SOLUTION (15.29)

Known: Three spur gears transmit power from a motor shaft to a machine shaft in a given geometric arrangement. The middle gear acts as an idler and is supported by two bearings.

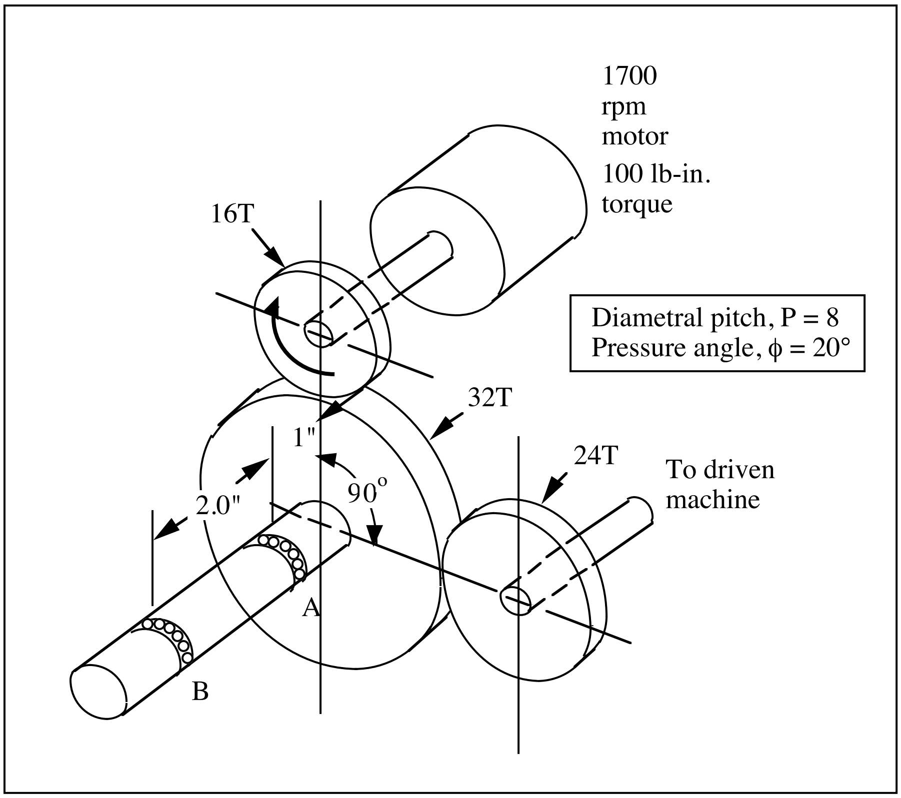
Find:

(a) Determine the radial load on idler shaft bearings for a given direction of motor shaft rotation.

(b) Determine the radial load on the bearings for the motor shaft rotation opposite to (a).

(c) Give an explanation as to why answers to (a) and (b) are different.

Schematic and Given Data:

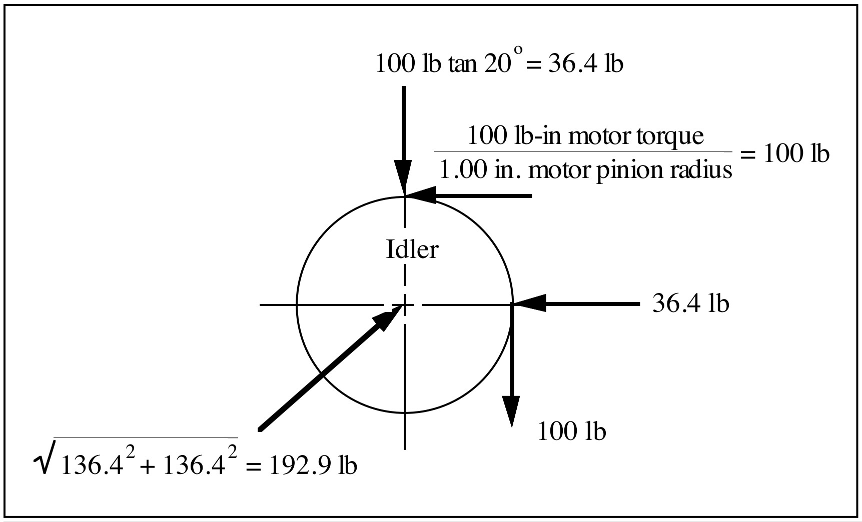


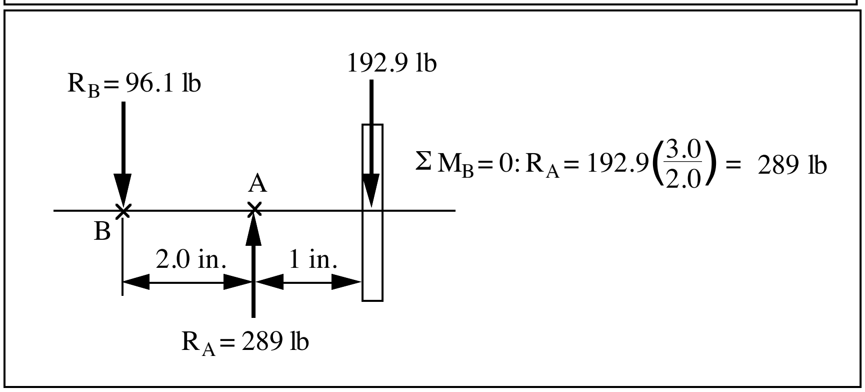
Assumptions:

1. The gears mesh along their pitch circles.
2. All the gear tooth loads are static and are transmitted at the pitch point.
3. Friction losses in the gears and bearings are negligible.
4. Shaft bending deflections can be neglected.

Analysis:

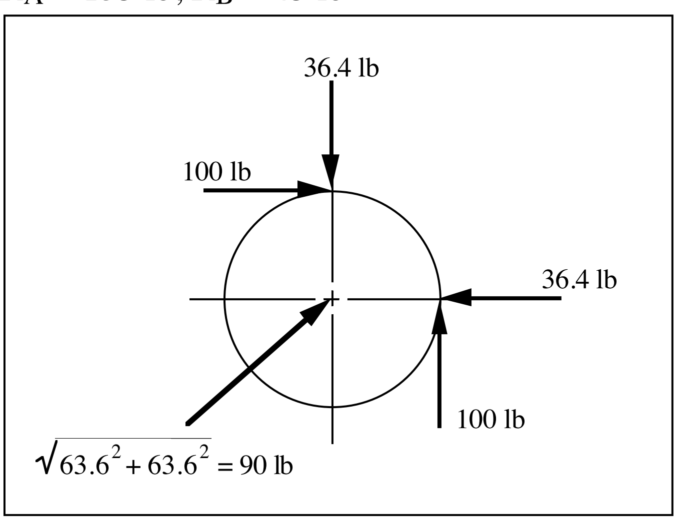
(a) Pitch diameter of idler $= \dfrac{N}{P} = \dfrac{32}{8} = 4.0$ in.





Bearing radial loads: R_A = 289 lb, R_B = 96.1 lb ■

(b) Bearing loads are reduced by factor of 90/192.9 to give:
R_A = 135 lb, R_B = 45 lb ■



(c) Reversing direction of rotation reversed tangential forces causes tangential and radial components to subtract, rather than add.

Comments:

1. This problem illustrates the use of gear trains for purposes other than strictly speed or torque changing. Idlers are frequently used to convey rotary motion short distances from driver to driven shafts or to drive multiple shafts.
2. The location of bearings for the idler shaft in this problem caused a large radial load on the bearing closer to the idler and a smaller radial load on the other bearing. Although location of bearings on either side of the idler could have equalized the radial loads, such an arrangement may or may not be allowed by space constraints or accessibility requirements in actual applications.
3. For cases in which the rotation of the shaft is reversed from that shown in the figure, the explanation in part (c) reveals that the arrangement of the gears can be changed, putting the driven gear to the left of the idler in the figure to obtain lower bearing loads.

SOLUTION (15.30)

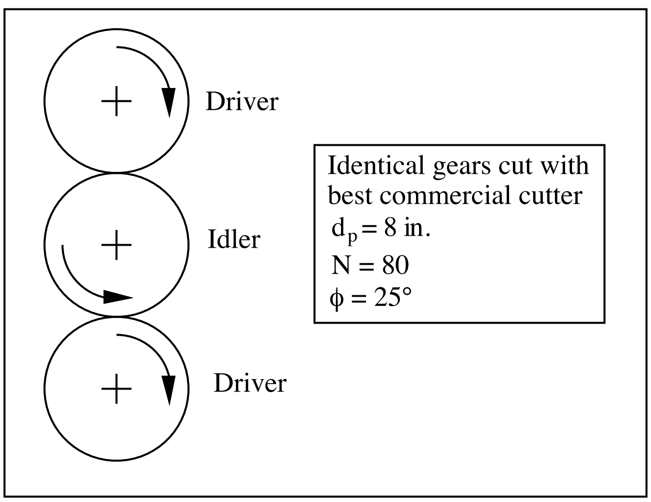
Known: Three identical spur gears are used to transmit power from a motor to a machine through an idler. Motor rpm is specified.

Find:

(a) Determine the gear most vulnerable to tooth bending fatigue failure.

(b) Determine the values for V, P, p, K_v, J

Schematic and Given Data:

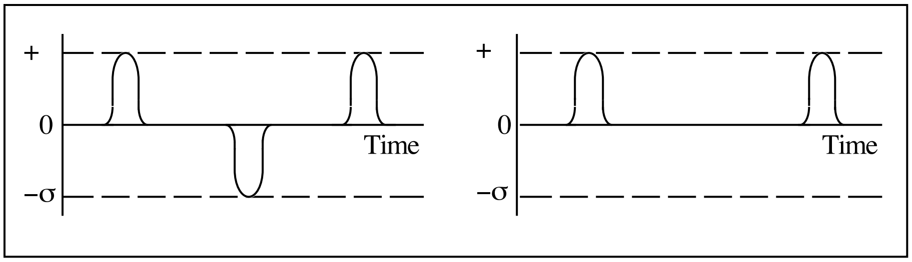


Assumption:

1. The gears mesh along their pitch circles and transmit all the load at the pitch point.
2. Friction losses can be neglected and load sharing is absent.

