## SOLUTION (8.19)

Known: A steel bar having known $S_{u}$ and $S_{y}$ has a fine ground surface.
Find: Determine the fatigue strength for bending corresponding to (1) $10^{6}$ or more cycles and (2) $2 \times 10^{5}$ cycles.

Schematic and Given Data:


## Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated $\mathrm{S}-\mathrm{N}$ curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, $\mathrm{C}_{\mathrm{s}}$.
4. The gradient factor, $\mathrm{C}_{\mathrm{G}}=0.9$.

## Analysis:

1. Endurance limits: $\left(10^{6}\right.$ cycle strength)
$\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$
For bending,

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.5(1200)=600 \mathrm{MPa} \text { (Fig. 8.5) } \\
& \mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \\
& \mathrm{C}_{\mathrm{G}}=0.9 \\
& \text { (Table 8.1) } \\
& \mathrm{C}_{\mathrm{s}}=0.86
\end{aligned} \quad \text { (Table 8.1) } \quad \text { (Fig. 8.13) } \quad \begin{aligned}
& \mathrm{S}_{\mathrm{n}}=(600)(1)(0.9)(0.86)(1)(1)=464.4 \mathrm{MPa}
\end{aligned}
$$

2. $10^{3}$ cycle strength

For bending,

$$
0.9 \mathrm{~S}_{\mathrm{u}}=0.9(1200)=1080 \mathrm{MPa}(\text { Table } 8.1)
$$

3. S-N curves

4. $\quad \underline{2 \times 10^{5}}$ cycle strength

Bending: 565.5 MPa

## Comments:

1. The surface factor, $\mathrm{C}_{\mathrm{s}}$ is not used for correcting the $10^{3}$-cycle strength because for ductile parts the $10^{3}$ strength is relatively unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.
3. Analytically the 200,000 cycle fatigue strength for bending may be determined by solving
$[\log (1080)-\log (565.5)] /(6-3)=[\log (S)-\log (565.5)] /(6-\log (200,000))$.
SOLUTION (8.20)
Known: A steel bar having known $\mathrm{S}_{\mathrm{u}}$ and $\mathrm{S}_{\mathrm{y}}$ has a hot rolled surface finish.
Find: Determine the fatigue strength at $2 \times 10^{5}$ cycles for reversed axial loading.

Schematic and Given Data:


## Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated $\mathrm{S}-\mathrm{N}$ curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, $\mathrm{C}_{\mathrm{s}}$.

## Analysis:

1. Endurance limit ( 106 cycle strength)

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}} \\
& \mathrm{For}^{2} \mathrm{axial} \\
& \mathrm{~S}_{\mathrm{n}}^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.5(950)=475 \mathrm{MPa} \\
& \mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \\
& \left.\mathrm{C}_{\mathrm{G}}=0.8 \quad \text { (between } 0.7 \text { and } 0.9\right) \\
& \mathrm{C}_{\mathrm{S}}=0.475 \quad \\
& \mathrm{~S}_{\mathrm{n}}=(475)(1)(0.8)(0.475)(1)(1)=180.5 \mathrm{MPa}
\end{aligned}
$$

2. $10^{3}$ cycle strength

For axial,

$$
0.75 \mathrm{~S}_{\mathrm{u}}=0.75(950)=712.5 \mathrm{MPa}
$$

## 3. S-N curves


4. $\quad 2 \times 10^{5}$ cycle strength

Axial: 248.7 MPa

## Comments:

1. The surface factor, $\mathrm{C}_{\mathrm{s}}$ is not used for correcting the $10^{3}$-cycle strength because for ductile parts the $10^{3}$ strength is relatively unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.
3. Analytically the 200,000 cycle strength for reverse axial loading may be determined by solving
$[\log (712.5)-\log (180.5)] /(6-3)=[\log (S)-\log (180.5)] /(6-\log (200,000))$.

## SOLUTION (8.21)

Known: A steel bar having known $\mathrm{S}_{\mathrm{u}}$ and $\mathrm{S}_{\mathrm{y}}$ has average machined surfaces.
Find: Plot on log-log coordinates estimated S-N curves for (a) bending, (b) axial, and (c) torsional loading. For each of the three types of loading, determine the fatigue strength corresponding to (1) $10^{6}$ or more cycles and (2) $5 \times 10^{4}$ cycles.

## Schematic and Given Data:



## Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, $\mathrm{C}_{\mathrm{s}}$.
4. The gradient factor, $\mathrm{C}_{\mathrm{G}}=0.9$, for axial and torsional loading.

## Analysis:

1. Endurance limits: $\left(10^{6}\right.$ cycle strength)
$\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$
For bending,

$$
\begin{array}{ll}
\mathrm{S}_{\mathrm{n}}^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.5(97)=48.5 \mathrm{ksi}(\text { Fig. 8.5) } \\
\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 & \text { (Table 8.1) } \\
\mathrm{C}_{\mathrm{G}}=0.9 & \text { (Table 8.1) } \\
\mathrm{C}_{\mathrm{s}}=0.76 & \text { (Fig. 8.13) } \\
\mathrm{S}_{\mathrm{n}}=(48.5)(1)(0.9)(0.76)(1)(1)=33.2 \mathrm{ksi}
\end{array}
$$

For axial,

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}{ }^{\prime}=48.5 \mathrm{ksi} \\
& \mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \\
& \mathrm{C}_{\mathrm{G}}=0.8(\text { between } 0.7 \text { and } 0.9) \\
& \mathrm{C}_{\mathrm{S}}=0.76 \\
& \mathrm{~S}_{\mathrm{n}}=48.5(1)(0.8)(0.76)(1)(1)=29.5 \mathrm{ksi}
\end{aligned}
$$

For torsion,

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}{ }^{\prime}=48.5 \mathrm{ksi} \\
& \mathrm{C}_{\mathrm{L}}=0.58 \\
& \mathrm{C}_{\mathrm{G}}=0.9 \\
& \mathrm{C}_{\mathrm{S}}=0.76 \\
& \mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \\
& \mathrm{~S}_{\mathrm{n}}=48.5(0.58)(0.9)(0.76)(1)(1)=19.2 \mathrm{ksi}
\end{aligned}
$$

2. $10^{3}$ cycle strength

For bending,

$$
0.9 \mathrm{~S}_{\mathrm{u}}=0.9(97)=87.3 \mathrm{ksi}(\text { Table } 8.1)
$$

For axial,

$$
0.75 \mathrm{~S}_{\mathrm{u}}=0.75(97)=72.8 \mathrm{ksi}
$$

For torsion,

$$
0.9 \mathrm{~S}_{\mathrm{us}}=0.9(0.8)(97)=69.8 \mathrm{ksi}
$$

## 3. S-N curves


4. $5 \times 10^{4}$ cycle strength

Bending: 50.5 ksi
Axial: $\quad 43.6$ ksi
Torsion: 33.6 ksi

## Comments:

1. The surface factor, $\mathrm{C}_{\mathrm{s}}$ is not used for correcting the $10^{3}$-cycle strength because for ductile parts the $10^{3}$ strength which is close to the static strength, is unaffected by surface finish.
2. For critical designs, pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.22)
Known: A steel bar having known Brinell hardness has machined surfaces.
Find: Determine the fatigue strength for bending corresponding to (1) $10^{6}$ or more cycles and (2) $2 \times 10^{5}$ cycles.

Schematic and Given Data:


## Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, $\mathrm{C}_{\mathrm{s}}$.
4. The gradient factor, $\mathrm{C}_{\mathrm{G}}=0.9$.

## Analysis:

1. Endurance limits: $\left(10^{6}\right.$ cycle strength $)$
$\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$
For bending,

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.25 \mathrm{Bhn}=0.25(375)=93.75 \mathrm{ksi} \text { (Fig. 8.5) } \\
& \mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \quad \text { (Table 8.1) } \\
& \mathrm{C}_{\mathrm{G}}=0.9 \\
& \mathrm{C}_{\mathrm{s}}=0.64 \quad \text { (Table 8.1) } \\
& \mathrm{S}_{\mathrm{n}}=(93.75)(1)(0.9)(0.64)(1)(1)=54 \mathrm{ksi}
\end{aligned}
$$

2. $10^{3}$ cycle strength

For bending,

$$
\mathrm{S} \approx 0.45 \mathrm{Bhn}=0.45(375)=168.75 \mathrm{ksi}(\text { Table } 8.1)
$$

3. S-N curves

4. $2 \times 10^{5}$ cycle strength

Bending: 70.4 ksi

## Comments:

1. $\mathrm{C}_{\mathrm{s}}$ is not used for correcting $10^{3}$-cycle strength because for ductile parts this is close to static strength, which is unaffected by surface finish.
2. For critical designs pertinent test data should be used rather than the preceding rough approximation.

