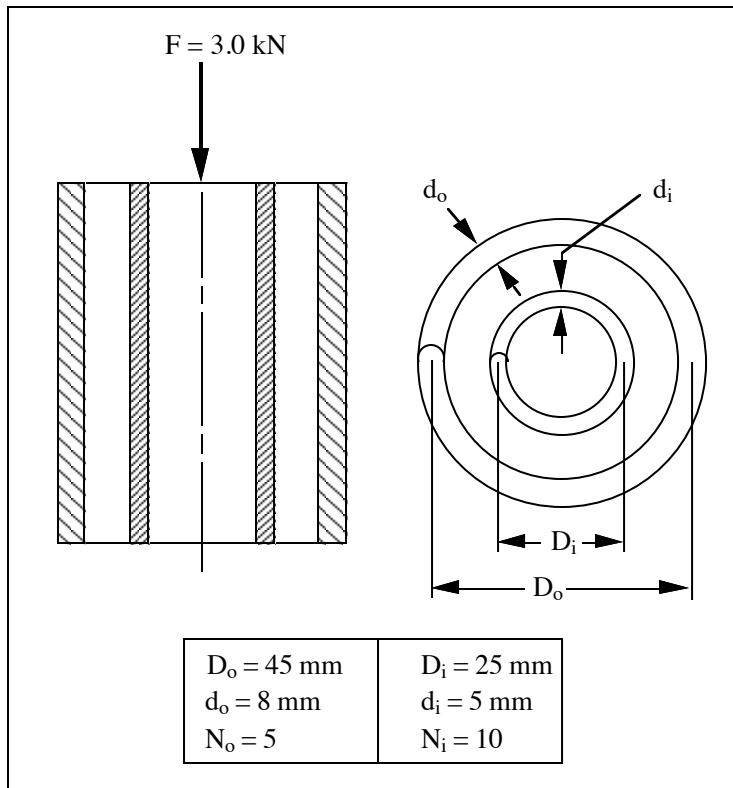Known: A machine uses a pair of concentric helical compression springs to support a known static load. Both springs are made of steel and have the same length when loaded and when unloaded.

Find: Calculate the deflection and the maximum stress in each spring.

Schematic and Given Data:

D_o = 45 mm	D_i = 25 mm
d_o = 8 mm	d_i = 5 mm
N_o = 5	N_i = 10

Assumptions:
1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

Analysis:

1. From Eq. (12.8), $k = \dfrac{d^4 G}{8 D^3 N}$

   where $G = 79 \times 10^9$ Pa for steel. (Appendix C-1)

   $$k_o = \frac{(8 \text{ mm})^4 (79{,}000 \text{ N/mm}^2)}{8(45 \text{ mm})^3 (5)} = 88.77 \text{ N/mm}$$

   $$k_i = \frac{(5 \text{ mm})^4 (79{,}000 \text{ N/mm}^2)}{8(25 \text{ mm})^3 (10)} = 39.50 \text{ N/mm}$$

2. From Eq. (12.8), $k = \dfrac{F}{\delta}$ or $\delta = \dfrac{F}{k}$

$$\delta = \frac{3{,}000}{(39.5 + 88.77)} = 23.39 \text{ mm}$$ ■

3. Using $F = k\delta$, we can calculate the force on each spring.
$F_o = k_o\delta = (88.77 \text{ N/mm})(23.39 \text{ mm}) = 2076 \text{ N}$
$F_i = k_i\delta = (39.50 \text{ N/mm})(23.39 \text{ mm}) = 924 \text{ N}$

4. Using Fig. 12.4, we can find K_s values for each spring.
For outer spring, $C = 45/8 = 5.63$, $K_s = 1.09$
For inner spring, $C = 25/5 = 5.00$, $K_s = 1.10$

5. From Eq. (12.6), $\tau = \dfrac{8FD}{\pi d^3} K_s$

$$\tau_o = \frac{8(2076)(45)}{\pi(8)^3}(1.09) = 506 \text{ MPa}$$ ■

$$\tau_i = \frac{8(924)(25)}{\pi(5)^3}(1.10) = 518 \text{ MPa}$$ ■

<mark>SOLUTION (12.22)</mark>
Known: A machine uses a pair of concentric helical compression springs to support a known static load. Both springs are made of steel and have the same length when loaded and when unloaded.

Find: Calculate the deflection and the maximum stress in each spring.

Schematic and Given Data:

F = 3.0 kN

d_o d_i

D_i

D_o

$D_o = 50$ mm	$D_i = 30$ mm
$d_o = 9$ mm	$d_i = 5$ mm
$N_o = 5$	$N_i = 10$
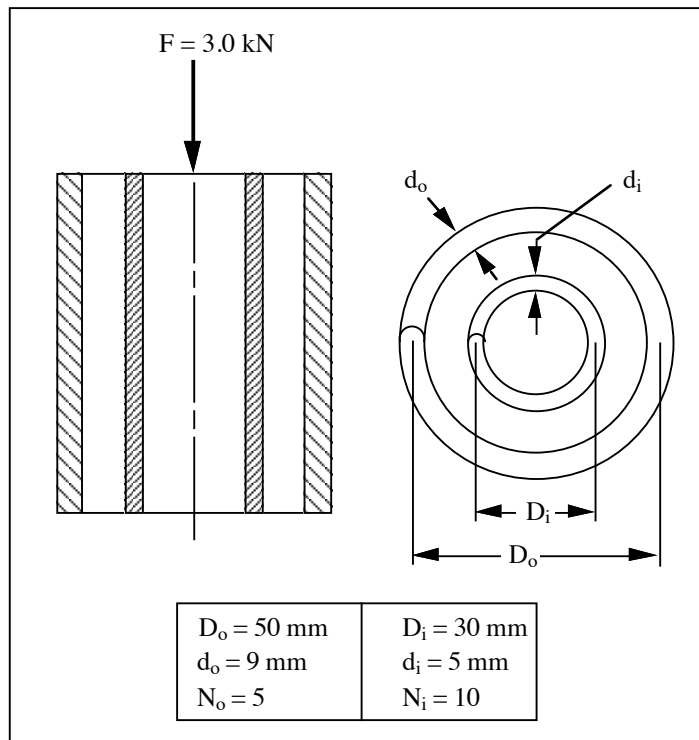
Assumptions:
1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

Analysis:

1. From Eq. (12.8), $k = \dfrac{d^4 G}{8D^3 N}$

   where $G = 79 \times 10^9$ Pa for steel. (Appendix C-1)

   $$k_o = \frac{(9 \text{ mm})^4 (79{,}000 \text{ N/mm}^2)}{8(50 \text{ mm})^3 (5)} = 103.66 \text{ N/mm}$$

   $$k_i = \frac{(5 \text{ mm})^4 (79{,}000 \text{ N/mm}^2)}{8(30 \text{ mm})^3 (10)} = 22.86 \text{ N/mm}$$

2. From Eq. (12.8), $k = \dfrac{F}{\delta}$ or $\delta = \dfrac{F}{k}$

   $$\delta = \frac{3{,}000}{(22.86 + 103.66)} = 23.71 \text{ mm} \qquad \blacksquare$$

3. Using $F = k\delta$, we can calculate the force on each spring.
   $F_o = k_o \delta = (103.66 \text{ N/mm})(23.71 \text{ mm}) = 2458.0 \text{ N}$
   $F_i = k_i \delta = (22.86 \text{ N/mm})(23.71 \text{ mm}) = 542 \text{ N}$

4. Using Fig. 12.4, we can find K_s values for each spring.
   For outer spring, $C = 50/9 = 5.56$
   $\qquad\qquad\qquad K_s = 1.09$
   For inner spring, $C = 30/5 = 6$
   $\qquad\qquad\qquad K_s = 1.08$

5. From Eq. (12.6), $\tau = \dfrac{8FD}{\pi d^3} K_s$

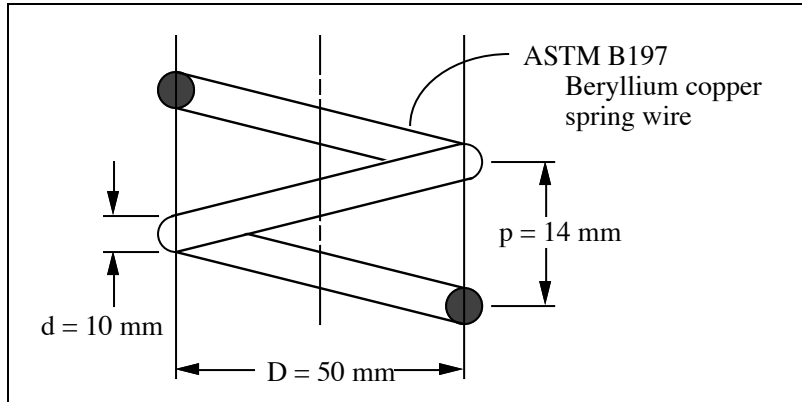 $$\tau_o = \frac{8(2458)(50)}{\pi(9)^3}(1.09) = 467.94 \text{ MPa} \qquad \blacksquare$$

   $$\tau_i = \frac{8(542)(30)}{\pi(5)^3}(1.08) = 357.75 \text{ MPa} \qquad \blacksquare$$

<mark>SOLUTION (12.23)</mark>
Known: A helical coil spring with given D and d is wound with a known pitch value. The material is ASTM B197 beryllium copper spring wire.