Analysis:

(a) The gear most vulnerable to tooth bending fatigue failure is the idler because it is subjected to 2-way bending; others are bent only 1-way, thus:



(b) Idler | Input and output

$V = \frac{\pi(8)}{12} \cdot 1000$: $V = 2094$ ft/min ■

$P = 80/8$: $P = 10$ teeth/in. ■

$p = \pi/10$: $p = 0.314$ in. ■

$K_v = \frac{50 + \sqrt{2094}}{50}$: $K_v = 1.92$ ■

$J = 0.375$ (Fig. 15.23b; no load sharing) ■

Comments:

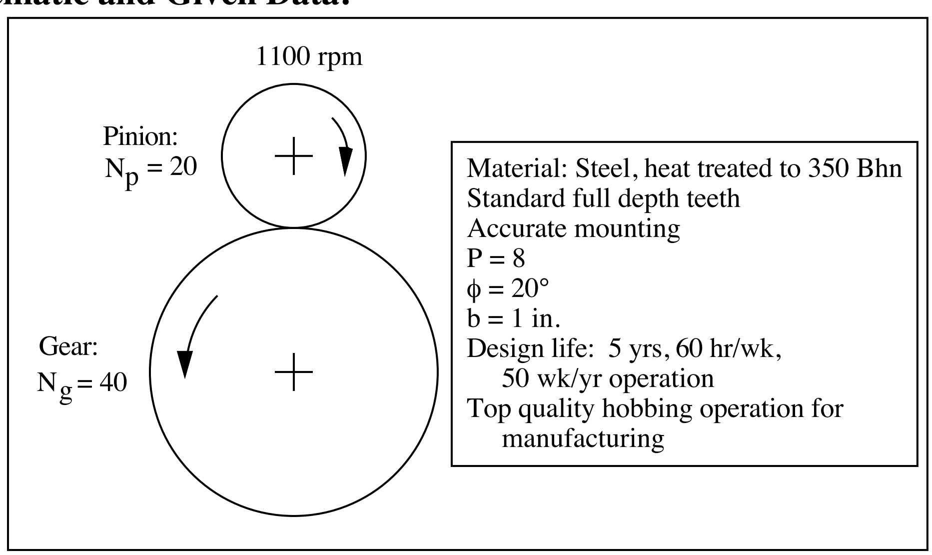
1. Larger diameters and higher rpm for gears produce larger values for K_v.
2. A pressure angle of 20° instead of 25° would reduce the value of the geometry factor from 0.375 to 0.31.

SOLUTION (15.31)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based only on bending fatigue.

Schematic and Given Data:



Assumptions:

1. The gears mesh along their pitch circles.
2. All the gear tooth loads are transmitted at the pitch point.
3. There is no load sharing between the teeth.
4. The electric motor and blower constitute uniform load driver and driven equipment.
5. Top quality hobbing operation for manufacturing corresponds to curve C in Fig. 15.24 (to estimate velocity factor K_v).

Analysis:

1. From Eq. (15.17): $\sigma = \dfrac{F_t P}{b\,J} K_v\, K_o\, K_m$

   K_v requires finding the pitch line velocity as,

   $$V = \frac{\pi\, d\, n}{12} = \frac{\pi(20/8)1100}{12} = 720 \text{ fpm}$$

   from Curve C of Fig. 15.24,

   $$K_v = \frac{50 + \sqrt{720}}{50} = 1.54$$

   from Fig. 15.23(a), J = 0.24 (for the pinion, as it is weaker - and with no load sharing)
   Also, K_m = 1.6 (from Table 15.2 - probably best judgment)
   and K_o = 1.0 (from Table 15.1- uniform driving and driven torque)
   Therefore,

   $$\sigma = \frac{F_t(8)}{(1.0)(0.24)}(1.54)(1.0)(1.6) = 82.1\, F_t$$

2. From Eq. (15.18):
   $S_n = S_n'\, C_L\, C_G\, C_s\, k_r\, k_t\, k_{ms}$
   $= (250 \times 350)(1)(1)(0.66)(0.814)(1)(1.4)$
   = 65,812 psi
   where
   $S_n' = 250$ (Bhn) $= 250 \times 350$ psi for infinite life,
   since design life = 5 yr $\times$ (50 wk/yr) $\times$ (60 hr/wk) $\times$ (60 min/hr) $\times$ 1100 rpm
   $= 9.9 \times 10^8 > 10^6$ cycles
   $C_L = 1.0$,
   $C_G = 1.0$ since $P > 5$
   $C_s = 0.66$ from Fig. 8.13
   $k_r = 0.814$ from Table 15.3
   $k_t = 1$ and $k_{ms} = 1.4$ since the pinion is not an idler

3. For SF = 1.5 : $82.1(1.5\, F_t) = 65{,}812$
   hence, $F_t = 534.4$ lb

   $$\dot{W} = \frac{F_t V}{33{,}000} = \frac{(534.4)(720)}{33{,}000} = 11.66 \text{ hp}$$

   Answer : approximately 11.7 hp ■

Comments:

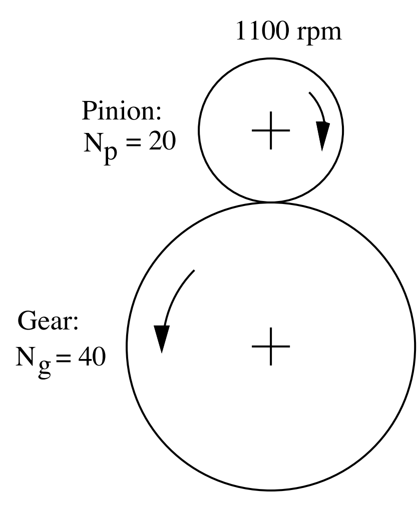
1. The bending stresses can be reduced for the specified rpm by decreasing P or increasing b. But these parameters as well as the factors K_v and J are closely interrelated. Decreasing P for the same number of teeth increases pitch diameter, which leads to larger pitch line velocity and hence to larger values of K_v and σ. Decreasing P for the same pitch diameter decreases the number of teeth resulting in a smaller value of J and a larger value of σ. Increasing the value of b requires accurate mounting and manufacturing to utilize the entire face width and ultimately tends to increase the value of K_m. Thus choice of suitable values for gear geometry parameters for specific applications requires balancing the parameter values with other side effects.
2. In this problem the design life of the gear pair did not enter into the solution except to determine whether the gears were to be rated for finite or infinite life.

SOLUTION (15.32)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the Bhn of gear so that gear and pinion teeth have the same factor of safety with respect to bending fatigue.

Schematic and Given Data:



Material: Steel, heat treated to 350 Bhn
Standard full depth teeth
Accurate mounting
$P = 8$
$\phi = 20°$
$b = 1$ in.
Design life: 5 yrs, 60 hr/wk, 50 wk/yr operation
Top quality hobbing operation for manufacturing

Assumptions:

1. The gears mesh along their pitch circles.
2. All the gear tooth loads are transmitted at the pitch point.
3. There is no load sharing between the teeth.
4. The electric motor and blower constitute uniform load driver and driven equipment.
5. Top quality hobbing operation for manufacturing corresponds to curve C in Fig. 15.24 (to estimate velocity factor K_v).

Analysis:

1. J for gear = 0.285 from Fig. 15.23a

2. From the analysis of Problem 15.31, gear tooth stress is only $\frac{0.24}{0.285}$ times pinion stress

3. Since all parameters and factors are identical for the pinion and gear except Bhn and C_s, (gear Bhn) • C_s could theoretically be

$$(350 \bullet 0.66)\left(\frac{0.24}{0.285}\right) = 194.5.$$ From Fig. 8.13 by trial and error:

gear Bhn = 274 (and C_s = 0.71) ■

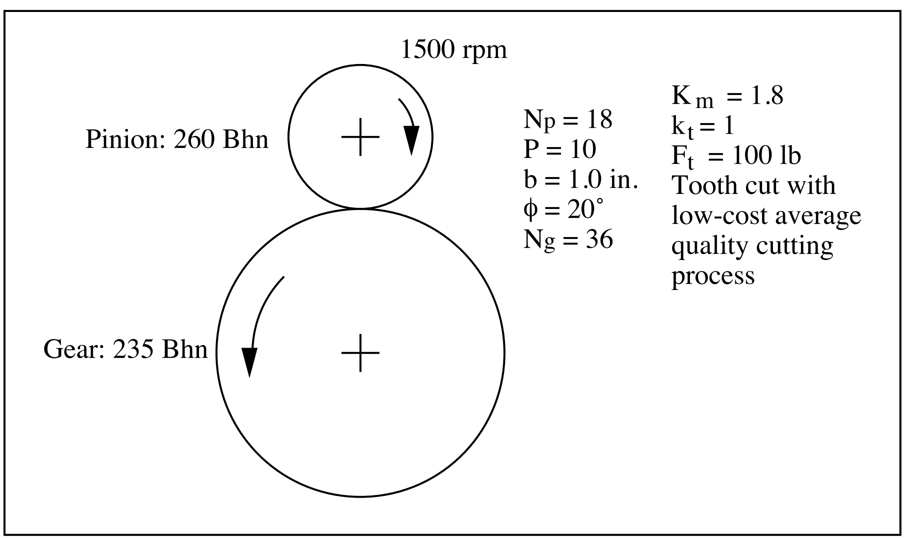
Comment: The gear material can have a lower strength than the pinion material because the stress concentrations at the root of the gear teeth are lower than at the root of the pinion teeth as a result of the gear having a larger diameter.

SOLUTION (15.33)

Known: A spur gear speed reducer is driven by an electric motor and drives a load involving "moderate shock". The gear teeth are standard full depth and of specified geometry and material. Required life is 10^6 pinion revolutions for a specified transmitted load.

Find: Determine an estimate of the reliability of the speed reducer with respect to bending fatigue failure.

Schematic and Given Data:



Assumptions:

1. The spur gears mesh at the pitch circles.
2. Load sharing is not expected since the cutting process is of average quality.
3. The effects corrected by the velocity factor, K_v, correspond to the middle of the range in Fig. 15.24 with manufacture by form cutters.

4. The pinion is driven by a uniform power motor while the gear drives a load involving "moderate shock" (given).
5. The tooth fillet radius is approximately equal to 0.35/P (to enable the use of Fig. 15.23 to estimate geometry factor J).

Analysis:

1. From Fig. 15.23(a), with no load sharing, J = 0.24.
   From Eq. (15.13a),

   $$V = \frac{\pi\, d\, n}{12} = \frac{\pi\, N_p n_p}{12P} = \frac{\pi(18)(1500)}{(12)(10)} = 706.8 \text{ ft/min}$$

   From Fig. 15.24, with V = 706.8 ft/min, $K_v = 2.0$
   From Table 15.1, $K_o = 1.25$
2. From Eq. (15.17) applied to the pinion:

   $$\sigma = \frac{F_t P}{b\, J} K_v\, K_o\, K_m = \frac{100(10)}{1.0(0.24)}(2.0)(1.25)(1.8)$$

   σ = 18,750 psi = 18.75 ksi
3. From Eq. (15.18) applied to the pinion:
   $S_n = S_n'\, C_L\, C_G\, C_s\, k_r\, k_t\, k_{ms}$
   $S_n = (65)(1)(1)(0.72)k_r(1)(1.4) = 65.52k_r$ ksi
   since, $S_u \cong 500(\text{Bhn}) = 500(260)$ psi = 130 ksi.
   $S_n' = S_u/2 = 65$ ksi and for bending loads, $C_L = 1.0$, for $P > 5$, $C_G = 1.0$,
   from Fig. 8.13, $C_s = 0.72$.
   Therefore $18.75 = 65.52k_r$; hence, $k_r = 0.29$
4. Similarly for the gear, J = 0.27, $S_n' = 58.75$ ksi,
   $C_s = 0.75$; hence, $k_r = 0.30$
5. From Table 15.3, reliability is >> 99.999% ■

Comments:

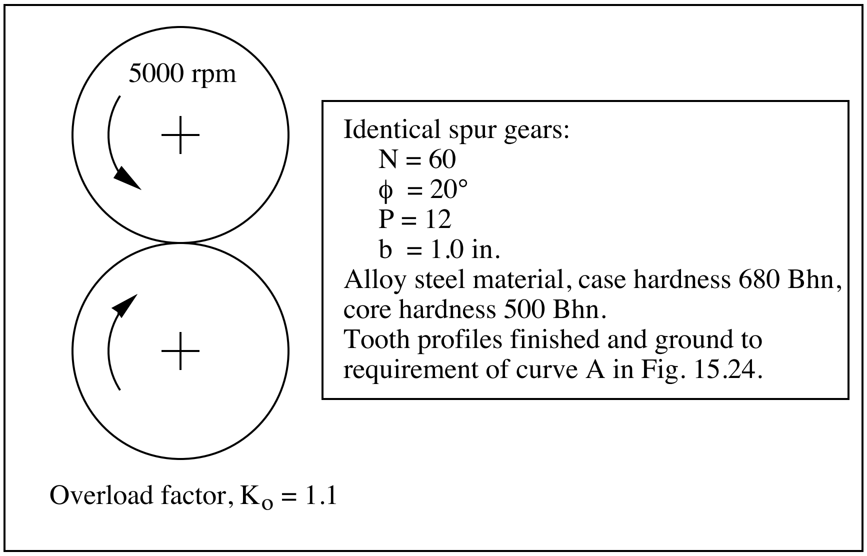
1. The reliability estimated in this problem is based on considering failure only by bending fatigue. A more accurate estimate of reliability must consider failure by surface fatigue also.
2. Increasing the hardness of the gears will result in new choices in transmitting a higher load and higher rpm or choosing a smaller face width or a larger diametral pitch (i.e., with finer teeth).
3. The choice of a harder material for the pinion gives approximately the same reliability for both the pinion and gear in this case. Thus choice of a harder material for the pinion reflects consistency in the strength design of the gears.