## SOLUTION (8.23)

Known: A steel bar having known $S_{u}$ and $S_{y}$ has machined surfaces.
Find: Plot on log-log coordinates estimated S-N curves for (a) bending, (b) axial, and (c) torsional loading. For each of the three types of loading, determine the fatigue strength corresponding to (1) $10^{6}$ or more cycles and (2) $6 \times 10^{4}$ cycles.

## Schematic and Given Data:



## Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, $\mathrm{C}_{\mathrm{s}}$.

## Analysis:

1. Endurance limits: $\left(10^{6}\right.$ cycle strength $)$
$\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$
For bending,

$$
\begin{array}{ll}
\mathrm{S}_{\mathrm{n}}^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.5(110)=55 \mathrm{ksi}(\text { Fig. 8.5 }) \\
\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 & \text { (Table 8.1) } \\
\mathrm{C}_{\mathrm{G}}=0.9 & \text { (Table 8.1) } \\
\mathrm{C}_{\mathrm{s}}=0.74 & \text { (Fig. 8.13) } \\
\mathrm{S}_{\mathrm{n}}=(55)(1)(0.9)(0.74)(1)(1)=36.6 \mathrm{ksi}
\end{array}
$$

For axial,

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}{ }^{\prime}=55 \mathrm{ksi} \\
& \mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \\
& \left.\mathrm{C}_{\mathrm{G}}=0.8 \text { (between } 0.7 \text { and } 0.9\right) \\
& \mathrm{C}_{\mathrm{s}}=0.74 \\
& \mathrm{~S}_{\mathrm{n}}=55(1)(0.8)(0.74)(1)(1)=32.6 \mathrm{ksi}
\end{aligned}
$$

For torsion,

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}{ }^{\prime}=55 \mathrm{ksi} \\
& \mathrm{C}_{\mathrm{L}}=0.58 \\
& \mathrm{C}_{\mathrm{G}}=0.9 \\
& \mathrm{C}_{\mathrm{s}}=0.74 \\
& \mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \\
& \mathrm{~S}_{\mathrm{n}}=55(0.58)(0.9)(0.74)(1)(1)=21.2 \mathrm{ksi}
\end{aligned}
$$

2. $10^{3}$ cycle strength

For bending,
$0.9 \mathrm{~S}_{\mathrm{u}}=0.9(110)=99.0 \mathrm{ksi}($ Table 8.1)
For axial,

$$
0.75 \mathrm{~S}_{\mathrm{u}}=0.75(110)=82.5 \mathrm{ksi}
$$

For torsion,

$$
0.9 \mathrm{~S}_{\mathrm{us}}=0.9(0.8)(110)=79.2 \mathrm{ksi}
$$

3. S-N curves

4. $\underline{6 \times 10^{4}}$ cycle strength

Bending: 55 ksi
Axial: 48 ksi
Torsion: 36 ksi

## Comments:

1. $\mathrm{C}_{\mathrm{s}}$ is not used for correcting $10^{3}$-cycle strength because for ductile parts this is close to static strength, which is unaffected by surface finish.
2. For critical designs pertinent test data should be used rather than the preceding rough approximation.

SOLUTION (8.26)
Known: A steel bar having known $\mathrm{S}_{\mathrm{u}}$ and $\mathrm{S}_{\mathrm{y}}$ has (i) fine ground surfaces or (ii) machined surfaces.

Find: Determine the fatigue strength corresponding to (1) $10^{6}$ or more cycles and (2) $6 \times 10^{4}$ cycles for the case of zero-to-maximum (rather than completely reversed) load fluctuations for bending, axial, and torsional loading.

## Schematic and Given Data:



## Assumptions:

1. Actual fatigue data is not available for this material.
2. The estimated S-N curves constructed using Table 8.1 are adequate.
3. Fig. 8.13 can be used to estimate surface factor, $\mathrm{C}_{\mathrm{s}}$.

## Analysis:

## Fine ground surface:

1. Bending


For $\infty$ life, $\sigma_{\text {max }}=630 \mathrm{MPa}$
For $6 \times 10^{4}$ cycles, $\sigma_{\max }=715 \mathrm{MPa}$ if no yielding is permitted; otherwise, $\sigma_{\max }=966 \mathrm{MPa}$
2. Axial


For $\infty$ life, $\sigma_{\text {max }}=578 \mathrm{MPa}$
For $6 \times 10^{4}$ cycles, $\sigma_{\max }=715 \mathrm{MPa}$ if no yielding is permitted; otherwise,
$\sigma_{\max }=720 \mathrm{MPa}$

## 3. Torsion



For $\infty$ life, $\tau_{\text {max }}=414 \mathrm{MPa}$
For $6 \times 10^{4}$ cycle life, $\tau_{\max }=415 \mathrm{MPa}$ if no yielding is permitted; otherwise, $\tau_{\text {max }}=800 \mathrm{MPa}$

## Machined surface:

1. Bending


For $\infty$ life, $\sigma_{\text {max }}=520 \mathrm{MPa}$
For $6 \times 10^{4}$ cycles, $\sigma_{\max }=710 \mathrm{MPa}$
2. Axial


For $\infty$ life, $\sigma_{\max }=470 \mathrm{MPa}$
For $6 \times 10^{4}$ cycles, $\sigma_{\max }=640 \mathrm{MPa}$
3. Torsion


For $\infty$ life, $\tau_{\text {max }}=330 \mathrm{MPa}$
For $6 \times 10^{4}$ cycle life, $\tau_{\max }=415 \mathrm{MPa}$ if no yielding is permitted; otherwise, $\tau_{\text {max }}=626 \mathrm{MPa}$

SOLUTION (8.27)
Known: A machined shaft having a known hardness experiences completely reversed torsion.

Find: With a safety factor of 2 , estimate the value of reversed torque that can be applied without causing eventual fatigue failure.

## Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical shaft geometry.

## Analysis:

1. For steel,

$$
\mathrm{S}_{\mathrm{u}}=0.5 \mathrm{Bhn}=0.5(150)=75 \mathrm{ksi}
$$

or, $\quad S_{u}=75 \mathrm{ksi}\left(\frac{6.890 \mathrm{MPa}}{\mathrm{ksi}}\right)=517 \mathrm{MPa}$
2. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$

$$
\mathrm{S}_{\mathrm{n}}^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.5(517) \quad \text { (Fig. 8.5) }
$$

$$
\mathrm{C}_{\mathrm{L}}=0.58 \quad \text { (Table 8.1) }
$$

$$
\mathrm{C}_{\mathrm{G}}=0.9 \quad \text { (Table 8.1) }
$$

$$
\mathrm{C}_{\mathrm{s}}=0.78 \quad \text { (Fig. 8.13) }
$$

$$
\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \quad \text { (Table 8.1) }
$$

$$
\mathrm{S}_{\mathrm{n}}=0.5(517)(0.58)(0.9)(0.78)(1)(1)=105.3 \mathrm{MPa}
$$

3. At the critical point ( 0.8 mm radius), $\mathrm{r} / \mathrm{d}=0.04$ and $\mathrm{D} / \mathrm{d}=1.2$

From Fig. 4.35 (c), $\mathrm{K}_{\mathrm{t}}=1.65$
From Fig. 8.23, $q=0.74$
Hence, $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q}[$ Eq.(8.2)]

$$
=1+(0.65)(0.74)=1.48
$$

4. Therefore, the nominal value of reversed torsional stress can be $\tau=105.3 / 1.48=$
71.1 MPa.

But, $\tau=\frac{16 T}{\pi d^{3}}$ or $T=\frac{\tau \pi d^{3}}{16}$

$$
\mathrm{T}=\frac{(71.1 \mathrm{MPa}) \pi(20 \mathrm{~mm})^{3}}{16}=111,700 \mathrm{~N} \cdot \mathrm{~mm}
$$

with $\mathrm{SF}=2, \mathrm{~T}=\frac{111.7 \mathrm{~N} \cdot \mathrm{~m}}{2}=55.8 \mathrm{~N} \cdot \mathrm{~m}$

## SOLUTION (8.28)

Known: An unnotched bar and a notched bar of known material have the same minimum cross section.

