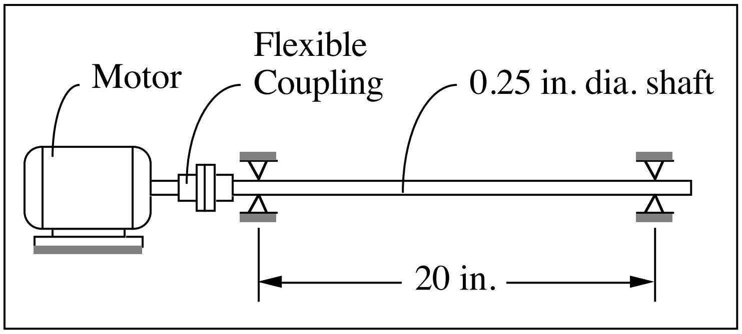
SOLUTION (17.3)

Known: A simply supported steel shaft is connected to an electric motor with a flexible coupling.

Find: Determine the value of the critical speed of rotation for the shaft.

Schematic and Given Data:

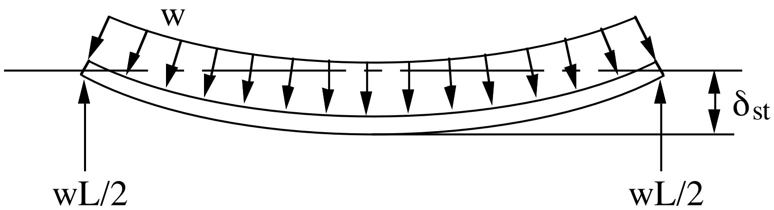


Assumptions:

1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.

Analysis:

1. For the simply supported uniform load case:



$$w = A\rho = \frac{\pi d^2}{4}\rho \quad \text{where } \rho = 0.28\ \frac{\text{lb}}{\text{in.}^3} \text{ for steel}$$

$$w = \frac{\pi(0.25)^2}{4}(0.28) = 0.0137\ \frac{\text{lb}}{\text{in.}}$$

2. From Appendix D-2,

$$\delta_{st} = \frac{5wL^4}{384EI} \text{ for a uniform load distribution}$$

where $E = 30 \times 10^6$ psi (Appendix C-1)

$$I = \frac{\pi d^4}{64} = \frac{\pi(0.25)^4}{64} = 1.92 \times 10^{-4} \text{ in.}^4 \text{ (Appendix B-1)}$$

$$\delta_{st} = \frac{5(0.0137)(20)^4}{384(30 \times 10^6)(1.92 \times 10^{-4})} = 4.98 \times 10^{-3} \text{ in.}$$

3. Using Fig. 17.5(c), to find the shaft critical speed

$$n_c \approx \sqrt{\frac{5g}{4\delta_{st}}} = \sqrt{\frac{5(32.2\ \frac{\text{ft}}{\text{s}^2})(12\ \frac{\text{in.}}{\text{ft}})}{4(4.98 \times 10^{-3}\text{ in.})}}$$

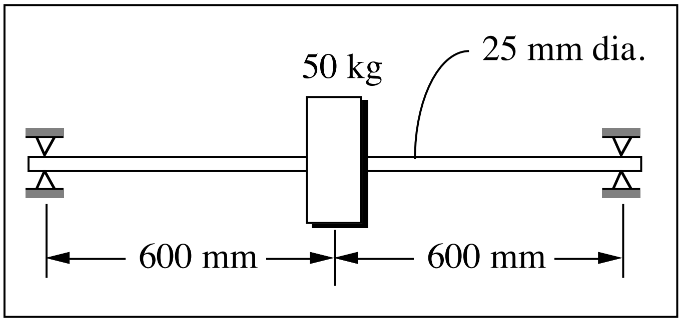
$n_c \approx 311$ rpm ■

SOLUTION (17.10)

Known: The dimensions of a steel shaft are given.

Find: Determine the critical speed of rotation for the steel shaft.

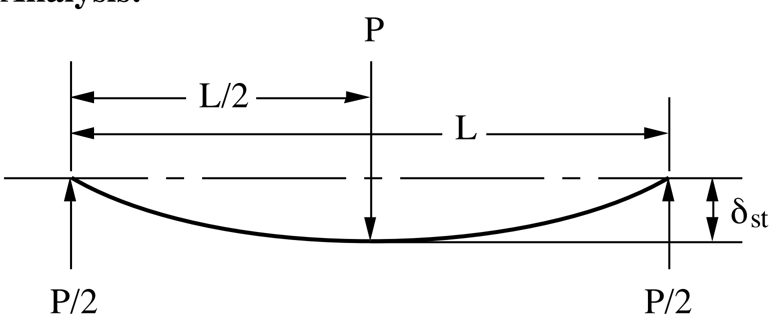
Schematic and Given Data:



Assumptions:

1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.
4. The shaft is simply supported.
5. The mass of the shaft is negligible.

Analysis:



1. Using Appendix D-2, for a concentrated center load on a simply supported beam

   we have $\delta_{st} = \dfrac{PL^3}{48EI}$ where

   $E = 207 \times 10^9$ Pa (Appendix C-1) and

   $I = \dfrac{\pi d^4}{64}$ (Appendix B-1)

   $= \dfrac{\pi(0.025)^4}{64} = 19.2 \times 10^{-9}\ m^4$

Thus, $\delta_{st} = \dfrac{50(9.8)(1.2)^3}{48(207 \times 10^9)(19.2 \times 10^{-9})} = 4.44 \times 10^{-3}$ m

2. Using Eq. (17.1) of Fig. 17.5(a) to find n_c

$$n_c = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}} = \frac{30}{\pi}\sqrt{\frac{9.8}{4.44 \times 10^{-3}}} = 449 \text{ rpm}$$

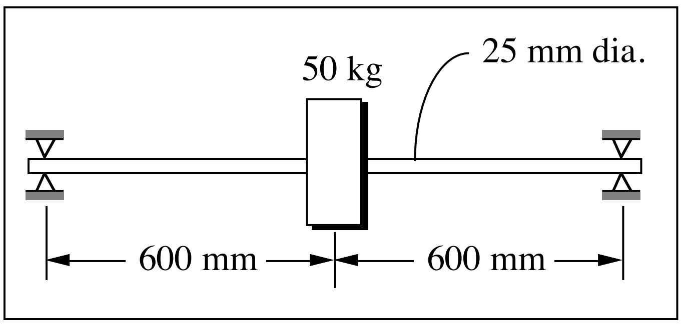
■

SOLUTION (17.11)

Known: The dimensions of a steel shaft are given.

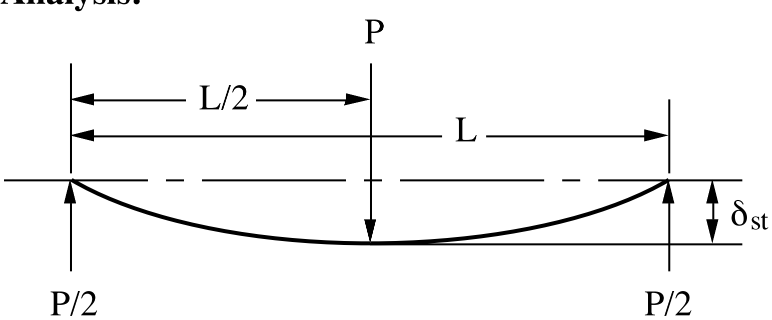
Find: Determine the critical speed of rotation for the steel shaft.

Schematic and Given Data:



Assumptions:
1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.
4. The shaft is simply supported.
5. The mass of the shaft is negligible.

Analysis:



1. Using Appendix D-2, for a concentrated center load on a simply supported beam

we have $\delta_{st} = \frac{PL^3}{48EI}$ where

$E = 127 \times 10^9$ Pa (Appendix C-1) and

$I = \frac{\pi d^4}{64}$ (Appendix B-1)

$= \frac{\pi(0.025)^4}{64} = 19.2 \times 10^{-9}\ m^4$

Thus, $\delta_{st} = \frac{50(9.8)(1.2)^3}{48(127 \times 10^9)(19.2 \times 10^{-9})} = 7.24 \times 10^{-3}$ m

2. Using Eq. (17.1) of Fig. 17.5(a) to find n_c

$$n_c = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}} = \frac{30}{\pi}\sqrt{\frac{9.8}{7.24 \times 10^{-3}}} = 351 \text{ rpm}$$

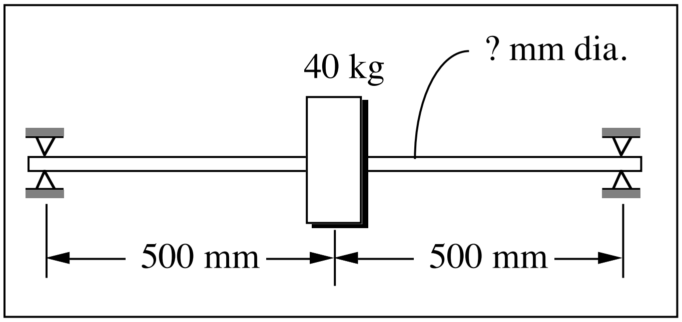
■

SOLUTION (17.14)

Known: The shaft is aluminum and the critical speed of rotation is given.