Find: If the spring is compressed solid, would you expect it to return to its original free-length when the force is removed?

Schematic and Given Data:

Assumptions:
1. There are no unfavorable residual stresses.
2. Both end plates are in contact with nearly a full turn of wire.
3. The end plate loads coincide with the spring axis.

Analysis:
1. Force to compress spring solid can be calculated by using Eq. (12.7).

$$F = \frac{d^4 G \delta}{8D^3 N}$$

where $\delta/N = p - d = 14 - 10 = 4$ mm

$\qquad G = 50 \times 10^9$ Pa (Appendix C-1)

$$F = \frac{(10 \times 10^{-3})^4 (50 \times 10^9)(4 \times 10^{-3})}{8(50 \times 10^{-3})^3} = 2000 \text{ N}$$ ∎

2. The corresponding stress can be calculated by using Eq. (12.6).

$$\tau = \frac{8FD}{\pi d^3} K_s$$

for $C = D/d = 50/10 = 5$

$\qquad K_s = 1.1$ (Fig. 12.4)

$$\tau = \frac{8(2000)(50 \times 10^{-3})}{\pi(10 \times 10^{-3})^3}(1.1) = 280.1 \text{ MPa}$$

3. For beryllium copper spring wire without presetting, $\tau_s \leq 0.35\, S_u$.
   Also, for beryllium copper spring wire without presetting, $S_u = 750$ MPa
   and $0.35\, S_u = 262.5$ MPa
4. Since 280.1 MPa > 262.5 MPa, set should occur; therefore, spring would not
   return to original length.   ∎

Comment: By considering the curvature (stress concentration) factor of the inner surface by using $K_w = 1.3$, the inner surface stress is $(1.3)(280.1) = 364.1$ MPa which is clearly even larger than 262.5 MPa.

**Known:** A helical compression spring is to be made with given wire diameter, outside diameter, and number of active coils. The least expensive steel spring wire is to be used, and presetting will not be used.

Find:
(a) Estimate the maximum static load that can be applied without encountering more than 2% long-term set.
(b) What is the spring rate?
(c) What should be the free-length in order that the spring becomes solid when load determined in (a) is applied?
(d) Would buckling problems be encountered if one end plate is free to tilt?

Schematic and Given Data:
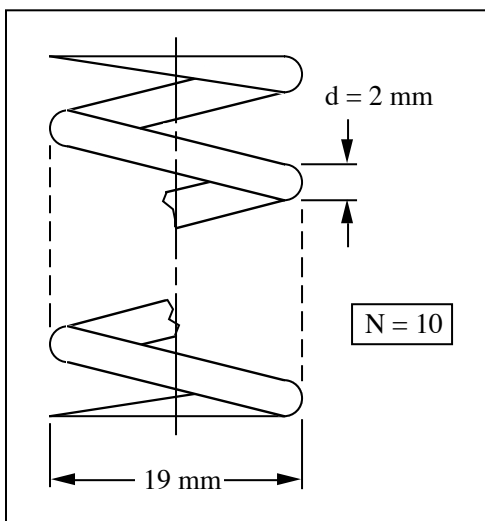
Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis:
1. For d = 2 mm and outside diameter = 19 mm, D = 17 mm, and C = D/d = 8.5.
2. From Table 12.1, the least costly steel spring wire is cold drawn ASTM A227.
3. From Eq. (12.9), $\tau_{max} = 0.45\ S_u$ where $S_u = 1550$ MPa (Fig. 12.7)
    $\tau_{max} = 0.45(1550) = 697.5$ MPa

4. From Eq. (12.6), $F_{max} = \dfrac{\pi d^2 \tau_{max}}{8\ CK_s}$ where $K_s = 1.06$ (Fig. 12.4)

    $F_{max} = \dfrac{\pi(2)^2(697.5)}{8(8.5)(1.06)} = 122$ N   ∎

5. From Eq. (12.8), $k = \dfrac{d^4 G}{8D^3 N}$ where G = 79 GPa (Appendix C-1)

$$k = \frac{2^4(79{,}000)}{8(17)^3(10)} = 3.22 \text{ N/mm}$$ ∎

6. The amount of deflection when $F_{max} = 122$ N is applied to the spring is

$$\delta_s = F_{max}/k = \frac{122}{3.22} = 37.89 \text{ mm}$$

From Fig. 12.8(d), $L_s = N_t d$
$L_s = (N + 2)d = (10 + 2)2 = 24$ mm
Therefore, $L_f = L_s + \delta_s$
$L_f = 24 + 37.89 = 61.89$ mm ∎
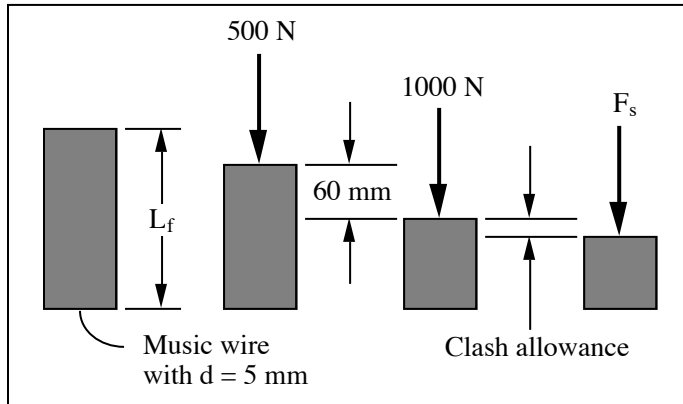
7. $L_f/D = 61.89/17 = 3.64$
$\delta_s/L_f = 37.89/61.89 = 0.612$
From Fig. 12.10, for the case B, no buckling should occur. ∎

Known: A helical compression spring with squared and ground ends is to be designed with given force and deflection requirement. Presetting is to be used. The loading is static.

Find: Determine appropriate values for D, N, and L_f. Check for possible buckling.

Schematic and Given Data:

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.
3. The clash allowance is 10% of the maximum deflection.

Analysis:
1. From Eq. (12.8), $k = \dfrac{F}{\delta}$

$$k = \frac{\Delta F}{\Delta \delta} = \frac{1000 - 500}{60} = 8.33 \text{ N/mm}$$

2. Since class allowance is 10% of maximum deflection,
   $F_s = 110\%$ of 1000 N = 1100 N

3. From Eq.(12.9), $\tau_s = 0.65\, S_u$

4. From Fig. 12.7, $S_u \approx 1650$ MPa for music wire with d = 5 mm.

5. Thus, $\tau_s = 0.65(1650) = 1073$ MPa.

6. From Eq. (12.6), $\tau = \dfrac{8F}{\pi d^2}\, CK_s$ or $CK_s = \dfrac{\tau \pi d^2}{8F}$

$$CK_s = \frac{(1073)(\pi)(5)^2}{8(1100)} = 9.58$$

Therefore, C = 9.0 (Fig. 12.4) and D = Cd = 9.0(5) = 45.0 mm ■

7. From Eq. (12.8), $N = \dfrac{dG}{8kC^3}$ where G = 79 GPa (Appendix C-1)

$$N = \frac{(5)(79,000)}{8(8.33)(9.0)^3} = 8.13$$

∎

8. For ends squared and ground,
    $L_s = N_t d = (N + 2)d$ [Fig. 12.8(d)]
    $L_s = (8.13 + 2)5 = 50.65$ mm
    $L_f = L_s + F_s/k = 50.65 + 1100/8.33 = 182.7$ mm

∎

9. $L_f/D = 182.7/45 = 4.1$

    $\delta_s/L_f = \left(\dfrac{1100}{8.33}\right)/182.7 = 0.72$
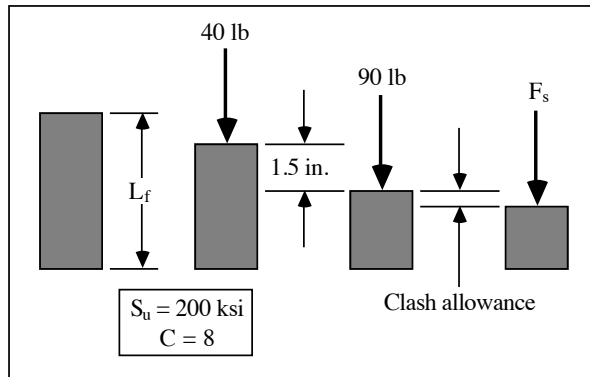
    From Fig. 12.10, we see that the end plates must be constrained parallel (case A) to avoid buckling.