Find: For each bar, estimate
(a) the value of static tensile load P causing fracture
(b) the value of alternating axial load $\pm \mathrm{P}$ that would be just on the verge of producing eventual fatigue fracture (after perhaps 1-5 million cycles).

Schematic and Given Data:


Assumption: The bar is manufactured as specified with regard to the critical fillet geometry and the bar surface finish.

## Analysis:

1. For a static fracture of a ductile material, the notch has little effect. Hence, for both bars,

$$
\mathrm{P} \approx \mathrm{~A} \cdot \mathrm{~S}_{\mathrm{u}}
$$

where $\mathrm{S}_{\mathrm{u}}=748.1 \mathrm{MPa}$ (Appendix C-4a)
$\mathrm{P}=(30 \mathrm{~mm})^{2}(748.1 \mathrm{MPa})=673 \times 10^{3} \mathrm{~N}$
$\mathrm{P}=670 \mathrm{kN}$
2. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{S}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$
where $\quad \mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.5(748.1) \mathrm{MPa}$
$\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1, \mathrm{C}_{\mathrm{G}}=0.8$
$\mathrm{C}_{\mathrm{s}}=0.74$
$\mathrm{S}_{\mathrm{n}}=0.5(748.1)(1)(0.8)(0.74)(1)(1)=221 \mathrm{MPa}$
From Fig. 4.39, $\mathrm{K}_{\mathrm{t}}=2.50$
Assuming Bhn $=217$ (Appendix C-4a), using Fig. 8.24, $q \approx 0.86$
Thus, $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q}$ [Eq. (8.2)]

$$
\mathrm{K}_{\mathrm{f}}=1+(1.50)(0.86)=2.29
$$

3. For the unnotched bar,

$$
\begin{aligned}
\mathrm{P} & =\mathrm{A} \cdot \mathrm{~S}_{\mathrm{n}}=(30 \mathrm{~mm})^{2}(221 \mathrm{MPa}) \\
& =199 \times 10^{3} \mathrm{~N}=199 \mathrm{kN}
\end{aligned}
$$

4. For the notched bar,
$\mathrm{P}=\mathrm{A} \cdot \mathrm{S}_{\mathrm{n}} / \mathrm{K}_{\mathrm{f}}=199 \mathrm{kN} / 2.29=87 \mathrm{kN}$
SOLUTION (8.29)
Known: A stepped shaft having known dimensions was machined from steel having known tensile properties.

## Find:

(a) Estimate the torque T required to produce static yielding.
(b) Estimate the value of reversed torque, $\pm \mathrm{T}$ required to produce eventual fatigue failure.

## Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical fillet.

## Analysis:

1. From Eqs. 4.3 and 4.4, for static yielding,

$$
\tau=\frac{\mathrm{Tc}}{\mathrm{~J}}=\frac{16 \mathrm{~T}}{\pi \mathrm{~d}^{3}}=\frac{16 \mathrm{~T}}{\pi}
$$

Equate this to shear yield,

$$
\mathrm{S}_{\mathrm{ys}} \approx 0.58 \mathrm{~S}_{\mathrm{y}}=0.58(75)=43.5 \mathrm{ksi}
$$

$$
\begin{aligned}
\tau & =\frac{16 \mathrm{~T}}{\pi}=43,500 \\
\mathrm{~T} & =\frac{\pi(43,500)}{16}=8540 \mathrm{lb} \mathrm{in} .
\end{aligned}
$$

2. For fatigue failure, the appropriate endurance limit is:

$$
\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}
$$

where $\quad \mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.5(90)$

$$
\begin{array}{ll}
\mathrm{C}_{\mathrm{L}}=0.58 & (\text { Table 8.1) } \\
\mathrm{C}_{\mathrm{G}}=0.9 & \text { (Table 8.1) } \\
\mathrm{C}_{\mathrm{S}}=0.77 & (\text { (Fig. 8.13) } \\
\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 & (\text { Table 8.1) }
\end{array}
$$

$$
\mathrm{S}_{\mathrm{n}}=0.5(90)(0.58)(0.9)(0.77)(1)(1)=18.1 \mathrm{ksi}
$$

3. From Fig. 4.35, $\mathrm{K}_{\mathrm{t}}=1.72$

From Fig. 8.24, $q=0.78$
Thus, $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q} \quad$ [Eq. (8.2)]

$$
=1+(0.72)(0.78)=1.56
$$

4. For fatigue failure,

$$
\mathrm{K}_{\mathrm{f}} \frac{16 \mathrm{~T}}{\pi}=18,100 ; \mathrm{T}=\frac{18,100 \pi}{1.56(16)}=2,280 \mathrm{lb} \text { in. }
$$

Comment: For static loading of a ductile material, the very first yielding at the notchroot is not significant; hence, ignore stress concentration.

## SOLUTION (8.30)

Known: A shaft rotates at high speed while the imposed loads remain static. The shaft is machined from AISI 1040 steel, oil quenched and tempered at $1000^{\circ} \mathrm{F}$. The loading is sufficiently great to produce a fatigue failure (after perhaps $10^{6}$ cycles).
Find: Determine where the failure would most likely occur.
Schematic and Given Data:


Assumption: The shaft is manufactured as specified.

## Analysis:

1. 


2. Only locations A, B, and C need to be investigated.

From Appendix C-5b, $\mathrm{S}_{\mathrm{u}} \approx 107 \mathrm{ksi}$
3. Since, $\sigma=\frac{M_{Z}}{Z} K_{f} \propto \frac{M^{3}}{d^{3}} K_{f}$
therefore, failure will occur at the highest value of $\frac{M}{d} K_{f}$
4.

| Point | Relative M | $\mathrm{d}^{3}$ | Kt | (Fig. 8.24) | [Eq. (8.2)] | M <br> $\mathrm{d}^{3}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 1 | Fig. 4.35 a <br> 1.85 | 0.82 | 1.70 | 1.70 |
| B | 3 | $(5 / 4)^{3} \approx 2$ | 1 | - | 1 | 1.50 |
| C | 2 | 1 |  |  |  |  |
| 1.8 |  |  |  |  |  |  |

A fatigue failure should occur at C

## SOLUTION (8.31)

Known: A grooved shaft having known dimensions is machined from steel of known hardness and yield strength. A commercial polish is given only to the surface of the groove. The shaft is to have a safety factor of 2 .

Find: Estimate the maximum value of torque T that can be applied for infinite life when the fluctuating torsional load consists of:
(a) completely reversed torsion, with the torque varying between +T and -T ,
(b) a steady torque of T with superimposed alternating torque of 2 T .

## Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical fillet geometry and the shaft surface finish.

## Analysis:

1. From Fig. 4.36(c), $\mathrm{K}_{\mathrm{t}}=1.63$

From Fig. 8.24, q $=0.79$
From Eq. (8.2), $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q}$

$$
\mathrm{K}_{\mathrm{f}}=1+(0.63)(0.79)=1.50
$$

2. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{S}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$ [Eq. (8.1)]
$\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}=0.25$ Bhn ksi (Fig. 8.5)
$\mathrm{C}_{\mathrm{L}}=0.58, \mathrm{C}_{\mathrm{G}}=0.9$
(Table 8.1)
$\mathrm{C}_{\mathrm{s}}=0.77$ (machined) to 0.90 (commercial polish); use $\mathrm{C}_{\mathrm{s}}=0.86$ (Fig. 8.13)
$\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1$
(Table 8.1)
$\mathrm{S}_{\mathrm{n}}=0.25(180)(0.58)(0.9)(0.86)(1)(1)=20.2 \mathrm{ksi}$
3. $\mathrm{S}_{\mathrm{us}}=0.8 \mathrm{~S}_{\mathrm{u}}$ where $\mathrm{S}_{\mathrm{u}}=0.5 \mathrm{Bhn} \mathrm{ksi}$
$\mathrm{S}_{\mathrm{us}}=0.8(0.5)(180)=72 \mathrm{ksi}$
$\mathrm{S}_{\mathrm{ys}}=0.58 \mathrm{~S}_{\mathrm{y}}=0.58(65)=37.7 \mathrm{ksi}$
4. $\tau_{\mathrm{a}}=\frac{16 \mathrm{~T}}{\pi \mathrm{~d}^{\mathrm{d}}} \mathrm{K}_{\mathrm{f}}=\mathrm{S}_{\mathrm{n}}$
$\mathrm{T}=\frac{\mathrm{S}_{\mathrm{n}} \pi \mathrm{d}^{3}}{16 \mathrm{~K}_{\mathrm{f}}}=\frac{20200(\pi)(1)^{3}}{16(1.50)}=2644 \mathrm{lb}$ in.
With $\mathrm{SF}=2, \mathrm{~T}=2644 / 2=1322 \mathrm{lb}$ in.
Thus, for completely reversed torsion,
$\mathrm{T}=1320 \mathrm{lb}$ in.
5. 