SOLUTION (15.34)

Known: An identical pair of standard full depth spur gears of given geometry and material rotate at a given rpm.

Find: Determine an estimate of the horsepower that can be transmitted with 99% reliability based on tooth bending fatigue.

Schematic and Given Data:



Assumptions:
1. The gears are mounted on accurate mountings to mesh along their pitch circles.
2. Loading of the gears involves only mild shock (given).
3. High precision gears with fine ground tooth profiles allow the use of curve A in Fig. 15.24 to estimate velocity factor K_v (given).
4. Load sharing between teeth can be assumed to estimate geometry factor J (given).
5. The core hardness will be used to estimate the strength of the tooth with respect to bending fatigue (given).
6. The tooth fillet radius is approximately equal to 0.35/P (to enable use of Fig. 15.23 to estimate geometry factor J).
7. Neither of the spur gears act as idler gears in the described application.
8. The operating temperature for the gears is less than 160 ^oF.

Analysis:
1. Pitch line velocity,

$$V = \frac{\pi\ d\ n}{12} = \frac{\pi(60/12)5000}{12} = 6545 \text{ fpm}$$

2. Velocity factor, $K_v = \sqrt{\frac{78 + \sqrt{6545}}{78}} = 1.43$
3. From Fig. 15.23(a), geometry factor, J = 0.451
4. From Table 15.2, the mounting factor,
   K_m =1.3 and K_o = 1.1 (given)

5. From Eq. (15.17):

$$\sigma = \frac{F_t\,(12)}{(1.0)(0.451)}(1.43)(1.1)(1.3) = 54.4\ F_t$$

6. From Eq. (15.18):
$S_n = (125\text{ ksi})(1)(1)(.73)(.814)(1)(1.4) = 104$ ksi
Since,

$$S_n' = \left(\frac{500}{4}\right) = 125\text{ ksi},$$

$C_L = 1.0$,
$C_G = 1$ for $P > 5$,
$C_s = 0.73$ from Fig. 8.13,
$k_r = 0.814$ from Table 15.3,
$k_t = 1$ and $k_{ms} = 1.4$ (for one-way bending).

7. Equating stress σ and strength S_n,
$54.4\ F_t = 104{,}000$ psi ; hence $F_t = 1911.76$ lb
Horsepower that can be transmitted,

$$\dot{W} = \frac{F_t V}{33{,}000} = \frac{1911.76(6545\text{ fpm})}{33{,}000} = 379\text{ hp.}$$ ■

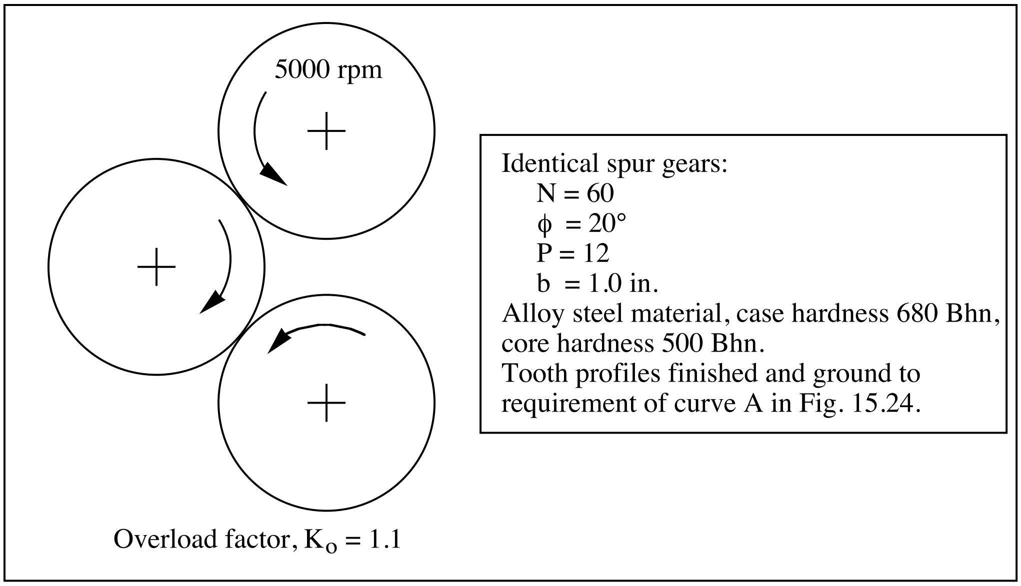
Comments:

1. The horsepower that can be transmitted was estimated here based only on tooth bending fatigue, a more accurate estimate of the horsepower rating must consider the possibility of failure by surface fatigue.
2. Use of the core hardness to estimate the bending strength resulted in a smaller horsepower rating. This is a conservative assumption since the highest bending stress occurs on the tooth surface which has a higher hardness.
3. If either gear were acting as an idler the teeth would have been loaded in two way bending for that gear and the effective bending strength would have reduced by 40% resulting in a lower horsepower rating.

SOLUTION (15.35)
Known: Three identical standard full depth spur gears of given geometry and material rotate at a given rpm.

Find: Determine an estimate of the horsepower that can be transmitted with 99% reliability based on tooth bending fatigue.

Schematic and Given Data:



Assumptions:
1. The gears are mounted on accurate mountings to mesh along their pitch circles.
2. Loading of the gears involves only mild shock (given).
3. High precision gears with fine ground tooth profiles allow the use of curve A in Fig. 15.24 to estimate velocity factor K_v (given).
4. Load sharing between teeth can be assumed to estimate geometry factor J (given).
5. The core hardness will be used to estimate the strength of the tooth with respect to bending fatigue (given).
6. The tooth fillet radius is approximately equal to 0.35/P (to enable use of Fig. 15.23 to estimate geometry factor J).
7. One of the spur gears acts as an idler gear in the described application.
8. The operating temperature for the gears is less than 160 ^oF.

Analysis:
1. Pitch line velocity,

$$V = \frac{\pi\ d\ n}{12} = \frac{\pi(60/12)5000}{12} = 6545 \text{ fpm}$$

2. Velocity factor, $K_v = \sqrt{\frac{78 + \sqrt{6545}}{78}} = 1.43$

3. From Fig. 15.23(a), geometry factor, J = 0.451

4. From Table 15.2, the mounting factor,
   $K_m = 1.3$ and $K_o = 1.1$ (given)
5. From Eq. (15.17):

$$\sigma = \frac{F_t\,(12)}{(1.0)(0.451)}(1.43)(1.1)(1.3) = 54.4\ F_t$$

6. The gear most vulnerable to tooth bending fatigue is the idler because it is subjected to 2-way bending; others are bent only 1-way, thus $k_{ms} = 1$. From Eq. (15.18):
   $S_n = (125\ \text{ksi})(1)(1)(.73)(.814)(1)(1.0) = 74.3\ \text{ksi}$
   Since,

$$S_n' = \left(\frac{500}{4}\right) = 125\ \text{ksi},$$

   $C_L = 1.0$,
   $C_G = 1$ for $P > 5$,
   $C_s = 0.73$ from Fig. 8.13,
   $k_r = 0.814$ from Table 15.3,
   $k_t = 1$
   and $k_{ms} = 1$ for the idler (two-way bending).
7. Equating stress σ and strength S_n,
   $54.4\ F_t = 74{,}300$ psi ; hence $F_t = 1365$ lb
   Horsepower that can be transmitted,

$$\dot{W} = \frac{F_t\,V}{33{,}000} = \frac{1365(6545\ \text{fpm})}{33{,}000} = 270.7\ \text{hp.} \qquad \blacksquare$$