Find: Determine the diameter of the aluminum shaft.

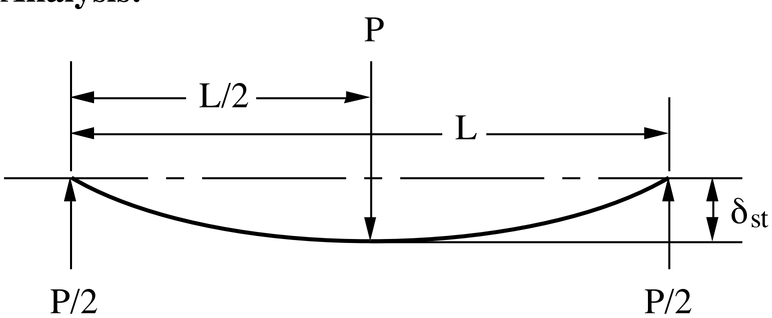
Schematic and Given Data:



Assumptions:

1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.
4. The shaft is simply supported.
5. The mass of the shaft is negligible.

Analysis:



1. Using Eq. (17.1) of Fig. 17.5(a) to find δ_{st}

$$n_c = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}} = \frac{30}{\pi}\sqrt{\frac{9.8}{\delta_{st}}} = 250 \text{ rpm}$$

$\delta_{st} = 0.0143$ m ■

2. Using Appendix D-2, for a concentrated center load on a simply supported beam

we have $\delta_{st} = \frac{PL^3}{48EI}$ where

$E = 72 \times 10^9$ Pa (Appendix C-1) or

$$I = \frac{PL^3}{48E\delta_{st}} \qquad I = \frac{(40)(9.8)(1.0)^3}{48(72\times10^9)(0.0143)} \qquad I = (7.93 \times 10^{-9})\ m^4$$

$I = \frac{\pi d^4}{64}$ (Appendix B-1)

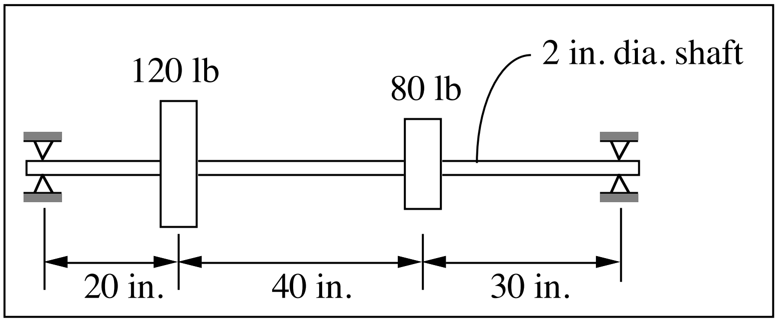
Solving for d in the above equation, gives d = 20 mm. ■

SOLUTION (17.15)

Known: The dimensions of a steel shaft are given.

Find: Determine the critical speed of rotation for the steel shaft.

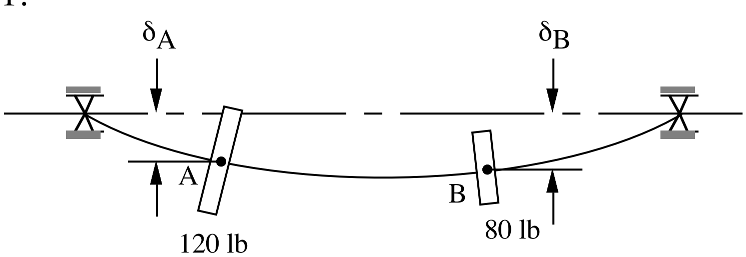
Schematic and Given Data:



Assumptions:
1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.
4. The shaft is simply supported.
5. The mass of the shaft is negligible.

Analysis:

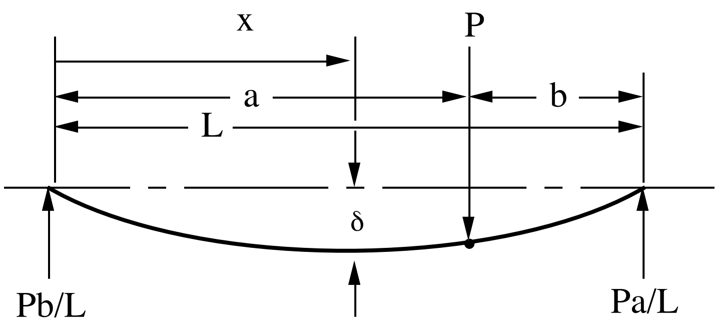
1.



2. Using the equation for a concentrated load at any point for a simply supported beam (Appendix D-2) and the method of superposition, the deflections δ_A and δ_B

can be determined using $\delta = \frac{Pbx}{6LEI}(L^2 - x^2 - b^2)$ for $0 \le x \le a$

where for steel $E = 30 \times 10^6$ psi and for a round shaft $I = \frac{\pi d^4}{64} = \frac{\pi (2)^4}{64} = 0.785 \text{ in.}^4$



Deflection at A due to 120 lb:

$$\delta = \frac{120(70)(20)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 20^2 - 70^2) = 0.0370 \text{ in.}$$

Deflection at A due to 80 lb:

$$\delta = \frac{80(30)(20)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 20^2 - 30^2) = 0.0257 \text{ in.}$$

Total deflection at A: $\delta_A = 0.0370 + 0.0257 = 0.0627$ in.

Deflection at B due to 120 lb:

$$\delta = \frac{120(20)(30)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 30^2 - 20^2) = 0.0385 \text{ in.}$$

Deflection at B due to 80 lb:

$$\delta = \frac{80(30)(60)}{6(90)(30 \times 10^6)(0.785)}(90^2 - 60^2 - 30^2) = 0.0408 \text{ in.}$$

Total deflection at B: $\delta_B = 0.0385 + 0.0408 = 0.0793$ in.

3. Using Eq. (17.2) in Fig. 17.5(b):

$$n_c \approx \frac{30}{\pi}\sqrt{\frac{g(w_A\delta_A + w_B\delta_B)}{w_A\delta_A^2 + w_B\delta_B^2}}$$

$$n_c \approx \frac{30}{\pi}\sqrt{\frac{(32.2)(12\frac{\text{in.}}{\text{ft}})\left[(120)(0.0627)+(80)(0.0793)\right]}{120(0.0627)^2 + 80(0.0793)^2}}$$

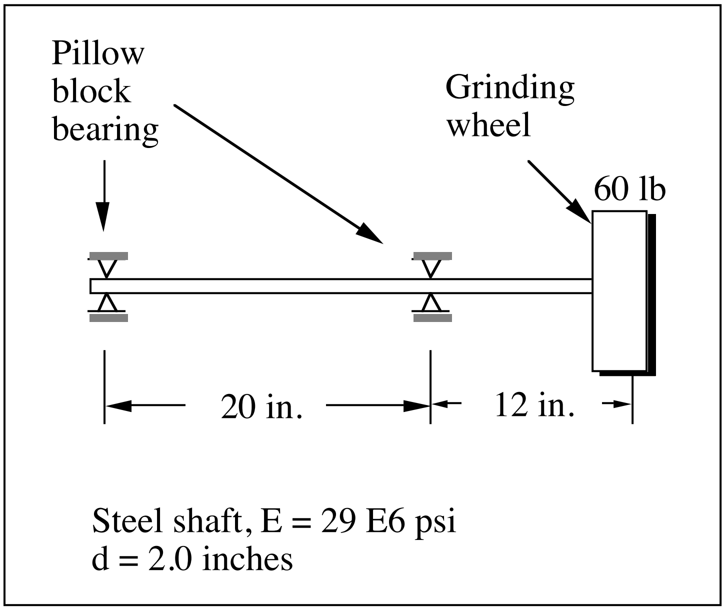
$= 708$ rpm ■

SOLUTION (17.22)

Known: An overhanging 2-in.-diameter steel shaft with attached 60 lbm grinding wheel is shown in Figure P17.22

Find: Determine the critical speed of rotation for the shaft.