6. From above figure, $\tau_{\mathrm{m}} \approx 9.0 \mathrm{ksi}$
$\tau_{\mathrm{m}}=\frac{16 \mathrm{~T}}{\pi \mathrm{~d}^{3}} \mathrm{~K}_{\mathrm{f}}=\tau_{\mathrm{m}}$
$\mathrm{T}=\frac{\tau_{\mathrm{m}} \pi \mathrm{d}^{3}}{16 \mathrm{~K}_{\mathrm{f}}}=\frac{9000(\pi)(1)^{3}}{16(1.50)}=1178 \mathrm{lb} \mathrm{in}$.
With $\mathrm{SF}=2, \mathrm{~T}=1178 / 2=589 \mathrm{lb}$ in.
Thus, for a steady torque of T with superimposed alternating torque of 2 T , $\mathrm{T}=590 \mathrm{lb} \mathrm{in}$.

SOLUTION (8.36)
Known: A cold-drawn rectangular steel bar has known hardness value and dimensions and is to have infinite life with $90 \%$ reliability and a safety factor of 1.3.

Find: Estimate the maximum tensile force that can be applied to the ends:
(a) if the force is completely reversed,
(b) if the force varies between zero and a maximum value.

## Schematic and Given Data:



Assumption: The hole is symmetrically machined in the plate.

## Analysis:

1. For $140 \mathrm{Bhn}, \mathrm{S}_{\mathrm{u}} \approx 0.5(140)=70 \mathrm{ksi}$ or
$\mathrm{S}_{\mathrm{u}}=6.890(70)=482.3 \mathrm{MPa} \quad($ Appendix A-1)
2. From Eq. (3.10a), $\mathrm{S}_{\mathrm{y}} \approx 525 \mathrm{Bhn}-30,000$

$$
=42,800 \mathrm{psi}=295 \mathrm{MPa}
$$

(May be higher for cold drawn, in any case, problem is not affected)
3. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{S}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$ [Eq. (8.1)]

$$
\begin{array}{ll}
\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}} & \text { (Fig. 8.5) } \\
\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 & \text { (Table 8.1) } \\
\mathrm{C}_{\mathrm{G}}=0.8 & \text { (Table 8.1) } \\
\mathrm{C}_{\mathrm{s}}=0.78 & \text { (Fig. 8.13) }
\end{array}
$$

$\mathrm{S}_{\mathrm{n}}=(0.5)(482.3)(1)(0.8)(0.78)(1)(1)=150 \mathrm{MPa}$
But this is (conservatively) for $50 \%$ reliability. For $90 \%$ reliability back off 1.3 standard deviations (Fig. 6.17) of $8 \%$ or $10.4 \%$. Therefore, $\mathrm{S}_{\mathrm{n}}$
$(90 \%$ reliability $) \approx 150(0.896)=134 \mathrm{MPa}$.
4. From Fig. 4.40, $\mathrm{K}_{\mathrm{t}}=2.5$

From Fig. 8.24, $\mathrm{q}=0.85$ (by extrapolation)
Thus, $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q} \quad$ [Eq. (8.2)]

$$
=1+(1.5)(0.85)=2.28
$$

5. $\quad \sigma=\frac{\mathrm{F}}{\mathrm{A}} \mathrm{K}_{\mathrm{f}}=\frac{\mathrm{F}}{(10)(48)}(2.28)=0.00475 \mathrm{~F}$
or $\mathrm{F}=0.21 \sigma$
6. For completely reversed load, $\sigma_{\max }=134 \mathrm{MPa}$;
$\mathrm{F}_{\max }=0.21 \sigma_{\max }$ or, with a safety factor of 1.3 ,
$\mathrm{F}_{\text {max }}=(0.21)(134) / 1.3=22 \mathrm{kN}$
7. 


8. From the figure above, for zero-to-max load,
$\sigma_{\max }=105+105=210 \mathrm{MPa}$. Therefore, with a safety factor of 1.3 , $\mathrm{F}_{\text {max }}=(0.21)(210) / 1.3=34 \mathrm{kN}$

## SOLUTION (8.37)

Known: A shaft with a transverse hole is made of cold-drawn steel having known $S_{u}$ and $\mathrm{S}_{\mathrm{y}}$. Surfaces in the vicinity of the hole have a machined finish.

Find: Estimate the safety factor with respect to infinite fatigue life for:
(a) torque fluctuations between 0 and $100 \mathrm{~N} \cdot \mathrm{~m}$,
(b) a completely reversed torque of $50 \mathrm{~N} \cdot \mathrm{~m}$,
(c) a mean torque of $60 \mathrm{~N} \cdot \mathrm{~m}$ plus a superimposed alternating torque of $40 \mathrm{~N} \cdot \mathrm{~m}$.

Schematic and Given Data:


Assumption: The shaft is manufactured as specified.
Analysis:

1. $\mathrm{S}_{\text {us }}=(0.8)(550)=440 \mathrm{MPa}$

$$
S_{y s}=(0.58)(462)=268 \mathrm{MPa}
$$

2. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{S}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}} \quad$ [Eq. (8.1)]

$$
\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}} \quad \text { (Fig. 8.5) }
$$

$\mathrm{C}_{\mathrm{L}}=0.58$
(Table 8.1)
$\mathrm{C}_{\mathrm{G}}=0.9$
(Table 8.1)
$\mathrm{C}_{\mathrm{s}}=0.78$
(Fig. 8.13)
$\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \quad$ (Table 8.1)
$\mathrm{S}_{\mathrm{n}}=0.50(550)(0.58)(0.9)(0.78)(1)(1)=112 \mathrm{MPa}$
3. From Fig. 4.37, $\mathrm{K}_{\mathrm{t}}=1.42$

From Fig. 8.24, $q=0.85$
From Eq. (8.2), $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q}$

$$
\mathrm{K}_{\mathrm{f}}=1+(0.42)(0.85)=1.36
$$

4. From Fig. 4.37,

$$
\tau=\frac{\mathrm{TK}_{\mathrm{f}}}{\left(\pi \mathrm{D}^{3} / 16\right)-\left(\mathrm{dD}^{2} / 6\right)}=\frac{\mathrm{T}(1.36)}{\left(\pi(20)^{3} / 16\right)-\left(6(20)^{2} / 6\right)}
$$

For $\mathrm{T}=100 \mathrm{~N} \cdot \mathrm{~m}, \tau=116.2 \mathrm{MPa}$
$\mathrm{T}=50 \mathrm{~N} \cdot \mathrm{~m}, \tau=58.1 \mathrm{MPa}$
$\mathrm{T}=60 \mathrm{~N} \cdot \mathrm{~m}, \tau=69.7 \mathrm{MPa}$
$\mathrm{T}=40 \mathrm{~N} \cdot \mathrm{~m}, \tau=46.5 \mathrm{MPa}$
5.

6. Fatigue safety factors:
(a) For torque fluctuations between 0 and $100 \mathrm{~N} \cdot \mathrm{~m}$,

$$
\begin{aligned}
& \tau_{\mathrm{m}}=\tau_{\mathrm{a}}=116.2 / 2=58.1 \\
& \mathrm{SF}=89 / 58.1=1.5
\end{aligned}
$$

(b) For a completely reversed torque of $50 \mathrm{~N} \cdot \mathrm{~m}$,

$$
\begin{aligned}
& \tau_{\mathrm{a}}=58.1, \tau_{\mathrm{m}}=0 \\
& \mathrm{SF}=112.4 / 58.1=1.9
\end{aligned}
$$

(c) For a mean torque of $60 \mathrm{~N} \cdot \mathrm{~m}$ plus a superimposed alternating torque of 40
$\mathrm{N} \cdot \mathrm{m}, \tau_{\mathrm{m}}=69.7, \tau_{\mathrm{a}}=46.5$
$\mathrm{SF}=122 / 69.7=1.7$

SOLUTION (8.38)
Known: A shaft with a transverse hole is made of cold-drawn steel having known $S_{u}$ and $S_{y}$. Surfaces in the vicinity of the hole have a machined finish.