∎

Known: A helical compression spring with squared and ground ends is to be made of steel, and presetting is to be used. The loading is static.

Find: Determine D, d, N, and L_f.

Schematic and Given Data:

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.
3. The clash allowance is 10% of the maximum deflection.

Analysis:
1. From Eq. (12.9), $\tau_s = 0.65\ S_u = 0.65(200) = 130$ ksi.
2. With 10% clash allowance, $F_s = 1.10(90) = 99$ lb.

3. $k = \dfrac{\Delta F}{\Delta \delta} = \dfrac{50}{1.5} = 33.3$ lb/in.

4. From Eq. (12.6), $\tau_s = \dfrac{8F}{\pi d^2}\ CK_s$ where $K_s = 1.06$ (Fig. 12.4)

$$d = \left[\frac{8F_s}{\pi \tau_s}CK_s\right]^{1/2}$$

$$d = \left[\frac{8(99)}{\pi(130,000)}(8)(1.06)\right]^{1/2} = 0.128 \text{ in.} \qquad \blacksquare$$

5. $D = Cd = 8(0.128) = 1.02$ in. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ∎

6. From Eq. (12.8), $N = \dfrac{d^4 G}{8D^3 k}$ where $G = 11.5 \times 10^6$ psi (Appendix C-1)

$$N = \frac{(0.128)^4(11.5 \times 10^6)}{8(1.02)^3(33.3)} = 10.92 \qquad\qquad\qquad\qquad \blacksquare$$

7. From Fig. 12.8, for squared and ground ends,

$L_s = N_t d$ or $L_s = (N + 2)d$

$L_s = (10.92 + 2)(0.128) = 1.65$ in.

$L_f = L_s + \delta_s = L_s + \dfrac{F_s}{k}$

$L_f = 1.65 + \dfrac{99}{33.3} = 4.62$ in.   ∎

Comment:

$L_f/D = \dfrac{4.62}{1.02} = 4.53$

$\delta_s/L_f = \left(\dfrac{99}{33.3}\right)/4.62 = 0.644$

From Fig. 12.10, we can see that the end plates should be constrained parallel to avoid buckling.

Known: A helical compression spring used for essentially static loading has known d, D, and N and squared and ground ends. It is made of ASTM A227 cold drawn steel wire.

Find:
(a) Compute the spring rate and the solid height.
(b) Estimate the greatest load that can be applied without causing long-term permanent set in excess of 2%.
(c) What spring free-length will result in the load determined in (b) just causing the spring to become solid?

Schematic and Given Data:
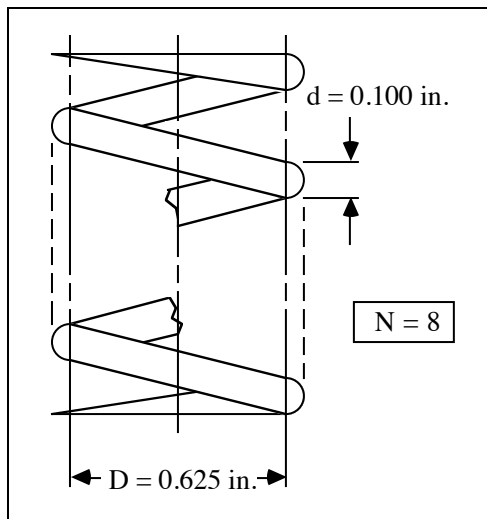
d = 0.100 in.

N = 8

D = 0.625 in.

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis:

1. From Eq. (12.8), $k = \dfrac{d^4 G}{8 D^3 N}$ where $G = 11.5 \times 10^6$ psi (Appendix C-1)

$$k = \frac{(0.1)^4 (11.5 \times 10^6)}{8(0.625)^3 (8)} = 73.6 \text{ lb in.} \qquad \blacksquare$$

2. To avoid set, use Eq. (12.9) $\tau = 0.45\, S_u$ where $S_u = 215$ ksi (Fig. 12.7)
$\tau = 0.45(215) = 97$ ksi

3. For static loads, $\tau = \dfrac{8FD}{\pi d^3} K_s$ [Eq. (12.6)] or $F = \dfrac{\tau \pi d^3}{8 D K_s}$

At $C = 6.25$, $K_s = 1.08$ (Fig. 12.4)

Thus, $F = \dfrac{(97,000)\pi(0.1)^3}{8(0.625)(1.08)} = 56.4\ \text{lb}$ ■

4. $L_s = (N + 2)d = (10)(0.1) = 1.0\ \text{in.}$ ■
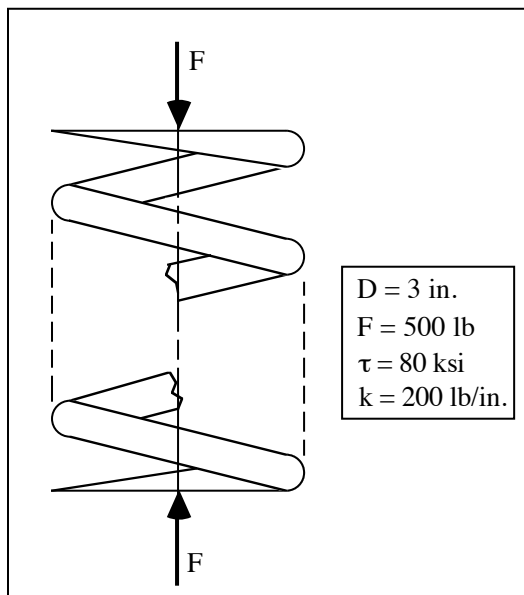 $L_f = L_s + F_s/k = 1.0 + 56.4/73.6 = 1.77\ \text{in.}$ ■

Known: A particular machine requires a helical compression spring, having ends squared and ground, to support a known essentially static load. The spring constant, the stress at the design load, and D are known. The clash allowance is to be 0.10 in.

Find: Determine N, d, and L_f.

Schematic and Given Data:

F

D = 3 in.
F = 500 lb
τ = 80 ksi
k = 200 lb/in.

F

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis:
1. From Eq. (12.6), $\tau = \dfrac{8FD}{\pi d^3}\ K_s$ assume $K_s = 1.06$, check later

    $80,000 = \dfrac{8(500)(3)}{\pi d^3}(1.06)$

    $d^3 = 0.0506$

    $d = 0.370\ \text{in.}$

2. $C = D/d = 3/0.370 = 8.1$

From Fig. 12.4, $K_s = 1.06$

Therefore, d = 0.370 in. ∎

3. From Eq. (12.8), $k = \dfrac{d^4 G}{8 D^3 N}$ or $N = \dfrac{d^4 G}{8 k D^3}$

where $G = 11.5 \times 10^6$ psi (Appendix C-1)

$$N = \frac{(0.370)^4 (11.5 \times 10^6)}{8(200)(3)^3} = 5.0$$ ∎

4. $L_s = (N + 2)d = (5.0 + 2)(0.370) = 2.59$ in.

Expansion from solid = 0.1 + 500/200 = 2.6 in.

$L_f = 2.59 + 2.6 = 5.19$ in. ∎

Known: A coil spring with squared and ground ends is to operate with a load which fluctuates between 45 and 90 lb, during which the deflection is to vary by 1/2 in. Mean coil diameter is 2 in. Steel wire corresponding to shot-peened wire in Fig. 12.16 is to be used. The effect of presetting is not to be taken into account.