Schematic and Given Data:

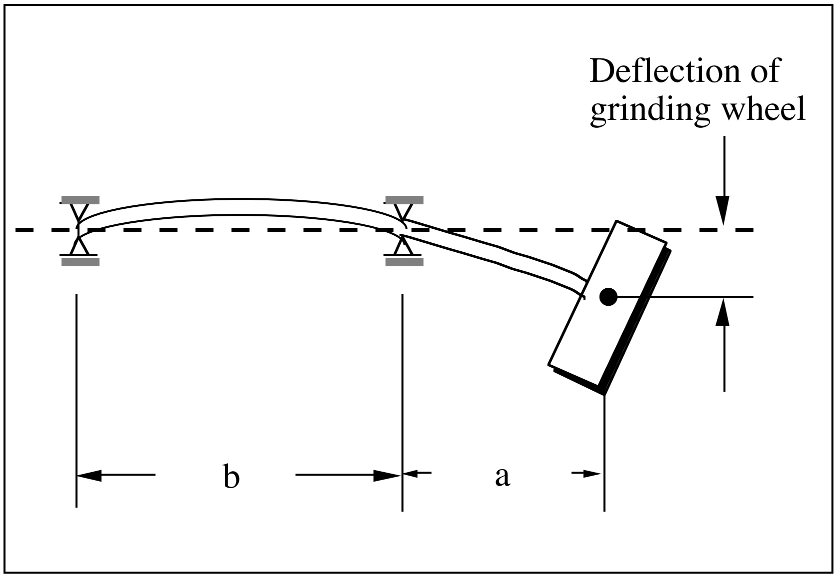


Assumptions:

1. Bearing friction is negligible.
2. The bearings supporting the shafts are accurately aligned.
3. The shaft remains linearly elastic.
4. The shaft is simply supported.
5. The mass of the shaft is negligible.
6. The ball bearings do not prevent angular deflection.

Analysis:

1. A drawing showing an exaggerated static shaft deflection:



2. The moment of inertia for a round steel shaft:

$$I = \frac{\pi d^4}{64} = \frac{\pi (2)^4}{64} = 0.785 \text{ in}^4$$

3. The deflection at the grinding wheel caused by the weight of the grinding wheel is found by:

$$\delta_{st} = \frac{Pa^2(a+b)}{3EI} = \frac{60(12)^2(12+20)}{3(29 \text{ x } 10^6)(.785)} = 0.004047 \text{ in}$$

where $E_{Steel} = 29 \text{ x } 10^6 \text{ psi}$.

4. The critical speed for a shaft with a single mass is found from Eq. 17.1a:

$$n_c = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}} = \frac{30}{\pi}\sqrt{\frac{386}{.004047}} = 2949 \text{ rpm}$$

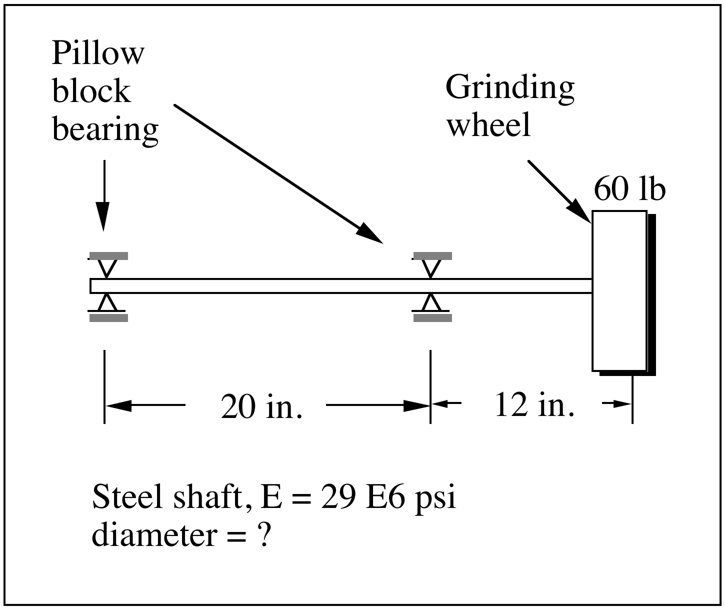
■

SOLUTION (17.23)

Known: An overhanging steel shaft with an attached 60 lbm grinding wheel is shown in P17.22. The minimum critical frequency for the shaft must be equal to or greater than 75Hz. (From the solution to the preceding problem we found that n_c = 2949 rpm for a 2 in. diameter shaft).

Find: Determine the diameter of the steel shaft that produces an acceptable critical frequency.

Schematic and Given Data:



Analysis:

1. We will determine the deflection of the grinding wheel which will produce a critical frequency by rearranging Eq. 17.1a and converting speed to frequency.

$$n_c = \frac{30}{\pi}\omega_c = 60 f_c$$

$$f_c = \frac{n_c}{60} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}$$

Solving the above equation for the static deflection of the guiding wheel yields:

$$\delta_{st} = \frac{g}{((2)(\pi)(f_c))^2} = \frac{386.4}{((2)(\pi)(75))^2} = .00174 \text{ in}$$

2. With the deflection known, the beam deflection equation used previously can be rearranged to solve for the required moment of inertia for the steel shaft:

$$I = \frac{Pa^2(a+b)}{3E\delta_{ST}} = \frac{(60)(12)^2(12+20)}{3(29 \text{ x } 10^6)(.00174)} = 18264 \text{ in}^4$$

3. The moment of inertia equation for a round shaft is rearranged to solve for diameter:

$$d = \sqrt[4]{\frac{64\ I}{\pi}} = \sqrt[4]{\frac{64(18264)}{\pi}} = 2.47 \text{ in}$$

4. Alternatively, the equation given below can be used to solve for the new shaft size directly.

$$d_{new} = d_{old}\sqrt{\frac{n_{new}}{n_{old}}} = d_{old}\sqrt{\frac{f_{new}}{f_{old}}} = (2 \text{ in})\sqrt{\frac{75\text{Hz}}{\frac{2949}{60}\frac{\text{rpm}}{\text{s/min}}}} = 2.47 \text{ in}$$

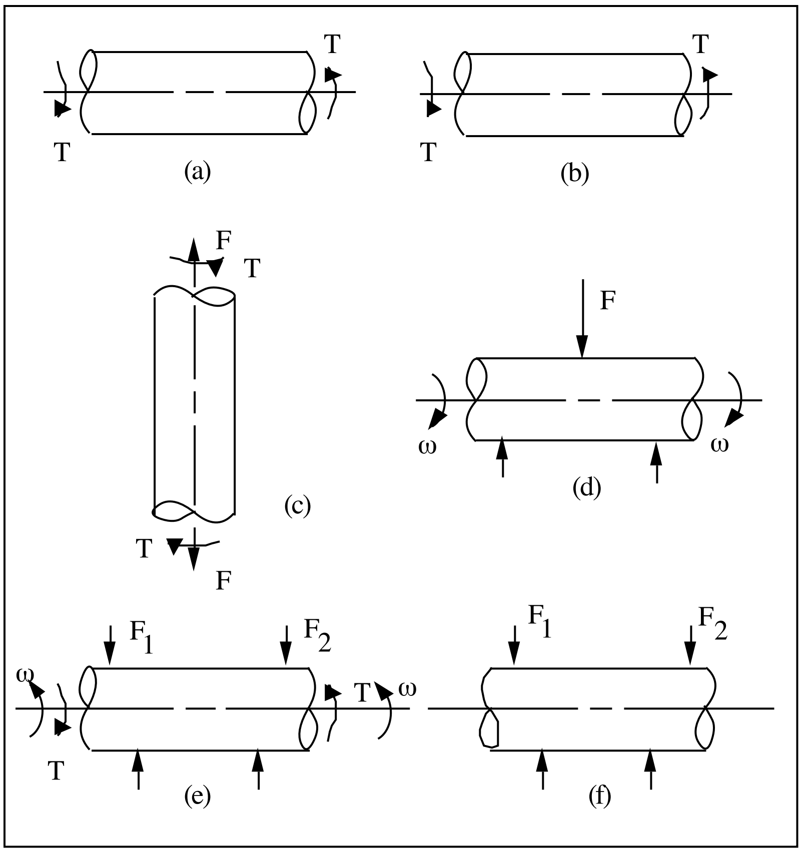
Comment: A shaft that is 2.47 inches in diameter is probably not commercially available which makes using a size, such as, 2.50 inches more attractive. From the analysis, we can tell that it would tend to increase the critical frequency which still allows us to meet the design constraint in the problem statement. ■

SOLUTION (17.24)

Known: Six shafts are used in six specified applications.