Find: Estimate the safety factor with respect to static yielding for:
(a) torque fluctuations between 0 and $100 \mathrm{~N} \cdot \mathrm{~m}$,
(b) a completely reversed torque of $50 \mathrm{~N} \cdot \mathrm{~m}$,
(c) a mean torque of $60 \mathrm{~N} \cdot \mathrm{~m}$ plus a superimposed alternating torque of $40 \mathrm{~N} \cdot \mathrm{~m}$.

## Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical hole geometry and its surface finish.

## Analysis:

1. Safety factor is the ratio of $S_{y s}$ to the nominal stress.
$S_{y s}=(0.58)(462)=268 \mathrm{MPa}$
$\tau_{\text {nominal }}=\frac{T}{\left(\pi D^{3} / 16\right)-\left(\mathrm{dD}^{2} / 6\right)}$
2. $\quad \tau_{\text {nominal }}=\frac{\mathrm{T}}{\left(\pi(20)^{3} / 16\right)-\left(6(20)^{2} / 6\right)}$

For $T=100 \mathrm{~N} \cdot \mathrm{~m}$,

$$
\tau=85.4 \mathrm{MPa}
$$

For $\mathrm{T}=50 \mathrm{~N} \cdot \mathrm{~m}$,

$$
\tau=42.7 \mathrm{MPa}
$$

For $\mathrm{T}_{\text {max }}=60+40=100 \mathrm{~N} \cdot \mathrm{~m}, \quad \tau=85.4 \mathrm{MPa}$
3. Safety factors:
(a) For torque fluctuations between 0 and $100 \mathrm{~N} \cdot \mathrm{~m}, \mathrm{SF}=268 / 85.4=3.1$
(b) For a completely reversed torque of $50 \mathrm{~N} \cdot \mathrm{~m}, \mathrm{SF}=268 / 42.7=6.3$
(c) For a mean torque of $60 \mathrm{~N} \cdot \mathrm{~m}$ plus a superimposed alternating torque of 40 $\mathrm{N} \cdot \mathrm{m}, \mathrm{SF}=268 / 85.4=3.1$

Comment: Stress concentration is usually neglected for static loading of a ductile material because the localized yielding at the notch can occur (once, or a few times) without harm and without significantly influencing the overall torque vs. deflection relationship.

SOLUTION (8.39)
Known: A 1/2-in. pitch roller chain plate is made of carbon steel heat-treated to give known values of $S_{u}$ and $S_{y}$. All surfaces are comparable to the "machined" category. The link is loaded in repeated axial tension by pins that go through the two holes. The safety factor is 1.2 .

Find: Estimate the value of maximum tensile force that would give infinite fatigue life.
Schematic and Given Data:


Assumption: The roller chain plate is manufactured as specified with regard to the surface finish and critical hole geometries.

## Analysis:

1. The net tensile area in a section through the hole axis is

$$
(0.382-0.125)(0.050)=0.0129 \mathrm{in}^{2}
$$

2. From Fig. 4.40, with $\mathrm{d} / \mathrm{b}=0.33, \mathrm{~K}_{\mathrm{t}}=3.3$

From Fig. 8.24, $q=0.87$
From Eq. (8.2), $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q}$

$$
\mathrm{K}_{\mathrm{f}}=1+(2.3)(0.87)=3.00
$$

3. $\sigma_{\mathrm{m}}=\sigma_{\mathrm{a}}=\frac{\mathrm{F}}{2 \mathrm{~A}} \mathrm{~K}_{\mathrm{f}}=\frac{3.00 \mathrm{~F}}{2(0.0129)}=116.3 \mathrm{~F}$
4. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$ [Eq. (8.1)]

$$
\begin{equation*}
\mathrm{S}_{\mathrm{n}}^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}} \tag{Fig.8.5}
\end{equation*}
$$

$\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \quad$ (Table 8.1)
$\mathrm{C}_{\mathrm{G}}=0.85$
(Table 8.1)
$\mathrm{C}_{\mathrm{s}}=0.70$
(Fig. 8.13)
$\mathrm{S}_{\mathrm{n}}=0.5(140)(1)(0.85)(0.70)(1)(1)=42 \mathrm{ksi}$
5.

6. From the graph, $\sigma_{m}=\sigma_{\mathrm{a}}=32 \mathrm{ksi}$.

With $\mathrm{SF}=1.2$,
(1.2)(116.3) $\mathrm{F}=32,000$

Therefore, $\mathrm{F}=229 \mathrm{lb}$
SOLUTION (8.40)
Known: A shaft is subjected to a fluctuating nominal stress. The shaft is made of steel having known $\mathrm{S}_{\mathrm{u}}$ and $\mathrm{S}_{\mathrm{y}}$.

Find: Estimate the safety factor with respect to eventual fatigue failure if:
(a) the stresses are bending,
(b) the stresses are torsional.

## Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to surface finish and critical fillet radii.

## Analysis:

1. For bending stresses,
$\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}} \quad$ [Eq. (8.1)]
$\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}$
$\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1$
(Fig. 8.5)
(Table 8.1)
$\mathrm{C}_{\mathrm{G}}=0.9$
(Table 8.1)
$\mathrm{C}_{\mathrm{s}}=0.77$
(Fig. 8.13)
$\mathrm{S}_{\mathrm{n}}=0.5(600)(1)(0.9)(0.77)(1)(1)=208 \mathrm{MPa}$
2. Highest stress is at the 1.5 mm fillet where

$$
\mathrm{D} / \mathrm{d}=1.2 \text { and } \mathrm{r} / \mathrm{d}=0.03
$$

From Fig. 4.35, $\mathrm{K}_{\mathrm{t}}=2.3$
From Fig. 8.24, $q=0.78$
From Fig. (8.2), $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q}$

$$
\mathrm{K}_{\mathrm{f}}=1+(1.3)(0.78)=2.01
$$

3. At the fillet
$\sigma_{\mathrm{m}}=2.01\left(\frac{80-16}{2}\right)=64 \mathrm{MPa}$
$\sigma_{a}=2.01\left(\frac{80+16}{2}\right)=96 \mathrm{MPa}$
4. Thus, for the bending stresses,


$$
\mathrm{SF}=168 / 96=1.8
$$

5. For torsional stresses,

$$
\begin{gathered}
\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}^{\prime} \\
\mathrm{S}_{\mathrm{n}}^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}} \\
\mathrm{C}_{\mathrm{L}}=0.58 \\
\mathrm{C}_{\mathrm{G}}=0.9 \\
\mathrm{C}_{\mathrm{s}}=0.77 \\
\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1
\end{gathered}
$$

$\mathrm{S}_{\mathrm{n}}=0.5(600)(0.58)(0.9)(0.77)(1)(1)=121 \mathrm{MPa}$
6. From Fig. $4.35, \mathrm{~K}_{\mathrm{t}}=1.78$

From Fig. 8.24, $q=0.81$
$\mathrm{K}_{\mathrm{f}}=1+(1.78-1)(0.81)=1.63$
7. At critical fillet,

$$
\begin{aligned}
& \tau_{\mathrm{m}}=1.63\left(\frac{80-16}{2}\right)=52 \mathrm{MPa} \\
& \tau_{\mathrm{a}}=1.63\left(\frac{80+16}{2}\right)=78 \mathrm{MPa}
\end{aligned}
$$

8. Thus, for torsional stresses,


$$
\mathrm{SF}=105 / 78=1.3
$$

## SOLUTION (8.41)

Known: A round shaft made of steel having known $S_{u}$ and $S_{y}$ is subjected to a torque fluctuation. All critical surfaces are ground.

Find: Estimate the safety factor for infinite fatigue life with respect to an overload that (a) increases both mean and alternating torque by the same factor,
(b) an overload that increases only the alternating torque.

Schematic and Given Data:


Assumption: The shaft is manufactured as specified with regard to critical radii, hole geometry, and surface finish.