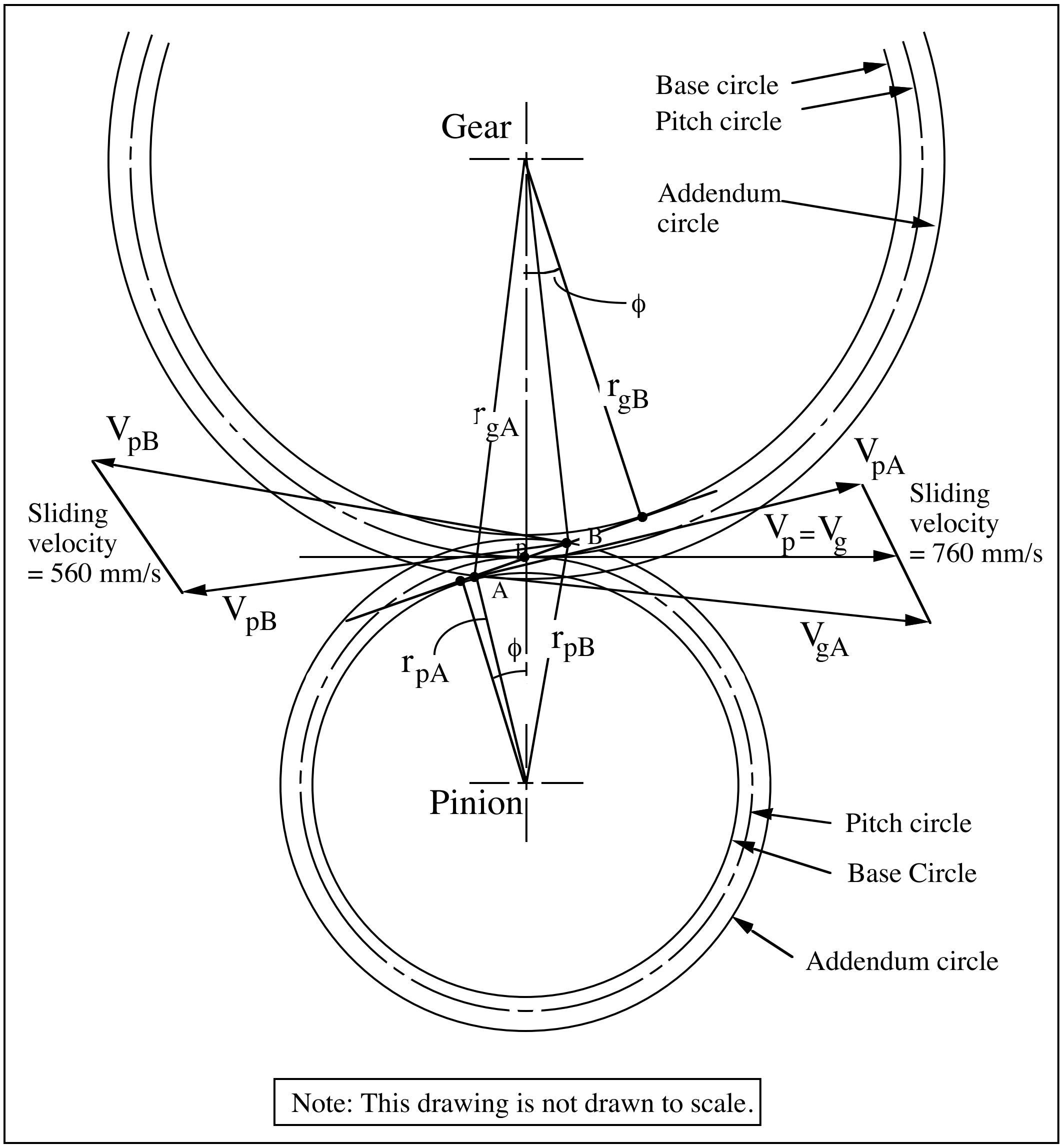
Comments:
1. The horsepower that can be transmitted was estimated here based only on idler tooth bending fatigue, a more accurate estimate of the horsepower rating must consider the possibility of failure of the idler gear tooth by surface fatigue.
2. Use of the core hardness to estimate the bending strength resulted in a smaller horsepower rating. This is a conservative assumption since the highest bending stress occurs on the tooth surface which has a higher hardness.
3. The input and the output gears would be loaded only in one way bending and the effective bending strength for each of these gears would be 1.4 times larger resulting in a higher horsepower capacity for these two gears.

SOLUTION (15.36)

Known: For a pair of spur gears the pressure angle, modulus, number of teeth, and the speed of the pinion are given.

Find: Determine graphically the sliding velocity between the teeth (a) at the start of contact, (b) at the pitch point, and (c) at the end of contact.

Schematic and Given Data:



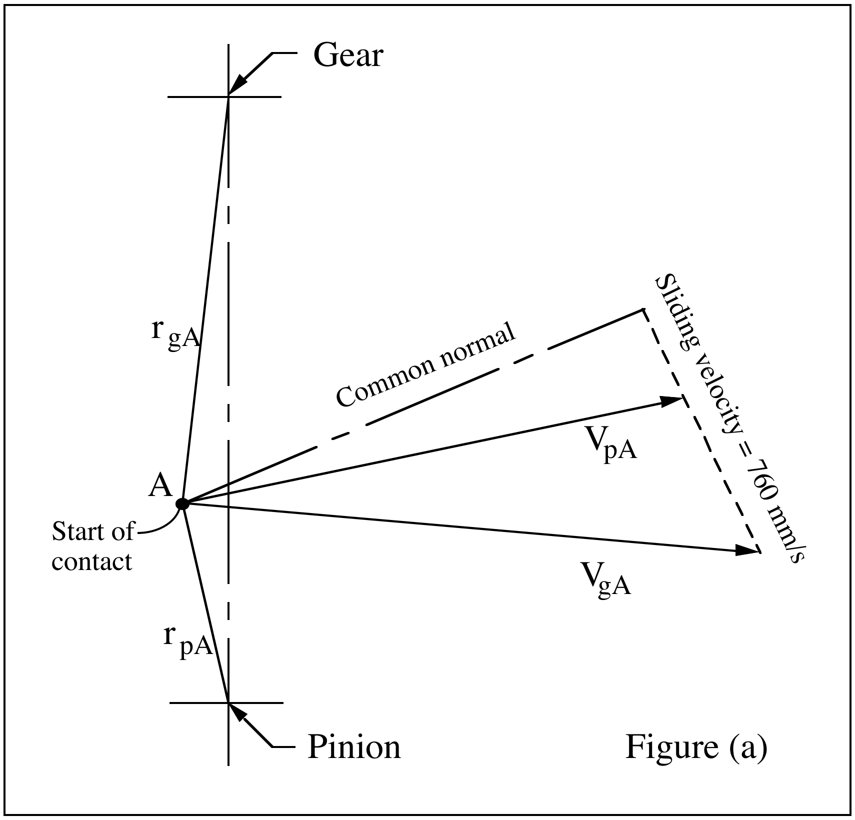
Assumption: The spur gears mesh along their pitch circles.

Analysis:

1. module, $m = d/N$
   Therefore, $r_p = 90$ mm, $r_g = 180$ mm
2. Addendum, $a = m$
   Therefore, $r_{ap} = 96$ mm, $r_{ag} = 186$ mm

Case (a): Start of contact

3.



4. From Fig. (a): $r_{pA} = 82$ mm, $r_{gA} = r_{ag} = 186$ mm

Hence, $V_{pA} = \omega_p r_{pA} = \dfrac{210(2\pi)}{60}(82) = 1803$ mm/s

$$V_{gA} = \omega_g r_{gA} = \frac{105(2\pi)}{60}(186) = 2045 \text{ mm/s}$$

5. From Fig. (a), the sliding velocity is 760 mm/s. ■

Case (b): Pitch point

6. Sliding velocity = 0 [See Fig. 15.26(b)] ■

Case (c): End of contact
7.

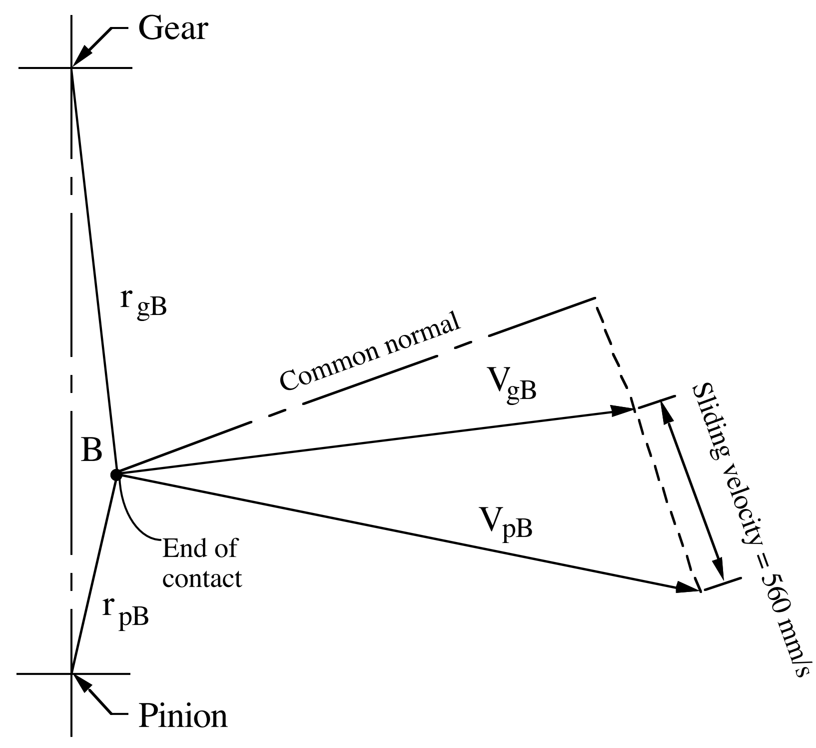


Figure (c)

8. From Fig. (c): $r_{gB} = 169$ mm, $r_{pB} = r_{ap} = 96$ mm

Hence, $V_{pB} = \omega_p r_{pB} = \dfrac{210(2\pi)}{60}(96) = 2111$ mm/s

$$V_{gB} = \omega_g r_{gB} = \frac{105(2\pi)}{60}(169) = 1858 \text{ mm/s}$$

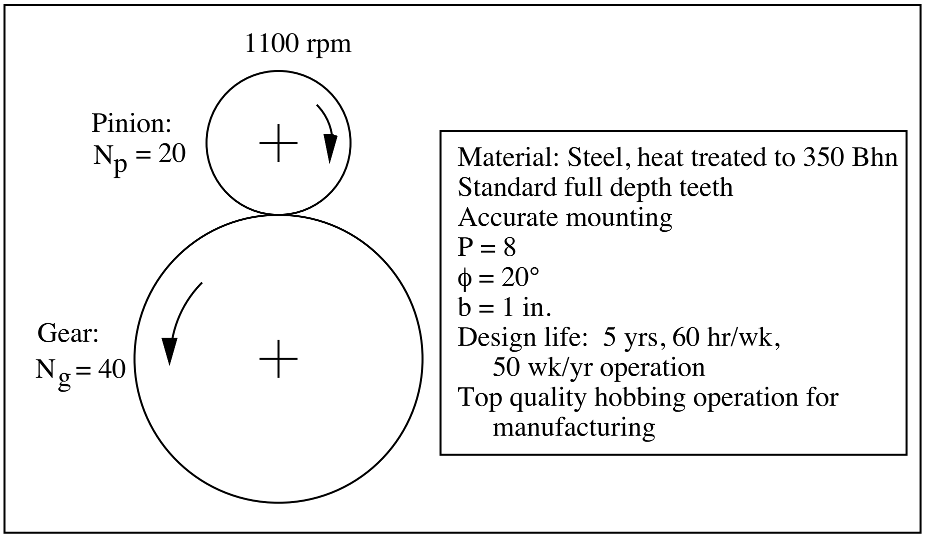
9. From Fig. (c), the sliding velocity is 560 mm/s. ■

SOLUTION (15.37)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based on surface durability.

Schematic and Given Data:



Assumptions:

1. The gears mesh along their pitch circles.
2. The gear tooth loads are transmitted at the pitch point.
3. Tooth contact surfaces are approximated by cylinders.
4. Surface stresses are unaffected by lubricant and sliding friction.

Analysis:

1. From Eq. (15.24): $\sigma_H = C_p \sqrt{\dfrac{F_t}{b\, d_p\, I} K_v\, K_o\, K_m}$

   with $I = \dfrac{\sin\phi \cos\phi}{2} \dfrac{R}{R+1} = \dfrac{\sin 20^o \cos 20^o}{2} \cdot \dfrac{2}{2+1} = 0.107$

   and b = 1 in., $K_v = 1.54$, $K_o = 1.0$, $K_m = 1.6$,

   $d_p = N_p/P = (20/8)$ in. (from the analysis of Problem 15.31),

   Therefore, $\sigma_H = 2300 \sqrt{\dfrac{F_t\,(1.54)(1)(1.6)}{(1.0)(20/8)(0.107)}} = 6980.5\sqrt{F_t}$

2. From Eq. (15.25): $S_H = S_{fe}\, C_{Li}\, C_R$

   $S_{fe} = 0.4$ (Bhn) - 10 ksi = (0.4)(350) - 10 = 130 ksi

   design life = 1100 cyl/min × 60 min/hr × 60 hr/wk

   × 50 wk/yr × 5 yr = 9.9×10^8 cycles

   hence, $C_{Li} = 0.8$

   $S_H = 130(0.8)(1) = 104$ ksi

3. For SF = 1.5 :

   $104{,}000 = 6980.5\sqrt{1.5\, F_t}$; $F_t = 148$ lb

   $\dot{W} = \dfrac{F_t\, V}{33{,}000} = \dfrac{148(720)}{33{,}000} = 3.23$ hp ■

   Therefore, the horsepower rating with respect to surface durability is approximately 3.2 hp.

