 45,000 \text{ psi}$ ,  $S_y = 63,300 \text{ psi}$ 

 $SF = S_v/\sigma_1 = 63,300/45,000 = 1.4$ 

2. Maximum-shear-stress theory For  $\sigma_1 = 45,000 \text{ psi}$ ,  $\sigma_2 = 25,000 \text{ psi}$ ,  $\sigma_3 = 0$ 

 $\tau_{\text{max}} = (0 + 45,000)/2 = 22,500 \text{ psi}$ 

 $SF = S_{sy}/\tau_{max} = 31,650/22,500 = 1.41$ 

3. Maximum-distortion-energy theory

 $\sigma_{\rm e} = (\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2)^{1/2}$  $=[(45,000)^{2}+(25,000)^{2}-(45,000)(25,000)]^{1/2}$ = 39,051 psi

The safety factor is  $SF = S_y/\sigma_e = 63,300/39,051 = 1.62$ 

#### SOLUTION (6.36)

**Known:** A lawn mower component has known stresses, an ultimate tensile strength of 97,000 psi, and a yield strength of 63,300 psi.

Find: Determine the safety factor according to:

- (a) the maximum-normal-stress theory
- (b) the maximum-shear-stress theory
- (c) the maximum-distortion-energy theory

#### **Schematic and Given Data:**



#### **Analysis:**

1. Maximum-normal-stress theory From, Eq. (4.16)

$$
\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2\right]^{\frac{1}{2}}
$$
  
=  $\left(\frac{45,000 + 25,000}{2}\right) + \left[\left(\frac{20,000}{2}\right)^2 + (15,000)^2\right]^{\frac{1}{2}}$   
= 53,028 psi

$$
SF = S_y / \sigma_1 = 63,300/53,028 = 1.19
$$

2. Maximum-shear-stress theory From, Eq. (4.18)

$$
\tau_{\text{max}} = \left[ \tau_{xy}^2 + \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 \right]^{1/2}
$$

$$
= \left[ (15,000)^2 + \left( \frac{20,000}{2} \right)^2 \right]^{1/2} = 18,028 \text{ psi}
$$

$$
SF = \frac{S_y}{2\tau_{\text{max}}} = \frac{63,300}{2(18,028)} = 1.8
$$

3. Maximum-distortion-energy theory From, Eq. (6.7)

$$
\sigma_e = [\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2]^{1/2}
$$
  
\n
$$
\sigma_e = [(45,000)^2 + (25,000)^2 - (45,000)(25,000) + 3(15,000)^2]^{1/2}
$$
  
\n= 46,904 psi

 $SF = S_y/\sigma_e = 63,300/46,904 = 1.35$