## Analysis:

1. $\quad \mathrm{S}_{\mathrm{us}}=0.8(162)=130 \mathrm{ksi}$
$S_{y s}=0.58(138)=80 \mathrm{ksi}$
2. At the hole,

$$
\text { from Fig. } 4.37, \mathrm{~K}_{\mathrm{t}}=1.75
$$

from Fig. 8.24, $\mathrm{q}=0.88$
$\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q} \quad$ [Eq. (8.2)]
$\mathrm{K}_{\mathrm{f}}=1+(0.75)(0.88)=1.66$
(At fillet, $\mathrm{K}_{\mathrm{t}}=1.33$; hence, not as critical as hole)
3. Using the equation in Fig. 4.37,

$$
\tau_{\mathrm{m}}=\frac{\mathrm{T}_{\mathrm{m}}}{\left(\pi \mathrm{D}^{3} / 16\right)-\left(\mathrm{dD}^{2} / 6\right)} \mathrm{K}_{\mathrm{f}}
$$

$$
\tau_{\mathrm{m}}=\frac{5000}{\pi(1 / 16)-(1 / 16)(1 / 6)}(1.66)=44,600 \mathrm{psi}
$$

$$
\tau_{\mathrm{a}}=\frac{2000}{\pi / 16-1 / 96}(1.66)=17,900 \mathrm{psi}
$$

4. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}} \quad$ [Eq. (8.1)]
$\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}$
(Fig. 8.5)
$\mathrm{C}_{\mathrm{L}}=0.58$
(Table 8.1)
$\mathrm{C}_{\mathrm{G}}=0.9$
(Table 8.1)
$\mathrm{C}_{\mathrm{s}}=0.89$
(Fig. 8.13)
$\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1$
(Table 8.1)
$\mathrm{S}_{\mathrm{n}}=0.5(162)(0.58)(0.9)(0.89)(1)(1)=38 \mathrm{ksi}$
5. 


6. For an overload that increases both the mean and the alternating torque by the same factor,

$$
\mathrm{SF}=22 / 17.9=1.2
$$

For an overload that increases only the alternating torque,

$$
\mathrm{SF}=25 / 17.9=1.4
$$

SOLUTION (8.42)
Known: A stepped shaft made of steel having known value of $\mathrm{S}_{\mathrm{u}}$ is finished by grinding the surface. In service, it is loaded with a fluctuating zero-to-maximum torque.

Find: Estimate the magnitude of maximum torque which would provide a safety factor of 1.3 with respect to a 75,000 cycle fatigue life.

Schematic and Given Data:


Assumption: The shaft is manufactured as specified with regard to the critical fillet geometry and shaft surface finish.

## Analysis:

1. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}} \quad$ [Eq. (8.1)]

| $\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}}$ | (Fig. 8.5) |
| :--- | :--- |
| $\mathrm{C}_{\mathrm{L}}=0.58$ | (Table 8.1) |
| $\mathrm{C}_{\mathrm{G}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1$ | (Table 8.1) |
| $\mathrm{C}_{\mathrm{s}}=0.88$ | (Fig. 8.13) |
| $=0.5(1200)(0.58)(1)(0.88)(1)(1)$ |  |

$\mathrm{S}_{\mathrm{n}}=0.5(1200)(0.58)(1)(0.88)(1)(1)=306 \mathrm{MPa}$
2. For $10^{3}$ cycle strength, from Table $8.1, S=0.9 S_{\text {us }}$ where $S_{u s}=0.8 S_{u}$ Therefore, $\mathrm{S}=0.9(0.8)(1200)=864 \mathrm{MPa}$
3.


4. From the $\tau_{\mathrm{m}}-\tau_{\mathrm{a}}$ plot, a 75,000 cycle life is expected at $\tau_{\mathrm{m}}=\tau_{\mathrm{a}}=310 \mathrm{MPa}$.

For $\mathrm{SF}=1.3$,
$\tau_{\text {max }}=2(310) / 1.3=477 \mathrm{MPa}$.
5. From Fig. 4.35(c), $\mathrm{K}_{\mathrm{t}}=1.33$

From Fig. 8.24, $q=0.88$
Thus, $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q} \quad$ [Eq. (8.2)]

$$
\mathrm{K}_{\mathrm{f}}=1+(0.33)(0.88)=1.29
$$

6. Using the equation from Fig. 8.35(c),

$$
\begin{aligned}
& \tau_{\max }=\frac{16 \mathrm{~T}_{\max }}{\pi \mathrm{d}^{3}} \mathrm{~K}_{\mathrm{f}} \\
& 477 \mathrm{MPa}=\frac{16 \mathrm{~T}_{\max }}{\pi(8 \mathrm{~mm})^{3}}(1.29) \\
& \mathrm{T}_{\max }=37,200 \mathrm{~N} \cdot \mathrm{~mm}=37.2 \mathrm{~N} \cdot \mathrm{~m}
\end{aligned}
$$

## SOLUTION (8.43)

Known: The critical portion of a machine part is shaped like a bar with known dimensions. The material is steel of known hardness. All surfaces are machined. The part is loaded in zero-to-maximum cyclic bending to give an infinite fatigue life with $99 \%$ reliability and a safety factor of 1 .

Find: Estimate the value of maximum bending moment.
Schematic and Given Data:


Assumption: The machine part is manufactured as specified with regard to fillet geometry and surface finish.

## Analysis:

1. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}} \quad$ [Eq. (8.1)]
$\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.25 \mathrm{Bhn}$
(Fig. 8.5)
$\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \quad$ (Table 8.1)
$\mathrm{C}_{\mathrm{G}}=0.9$
(Table 8.1)
$\mathrm{C}_{\mathrm{s}}=0.78$
(Fig. 8.13)
$\mathrm{S}_{\mathrm{n}}=0.25(160)(1)(0.9)(0.78)(1)(1)=28 \mathrm{ksi}$ or 193 MPa
2. For $99 \%$ reliability, $\mathrm{S}_{\mathrm{n}}$ must be reduced by 2.3 standard deviations (Fig. 6.17). Assuming each standard deviation to be $8 \%$ (Sec. 8.3), the "reliability factor," $C_{R}=1-(2.3)(0.08)=0.82$. Thus, for $99 \%$ reliability, use $S_{n}=193(0.82)=158$ MPa.
3. 


4. For zero-to-maximum fluctuation $\sigma_{\mathrm{a}}=\sigma_{\mathrm{m}}=121 \mathrm{MPa}$, or $\sigma_{\max }=242 \mathrm{MPa}$.
5. From Fig. 4.38(a), $\mathrm{H} / \mathrm{h}=1.4, \mathrm{r} / \mathrm{h}=0.08, \mathrm{~K}_{\mathrm{t}}=1.87$

From Fig. 8.24, $q=0.78$
Thus, $\mathrm{K}_{\mathrm{f}}=1+\left(\mathrm{K}_{\mathrm{t}}-1\right) \mathrm{q} \quad$ [Eq. (8.2)]
$\mathrm{K}_{\mathrm{f}}=1+(0.87)(0.78)=1.68$
6. Using the equation from Fig. 4.38(a),

$$
\sigma_{\max }=\frac{\mathrm{M}_{\max } \mathrm{c}}{\mathrm{I}} \mathrm{~K}_{\mathrm{f}}=\frac{6 \mathrm{M}_{\max } \mathrm{K}_{\mathrm{f}}}{\mathrm{bh}^{2}}
$$

$$
242 \mathrm{MPa}=\frac{6 \mathrm{M}_{\max }}{(20 \mathrm{~mm})(25 \mathrm{~mm})^{2}}(1.68)
$$

Therefore, $\mathrm{M}_{\text {max }}=300,000 \mathrm{~N} \cdot \mathrm{~mm}=300 \mathrm{~N} \cdot \mathrm{~m}$

## SOLUTION (8.44)

Known: A solid round shaft has a shoulder with known D and d. The shaft is made of steel having known values of $\mathrm{S}_{\mathrm{u}}$ and $\mathrm{S}_{\mathrm{y}}$. All surfaces are machined. In service the shaft is subjected to a fluctuating torsional load and is to have an infinite life (with safety factor $=1$ ).

Find: Estimate the smallest fillet radius.
Schematic and Given Data:


Assumption: The shaft is manufactured as specified with regard to the critical fillet geometry and the shaft surface finish.