Find: Choose an appropriate clash allowance, and determine appropriate values for N, d, and L_f.

Schematic and Given Data:
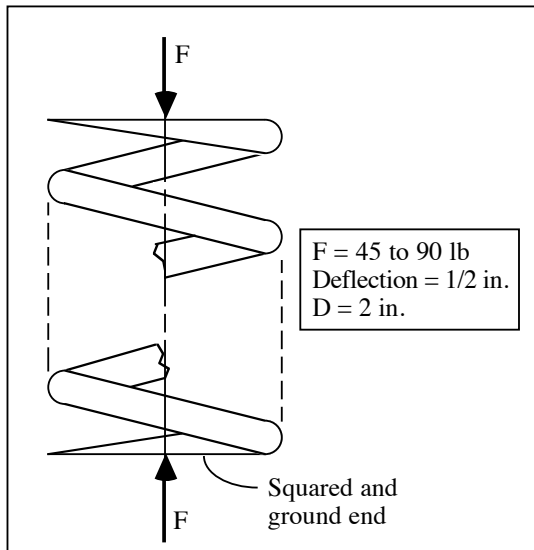
F

F = 45 to 90 lb
Deflection = 1/2 in.
D = 2 in.

Squared and
ground end

F

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.
3. The clash allowance is 10% of the maximum deflection.

Analysis--Case A--with shot peening:
1. See Fig. 12.16:
   For $\tau_{max}/\tau_{min} = 2$, $\tau_{max} = 800$ MPa = 116 ksi
2. $k = F/\delta = 45/0.5 = 90$ lb/in.

3. From Eq. (12.5), $\tau_{max} = \dfrac{8F_{max}D}{\pi d^3} K_w$

   Assume $K_w = 1.2$

   $116,000 = \dfrac{8(90)(2)}{\pi d^3}(1.2)$

   $d = 0.168$ in.
4. $C = D/d = 2/0.168 = 11.9$
   $K_w = 1.125$ (Fig. 12.4)
5. Assume $K_w = 1.125$

   $116,000 = \dfrac{8(90)(2)}{\pi d^3}(1.125)$

d = 0.164 in.
6. $C = D/d = 12.19, K_w \approx 1.12$
   From Eq. (12.5), d = 0.164 in. ∎

7. From Eq. (12.8), $k = \dfrac{d^4 G}{8D^3 N}$ or $N = \dfrac{d^4 G}{8D^3 k}$

   where $G = 11.5 \times 10^6$ psi (Appendix C-1)

   $N = \dfrac{(0.164)^4 (11.5 \times 10^6)}{8(2)^3(90)} = 1.44$ ∎

8. Clash allowance = 10%

   $\delta_{solid} = (1.1)\dfrac{90 \text{ lb}}{90 \text{ lb/in.}} = 1.1$ in.

   Solid height = $(N + 2)d = (1.44 + 2)(0.164) = 0.565$ in.

   $L_f = 0.565 + 1.1 = 1.665$ in. ∎

Analysis--Case B--without shot peening:
1. See Fig. 12.16:
   For $\tau_{max}/\tau_{min} = 2$, $\tau_{max} = 640$ MPa = 93 ksi
2. $k = F/\delta = 45/0.5 = 90$ lb/in.

3. From Eq. (12.5), $\tau_{max} = \dfrac{8 F_{max} D}{\pi d^3} K_w$

   Assume $K_w = 1.2$

   $93,000 = \dfrac{8(90)(2)}{\pi d^3}(1.2)$

   d = 0.181 in.
4. $D/d = 2/0.181 = 11.1$
   $K_w = 1.13$ (Fig. 12.4)
5. Assume $K_w = 1.13$

   $93,000 = \dfrac{8(90)(2)}{\pi d^3}(1.13)$

   d = 0.177 in.
6. $D/d = 11.3, K_w \approx 1.13$
   Thus, d = 0.177 in. ∎

7. From Eq. (12.8), $k = \dfrac{d^4 G}{8D^3 N}$ or $N = \dfrac{d^4 G}{8D^3 k}$

   where $G = 11.5 \times 10^6$ psi (Appendix C-1)

   $N = \dfrac{(0.177)^4 (11.5 \times 10^6)}{8(2)^3(90)} = 1.96$ ∎

8. Clash allowance = 10 %

$$\delta_{solid} = (1.1)\frac{90\ lb}{90\ lb/in.} = 1.1\ in.$$

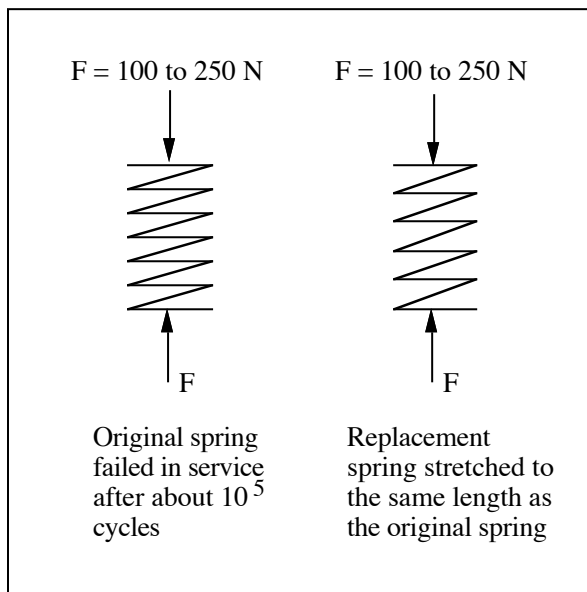Solid height $= (N + 2)d = (1.96 + 2)(0.186) = 0.737$ in.
$L_f = 0.737 + 1.1 = 1.837$ in.
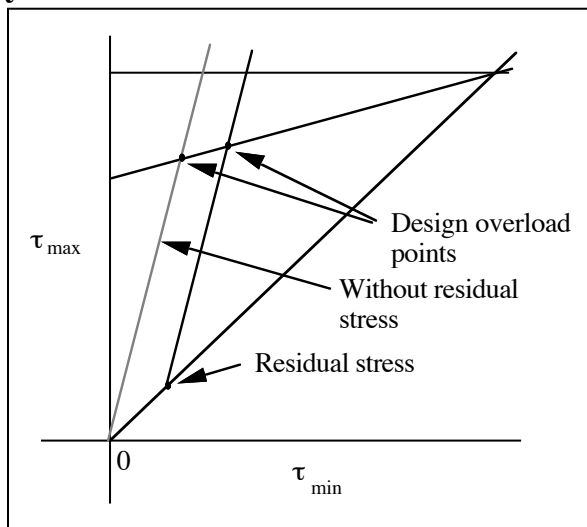

SOLUTION (12.40)

Known: A helical compression spring is subjected to a load fluctuating between 100 and 250. Fatigue properties of the spring wire correspond to the curve for shot-peened wire given in Fig. 12.16. The spring failed in service after about 10^5 cycles. A replacement spring was found which was identical in all respect except that its free-length was slightly shorter. To correct for this, the spring was stretched slightly to increase its free-length to exactly that of the original spring.

Find: Show by means of a τ_{max} - τ_{min} plot, whether you would expect the life of the replacement spring to be the same as, less than, or greater than that of the original.

Schematic and Given Data:

F = 100 to 250 N   F = 100 to 250 N

F   F

Original spring failed in service after about 10^5 cycles

Replacement spring stretched to the same length as the original spring

Analysis:

τ_{max}

Design overload points

Without residual stress

Residual stress

0   τ_{min}

12-55

In this problem, the residual stress causes the load stresses to begin closer to the fatigue limit line. Thus, the life of the replacement spring will be shorter than the original. ∎
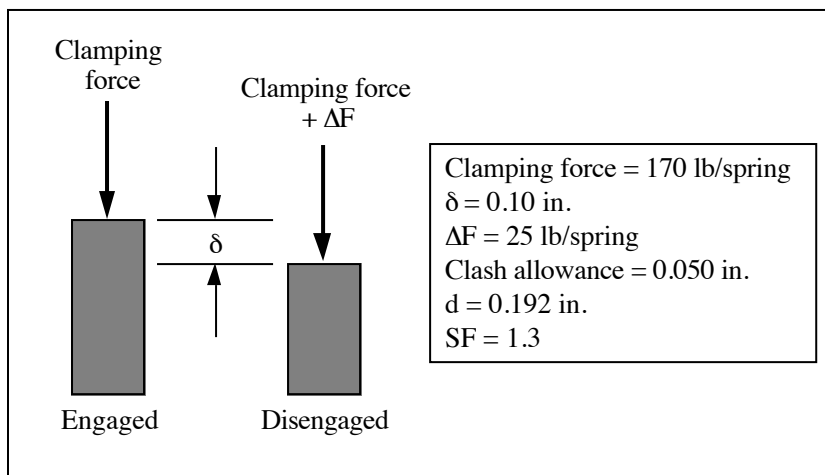
Comment: Stretching the spring in tension gives residual stresses beneficial for future tensile load but harmful for future compressive load.