Find: Determine the shaft loadings (bending, axial and torsion) involved in each application and give a short explanation of the cause of each loading.

Schematic and Given Data:



Assumptions:

1. The shafts are shown in the physical arrangement of the application (with the gravitation force on the turbine in case (c) being significant and vertically downward).
2. Bearing friction is negligible.
3. The weights of the shafts are negligible and connected elements are separately supported.
4. The bearings supporting the shafts are accurately aligned.
5. The gears are all spur gears of the same pressure angle and mounted to mesh properly with each other.

Analysis:

(a) Static torsion only. Shaft weight would normally be negligible.

(b) Same as (a). (Note that gear tooth forces balance to produce pure torque).

(c) Static torsion and static axial load.

(d) Alternating bending only (forces remain fixed while shaft rotates--as in Fig. 8.3).

(e) Static torsion plus alternating bending (forces are fixed while shaft rotates). Possible static axial load if gears are helical.

(f) Static bending only. (Bearings can apply only radial loads to the shaft.)

Comments:

(1) Minor misalignments of the bearings, improper mounting and meshing of gears and large shaft weights can cause significant additional shaft bending.

(2) Dynamic loads can cause substantially higher stresses in the shaft than static or low cycle loads.

SOLUTION (17.25)

Known: A helical gear mounted on a shaft is simply supported by bearing A and bearing B and has specified load components acting on it. The left end of the shaft is free, the right end, near the bearing B, is attached to a flexible coupling to transmit torque. Bearing B takes thrust. Shaft material, its ultimate and yield strength, and stress concentration factors are given.

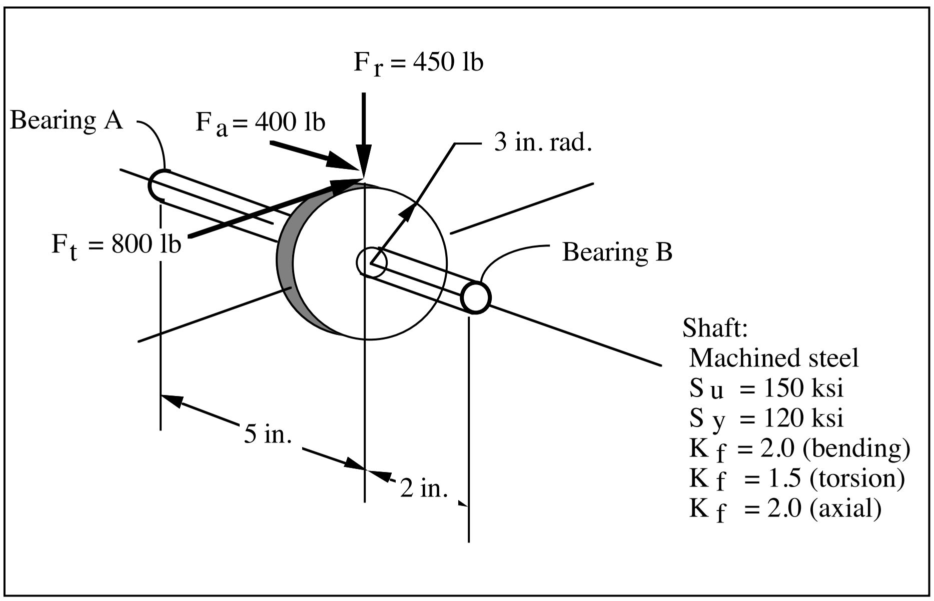
Find:

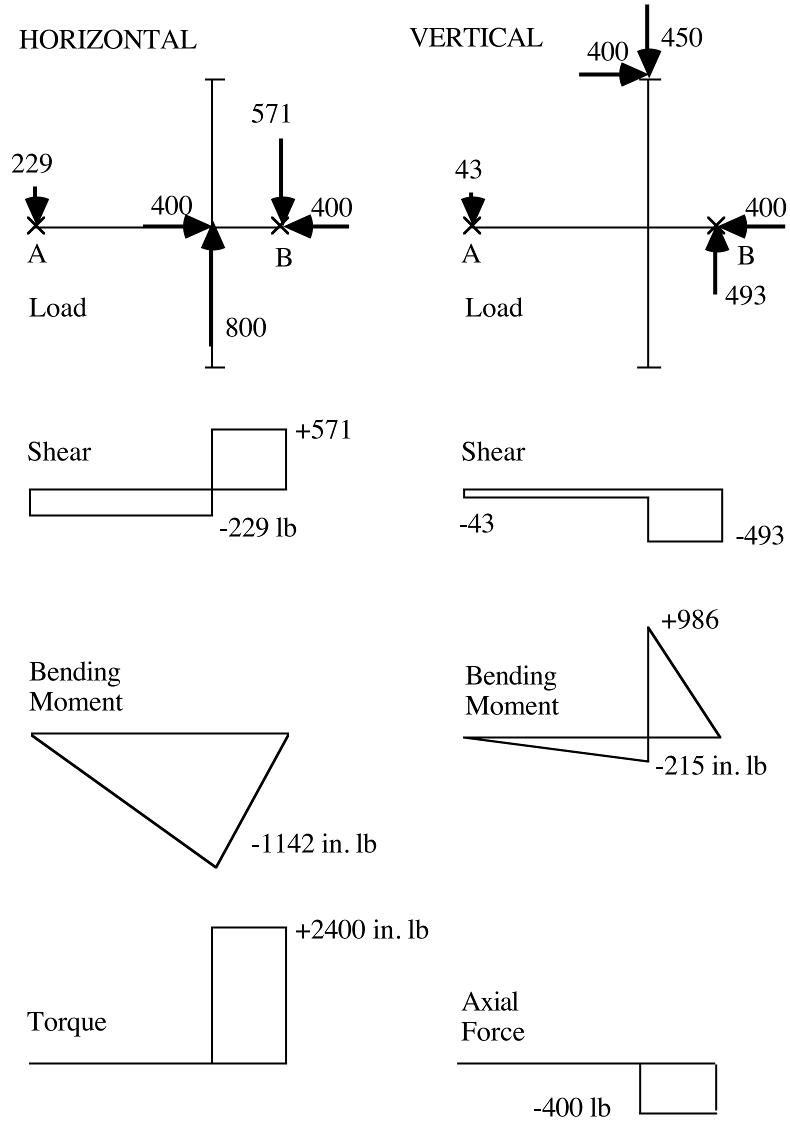
(a) Determine load, shear force and bending moment diagrams for the shaft in the vertical and horizontal planes and also diagrams for torsional and axial loading.

(b) Determine the radial and thrust loads on the bearings.

(c) Identify the critical cross section of the loaded shaft and for this location determine the cross sectional diameter required for infinite design life.

Schematic and Given Data:





Assumptions:

1. The bearing widths are small relative to the length of the shaft so that they can be idealized as point supports.
2. Bearing friction is negligible.
3. Shaft deflection is small so that locations and directions of loads are constant with respect to the shaft.
4. The gear is rigidly connected to the shaft.
5. The weights of the shaft and gear can be neglected.
6. Axial stresses are negligible compared to torsion stresses (to be verified).
7. The diameter required at the critical section is between 0.4 in. and 2 in. so that the gradient factor, $C_G = 0.9$ according to Table 8.1.