## Analysis:

1. $\frac{\tau_{\mathrm{a}}}{\tau_{\mathrm{m}}}=\frac{\mathrm{T}_{\mathrm{a}}}{\mathrm{T}_{\mathrm{m}}}=\frac{(123-82) / 2}{(123+82) / 2}=\frac{20.5}{102.5}=0.20$
2. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$ [Eq. (8.1)]
$\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.5 \mathrm{~S}_{\mathrm{u}} \quad$ (Fig. 8.5)
$\mathrm{C}_{\mathrm{L}}=0.58$
(Table 8.1)
$\mathrm{C}_{\mathrm{G}}=0.9$
$\mathrm{C}_{\mathrm{s}}=0.69$
(Table 8.1)
$\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1$
(Fig. 8.13)
$\mathrm{S}_{\mathrm{n}}=0.5(150)(0.58)(0.9)(0.69)(1)(1)=27 \mathrm{ksi}$
3. $\mathrm{S}_{\mathrm{us}}=0.8 \mathrm{~S}_{\mathrm{u}}=0.8(150)=120 \mathrm{ksi}$
$\mathrm{S}_{\mathrm{ys}}=0.58 \mathrm{~S}_{\mathrm{y}}=0.58(120)=70 \mathrm{ksi}$
4. 


5. From $\tau_{\mathrm{m}}-\tau_{\mathrm{a}}$ plot, $\tau_{\mathrm{a}}=14.5 \mathrm{ksi}$
6. Using the equation in Fig. 4.35(c),

$$
\begin{aligned}
& \tau_{\mathrm{a}}=\frac{16 \mathrm{~T}_{\mathrm{a}}}{\pi \mathrm{~d}^{3}} \mathrm{~K}_{\mathrm{f}} \\
& 14,500 \mathrm{psi}=\frac{16(20.5)(12)}{\pi(0.5)^{3}} \mathrm{~K}_{\mathrm{f}}
\end{aligned}
$$

$$
\mathrm{K}_{\mathrm{f}}=1.45
$$

7. From Fig. 8.24, estimate $\mathrm{q} \approx 0.85$.

Then, $K_{f}=1+\left(K_{t}-1\right) q$

$$
\begin{aligned}
\mathrm{K}_{\mathrm{t}} & =\frac{\left(\mathrm{K}_{\mathrm{f}}-1\right)}{\mathrm{q}}+1 \\
& =\frac{(1.45-1)}{0.85}+1=1.53
\end{aligned}
$$

8. From Fig. 4.35, for $\mathrm{D} / \mathrm{d}=2$ and $\mathrm{K}_{\mathrm{t}}=1.53$,
$\mathrm{r} / \mathrm{d}=0.08$; then, $\mathrm{r}=(0.08)(0.5)=0.04 \mathrm{in}$.
for which $\mathrm{q} \approx 0.88$. Hence, r is slightly greater than 0.04 in .

SOLUTION (8.45)
Known: A steel shaft used in a spur gear reducer is subjected to a constant torque together with lateral forces that tend always to bend it downward in the center. The stresses are known, but these values do not take into account stress concentration caused by a shoulder with known dimensions. All surfaces are machined and the strength values and hardness of the steel are known.

Find: Estimate the safety factor with respect to infinite life.
Schematic and Given Data:


Assumption: The shaft is manufactured as specified with regard to the critical fillet and shaft surface finish.

## Analysis:

1. We use the Fig. 8.16 relationship for "general biaxial loads":

- Bending provides an alternating stress:
$\sigma_{\mathrm{a}}=\sigma_{\text {ea }}=60 \mathrm{~K}_{\mathrm{f}} \mathrm{MPa}$
From Fig. 4.35(a), $\mathrm{K}_{\mathrm{t}}=1.63$
From Fig. 8.24, $\mathrm{q}=0.84$
From Eq. 8.2, $\mathrm{K}_{\mathrm{f}}=1+(0.63)(0.84)=1.53$

$$
\sigma_{\mathrm{ea}}=60(1.53)=91.8 \mathrm{MPa}
$$

- Torsion provides a mean stress:

$$
\tau_{\mathrm{m}}=\sigma_{\mathrm{em}}=80 \mathrm{~K}_{\mathrm{f}} \mathrm{MPa}
$$

From Fig. 4.35(c), $\mathrm{K}_{\mathrm{t}}=1.33$
From Fig. 8.24, $\mathrm{q}=0.86$
From Eq. 8.2, $\mathrm{K}_{\mathrm{f}}=1+(0.33)(0.86)=1.28$

$$
\sigma_{\mathrm{em}}=80(1.28)=102.4 \mathrm{MPa} .
$$

2. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$ [Eq. (8.1)]

$$
\mathrm{S}_{\mathrm{n}^{\prime}}=0.5 \mathrm{~S}_{\mathrm{u}} \quad \text { (Fig. 8.5) }
$$

$$
\mathrm{C}_{\mathrm{L}}=\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \quad \text { (Table 8.1) }
$$

$$
\mathrm{C}_{\mathrm{G}}=0.9 \quad \text { (Table 8.1) }
$$

$$
\mathrm{C}_{\mathrm{s}}=0.76 \quad \text { (Fig. 8.13) }
$$

$$
\mathrm{S}_{\mathrm{n}}=0.5(700)(1)(0.9)(0.76)(1)(1)=239 \mathrm{MPa}
$$

3. 


4. $\mathrm{SF}=175 / 91.8=1.9$

## SOLUTION (8.51)

Known: A countershaft has helical gear (B), bevel gear (D), and two supporting bearings (A and C). Loads acting on the bevel gear are known. Forces on the helical gears can be determined. Shaft dimensions are known. All shoulder fillets have a radius of 5 mm . Only bearing A takes thrust. The shaft is made of hardened steel having known values of $\mathrm{S}_{\mathrm{u}}$ and $\mathrm{S}_{\mathrm{y}}$. All important surfaces are finished by grinding.

## Find:

(a) Draw load, shear force, and bending moment diagrams for the shaft in the $x y$ - and xz- planes. Also draw diagrams showing the intensity of the axial force and torque along the length of the shaft.
(b) At points B, C, and E of the shaft, calculate the equivalent stresses in preparation for making a fatigue safety factor determination. (Note: Refer to Table 8.2.)
(c) For a reliability of $99 \%$ (and assuming $\sigma=0.08 \mathrm{~S}_{\mathrm{n}}$ ), estimate the safety factor of the shaft at points B, C, and E.

## Schematic and Given Data:



Assumption: The shaft is manufactured as specified with regard to the critical shaft geometry and surface finish.

## Analysis:

1. Load determination
(a) Helical gear forces:

For $\sum \mathrm{M}_{\mathrm{x}}=0$, the torque at the two gears must be equal. Therefore, $\mathrm{F}_{\mathrm{y}}(250 \mathrm{~mm})=$ $5.33(187.5 \mathrm{~mm})$. Hence, $\mathrm{F}_{\mathrm{y}}=4.00 \mathrm{kN}$.
From the given data, $\mathrm{F}_{\mathrm{x}}=.2625 \mathrm{~F}_{\mathrm{y}}=1.05 \mathrm{kN} ; \mathrm{F}_{\mathrm{z}}=.3675 \mathrm{~F}_{\mathrm{y}}=1.47 \mathrm{kN}$.
(b) Determine shaft loads in the xy and xz planes


Vertical forces:
$\sum \mathrm{M}_{\mathrm{A}}=0: \mathrm{C}_{\mathrm{v}}=\frac{4(550)+1.37(187.5)-1.37(1400)}{1000}$

$$
=0.54 \mathrm{kN} \text { downward }
$$

$\sum \mathrm{F}=0: \quad \mathrm{A}_{\mathrm{v}}=4-0.54-1.37=2.09 \mathrm{kN}$ downward
Horizontal forces:

$$
\begin{gathered}
\sum \mathrm{M}_{\mathrm{A}}=0: \mathrm{C}_{\mathrm{H}} \\
=\frac{1.05(250)-1.47(550)+5.33(1400)}{1000} \\
\\
=6.92 \mathrm{kN} \text { upward } \\
\Sigma \mathrm{F}=0: \quad \begin{aligned}
\mathrm{A}_{\mathrm{H}} & =1.47+6.92-5.33 \\
& =3.06 \mathrm{kN} \text { downward }
\end{aligned}
\end{gathered}
$$