Known: When the clutch is engaged, the springs must provide a clamping force of 1020 lb (170 lb per spring). When the clutch is disengaged the springs are all 0.10 in. shorter where a 25 lb increase of the force of each spring is considered satisfactory. The clash allowance is to be 0.050 in. The wire size is 0.192 in. The material is to be shot-peened wire having fatigue properties as indicated in Fig. 12.16. Use a safety factor for eventual fatigue failure of 1.3. Presetting is to be used but not taken into account in the calculations.

Find: Determine a suitable combination of D, N, L_s, and L_f.

Schematic and Given Data:

```
Clamping
force          Clamping force
                   + ΔF

                             ┌──────────────────────────────────┐
                             │ Clamping force = 170 lb/spring     │
   ┌──┐          ┌──┐        │ δ = 0.10 in.                       │
   │  │    δ     │  │        │ ΔF = 25 lb/spring                  │
   │  │   ═══    │  │        │ Clash allowance = 0.050 in.        │
   │  │          │  │        │ d = 0.192 in.                      │
   │  │          └──┘        │ SF = 1.3                           │
   └──┘                      └──────────────────────────────────┘
 Engaged      Disengaged
```
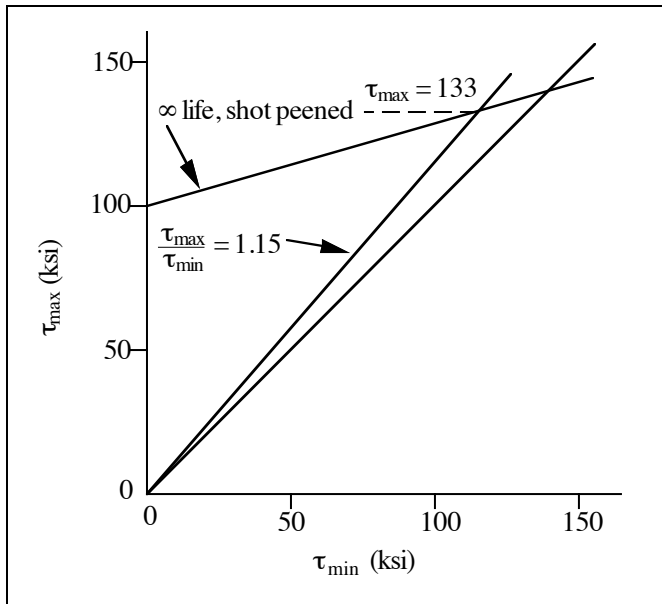
Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.
3. The springs have squared and ground ends.

Analysis:
1. For $\dfrac{\tau_{max}}{\tau_{min}} = \dfrac{F_{max}}{F_{min}} = \dfrac{170 + 25}{170} = 1.15$

2. See Fig. 12.16

3. From above graph, $\tau_{max} = 133$ ksi

   For SF = 1.3, use $\tau_{max} = \dfrac{133}{1.3} = 102$ ksi

4. From Eq. (12.5), $\tau_{max} = \dfrac{8F_{max}}{\pi d^2} CK_w$

   $102,000 = \dfrac{8(195)}{\pi(0.192)^2} CK_w$

   $CK_w = 7.57$

5. From Fig. 12.4, C = 6.0
   D = Cd = (6.0)(0.192) = 1.15 in. ∎

6. $k = \dfrac{F}{\delta} = \dfrac{25 \text{ lb}}{0.1 \text{ in.}} = 250$ lb/in.

7. From Eq. (12.8), $k = \dfrac{dG}{8C^3 N}$ or $N = \dfrac{dG}{8C^3 k}$

   where $G = 11.5 \times 10^6$ ksi (Appendix C-1)

   $N = \dfrac{(0.192)(11.5 \times 10^6)}{8(6.0)^3(250)} = 5.1$ ∎

8. Assuming squared and ground ends,
 $L_s = (N + 2)d = (7.1)(0.192) = 1.36$ in. ∎

   $L_f = L_s + \text{clash allowance} + \dfrac{F_{max}}{k} = 1.36 + 0.05 + \dfrac{195}{250} = 2.19$ in. ∎

Known: A force of 4.45 kN is required to engage a clutch similar to the one shown in Fig. 18.2. This force is to be provided by nine identical springs equally spaced around the pressure plate of the clutch. The outside diameter of the coils can be no more than 40 mm, and the length of the springs when the clutch is engaged cannot exceed 52 mm. The pressure plate must move 3 mm to disengage the friction surfaces, and the lowest spring rate reasonably possible is desired.

Find: Design the springs, determining a satisfactory combination of D, d, N, wire material, type of ends, L_s, and L_f.

Schematic and Given Data:

See Fig. 18.2 where total clamping force = 4.45 kN
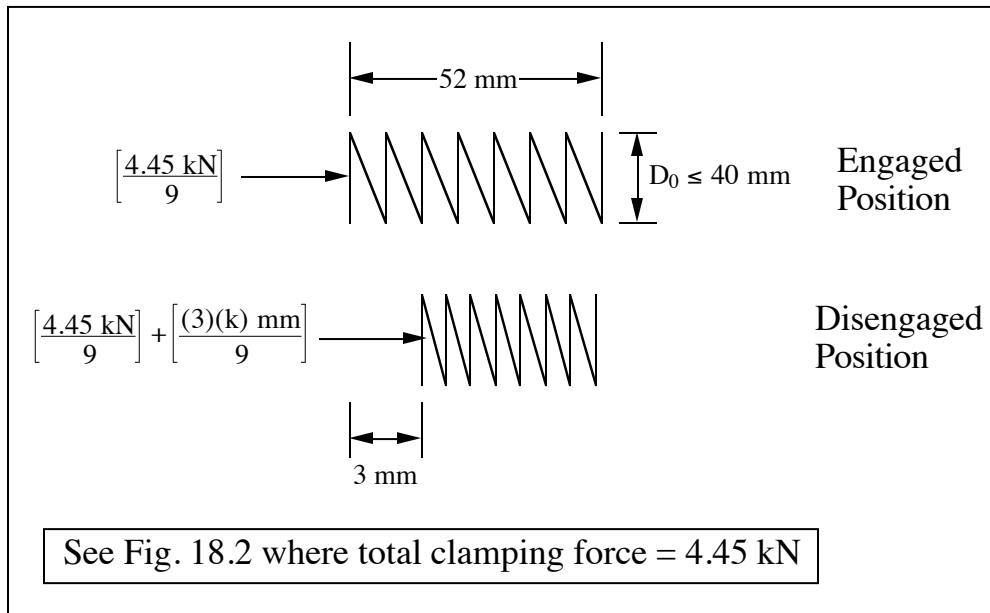
Decisions/Assumptions:
1. The clash allowance is 2 mm.
2. The safety factor is 1.3.
3. The ends are squared and ground.
4. Presetting is to be used but not taken into account in the calculations.
5. The spring is made of shot-peened steel wire having fatigue properties as indicated in Fig. 12.16.

Analysis:
1. Design constraints are:
(a) Since the lowest spring rate reasonably possible is desired, from Eq. 12.8

$$k = \frac{d^4 G}{8D^3 N}$$ the largest D, largest N, and smallest d is desired.