Analysis:

1. From the free body diagrams in the horizontal and vertical planes,
   $\sum M_A = 0$; $800(5) = B_H\,(7)$
   hence, $B_H = 571$ lb
   $\sum M_A = 0$; $450(5) + 400(3) = B_v\,(7)$
   hence, $B_v = 493$ lb
   Therefore, the loads on the bearings A and B are:
   $A_r = \sqrt{229^2 + 43^2}$: $A_r = 233$ lb
   $B_r = \sqrt{571^2 + 493^2}$: $B_r = 754.4$ lb $B_t = 400$ lb (thrust)

2. The most critical section is just to right of the gear.
   For the most critical section:

$$\tau_m = \frac{16T}{\pi d^3} K_f = \frac{16(2400)}{\pi d^3}(1.5) = \frac{18{,}335}{d^3}$$

$$\sigma_{a,m} \text{ (axial mean stress)} = \frac{P}{A} K_f = -\frac{400(4)}{\pi d^2}(2) = -\frac{1019}{d^2}$$

$$\sigma_{b,a} \text{ (bending alt. stress)} = \frac{32M}{\pi d^3} K_f = \frac{32\sqrt{1142^2 + 986^2}}{\pi d^3}(2)$$

$$\sigma_{b,a} = \frac{30{,}736}{d^3}$$

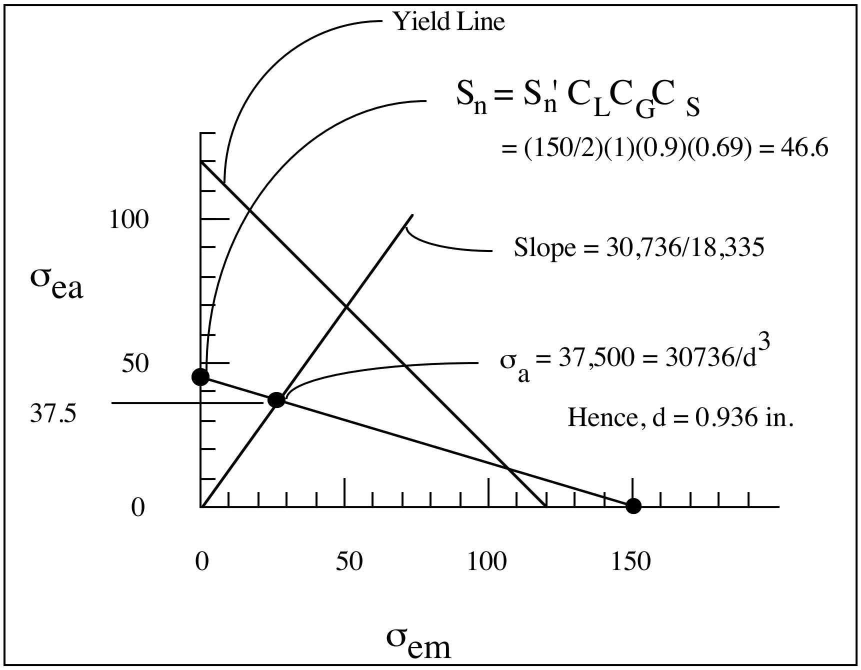
3. Applying the Fig. 8.16 procedure:

$$\sigma_{e,a} = \sqrt{\sigma_{b,a}{}^2 + 0} = \sigma_{b,a} = \frac{30{,}736}{d^3}$$

Assume σ_{am} is negligible, then $\sigma_{em} = 0 + \sqrt{\tau_{am}{}^2 + 0} = \tau_m = \dfrac{18{,}335}{d^3}$

The slope of the load line is $\sigma_{ea}/\sigma_{em} = 30{,}736/18{,}335$.

4. We now construct a Goodman diagram:



5. From the Goodman diagram, d = 0.936 in. ■

6. Note: for $d \approx 0.94$, $\sigma_{am} = 1019/0.94^2 < 1.2$ ksi and is therefore negligible, and $C_G = 0.9$. Hence earlier assumptions are appropriate.

Comments:

(1) Consideration of the weight of the gear and the shaft will produce additional radial and/or thrust loads on the bearing depending on the orientation of the shaft axis in the application.

(2) If the bearing friction forces are high enough to warrant consideration, they will change the torque diagram and will reduce the maximum torque value at the critical section by the friction torque of the left bearing.

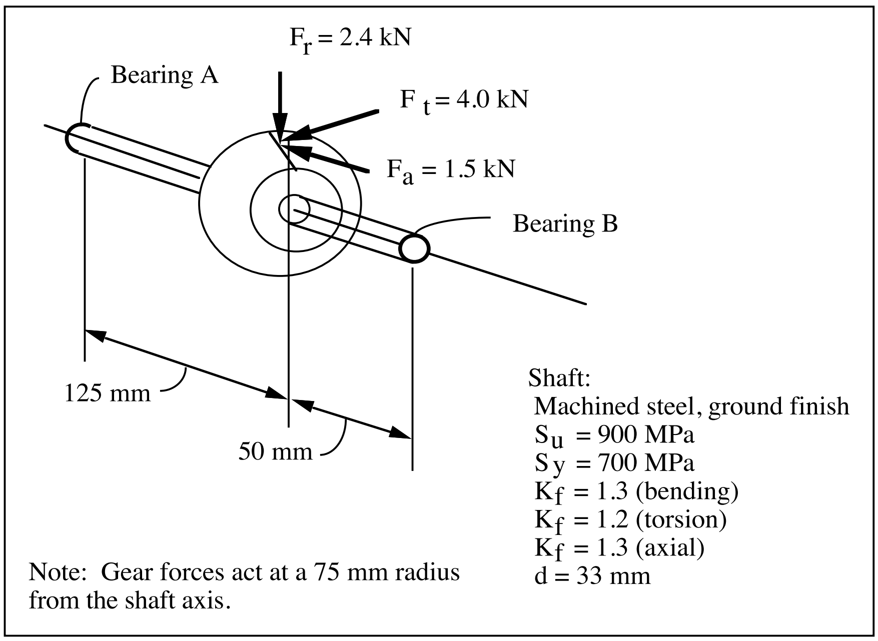
(3) Although the axial load is of the same order of magnitude as the radial load the axial stress is very much smaller than the bending stress in this case because the bending moment is fairly large. For short shafts the same axial load can cause stresses comparable in magnitude to the bending stresses since the maximum loading moment will be smaller for a shorter shaft.

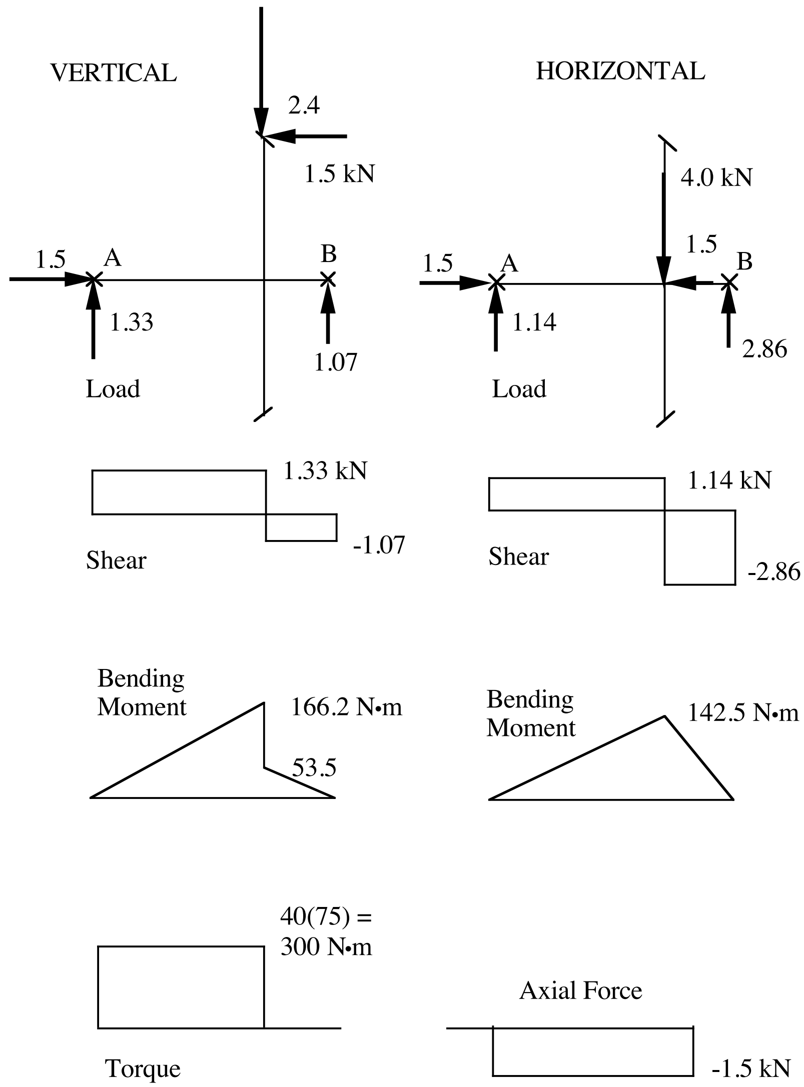
SOLUTION (17.27)

Known: A bevel pinion mounted on a shaft is simply supported by bearing A and bearing B, and has specified load components acting on it. The left end of the shaft, closer to bearing A, is coupled to an electric motor while the right end is free. Bearing A takes thrust. Shaft diameter, stress concentration factors, material strengths and surface finish are specified.