2. Stress determination:
(a) At E, the loading is:

Compression of $1.37 \mathrm{kN}, \mathrm{K}_{\mathrm{t}}=2.2, \mathrm{q}=.94$,
$\mathrm{K}_{\mathrm{f}}=2.13$. Axial stress (mean or constant) $=$

$$
\frac{4 \mathrm{PK}_{f}}{\pi \mathrm{~d}^{2}}=\frac{4(-1.37)(2.13)}{\pi(80)^{2}}=-0.581 \mathrm{MPa}
$$

The tension stress is zero.
$\mathrm{M}=\sqrt{(2.09 \times 400)^{2}+(3.06 \times 400)^{2}}$

$$
=1482 \mathrm{kN} \cdot \mathrm{~mm}
$$

$\mathrm{K}_{\mathrm{t}}=1.9, \mathrm{q}=.94$. Therefore, $\mathrm{K}_{\mathrm{f}}=1.85$
Bending stress (alternating) $=\frac{32 M^{3}}{\pi \mathrm{~d}^{3}}$

$$
=\frac{32\left(1482 \times 10^{3}\right)}{\pi(80)^{3}}(1.85)=54.5 \mathrm{MPa}
$$

From Eq. (a) and Eq. (b) in the figure caption of Fig. 8.16, $\sigma_{\mathrm{em}}=0$;
$\sigma_{\mathrm{ea}}=54.5 \mathrm{MPa}$
(b) At B, the loading is:

Axial, $\mathrm{P}=-1.37 \mathrm{kN}, \mathrm{K}_{\mathrm{f}}=1.0, \sigma=-0.27 \mathrm{MPa}$
Torsion $=(4.0)(250)=1000 \mathrm{kN} \cdot \mathrm{mm}$
Bending : $\mathrm{M}=\sqrt{(2.09 \times 550)^{2}+(3.06 \times 550)^{2}}=2038 \mathrm{kN} \cdot \mathrm{mm}$
$\mathrm{K}_{\mathrm{f}}=1.6$ for bending and torsion
Bending stress (alternating) $=\frac{32 M_{1}}{\pi \mathrm{~d}^{3}} \mathrm{~K}_{\mathrm{f}}$

$$
=\frac{32\left(2038 \times 10^{3}\right)}{\pi(80)^{3}}(1.6)=64.9 \mathrm{MPa}
$$

Torsional stress (mean) $=\frac{16 \mathrm{~T}}{\pi \mathrm{~d}^{3}} \mathrm{~K}_{\mathrm{f}}=\frac{16(10)^{6}}{\pi(80)^{3}}(1.6)=15.9 \mathrm{MPa}$
$\sigma_{\mathrm{em}}=\frac{-0.27}{2}+\sqrt{(15.9)^{2}+\left(\frac{-0.27}{2}\right)^{2}}=15.76 \mathrm{MPa} ; \quad \sigma_{\mathrm{ea}}=64.9 \mathrm{MPa}$
(c) At C, the loading is:

Bending:
$\mathrm{M}=\sqrt{(5.33 \times 400)^{2}+[1.37 \times(400-187.5)]^{2}}=2152 \mathrm{kN} \cdot \mathrm{mm}$
Bending stress (alternating) $=\frac{32(2152) \times 10^{3}}{\pi(80)^{3}}=42.8 \mathrm{MPa}$
$\sigma_{\text {ea }}=42.8 \mathrm{MPa}$

Torsional stress - same as (b) except no stress concentration factor; axial same as (b).
$\sigma_{\mathrm{em}}=\frac{-0.27}{2}+\sqrt{\left(\frac{15.9}{1.6}\right)^{2}+\left(\frac{.27}{2}\right)^{2}}=9.80 \mathrm{MPa}$
3. Strength and safety factor determination
$\mathrm{S}_{\mathrm{u}}=155 \mathrm{ksi}=1069 \mathrm{MPa} ; \quad \mathrm{S}_{\mathrm{y}}=130 \mathrm{ksi}=896 \mathrm{MPa}$
For working with equivalent bending stress, $\mathrm{S}_{\mathrm{n}}$ is

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}=\left|\frac{1069}{2}\right|(1)(0.8)^{*}(0.9)(1)(1) \\
&=385 \mathrm{MPa} \text { for } \mathrm{C}_{\mathrm{s}}=0.9 \\
& *(\text { See note b, Table } 8.1)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{S}_{\mathrm{n}} & =\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{s}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}=\left(\frac{1069}{2}\right)(1)(0.8)(1.0)(1)(1) \\
& =428 \mathrm{MPa} \text { for } \mathrm{C}_{\mathrm{s}}=1.0
\end{aligned}
$$

But for $99 \%$ reliability, reduce this by 2.3 standard deviations, which amounts to multiplying by a factor of $(1-2.3 \times .08)=.816$
Thus, for $99 \%$ reliability,
$\mathrm{S}_{\mathrm{n}}=385(.816)=314 \mathrm{MPa}\left(\right.$ for $\left.\mathrm{C}_{\mathrm{s}}=.9\right)$
$\mathrm{S}_{\mathrm{n}}=428(.816)=349 \mathrm{MPa}\left(\right.$ for $\left.\mathrm{C}_{\mathrm{s}}=1.0\right)$
4.

5. Safety factors: (B) $\mathrm{SF}=325 / 64.9=5.0$
(C) $\mathrm{SF}=290 / 42.8=6.8$
(E) $\mathrm{SF}=314 / 54.5=5.8$

SOLUTION (8.52)
Known: A stepped shaft having known dimensions was machined from AISI steel of known hardness. The loading is one of completely reversed torsion. During a typical 30 seconds of operation under overload conditions the nominal ( $\mathrm{Tc} / \mathrm{J}$ ) stress in the 1-in.dia. section was measured.

Find: Estimate the life of the shaft when operating continuously under these conditions.
Schematic and Given Data:


Assumption: The shaft is manufactured as specified with regard to the critical fillet geometry and surface finish.

## Analysis:

1. At the fillet,
from Fig. 4.35(c), $\mathrm{K}_{\mathrm{t}}=1.46$
from Fig. 8.24, $\mathrm{q}=0.86$
Thus, using Eq. (8.2), $\mathrm{K}_{\mathrm{f}}=1+(0.46)(0.86)=1.40$
2. $\mathrm{S}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}{ }^{\prime} \mathrm{C}_{\mathrm{L}} \mathrm{C}_{\mathrm{G}} \mathrm{C}_{\mathrm{S}} \mathrm{C}_{\mathrm{T}} \mathrm{C}_{\mathrm{R}}$ [Eq. (8.1)]

$$
\mathrm{S}_{\mathrm{n}}{ }^{\prime}=0.25 \mathrm{Bhn} \quad \text { (Fig. 8.5) }
$$

$\mathrm{C}_{\mathrm{L}}=0.58$
(Table 8.1)
$\mathrm{C}_{\mathrm{G}}=0.9$
(Table 8.1)
$\mathrm{C}_{\mathrm{s}}=0.76$
(Fig. 8.13)
$\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{R}}=1 \quad$ (Table 8.1)
$\mathrm{S}_{\mathrm{n}}=0.25(200)(0.58)(0.9)(0.76)(1)(1)=19.8 \mathrm{ksi}$
3. From Table 8.1,
$10^{3}$ cycle strength $=0.9 \mathrm{~S}_{\text {us }}=0.9(0.8) \mathrm{S}_{\mathrm{u}}$
$=0.9(0.8)(0.5) \mathrm{Bhn}=0.9(0.8)(0.5)(200)=72 \mathrm{ksi}$
4.

5. The 30 second test involves these stresses (in the fillet) above the endurance limit (see graph):
1 cycles at $\tau_{\mathrm{a}}=35(1.4)=49 \mathrm{ksi}$

$$
\left(\mathrm{N}=8 \times 10^{3} \text { cycles }\right)
$$

2 cycles at $\tau_{a}=25(1.4)=35 \mathrm{ksi}$

$$
\left(\mathrm{N}=4.8 \times 10^{4} \text { cycles }\right)
$$

4 cycles at $\tau_{a}=20(1.4)=28 \mathrm{ksi}$

$$
\left(\mathrm{N}=1.5 \times 10^{5} \text { cycles }\right)
$$

Life used in 30 seconds $=\frac{1}{8 \times 10^{3}}+\frac{2}{4.8 \times 10^{4}}+\frac{4}{1.6 \times 10^{5}}=1.916 \times 10^{-4}$
Estimated life $=\frac{1}{1.916 \times 10^{-4}}=5217$ periods of 30 seconds
Estimated life $\approx 43$ hours