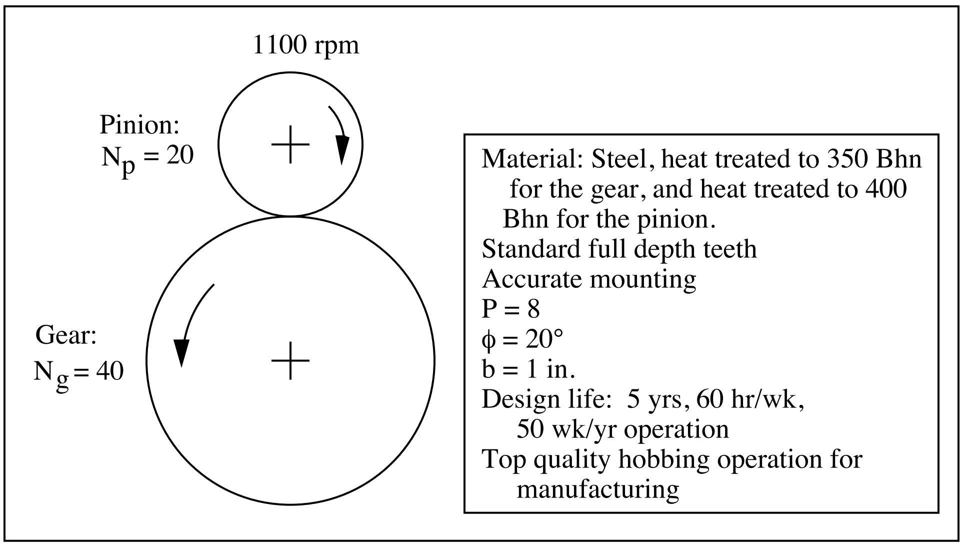
Comment: The horsepower rating of the gear pair is much lower when analyzed with respect to surface durability than with respect to bending fatigue (Problem 15.31). With other choices of material and geometry the opposite result can also occur. This problem illustrates the need for considering both bending fatigue and surface durability in the design and analysis of gears.

SOLUTION (15.38)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based on surface durability and bending fatigue.

Schematic and Given Data:



Assumptions:
1. The gears mesh along their pitch circles.
2. The gear tooth loads are transmitted at the pitch point.
3. Tooth contact surfaces are approximated by cylinders.
4. Surface stresses are unaffected by lubricant and sliding friction.

Analysis:

For surface durability:

For the pinion:

1. From Eq. (15.24): $\sigma_H = C_p \sqrt{\dfrac{F_t}{b\, d_p\, I} K_v\, K_o\, K_m}$

   with $I = \dfrac{\sin\phi \cos\phi}{2} \dfrac{R}{R+1} = \dfrac{\sin 20^\circ \cos 20^\circ}{2} \cdot \dfrac{2}{2+1} = 0.107$

   and b = 1 in., $K_v = 1.54$, $K_o = 1.0$, $K_m = 1.6$,
   $d_p = N_p/P = (20/8)$ in. (from the analysis of Problem 15.31),

Therefore, $\sigma_H = 2300 \sqrt{\dfrac{F_t\,(1.54)(1)(1.6)}{(1.0)(20/8)(0.107)}} = 6980.5\sqrt{F_t}$

2. From Eq. (15.25): $S_H = S_{fe}\, C_{Li}\, C_R$
   $S_{fe} = 0.4$ (Bhn) - 10 ksi = (0.4)(400) - 10 = 150 ksi
   design life = 1100 cyl/min $\times$ 60 min/hr $\times$ 60 hr/wk
   $\times$ 50 wk/yr $\times$ 5 yr = 9.9×10^8 cycles
   hence, $C_{Li} = 0.8$
   $S_H = 150(0.8)(1) = 120$ ksi
3. For SF = 1.5 :
   $120{,}000 = 6980.5\sqrt{1.5\,F_t}$; $F_t = 197$ lb

   $$\dot{W} = \frac{F_t\,V}{33{,}000} = \frac{148(720)}{33{,}000} = 4.3 \text{ hp}$$

<u>For the gear:</u>

4. From Eq. (15.24); $\sigma_H = C_p \sqrt{\dfrac{F_t}{bd_gI}}\; K_vK_oK_m$

   $d_g = 40/8$. Hence, $\sigma_H = 4936\sqrt{F_t}$
5. From Eq. (15.25); $S_H = S_{fe}\, C_{Li}\, C_R$
   $S_{fe} = 0.4$ (Bhn) - 10 ksi = 0.4(350) - 10 = 130 ksi.
   Hence, $S_H = 130\,(0.8)(1) = 104$ ksi.
6. For SF = 1.5:
   $104{,}000 = 4936\sqrt{1.5\,F_t}$; $F_t = 295.95$ lb
   $$\dot{W} = \frac{F_tV}{33{,}000} = \frac{295.95(720)}{33{,}000} = 6.46 \text{ hp}$$

It is evident that the gear is stronger than the pinion based on surface durability. Therefore, the horsepower rating with respect to surface durability is approximately 4.3 hp.

For bending fatigue:
<u>For the pinion:</u>

1. From Eq. (15.17): $\sigma = \dfrac{F_tP}{b\,J}\, K_v\, K_o\, K_m$
   K_v requires finding the pitch line velocity as,

   $$V = \frac{\pi\, d\, n}{12} = \frac{\pi(20/8)1100}{12} = 720 \text{ fpm}$$

   from Curve C of Fig. 15.24,

   $$K_v = \frac{50 + \sqrt{720}}{50} = 1.54$$

   from Fig. 15.23(a), J = 0.24 (for the pinion - and with no load sharing)
   Also, $K_m = 1.6$ (from Table 15.2 - probably best judgment)

and $K_o = 1.0$ (from Table 15.1- uniform driving and driven torque)
Therefore,

$$\sigma = \frac{F_t\,(8)}{(1.0)(0.28)}(1.54)(1.0)(1.6) = 70.37\ F_t$$

2. From Eq. (15.18):
$S_n = S_n'\ C_L\ C_G\ C_s\ k_r\ k_t\ k_{ms}$
$= (250 \times 350)(1)(1)(0.66)(0.814)(1)(1.4)$
$= 65{,}812$ psi
where
$S_n' = 250$ (Bhn) $= 250 \times 350$ psi for infinite life,
since design life $= 5$ yr $\times$ (50 wk/yr) $\times$ (60 hr/wk)
$\times$ (60 min/hr) $\times$ 1100 rpm
$= 9.9 \times 10^8 > 10^6$ cycles
$C_L = 1.0$,
$C_G = 1.0$ since $P > 5$
$C_s = 0.66$ from Fig. 8.13
$k_r = 0.814$ from Table 15.3
$k_t = 1$ and $k_{ms} = 1.4$ since the pinion is not an idler
3. For SF = 1.5: $70.37(1.5F_t) = 65{,}812$
hence, $F_t = 623.5$ lb

$$\dot{W} = \frac{F_t\,V}{33{,}000} = \frac{(623.5)(720)}{33{,}000} = 13.6 \text{ hp}$$

<u>For the gear:</u>

4. From Eq. (15.17): $\sigma = \frac{F_t\,P}{b\,J} K_v\ K_o\ K_m$

K_v requires finding the pitch line velocity as,

$$V = \frac{\pi\, d\, n}{12} = \frac{\pi(20/8)1100}{12} = 720 \text{ fpm}$$

from Curve C of Fig. 15.24,

$$K_v = \frac{50 + \sqrt{720}}{50} = 1.54$$

from Fig. 15.23(a), $J = 0.28$ (for the gear - and with no load sharing)
Also, $K_m = 1.6$ (from Table 15.2 - probably best judgment)
and $K_o = 1.0$ (from Table 15.1- uniform driving and driven torque)
Therefore,

$$\sigma = \frac{F_t\,(8)}{(1.0)(0.24)}(1.54)(1.0)(1.6) = 82.1\ F_t$$

5. From Eq. (15.18):
$S_n = S_n'\ C_L\ C_G\ C_s\ k_r\ k_t\ k_{ms}$
$= (250 \times 400)(1)(1)(0.66)(0.814)(1)(1.4)$
$= 75{,}214$ psi

where
$S_n' = 250$ (Bhn) $= 250 \times 400$ psi for infinite life,
since design life = 5 yr $\times$ (50 wk/yr) $\times$ (60 hr/wk)
$\times$ (60 min/hr) $\times$ 1100 rpm
$= 9.9 \times 10^8 > 10^6$ cycles
$C_L = 1.0$,
$C_G = 1.0$ since $P > 5$
$C_s = 0.66$ from Fig. 8.13
$k_r = 0.814$ from Table 15.3
$k_t = 1$ and $k_{ms} = 1.4$ since the pinion is not an idler

6. For SF = 1.5 : $82.1(1.5\ F_t) = 75{,}214$
hence, $F_t = 610.75$ lb

$$\dot{W} = \frac{F_t V}{33{,}000} = \frac{(610.75)(720)}{33{,}000} = 13.325 \text{ hp} \qquad \blacksquare$$

It is evident that the gear is stronger than the pinion based on the surface durability. Therefore, the horsepower rating with respect to bending fatigue failure is approximately 13.325 hp.

Comments:

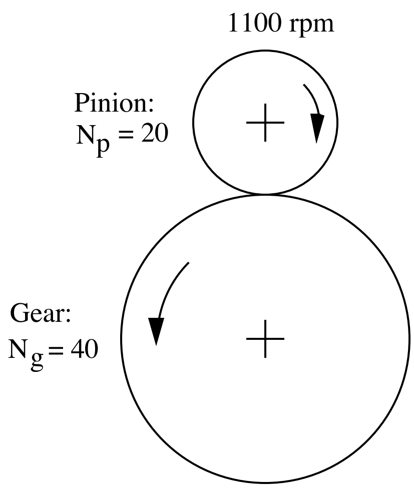
1. The horsepower rating with respect to surface durability is much less than with respect to bending fatigue. This homework problem illustrates the need to consider both bending fatigue and surface durability in the design and analysis of gears. It also shows the need for calculating the strengths of both the gear and the pinion for comparison when the hardness of the gear materials differ, unlike the case where the hardness of both the materials is the same, and we could carry out our design calculations for the smaller of the two gears.
2. The bending stresses can be reduced for the specified rpm by decreasing P or increasing b. But these parameters as well as the factors K_v and J are closely interrelated. Decreasing P for the same number of teeth increases pitch diameter, which leads to larger pitch line velocity and hence to larger values of K_v and s. Decreasing P for the same pitch diameter decreases the number of teeth resulting in a smaller value of J and a larger value of s. Increasing the value of b requires accurate mounting and manufacturing to utilize the entire face width and ultimately tends to increase the value of K_m. Thus choice of suitable values for gear geometry parameters for specific applications requires balancing the parameter values with other side effects.
3. It is evident that the gear is stronger than the pinion based on surface durability. Therefore, the horsepower rating based on surface fatigue is 4.3 hp.
4. The horsepower rating of the gear pair is much lower when analyzed with respect to surface durability than with respect to bending fatigue (Problem 15.31). With other choices of material and geometry the opposite result can also occur. This problem illustrates the need for considering both bending fatigue and surface durability in the design and analysis of gears.