(b) $D + d \le 40$ mm
(c) $L_s \ge$ Engaged length – Working deflection – Clash allowance
Assuming the clash allowance to be 2 mm,
$L_s \le (52 - 3 - 2) = 47$ mm

2. First, choose $C = 8.0$, $N = 9$, and $D = 35$ mm. Then, $d = 4.375$ mm and $D + d < 40$ mm.
    $L_s = (N + 2)d = 48.1$ mm which is too long. Thus, choose a smaller N.
3. Choose $N = 7.5$. Then, $L_s = (7.5 + 2)4.375 = 41.6$ mm which is satisfactory.
4. From Eq. 12.8, where $G = 79$ MPa for a steel wire.

$$k = \frac{(4.375)^4(79{,}000)}{8(35)^3(7.5)} = 11.25 \text{ N/mm}$$

5. $L_f = \dfrac{F_{engaged}}{k} + L_{engaged} = \dfrac{(4450/9)}{11.25} + 52 = 95.95$ mm

6. $F_{engaged} = 4450/9 = 494$ N, $F_{disengaged} = (95.95 - 49)11.25 = 528$ N

    Thus, $\dfrac{F_{max}}{F_{min}} = \dfrac{\tau_{max}}{\tau_{min}} = \dfrac{528}{494} = 1.07 \approx 1$

7. From Fig. 12.16, for infinite-life with shot-peening, $\tau_{max} = 965$ MPa.
    With a safety factor of 1.3,
    $\tau_{max} = 965/1.3 = 742$ MPa.
8. Using Eq. 12.5,

$$\tau_{disengaged} = \frac{8F_{disengaged}}{\pi d^2} CK_w \text{ where } K_w = 1.18 \text{ (Fig. 12.4)}$$

$$\tau_{diseng.} = \frac{8(528)}{\pi(4.375 \times 10^{-3})^2}(8)(1.18) = 663 \text{ MPa}$$

    Therefore, $\tau_{diseng.} < \tau_{max}$
9. Checking for buckling using Fig. 12.10, $L_f/D = 95.95/35 = 2.74$. Thus, the spring will not buckle.
10. In summarizing the answers, the spring is made of shot-peened steel wire having properties as indicated in Fig. 12.16, has squared and ground ends, and
    $d = 4.375$ mm
    $D = 35$ mm
    $N = 7.5$
    $L_s = 41.6$ mm
    $L_f = 95.95$ mm ∎

Comment: If a lower spring rate is desired, the above steps can be repeated with different combinations of C, N, and D.
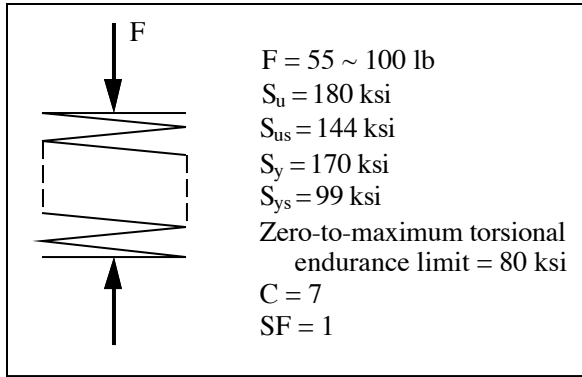
Known: A helical compression spring is to be designed for infinite life when subjected to a known fluctuating load. S_u, S_{us}, S_y, S_{ys}, and a zero-to-maximum torsional endurance limit are known for the steel spring wire. The value of $C = 7$ and safety factor $= 1$.

Find: Determine the wire diameter theoretically required:
(a) if presetting is not used
(b) if presetting is used to maximum advantage.

Schematic and Given Data:

F = 55 ~ 100 lb
$S_u = 180$ ksi
$S_{us} = 144$ ksi
$S_y = 170$ ksi
$S_{ys} = 99$ ksi
Zero-to-maximum torsional
  endurance limit = 80 ksi
C = 7
SF = 1

Assumptions:
1.   Both end plates are in contact with nearly a full turn of wire.
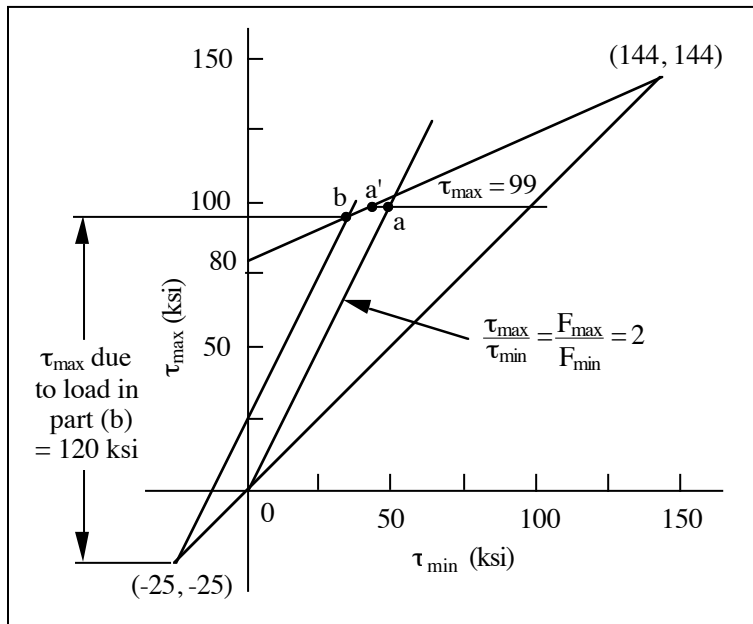2.   The end plate loads coincide with the spring axis.

Analysis:
1.   For C = D/d = 7, from Fig. 12.4 $K_w = 1.21$

2.   From Eq. (12.5), $\tau_{max} = \dfrac{8F_{max}}{\pi d^2} CK_w$

$$\tau_{max} = \frac{8(110)}{\pi d^2}(7)(1.21) = \frac{2373}{d^2}$$

3.

4.   From the above graph, $\tau_{max} = 99,000$ psi; therefore,

$$99,000 = \frac{2373}{d^2} \text{ or } d = 0.155 \text{ in.}$$ ∎

(Note: This assumes no yielding is permitted. If slight yielding is okay, the operating point moves from a to a′ on the graph, giving slightly higher τ_a, which is permitted; therefore, slightly smaller d.)

5. The theoretical maximum $\tau_{residual} = S_{ys}/3$. Assume practical limit is about $S_{ys}/4 \approx 25$ ksi. Then from graph, τ_{max} due to load $= 120$ ksi.
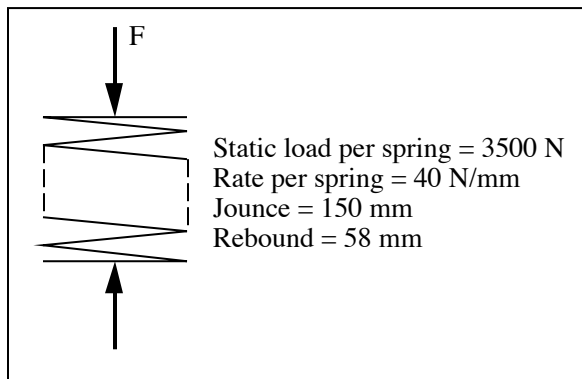
   Therefore, $120{,}000 = \dfrac{2373}{d^2}$ or d $= 0.141$ in. ∎

Known: The specifications for a coil spring are known. Infinite fatigue life is required, using a safety factor of 1.3 applied to the maximum load only. The fatigue strength can be represented by a line between $\tau_{max} = 600$, $\tau_{min} = 0$ and $\tau_{max} = \tau_{min} = 900$ MPa.

Find: Determine a suitable combination of d, D, and N.