Find: Estimate the factor of safety for the shaft.

Schematic and Given Data:





Assumptions:

1. The bearing widths are small relative to the length of the shaft so that they can be idealized as point supports.
2. Bearing friction is negligible.
3. Shaft deflection is small so that locations and directions of loads are constant with respect to the shaft.
4. The gear is rigidly connected to the shaft.
5. The weights of the shaft and gear can be neglected.

Analysis:

1. From the free body diagrams in the horizontal and vertical planes,
   $\sum M_A = 0;\ 2.4(125) - 1.5(75) = B_v(175)$
   $\sum M_A = 0;\ 4.0(125) = B_H(175)$
   $B_v = 1.07$ kN; $B_H = 2.86$ kN
   Therefore, the loads on the bearings A and B are:

$A_r = \sqrt{1.33^2 + 1.14^2}$: $A_r = 1.75$ kN, $A_t = 1.5$ kN (thrust)
$B_r = \sqrt{1.07^2 + 2.86^2}$: $B_r = 3.05$ kN

2. The most critical section is just to the left of the gear. For the most critical section:

$$\tau_m = \frac{16T}{\pi d^3} K_f = \frac{16(300)}{\pi(33)^3}(1.2) = 0.0510 \text{ GPa} = 51.0 \text{ MPa}$$

$$\sigma_{a,m} \text{ (axial mean stress)} = \frac{P}{A} K_f = \frac{-1.5(4)}{\pi(33)^2}(1.3) = -0.00228 \text{ GPa} = -2.28 \text{ MPa}$$

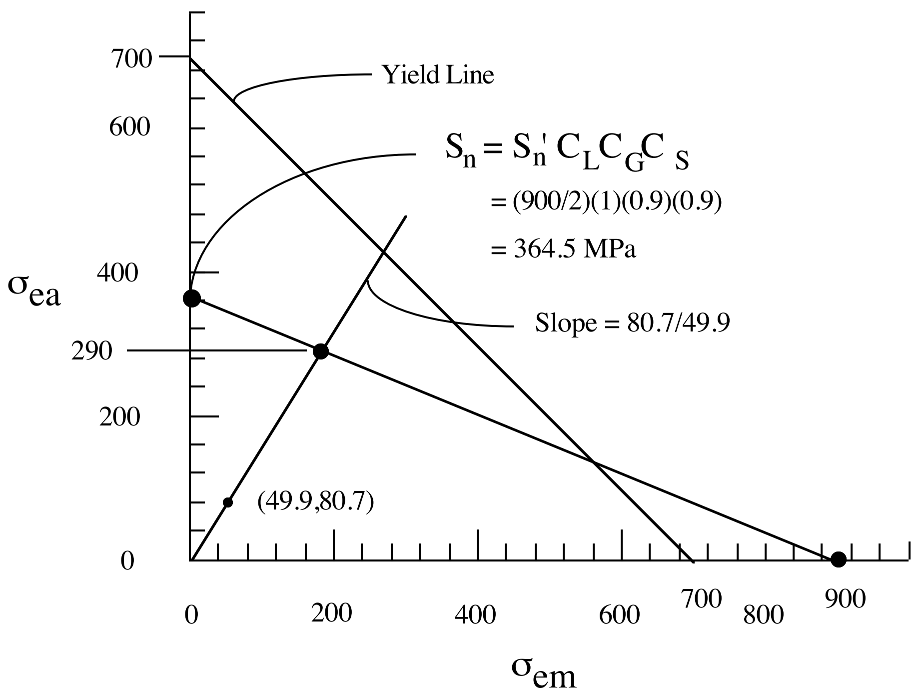
$$\sigma_{b,a} \text{ (bending alt. stress)} = \frac{32M}{\pi d^3} K_f = \frac{32\left(\sqrt{166.2^2 + 142.5^2}\right)}{\pi(33)^3}(1.3)$$
$= 0.0807$ GPa $= 80.7$ MPa

3. Applying the Fig. 8.16 procedure:

$$\sigma_{e,a} = \sqrt{\sigma_{b,a}{}^2 + 0} = \sigma_{b,a} = 80.7 \text{ MPa}$$

$$\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau_m{}^2 + \left(\frac{\sigma_m}{2}\right)^2} = -\frac{2.28}{2} + \sqrt{51.0^2 + \left(\frac{-2.28}{2}\right)^2} = 49.9 \text{ MPa}$$

$$SF \approx \frac{290}{80.7} = 3.59, \text{ say } SF \approx 3.6$$



Comments:

(1) In this problem, the (mean) axial stresses are small compared to the (mean) torsion stresses but are still included in the analysis (σ_{em} changes by about 2% when axial stresses are considered).

(2) It is important to estimate the stress concentration factors accurately since they inversely affect the safety factor.

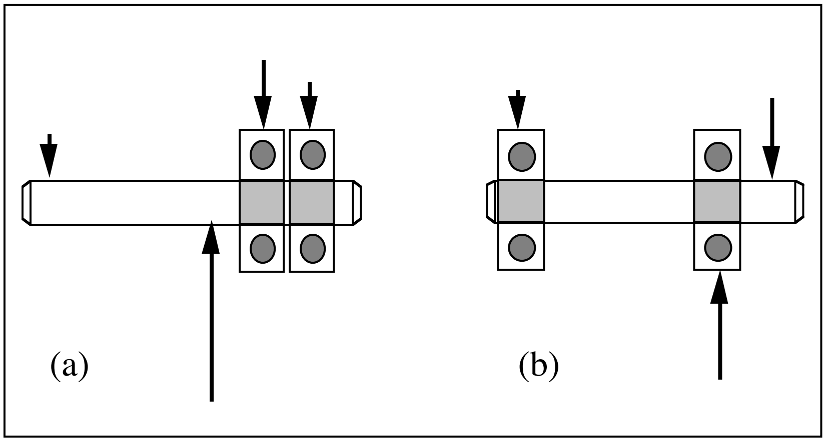
(3) The maximum bending moment can be reduced by decreasing the length of the shaft and/or placing the gear closer to one of the support bearings. Placing the gear closer to a bearing, however, has the undesirable side-effect of producing higher loads on the bearing.

SOLUTION (17.28D)

Known: Two alternative approaches to supporting an overhung chain idler sprocket (or spur gear or belt sheave) are given.

Find: Determine the fundamental differences between the two approaches with respect to shaft loading and bearing loading. Also, determine how this comparison would change if a bevel gear were substituted for the chain sprocket.

Schematic and Given Data:



Assumptions:

1. The chain sprocket (or spur gear or belt sheave) does not experience any significant axial load at its periphery.
2. The bevel gear substituted for the chain sprocket has a substantial cone angle so as to produce a significant axial load.

Analysis:

1. In (a) the bearings each take a radial load equal to half the total chain tension, and the shaft bending load is static. In (b) the bearing next to the sprocket takes a radial load of about 1.5 times the total chain tension, and the shaft bending load is alternating (as in Fig. 8.3).
2. With a bevel gear, a thrust load and a cocking moment would be added to the shaft due to an axial load on the gear tooth. Since the distance between bearings is much closer in (a), the added bearing radial loads needed to resist the cocking moment would be much greater in (a) than in (b).

Comment: It is clear from the solution of this problem that the geometric arrangement of the shafts and bearings must be chosen carefully after consideration of the types and magnitudes of loadings that are applied to the shaft through other elements connected to the shaft.

SOLUTION (17.33)

Known: A countershaft has helical gear (B), bevel gear (D), and two supporting bearings (A and C) as shown in FIGURE P17.33 of the textbook. Loads acting on the bevel gear are known. Forces on the helical gears can be determined. Shaft dimensions are known. All shoulder fillets have a radius of 5 mm. Only bearing (A) takes thrust. The shaft is made of hardened steel having known values of S_u = 1069 MPa and S_y = 896 MPa. Important surfaces are finished by grinding.