SOLUTION (15.39)

Known: A pair of mating spur gears of specified geometry, material and manufacturing quality is given. The pinion is driven by an electric motor of specified rpm and the gear drives a blower. Design life is specified.

Find: (a) Determine the horsepower rating of the gear set for a safety factor of 1.5 and 99% reliability based on surface durability. (b) Estimate the value to which the gear hardness can be reduced without making the gear teeth weaker than the pinion teeth based on surface fatigue.

Schematic and Given Data:



Material: Steel, heat treated to 350 Bhn for the gear, and heat treated to 400 Bhn for the pinion.
Standard full depth teeth
Accurate mounting
$P = 8$
$\phi = 20°$
$b = 1$ in.
Design life: 5 yrs, 60 hr/wk, 50 wk/yr operation
Top quality hobbing operation for manufacturing

Assumptions:
1. The gears mesh along their pitch circles.
2. The gear tooth loads are transmitted at the pitch point.
3. Tooth contact surfaces are approximated by cylinders.
4. Surface stresses are unaffected by lubricant and sliding friction.

Analysis:

1. <u>For the pinion:</u> From Eq. (15.24): $\sigma_H = C_p \sqrt{\dfrac{F_t}{b\, d_p\, I} K_v K_o K_m}$

with $I = \dfrac{\sin\phi \cos\phi}{2} \dfrac{R}{R+1} = \dfrac{\sin 20^\circ \cos 20^\circ}{2} \cdot \dfrac{2}{2+1} = 0.107$

and $b = 1$ in., $K_v = 1.54$, $K_o = 1.0$, $K_m = 1.6$,
$d_p = N_p/P = (20/8)$ in. (from the analysis of Problem 15.31),

Therefore, $\sigma_H = 2300 \sqrt{\dfrac{F_t (1.54)(1)(1.6)}{(1.0)(20/8)(0.107)}} = 6980.5\sqrt{F_t}$

2. From Eq. (15.25): $S_H = S_{fe}\, C_{Li}\, C_R$
$S_{fe} = 0.4\ (\text{Bhn}) - 10\ \text{ksi} = (0.4)(350) - 10 = 130\ \text{ksi}$
design life = 1100 cyl/min $\times$ 60 min/hr $\times$ 60 hr/wk
$\times$ 50 wk/yr $\times$ 5 yr = 9.9 $\times$ 10^8 cycles
hence, $C_{Li} = 0.8$
$S_H = 130(0.8)(1) = 104\ \text{ksi}$
3. For SF = 1.5 :
$104{,}000 = 6980.5\sqrt{1.5\ F_t}$; $F_t = 148$ lb

$$\dot{W} = \frac{F_t\, V}{33{,}000} = \frac{148(720)}{33{,}000} = 3.23\ \text{hp}$$

Therefore, the horsepower rating with respect to surface durability is approximately 3.2 hp. ■

4. <u>For the gear:</u> From Eq. (15.24); $\sigma_H = C_p \sqrt{\dfrac{F_t}{b d_g I} K_v K_o K_m}$

$$I = \frac{\sin\phi\, \cos\phi}{2}\, \frac{R}{R+1} = \frac{\sin 20^\circ\, \cos 20^\circ}{2}\, \frac{2}{2+1} = 0.107$$

$b = 1$ in., $K_v = 1.54$, $K_o = 1.0$, $K_m = 1.6$,
$d_g = N_g/P = 40/8$ in. (from the analysis of Problem 15.31). Therefore,

$$\sigma_H = 2300 \sqrt{\frac{148(1.54)(1)(1.6)}{1.0(40/8)(0.107)}} = 60049\ \text{psi} = 60\ \text{ksi}$$

5. From Eq. (15.25); $S_H = S_{fe}\, C_{Li}\, C_R$
$S_{fe} = 0.4\ (\text{Bhn}) - 10\ \text{ksi}$
6. $S_H = [0.4\ (\text{Bhn}) - 10\ \text{ksi}]\ (0.8)\ (1) = 0.32\ (\text{Bhn}) - 8 = 60\ \text{ksi}$. Solving for Bhn, Bhn = 187.65. ■

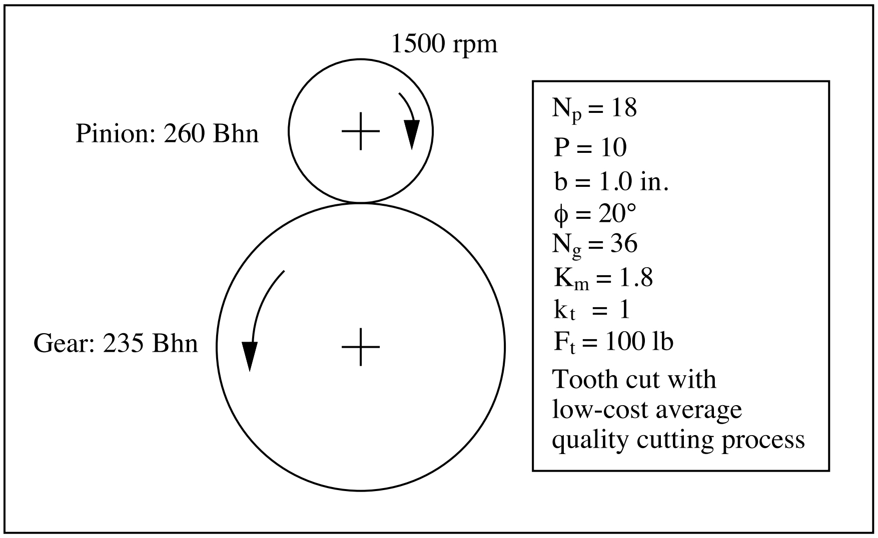
Comment: The gear hardness can be reduced to 187.65 Bhn without making the gear teeth weaker than the pinion teeth based on surface fatigue.

SOLUTION (15.40)

Known: A spur gear speed reducer is driven by an electric motor and drives a load involving "moderate shock". The gear teeth are standard full depth and of given geometry and material. Required life is 10^6 pinion revolutions for a specified transmitted load.

Find: Determine an estimate of the reliability of the speed reducer with respect to surface durability.

Schematic and Given Data:



Assumptions:
1. The gears mesh at the pitch circles.
2. Load sharing is not expected since the cutting process is of average quality.
3. The effects corrected by the velocity factor, K_v, correspond to the middle of the range in Fig. 15.24 with manufacture by form cutters.
4. The pinion is driven by a uniform power motor while the gear drives a load involving "moderate shock" (given).
5. The surfaces at the contact region of the teeth can be approximated by cylinders.
6. The surface stress distribution is unaffected by the presence of the lubricant.
7. Surface loads due to sliding in the tooth contact region are negligible.

Analysis:
1. The pitch diameters of the gears are,
   $d_g = N_g/P = 36/10 = 3.6$ in.
   $d_p = N_p/P = 18/10 = 1.8$ in.

2. Ratio of pitch diameters, $R = \dfrac{d_g}{d_p} = \dfrac{3.6}{1.8} = 2.$

3. Pitch line velocity,

$$V = \frac{\pi\, d\, n}{12} = \frac{\pi\, N_p\, n_p}{12P} = \frac{\pi(18)(1500)}{12(10)} = 706.8 \text{ ft/min}$$

from Fig. 15.24, with V = 706.8 ft/min, $K_v = 2.0$
from Table 15.1, $K_o = 1.25$

4. From Eq. (15.23): $I = \frac{\sin 20^\circ \cos 20^\circ}{2} \bullet \frac{2}{3} = 0.107$
5. From Eq. (15.24):

$$\sigma_H = C_p\sqrt{\frac{F_t}{b\, d_p\, I} K_v\, K_o\, K_m}$$

$$= 2300\sqrt{\frac{100}{(1.0)(1.8)(0.107)}(2.0)(1.25)(1.8)}$$

= 111,174 psi = 111.2 ksi

6. From Table 15.5, an estimate of surface fatigue strength,

$$S_{fe} = 0.4\left(\frac{\text{Bhn of gear + Bhn of pinion}}{2}\right) - 10 \text{ ksi}$$

and from Fig. 15.27, $C_{Li} = 1.12$

7. Therefore, from Eq. (15.25):
$S_H = S_{fe}\, C_{Li}\, C_R$

$$= \left[0.4\left(\frac{235 + 260}{2}\right) - 10\right](1.12)\, C_R = 99.68\, C_R$$

Equating stress and strength $111.2 = 99.68C_R$; hence $C_R = 1.11$

8. Interpolating the rough data from Table 15.6 of the text:

C_R	Reliability
1.25	50%
1.00	99%
0.80	99.9%

A rough approximation of the reliability for
$C_R = 1.11$ is reliability ≈ 80% ■

Comments:

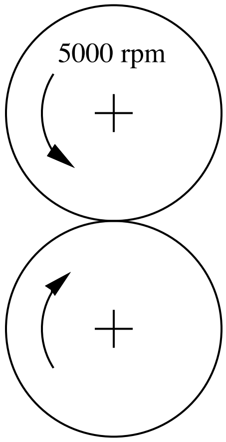
1. The estimate of reliability from the value of C_R is very approximate. As the textbook suggests, data on reliability factors for surface durability are scarce.
2. In comparing the reliability obtained in Problem 15.33 for bending fatigue for the same speed reducer, we find that the estimate of reliability of the gears is much lower for surface fatigue. This indicates that the speed reducer is more likely to fail due to surface damage than bending fatigue.
3. The change in contact pressure distribution due to the presence of a lubricant and sliding loads on the tooth are not explicitly considered in the Hertz equation used for calculating maximum stress in the contact region. These considerations are by implication absorbed in the surface strength data obtained from experiments. It is thus important to judge if the lubrication and sliding effects in the application are similar to those in experiments from which this data is obtained.

SOLUTION (15.41)

Known: An identical pair of standard full depth spur gears of given geometry and material rotate at a given rpm.

Find: Determine an estimate of the horsepower that can be transmitted for 10^9 cycles with 90% reliability based on surface fatigue.

Schematic and Given Data:



Identical spur gears:
- $N = 60$
- $\phi = 20°$
- $P = 12$
- $b = 1.0$ in.

Alloy steel material, case hardness 680 Bhn, core hardness 500 Bhn.
Tooth profiles finished and ground to requirement of curve A in Fig. 15.24.

Overload factor, $K_o = 1.1$

Assumptions:
1. The spur gears mesh at the pitch circles using accurate mountings.
2. Loading on the gears involves only mild shock (given).
3. High precision gears with fine ground tooth profiles allow the use of curve A in Fig. 15.24 to estimate velocity factor K_v (given).
4. Load sharing between teeth is expected in these high precision gears but the full transmitted load will be applied to a tooth to make a conservative estimate of horsepower.
5. The surfaces at the contact region is approximated by cylinders.
6. The surface stress distribution is unaffected by the presence of lubricant.
7. Surface loads due to sliding in the tooth contact region are negligible.
8. The operating temperature for the gears is below 160 °F.