Schematic and Given Data:

Static load per spring = 3500 N
Rate per spring = 40 N/mm
Jounce = 150 mm
Rebound = 58 mm

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis:
1. First, choose a reasonable value of C, say C = 8. Thus, from Fig. 12.4 $K_w = 1.18$.
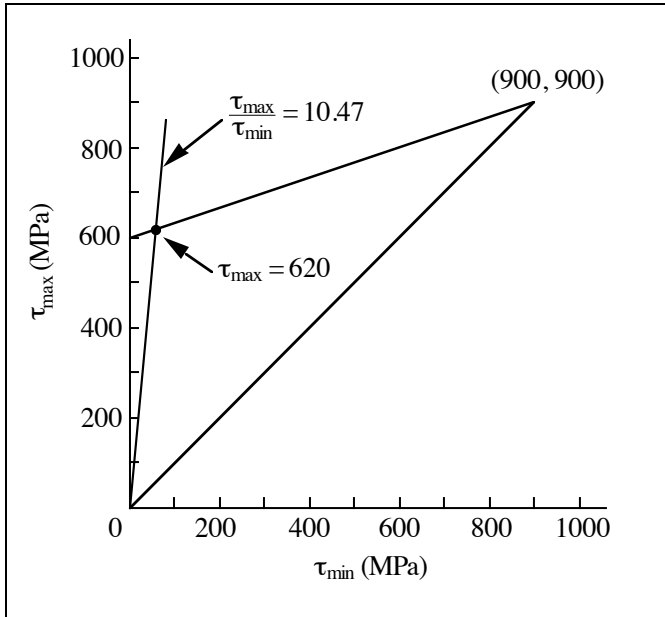2. At the "design overload":
   Using Eq. (12.5),
   $$\tau_{max} = \frac{8F_{max}}{\pi d^2} CK_w = \frac{8[(3500 + (40)(150))1.3]}{\pi d^2}(8)(1.18) = 296{,}879/d^2$$

   $$\tau_{min} = \frac{8F_{min}}{\pi d^2} CK_w = \frac{8[(3500 - (40)(58)]}{\pi d^2}(8)(1.18) = 28{,}366/d^2$$

   $\tau_{max}/\tau_{min} = 296{,}879/28{,}366 = 10.47$

3.

4. From graph, τ_{max} = 620 MPa.

$$620 = \frac{296,879}{d^2} \text{ or } d = 21.88 \text{ mm}^*$$ ■

5. D = Cd = 8(21.88) = 175.0 mm ■

6. From Eq. (12.8), $k = \dfrac{dG}{8NC^3}$ where G = 79 GPa (Appendix C-1)
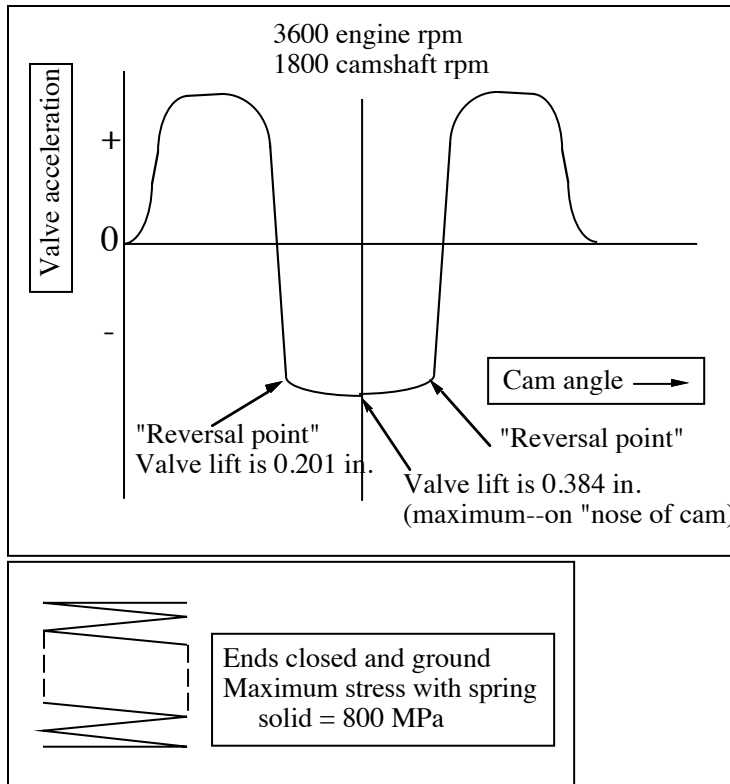
$$N = \frac{(21.88)(79,000)}{8(8)^3(40)} = 10.55$$ ■

Comment: *A slightly modified solution with d = 22 mm would probably be preferred.

Known: A valve spring is subjected to the accelerations shown. The critical point for the spring is the valve lift of 0.201 in. A larger spring force is required at maximum valve lift (0.384 in.) but this is easily satisfied because the spring is further compressed. The specifications of the valve spring are known. High quality valve spring wire is to be used, and full advantage taken of both shot-peening and presetting. The fatigue failure will not occur if the calculated stress with spring solid is limited to 800 MPa. Ends are to be closed and ground.

Find: Determine a suitable combination of d, D, N, and L_f.

Schematic and Given Data:

3600 engine rpm
1800 camshaft rpm

Valve acceleration

+

0

−

Cam angle ⟶

"Reversal point"
Valve lift is 0.201 in.

"Reversal point"

Valve lift is 0.384 in.
(maximum--on "nose of cam)

Ends closed and ground
Maximum stress with spring
solid = 800 MPa

Valve spring specifications:
1. Spring length when valve is closed: not over 1.50 in.
2. Spring force when valve is closed: at least 45 lb.
3. Spring force when valve lift is 0.201 in.: at least 70 lb.
4. Spring force at maximum valve lift of 0.384 in.: at least 86 lb but not over 90 lb.
5. Spring outside diameter: not over 1.65 in.
6. Clash allowance: 0.094 in.
7. Natural frequency: at least as high as the 13th harmonic at 1800 camshaft rpm (i.e., at least 390 Hz).

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis:
1. For maximum f_n, select highest allowable stiffness:

$$k = \frac{\Delta F}{\delta} = \frac{90 - 70}{0.384 - 0.201} = 109.29 \text{ lb/in.}$$

This gives a valve closed force of $70 - (109.29)(0.201) = 48.03$ lb which is satisfactory.
2. Spring rate requirement:

From Eq. (12.8), $k = \dfrac{d^4 G}{8D^3 N}$

where $G = 11.5 \times 10^6$ psi (Appendix C-1)

Thus, $N = \dfrac{d^4 G}{8D^3 k} = \dfrac{d^4(11.5 \times 10^6)}{8(109.29)D^3}$

Since $C = D/d$, $N = \dfrac{13{,}153 d}{C^3}$　　　　(1)

3. Spring length requirement:
   Valve open length $= L_s + 0.384 + \text{clash allowance} < 1.50$
   $(N + 2)d + 0.384 + 0.094 < 1.50$
   $(N + 2)d < 1.022$　　　　(2)
4. Spring diameter requirement:
   $D + d \le 1.65$　　　　(3)
5. Spring stress requirement:
   $F_s = 90 + (0.094)(109.29) = 100.27$ lb

   From Eq. (12.5), $\tau_s = \dfrac{8 F_s}{\pi d^2} C K_w$
   $800 \text{ MPa} = 800(0.145) = 116$ ksi
   $116{,}000 = \dfrac{8(100.27)}{\pi d^2} C K_w$

   $d = \sqrt{\dfrac{C K_w}{454.74}}$　　　　(4)
6. Spring natural frequency requirement:

   From Eq. (12.11), $f_n = \dfrac{13{,}900 d}{N D^2}$ Hz where d and D are in inches.

   $f_n = \dfrac{13{,}900 d}{N D^2} = \dfrac{13{,}900}{N D C} \ge 390$ Hz　　　　(5)
7. For maximum nominal wire stress (for minimum wire mass and high f_n), try to satisfy Eq. (4) with smallest K_w.
8. From Fig. 12.4, choose $C = 9$ for first trial.

   From (4), $d = \sqrt{\dfrac{10.46}{454.74}} = 0.152$ in.　　　　■

   　　　$D = Cd = (9)(0.152) = 1.365$ in.　　　　■
   From (3), $D + d = 1.52$ in. < 1.65 in.; therefore, OK.

   From (1), $N = \dfrac{13{,}153(0.152)}{9^3} = 2.74$　　　　■

   From (2), $(2.74 + 2)(0.152) = 0.72$ in. < 1.022 in.; therefore OK.

   From (5), $f_n = \dfrac{13{,}900(0.152)}{(2.74)(1.365)^2} = 414 > 390$ Hz; therefore OK.

$L_f = L_s$ + clash allowance + 90 lb/k
    $= (N + 2)d + 0.094 + 90/109.23$
    $= (4.74)(0.152) + 0.094 + 0.824$
    $= 1.638$ in. ∎
$L_f/D = 1.20$, Fig. 12.10 shows no buckling concern.

9. For comparison, try solution with $C = 7$.

From (4), $d = \sqrt{\dfrac{8.49}{454.74}} = 0.137$ in. ∎

$D = Cd = (7)(0.137) = 0.956$ in. ∎

From (3), $D + d = 1.093$ in. < 1.65 in.; therefore, OK.