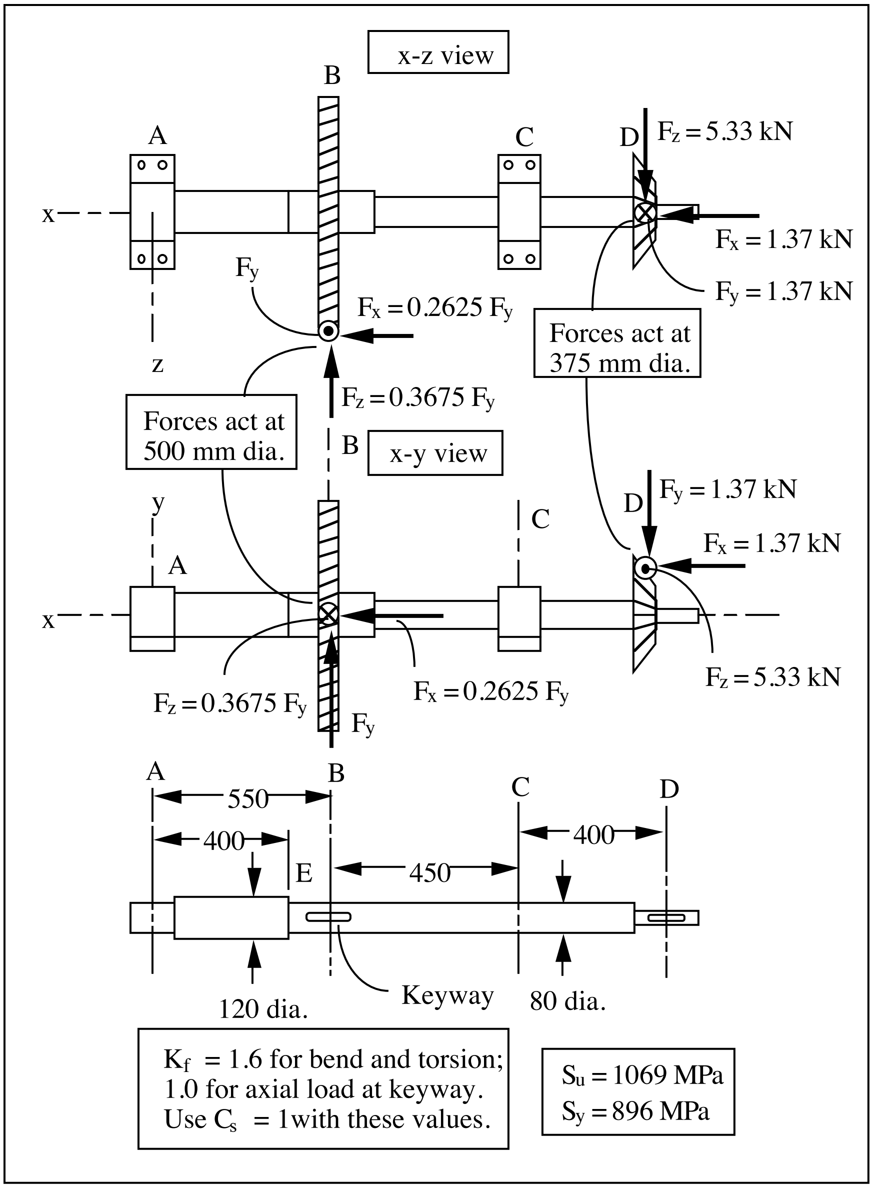
Find:

(a) Draw load, shear force, and bending moment diagrams for the shaft in the xy- and xz- planes. Also draw diagrams showing the intensity of the axial force and torque along the length of the shaft.

(b) At points B, C, and E of the shaft, calculate the equivalent stresses in preparation for making a fatigue safety factor determination. (Note: Refer to Figure 8.16 and Table 8.2 of the textbook.)

(c) For a reliability of 99% (and assuming $\sigma = 0.08\ S_n$), estimate the safety factor of the shaft at points B, C, and E.

Schematic and Given Data:



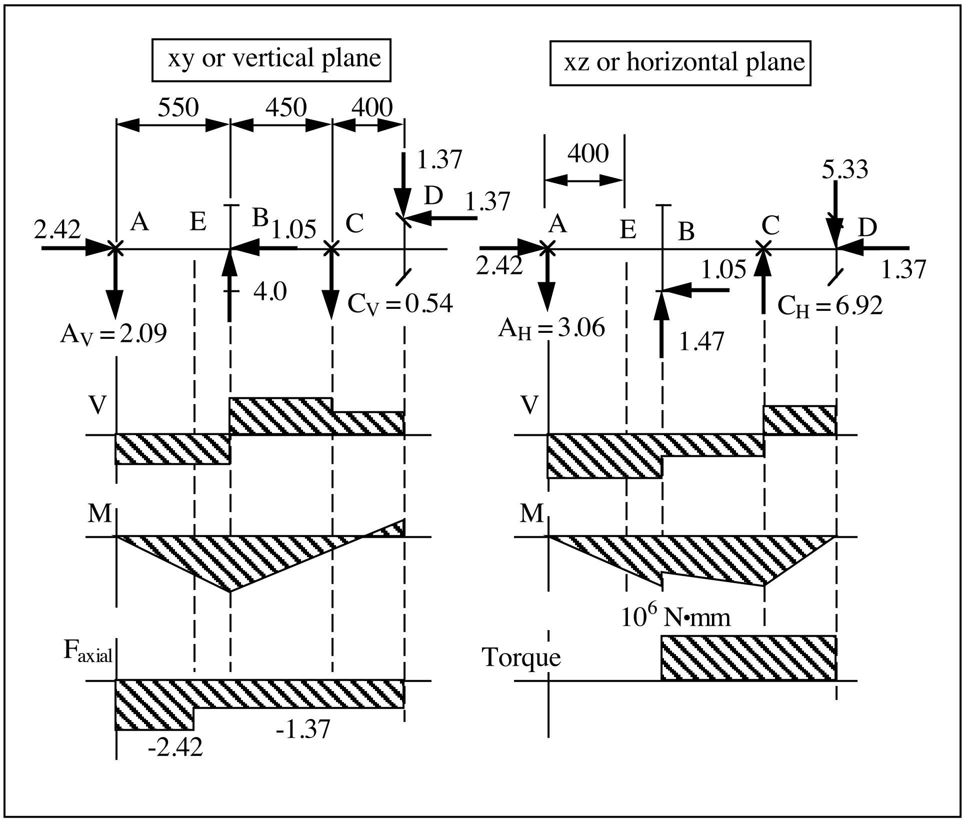
Assumption: The shaft is manufactured as specified with regard to the critical shaft geometry and surface finish.

Analysis:

1. Load determination

(a) Helical gear forces:
For $\sum M_x = 0$, the torque at the two gears must be equal. Therefore, F_y (250 mm) = 5.33(187.5 mm). Hence, $F_y = 4.00$ kN.
From the given data, $F_x = .2625 F_y = 1.05$ kN; $F_z = .3675\ F_y = 1.47$ kN.

(b) Determine shaft loads in the xy and xz planes



Vertical forces:

$$\sum M_A = 0: \; C_v = \frac{4(550) + 1.37(187.5) - 1.37(1400)}{1000}$$
$$= 0.54 \text{ kN downward}$$

$\sum F = 0: \quad A_v = 4 - 0.54 - 1.37 = 2.09$ kN downward

Horizontal forces:

$$\sum M_A = 0: \; C_H = \frac{1.05(250) - 1.47(550) + 5.33(1400)}{1000}$$
$$= 6.92 \text{ kN upward}$$

$\sum F = 0: \quad A_H = 1.47 + 6.92 - 5.33$
$= 3.06$ kN downward

2. Stress determination

(a) At E, the loading is:
Compression of 1.37 kN, $K_t = 2.2$, $q = .94$,

$K_f = 2.13$. Axial stress (mean or constant) =

$$\frac{4PK_f}{\pi d^2} = \frac{4(-1.37)(2.13)}{\pi(80)^2} = -0.581 \text{ MPa}$$

The tension stress is zero.

$$M = \sqrt{(2.09 \times 400)^2 + (3.06 \times 400)^2}$$
$$= 1482 \text{ kN·mm}$$

$K_t = 1.9$, $q = .94$. Therefore, $K_f = 1.85$

$$\text{Bending stress (alternating)} = \frac{32M}{\pi d^3}K_f$$

$$= \frac{32(1482 \times 10^3)}{\pi(80)^3}(1.85) = 54.5 \text{ MPa}$$

From Eq. (a) and Eq. (b) in the figure caption of Fig. 8.16, $\sigma_{em} = 0$; $\sigma_{ea} = 54.5$ MPa

(b) At B, the loading is:

Axial, $P = -1.37$ kN, $K_f = 1.0$, $\sigma = -0.27$ MPa
Torsion = (4.0)(250) = 1000 kN·mm
Bending : $M = \sqrt{(2.09 \times 550)^2 + (3.06 \times 550)^2} = 2038$ kN·mm
$K_f = 1.6$ for bending and torsion

$$\text{Bending stress (alternating)} = \frac{32M}{\pi d^3}K_f$$

$$= \frac{32(2038 \times 10^3)}{\pi(80)^3}(1.6) = 64.9 \text{ MPa}$$

$$\text{Torsional stress (mean)} = \frac{16T}{\pi d^3}K_f = \frac{16(10)^6}{\pi(80)^3}(1.6) = 15.9 \text{ MPa}$$

$$\sigma_{em} = \frac{-0.27}{2} + \sqrt{(15.9)^2 + \left(\frac{-0.27}{2}\right)^2} = 15.76 \text{ MPa}; \qquad \sigma_{ea} = 64.9 \text{ MPa}$$

(c) At C, the loading is:

Bending:
$$M = \sqrt{(5.33 \times 400)^2 + \left[1.37 \times (400 - 187.5)\right]^2} = 2152 \text{ kN·mm}$$

$$\text{Bending stress (alternating)} = \frac{32(2152) \times 10^3}{\pi(80)^3} = 42.8 \text{ MPa}$$

$\sigma_{ea} = 42.8$ MPa

Torsional stress - same as (b) except no stress concentration factor; axial same as (b).

$$\sigma_{em} = \frac{-0.27}{2} + \sqrt{\left(\frac{15.9}{1.6}\right)^2 + \left(\frac{.27}{2}\right)^2} = 9.80 \text{ MPa}$$

3. Strength and safety factor determination

$S_u = 155$ ksi $= 1069$ MPa; $S_y = 130$ ksi $= 896$ MPa
For working with equivalent bending stress, S_n is

$$S_n = S_n' C_L C_G C_s C_T C_R = \left(\frac{1069}{2}\right)(1)(0.8)^*(0.9)(1)(1)$$

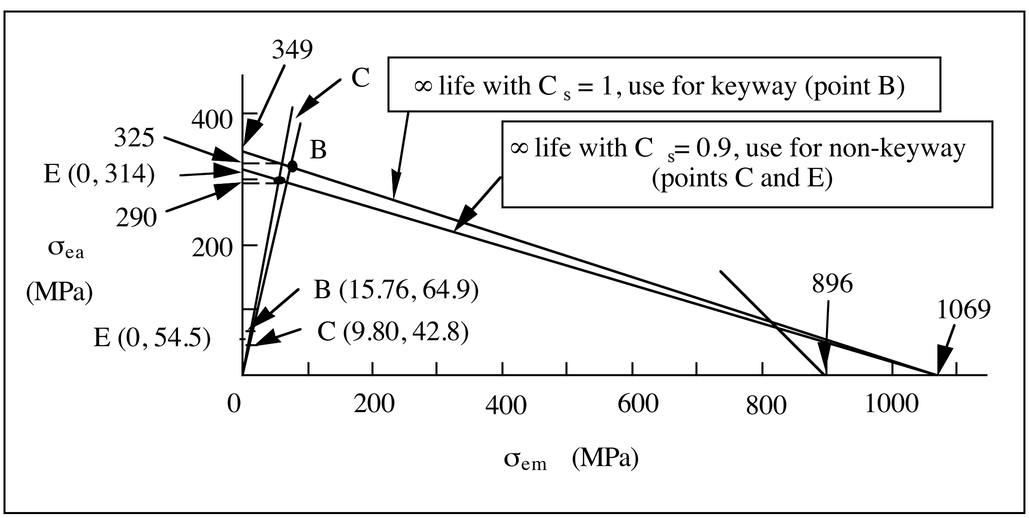
$= 385$ MPa for $C_s = 0.9$

*(See note b, Table 8.1)

$$S_n = S_n' C_L C_G C_s C_T C_R = \left(\frac{1069}{2}\right)(1)(0.8)(1.0)(1)(1)$$
$= 428$ MPa for $C_s = 1.0$

But for 99% reliability, reduce this by 2.3 standard deviations, which amounts to multiplying by a factor of $(1 - 2.3 \times .08) = .816$
Thus, for 99% reliability,
$S_n = 385(.816) = 314$ MPa (for $C_s = .9$)
$S_n = 428(.816) = 349$ MPa (for $C_s = 1.0$)

4.



5. Safety factors: (B) SF = 325/64.9 = 5.0 ■
(C) SF = 290/42.8 = 6.8 ■
(E) SF = 314/54.5 = 5.8 ■

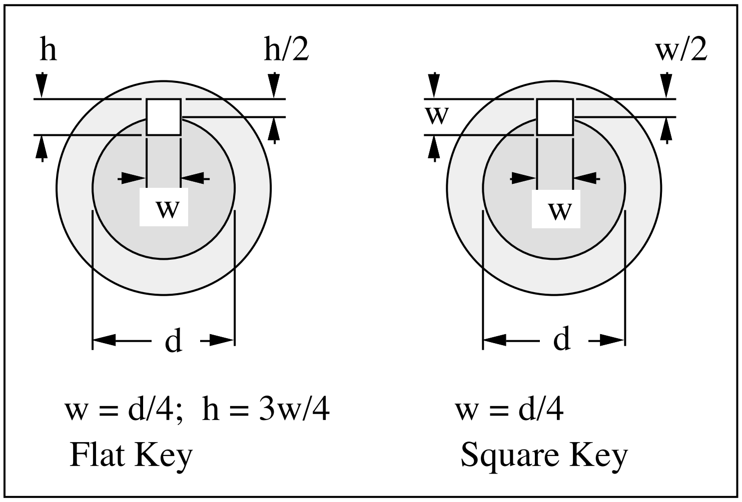
Comment: This problem is the same as one in Chapter 8 but probably still appropriate for the student to work in this chapter.

SOLUTION (17.35D)

Known: A flat key is to be used with a round shaft to transmit a torque equal to the elastic torque capacity of the shaft. Key and shaft material are made of the same ductile material and the key in tightly fitted at its top and bottom.

Find: Estimate the length of flat key required. Also, compare the estimate with the length of square key required and suggest a possible reason why a flat key might be preferred in some cases.

Schematic and Given Data:



Decisions:

1. The flat and square keys to be considered are of standard proportions.
2. The key and shaft materials are identical ductile steels (given).
3. Key clearances with the shaft and hub are small.

Assumptions:

1. Forces on the key sides are uniformly distributed.
2. The loading on the shaft is steady (no shock or fatigue).

Design Analysis:

1. From Eq. (4.4), with $\tau = S_{ys} = 0.58S_y$, shaft torque capacity is:

$$T = \frac{\pi d^3}{16}(0.58S_y) \qquad \text{----(a)}$$

2. For a standard proportioned flat key, key torque capacity limited by compression is (see Fig. 17.1b): (limiting stress)(contact area)(radius),

$$\text{hence, } T = (S_y)\left(\frac{L \cdot 3d}{32}\right)\left(\frac{d}{2}\right) = 0.047S_y\,Ld^2 \qquad \text{----(b)}$$

3. For the flat key, key torque capacity limited by key shear is

$$T = 0.58S_y\,Ld^2/8 \qquad \text{----(c)}$$

This torque capacity is the same for a square key.

4. Equating (a) and (b):

$$\frac{\pi d^3}{16}(0.58S_y) = 0.047S_y\,Ld^2$$

$$\frac{0.58\pi}{16}d = 0.047L. \text{ Hence, } L = 2.4d \qquad \blacksquare$$

Equating (a) and (c) gives L = 1.57d

5. The flat key weakens the shaft less than does the square key since a shallower seat is required for the flat key.

Comment: The torque capacity with respect to shearing of the key is the same whether a square key or a flat key of the same length (and of standard proportions) is used because their widths are both equal to d/4. The torque capacity with respect to compressive failure is, however, higher for a square key than a flat key since the height of a square key is greater. But, both the standard proportioned square key and flat key have the same torque capacity here because shear failure limits torque capacity.

SOLUTION (17.36D)

Known: Web site addresses are given as **http://www.pddnet.com** and **http://www.powertransmission.com**.

Find: Identify and discuss methods of coupling rotating shafts.

Analysis:

1. The web site search is left as an exercise for the student.
2. The book, Mechanical Details for Product Design, edited by Douglas C. Greenwood, McGraw-Hill, Inc., 1964, p. 288-291, illustrates typical methods of coupling rotating shafts. Methods of coupling rotating shafts vary from simple bolted flange constructions to complex spring and synthetic rubber mechanisms. Some types incorporating chain belts, splines, bands, and rollers are described and illustrated. Shaft couplings that utilize internal and external gears, balls, pins, and nonmetallic parts to transmit torque are also shown.