Analysis:
1. Pitch line velocity,

$$V = \frac{\pi\, d\, n}{12} = \frac{\pi(60/12)5000}{12} = 6545 \text{ fpm}$$

2. Velocity factor, $K_v = \sqrt{\dfrac{78 + \sqrt{6545}}{78}} = 1.43$

3. Pitch diameters are, $d = \dfrac{N}{P} = \dfrac{60}{12} = 5$ in.
   Ratio of pitch diameters, $R = 1$
4. From Table 15.2, the mounting factor, $K_m = 1.3$
   and $K_o = 1.1$ (given)

5. From Eq. (15.23): the geometry factor,

$$I = \frac{\sin 20^\circ \cos 20^\circ}{2}\left(\frac{1}{1+1}\right) = 0.080$$

6. From Eq. (15.24): the surface stress,

$$\sigma_H = 2300\sqrt{\frac{F_t}{(1.0)(5)(0.08)}(1.43)(1.1)(1.3)} = 5200\sqrt{F_t}$$

7. From Table 15.6: $C_R \approx 1.06$ for 90% reliability
   From Fig. 15.27, $C_{Li} = 0.8$
8. From Eq. (15.25) and Table 15.5:
   $S_H = \left|0.4(680) - 10\right|(0.8)(1.06) = 222.222$ ksi
   Equating stress and strength $5200\sqrt{F_t} = 222{,}222$ psi : $F_t = 1826.3$ lb
9. Horsepower that can be transmitted,

$$\dot{W} = \frac{F_t V}{33{,}000} = \frac{1826.3\text{ lb}(6545\text{ fpm})}{33{,}000} = 362.2\text{ hp}$$ ■

Comments:

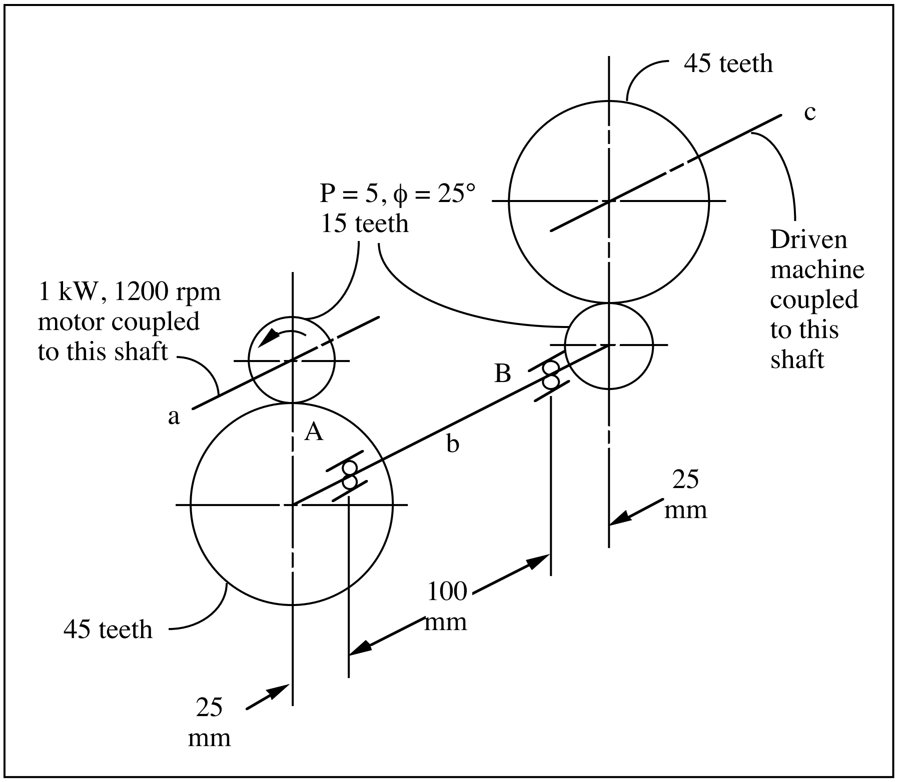
1. Comparing the horsepower rating of the drive unit with respect to bending fatigue estimated in Problem 15.34, it is evident that the horsepower rating determined by consideration of surface fatigue is more critical. While the horsepower rating is 379 hp with a 99% reliability with respect to bending fatigue failure, the horsepower rating is only 362 hp with a 90% reliability with respect to surface fatigue failure. However, the process of failure and its consequence are substantially different in bending fatigue and surface fatigue. Bending fatigue failure is sudden and drastic while surface fatigue failure is gradual and provides easily observable indications of failure.
2. If the surface of the teeth were not made harder than the core, the durability of the teeth with respect to surface fatigue would be further reduced and the horsepower rating would drop by a factor of approximately 1.36 $\left(= \frac{\text{surface hardness}}{\text{core hardness}}\right)$ to about 266 hp.
3. The horsepower rating is unaffected whether the gears act as idlers or not as far as surface fatigue failure is concerned. This is in contrast to the case of bending fatigue where the strength is less by a factor of 1.4 for idler gears due to two way tooth bending.
4. If the solution of SAMPLE PROBLEM 15.5 is followed rather than the solution of SAMPLE PROBLEM 15.4, Figure 9.21 gives $S_H = 120{,}000$ psi, and a smaller transmitted horsepower rating is calculated (estimated).

SOLUTION (15.42)

Known: A two stage gear speed reducer is given which uses a countershaft and has identical gear pairs in each stage. Gear and shaft geometry is specified such that the input and output shafts are collinear.

Find: Determine the relative strengths of the gears for serving in the high-speed and low-speed positions considering both bending fatigue and surface durability with 10^7 cycles life for the high speed gear.

Schematic and Given Data:



Assumptions:
1. The high-speed and low-speed gears are mounted to mesh identically.
2. Load sharing need not be considered.
3. The high speed and low speed gears operate at the same temperature.
4. The countershaft can be considered to be rigid so that both gears have the same overload conditions determining the value of the overload factor, K_o.
5. The friction in the gears and bearings can be neglected.

Analysis:
1. Pitch line velocity of gear in high speed position is,

$$V_{high} = \frac{\pi d_g n_g}{12} = \frac{\pi d_p n_p}{12} = \frac{\pi N_p n_p}{12P} = \frac{\pi(15)(1200)}{12(5)} = 942.4 \text{ ft/min}$$

Pitch line velocity of gear in low speed position is,

$$V_{low} = \frac{\pi d_g n_g}{12} = \frac{\pi N_g n_g}{12P} = \frac{\pi(45)(1200/9)}{12(5)} = 314.1 \text{ ft/min}$$

Ratio of tangential tooth loads in low-speed and high-speed positions for the gears is, $\frac{F_{t,\ low}}{F_{t,\ high}} = \frac{V_{high}}{V_{low}} = 3$

2. For bending fatigue, using Eq. (15.17):

$$\frac{\sigma_{low\ sp}}{\sigma_{hi\ sp}} = \left(\frac{F_{t,low}}{F_{t,high}}\right) \bullet \left(\frac{K_{v,\ low}}{K_{v,\ high}}\right)$$

$= (3)\left(\frac{1.10}{1.18}\right) = 2.8$ with K_v from curve A in Fig. 15.24. ■

$= (3)\frac{1.26}{1.78} = 2.12$ with K_v from curve D in Fig. 15.24. ■

3. Eq. (15.18) for S_n is the same for both applications
Thus for bending fatigue, the low speed application is more severe by factor of 2.12 to 2.8 depending on the manufacturing accuracy. ■
4. For surface fatigue, using Eq. (15.24):

$$\frac{\sigma_{H\ low\ sp}}{\sigma_{H\ hi\ sp}} = \sqrt{2.8} = 1.67 \text{ with } K_v \text{ from curve A and}$$ ■

$$\frac{\sigma_{H\ low\ sp}}{\sigma_{H\ hi\ sp}} = \sqrt{2.12} = 1.46 \text{ with } K_v \text{ from curve D}$$ ■

5. From Eq. (15.25): $\frac{S_{H\ low\ sp}}{S_{H\ hi\ sp}} = \frac{C_{Li\ low}}{C_{Li\ high}} \approx \frac{1.03}{1.0} = 1.03$ ■

because the low speed gears accumulate fatigue cycles only a third as rapidly at the high speed gears and the high speed gear must have a life of 10^7 cycles.
6. Thus, for surface fatigue, the low speed application is more severe by a

factor of about $\frac{1.46}{1.03} = 1.42$ to $\frac{1.67}{1.03} = 1.62$, depending on manufacturing accuracy. ■

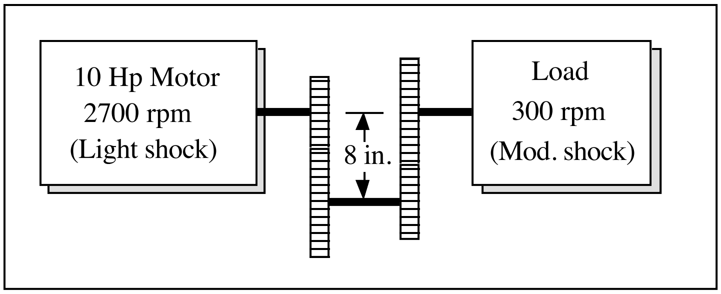
Comments:
1. Unlike the surface fatigue strength, the bending strength is unaffected by the fact that the low-speed gear accumulates fatigue cycles only a third as rapidly as the high-speed gear. This is because, while the endurance limit for bending stress is reached at 10^6 cycles the surface strength continues to decrease well past 10^7 cycles.
2. Since the surface fatigue stress is proportional to the root of the velocity factor, K_v, the relative severity of service for the low-speed gear is not as high as in the case of bending fatigue.
3. If friction forces in the gears and bearings were taken into consideration, the tangential tooth loads for the low-speed and high-speed gears would not be precisely in the inverse ratio of their speeds.

SOLUTION (15.43D)

Known: A two stage spur gear speed reducer is given which uses a countershaft and has identical gear pairs in each stage. Gear and shaft geometry is specified such that the input and output shafts are collinear. Shafts and mountings correspond to good industrial practice but not "high precision".

Find: Determine a design of the gears for 10^7 cycles with 99% reliability and safety factor of 1.2.

Schematic and Given Data:



Decisions:

1. Choose steel for gear material with pinion material 10% harder than the gear material.
2. Choose standard full depth teeth with pressure angle, $\phi = 20^o$.
3. Select number of teeth for pinion, $N_p = 20$.
4. Choose manufacturing precision between curves C and D in Fig. 15.24 to estimate velocity factor, K_v.
5. Choose face width for gears as $b = \frac{12}{P}$.
6. The surface hardness and core hardness for the teeth are equal.
7. The tooth fillet radius is 0.35/P (to enable use of Fig. 15.23(a) to estimate J).

Assumptions:

1. The gears are mounted at their theoretical center distance.
2. Friction losses in gears and bearings can be neglected.
3. No load sharing is expected and all tooth loads are transmitted at the pitch point.
4. The operating temperature for the gears is below 160 ^oF.
5. Surface stress can be estimated by approximating tooth contact region by cylinders.
6. Surface stress distribution is unaffected by lubricant.
7. Surface stress due to sliding friction is negligible.