From (1), $N = \dfrac{13{,}153(0.137)}{7^3} = 5.25$ ∎

From (2), $(5.25 + 2)(0.137) = 0.993$ in. < 1.022 in.; therefore OK.

From (5), $f_n = \dfrac{13{,}900}{(5.25)(0.956)(7)} = 396 > 380$ Hz; therefore OK. (But not as good as $C = 9$)

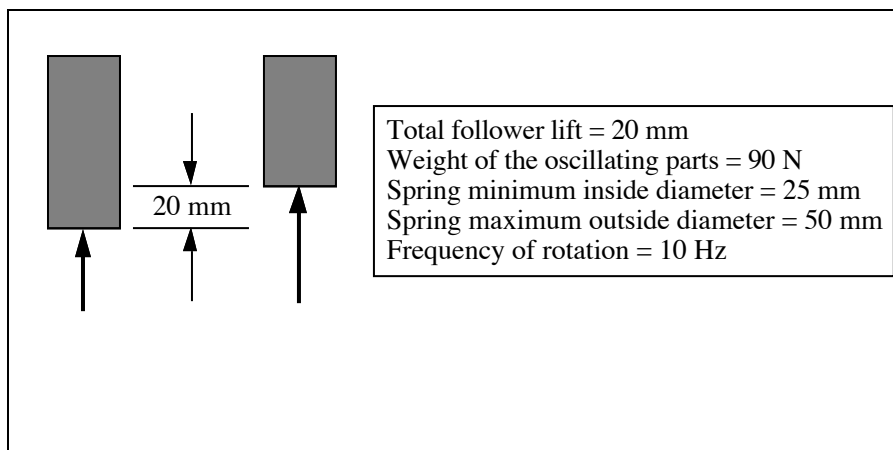$L_f = (7.25)(0.137) + 0.094 + 0.824$
    $= 1.911$ in. ∎
$L_f/D = 2.0$, Fig. 12.10 shows no buckling concern.

SOLUTION (12.46)

Known: A cam and follower is to rotate at 10 Hz and impart a harmonic or sinusoidal oscillation to the follower. Total follower lift and weight of the oscillating parts are known. The spring diameter is limited by the available space.

Find: Determine a satisfactory combination of D, d, N, material, L_s, L_f. Determine the natural frequency of the spring proposed.

Schematic and Given Data:

Total follower lift = 20 mm
Weight of the oscillating parts = 90 N
Spring minimum inside diameter = 25 mm
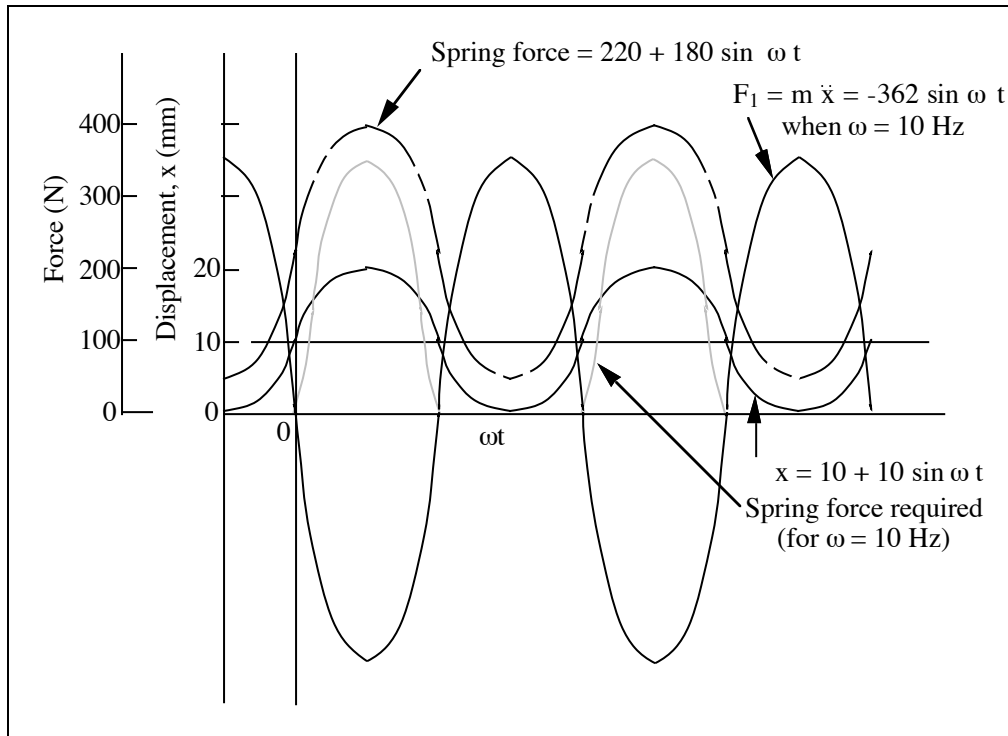Spring maximum outside diameter = 50 mm
Frequency of rotation = 10 Hz

Assumptions:
1. Both end plates are in contact with nearly a full turn of wire.
2. The end plate loads coincide with the spring axis.

Analysis: [The following is only one of many possible solutions.]

1.

2. The above graph shows follower displacement, x, vs. cam angle, wt. The equation with x in meters is: $x = 0.01 + 0.01 \sin \omega t$

3. At a rotating speed of $\omega = 10$ Hz $= 20\pi$ rad/s, the inertial force, F_i is:

$$F_i = m\ddot{x} = \frac{90 \text{ N}}{9.81 \text{ m/s}^2}\left[-0.01(20\pi)^2 \sin \omega t\right]$$
$$= -362 \sin \omega t \text{ (in newtons)}$$

4. The spring is needed only to oppose the negative acceleration. Both F_i and spring force are plotted above.

5. To compensate for small friction and gravity forces plus provide a small safety factor, select minimum and maximum spring forces of 40 N and 400 N respectively (as shown on graph). Thus, $k = \dfrac{\Delta F}{\delta} = \dfrac{400 - 40}{20} = 18$ N/mm

   And the expression for the spring force plotted is:
   $F = 40 + kx = 40 + 18(10 + 10\sin \omega t) = 220 + 180 \sin \omega t$

6. Select spring wire having fatigue properties shown in Fig. 12.16, and specify shot-peening.

$$\frac{\tau_{max}}{\tau_{min}} = \frac{F_{max}}{F_{min}} = \frac{400}{40} = 10$$

7. From Fig. 12.16, $\tau_{max} = 710$ MPa

8. Based on information on Fig. 12.4, tentatively select C = 8. Since a high f_n is not required and since space is ample, select a generous safety factor of 1.4. Hence, design for $\tau_{max} = 710/1.4 = 507$ MPa.

9. From Eq. (12.5), $\tau_{max} = \dfrac{8F_{max}}{\pi d^2} CK_w$

$$507 = \frac{8(400)}{\pi d^2}(9.47)$$

Therefore, d = 4.36 mm ∎
D = Cd = 8(4.36) = 34.9 mm ∎

10. From Eq. (12.8), $k = \dfrac{dG}{8NC^3}$ where G = 79 GPa (Appendix C-1)

$$N = \frac{(4.36)(79{,}000)}{8(18)(8)^3} = 4.67$$ ∎

11. Choose a dash allowance of 10 % of deflection at 400 N load; therefore,
    $F_s = 400(1.1) = 440$ N.

12. $L_s = (N + 2)d = 6.67(4.36) = 29.08$ mm ∎
    $L_f = L_s + F_s/k = 29.08 + 440/18 = 53.52$ mm ∎

13. From Eq. (12.11a), $f_n = \dfrac{353{,}000d}{ND^2}$ Hz where d and D are in mm

$$f_n = \frac{353{,}000d}{4.67(34.9)^2} = 271 \text{ Hz}$$

$L_f/D = 53.52/34.9 = 1.53$; therefore, no buckling problem.

14. For rotation at 10 Hz, only harmonics would be in resonance during starting and stopping. Since this cam should give no harmonics, dynamics should be of no concern.

Comment: Since f_n and size limitations were easily met, one should consult the spring supplier in search of a lowest cost solution, retaining high reliability.

Known: The internet has various sites that contain calculators useful for designing compression springs.

Find: Select a calculator that is (a) potentially useful, (b) easy to use, and (c) accurate. Write a short description of the spring calculator.

Analysis: This exercise is left for the student.