Design Analysis:

1. With a center distance of 8 in., the 9:1 reduction requires 3:1 reduction by each gear set.
   Hence, $d_p = 4$ in., $d_g = 12$ in.
2. With 20 pinion teeth, $P = \frac{20}{4}$, $P = 5$ ■
3. With $b = \frac{12}{P}$, $b = 2.4$ in. ■
4. Pitch line velocities are,
   $V = (4/12)\pi \bullet 2700 = 2827$ fpm (high speed set)
   and similarly $V = 942$ fpm (low speed set)
5. With manufacturing precision between curves C and D, from Fig. 15.24,
   $K_v = 1.7$ (low speed) and 2.7 (high speed) ■

6. Since $F_t \cdot K_v$ is the greatest on the low speed set, we design the gears for this application.

$$\dot{W} = \frac{F_t V}{33{,}000} : 10 = \frac{F_t(942)}{33{,}000} : F_t = 350 \text{ lb},$$

with a safety factor of 1.2, $F_t = 350(1.20) = 420$ lb

7. Having chosen steel gears with $\phi = 20^o$, we find hardness needed for surface fatigue criterion:
from Eq. (15.24):

$$\sigma_H = 2300\sqrt{\frac{420}{2.4(4)(0.12)}(1.7)(1.5)(1.6)}$$

= 88,707 psi
where, from Eq. (15.23),

$$I = \frac{\sin 20^o \cos 20^o}{2} \cdot \frac{3}{4} = 0.12$$

since, from Table 15.4a, $C_p = 2300$ psi,
from Table 15.1, $K_o = 1.5$,
from Table 15.2, $K_m = 1.6$ and $K_v = 1.7$.

8. Equating stress and strength,
88.7 ksi = $S_H = S_{fe}\, C_{Li}\, C_R$
88.7 = (0.4 Bhn - 10)(1)(1) : hence, Bhn = 247
specify gear hardness as 250 Bhn, pinion hardness as 275 Bhn ■

9. To verify that the above solution is adequate for bending fatigue:
from Eq. (15.17):

$$\sigma = \frac{420(5)}{2.4(0.24)}(1.7)(1.5)(1.6) = 14{,}875 \text{ psi}$$

where, from Fig. 15.23(a), J = 0.24,
and $K_v = 1.7$, $K_o = 1.5$, $K_m = 1.6$.
from Eq. (15.18):

$$S_n = \frac{275}{2}(1)(0.85)(0.71)(.814)(1.4) = 47.283 \text{ ksi} = 47{,}283 \text{ psi}$$

where, $S_n' = \frac{1}{4}(\text{Bhn})$ ksi
$C_L = 1.0$, $C_G = 0.85$, $C_s = 0.71$

for $S_u = \frac{275}{2} = 137.5$ ksi, $k_r = 0.814$ from Table 15.3,
$k_t = 1$, $k_{ms} = 1.4$ from Eq. (15.19)
hence the gears are more than adequate to resist bending fatigue.

10. Design results:
c = 8 in., b = 2.4 in.
$N_p = 20$, $N_g = 60$, standard full depth teeth.
P = 5, $\phi = 20^o$, steel gears, pinion hardness 275 Bhn, gear hardness 250 Bhn. ■
$d_p = 4$ in., $d_g = 12$ in.

Tooth fillet radius = $\frac{0.35}{P}$ = 0.07 in.

Manufacturing precision between curves C and D in Fig. 15.24. Shafts and mountings of ordinary good engineering practice. ■

Comments:

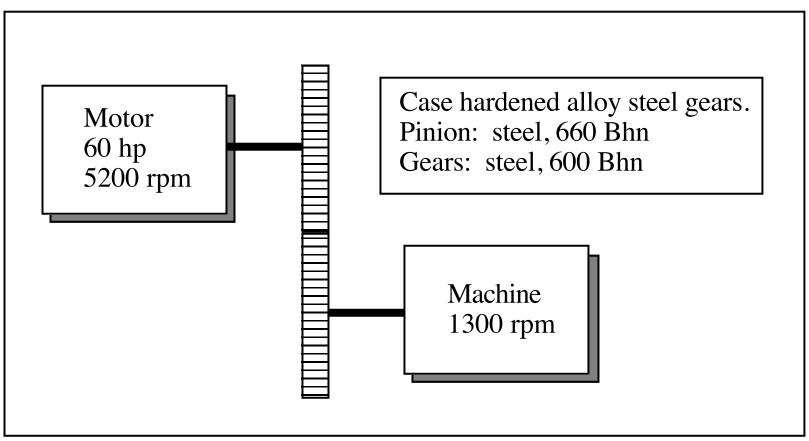
1. Specifying a higher surface hardness and a lower core hardness for the gear teeth would have resulted in a more balanced factor of safety for bending fatigue and surface fatigue.
2. By specifying a material of 250 Bhn using design calculations which required a hardness of 247 Bhn for the pinion, separate design calculations for the gear were avoided. The pinion is always more severely stressed than the corresponding gear. By selecting pinion material 10% harder than the gear an additional factor of safety is provided.
3. If a larger number of teeth for the pinion were selected, the diametral pitch would have been larger and a proportionately smaller face width could be selected. These decisions would result in a higher value of bending stress as well as a higher value of surface stress thus requiring harder gear material specifications.

SOLUTION (15.44D)

Known: A pair of standard spur gears are to transmit a specified hp from an electric motor to a machine with minimum size and weight. Case hardened alloy steel gears of specified Bhn are to be used.

Find: Determine a design of the spur gears for 10^7 pinion revolutions at full load and 99% reliability and a factor of safety of 1.2.

Schematic and Given Data:



Decisions:

1. Select curve A in Fig. 15.24 for high precision manufacturing accuracy with shaved and ground teeth.
2. Select standard full depth teeth with pressure angle, $\phi = 20^o$.
3. Choose number of teeth on pinion, $N_p = 18$.
4. Choose face width, b = 14/P.
5. Choose velocity factor, $K_v = 1.4$, $K_m = 1.3$ (to be verified later).
6. Core hardness of the teeth will be specified from bending fatigue considerations.
7. Choose tooth fillet radius as 0.35/P (to enable use of Fig. 15.22(a) to estimate J).

Assumptions:

1. No significant shock load is present and thus the overload factor, $K_o = 1$.
2. The operating temperature of the gear set is less than 160 ^oF.
3. The gears are mounted at the theoretical center distance.
4. Load sharing between the teeth can be expected since the gears are of high precision.
5. Surface stress can be estimated by approximating tooth contact region by cylinders.
6. Surface stresses are unaffected by sliding friction and presence of lubricant.

Design Analysis:

1. With $N_p = 18$, $N_g = N_p\left(\frac{n_p}{n_g}\right) = 18\left(\frac{5200}{1300}\right) = 72$ ■
2. Pitch line velocity,

   $$V = \pi d_p(5200)/12 = \pi\left(\frac{18}{P}\right)(5200)/12 = \frac{24504}{P} \text{ ft/min}$$
3. Tangential tooth load,

   $$F_t = \frac{(60 \text{ hp})(33{,}000)}{V}(1.2) = 96.96P$$

   at "design overload"
4. We solve for P with $\sigma_H = S_H$ at design overload conditions:

   $$C_p\sqrt{\frac{F_t}{b\, d_p\, I} K_v\, K_o\, K_m} = S_{fe}\, C_{Li}\, C_R$$

   with $I = (\sin 20^o \cos 20^o/2)(4/5) = 0.128$ from Eq. (15.23)
   $K_v = 1.4$, $K_m = 1.3$ (must be checked later)
   $K_o = 1$,

   $$S_{fe} = 0.4\left(\frac{600 + 660}{2}\right) - 10 = 242 \text{ ksi}$$

   $$C_{Li} = 1,\ C_R = 1,\ C_p = 2300,\ b = \frac{14}{P},\ d_p = \frac{18}{P}$$

   Therefore, $2300\sqrt{\frac{96.96P}{(14/P)(18/P)(0.128)}(1.4)(1)(1.3)}$
   $= 242{,}000$ from which $P = 12.6$. We choose
   $P = 12$. (note: choosing $P = 14$ would require $b > 14/P$) ■
5. Then, $V = \frac{24504}{12} = 2042$ fpm
   for which $K_v = 1.25$ (therefore, decision of $K_v = 1.4$ is conservative)
6. To solve for b:

   $$2300\sqrt{\frac{96.96(12)}{b(18/12)(0.128)}(1.4)(1)(1.3)} = 242{,}000$$

   $b = 0.996$ in. specify $b = 1$ in. ■

   (note: $K_m = 1.3$ is satisfactory, and $b = \frac{12}{P}$ which is satisfactory)

7. To check contact ratio:

$$r_p = \frac{1}{2}\frac{N_p}{p} = \frac{9}{12} = 0.75 \text{ in. and similarly } r_g = 3.0 \text{ in.}$$

adding addendum $= \frac{1}{P}$: $r_{ap} = 0.833$ in.,
$r_{ag} = 3.083$ in.
from Eq. (15.11): $r_{bp} = 0.75 \cos 20^o = 0.7048$ in.

$r_{bg} = 3.0 \cos 20^o = 2.8190$ in.

from Eq. (15.11): $p_b = \frac{\pi}{12} \cos 20^o = 0.2460$ in.
center distance, $c = r_p + r_g = 3.75$ in. ■
from Eq. (15.9):

$$CR = \frac{\sqrt{0.833^2 - 0.7048^2} + \sqrt{3.083^2 - 2.8190^2} - 3.75 \sin 20^o}{0.2460} = 1.66$$

CR = 1.66 is satisfactory.

8. To check core hardness for bending fatigue
from Eq. (15.17):

$$\sigma = \frac{(96.96 \times 12)(12)}{(1)(0.34)}(1.25)(1.0)(1.3) = 66731.3 \text{ psi}$$

where from Fig. 15.23(a), J = 0.34.
Equating this value to S_n in Eq. (15.18),
$66731 = S_n'\ (1)(1)(1)(0.814)(1)(1.4)$;
where $C_L = 1$, $C_G = 1$, $C_s = 1$, $k_r = 0.814$, $k_t = 1$, and $k_{ms} = 1.4$
$S_n' = 58{,}556$ psi
This requires $S_u = 117$ ksi, or approximately 235 Bhn.
We specify core hardness ≥ 235 Bhn

9. Design results:
Case hardened alloy steel gear and pinion.
Pinion: surface hardness is 660 Bhn
Gear: surface hardness is 600 Bhn
Core hardness for gears is at least 235 Bhn.
$N_p = 18$, $N_p = 72$, $\phi = 20^o$ FD.
$P = 12$, $b = 1$ in., $c = 3.75$ in.,
$d_p = 1.5$ in., $d_g = 6.0$ in.
Tooth fillet radius = 0.35/P = 0.03 in.
Manufacturing accuracy is high precision with shaved and ground teeth. ■

Comment: Choice of a lower surface hardness for the pinion and gear would have resulted in a lower surface strength and consequently a lower value of diametral pitch. A smaller diametral pitch implies thicker teeth and a smaller contact ratio leading to less quieter and less smoother operation. A smaller diametral pitch also implies larger pitch diameters for the same numbers of teeth requiring a larger center distance.